SUPPLEMENT TO "PARTIAL AMBIGUITY"

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APPENDIX A: FURTHER THEORETICAL ANALYSIS

WE FIRST ANALYZE THE THEORETICAL PREDICTION for maxmin expected utility and follow-up models under multiple-prior perspective in the domain of partial ambiguity. Next, we derive the detailed analysis for recursive rank-dependent utility for the three forms of partial ambiguity. Last, we analyze the theoretical prediction for Chew and Sagi's (2008) source preference model.

A.1. *Maxmin Expected Utility and Follow-up Models*

Maxmin Expected Utility. The maxmin expected utility (MEU) in Gilboa and Schmeidler (1989) evaluates an ambiguous lottery L_i^A with the expected utility corresponding to the worst prior in a convex set of priors $\Pi_{L_i^A}$ as follows:

$$U_{\text{MEU}}(L_i^A) = \min_{\mu \in \Pi_{L_i^A}} \mu(R_{L_i^A})u(w).$$

As indifference between betting on red and black implies that $\Pi_{L_i^A}$ is symmetric, MEU exhibits global ambiguity aversion: {50} is preferred to any ambiguous lottery L_i. It should be noted that the behavior of the set of priors Π_{L_i} is inherently flexible, and MEU can account for a wide range of ambiguity averse choice behavior with a judicious choice of the worst prior in each $\Pi_{L_i^A}$.

Variational Preference. Maccheroni, Marinacci, and Rustichini (2006) proposed an alternative and more flexible generalization of MEU as follows:

$$U_{VP}(L_i^A) = \min_{\mu \in \Delta}\{\mu(R_{L_i^A})u(w) + a_{L_i^A}(\mu)\},$$

where Δ refers to the set of all possible priors and $a_{L_i}(\mu) : \Delta \to [0, \infty)$ is an index of ambiguity aversion. Notice that VP reduces to MEU if $a_{L_i^A}$ is an indicator function for $\Pi_{L_i^A}$, and it follows that VP inherits the predictions of MEU in the domain of partial ambiguity. The same qualitative behavior also applies to the contraction model

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([Gajdos, Hayashi, Tallon, and Vergnaud](2008)), which delivers a weighted combination between SEU and MEU with built-in ambiguity aversion.

α-Maxmin Expected Utility. [Ghirardato, Maccheroni, and Marinacci](2004) axiomatized α-maxmin expected utility (α-MEU) that delivers a linear combination of maxmin EU and maxmax EU as follows:

$$U_{\alpha\text{-MEU}}\big(L_i^A\big) = \alpha \min_{\mu \in \Pi_{L_i^A}} \mu(R_{L_i^A})u(w) + (1 - \alpha) \max_{\mu \in \Pi_{L_i^A}} \mu(R_{L_i^A})u(w).$$

Depending on the value of α, this model is highly flexible and can selectively exhibit ambiguity tolerance. The same predictions apply to [Siniscalchi](2009)'s vector expected utility model which incorporates an adjustment function in addition to SEU.

A.2. *Recursive Rank-Dependent Utility*

Interval Ambiguity $\mathbf{I}_n^A$. The utility for an interval ambiguity lottery $\mathbf{I}_n^A$ under uniform prior is given by

$$U(\mathbf{I}_n^A) = \sum_{i=50-n}^{50+n} f\left(1 - \frac{i}{100}\right)\left[f\left(\frac{i+1-(50-n)}{2n+1}\right) - f\left(\frac{i-(50-n)}{2n+1}\right)\right].$$

This can be approximated using a uniform random variable over $[0, 1]$ with cumulative distribution function F as follows:

$$U\big(\mathbf{I}_n^A\big) = \int_{0.5-\widehat{n}}^{0.5+\widehat{n}} f(s)\,d\big(-f\big(1 - F(s)\big)\big),$$

where $\widehat{n}$ takes the values between 0 and 0.5, and $F = \frac{s+\widehat{n}-0.5}{2\widehat{n}}$ for $s \in [0.5 - \widehat{n}, 0.5 + \widehat{n}]$. Let $x = \frac{s+\widehat{n}-0.5}{2\widehat{n}}$. We have

$$U = \int_0^1 -f(2\widehat{n}x + 0.5 - \widehat{n})\,df(1 - x).$$

Differentiating with respect to $\widehat{n}$ yields

$$U' = \int_0^1 (2x - 1)f'\big((1 - 2\widehat{n})x + \widehat{n}\big)f'(1 - x)\,dx.$$

Evaluating U' at $\widehat{n} = 0$ gives

$$U'|_{\widehat{n}=0} = \int_0^1 (2x - 1)f'(x)f'(1 - x)\,dx,$$

which can be rewritten as

$$-\int_0^{0.5} (1 - 2x)f'(x)f'(1 - x)\,dx + \int_{0.5}^1 (2x - 1)f'(x)f'(1 - x)\,dx, \qquad \text{(A.1)}$$

which equals 0 given the symmetry of the two terms after changing the variable in the second term to $1 - x$.

Observe that $f(2\widehat{n}x + 0.5 - \widehat{n}) > f(x)$ when $x < 0.5$ since $2\widehat{n}x + 0.5 - \widehat{n} > x$. Similarly, we have $f(2\widehat{n}x + 0.5 - \widehat{n}) < f(x)$ when $x > 0.5$. It follows that when f is convex, $U' < 0$, that is, aversion to increasing the number of possible compositions in interval ambiguity, since changing $f'(x)$ to $f'(2\widehat{n}x + 0.5 - \widehat{n})$ will increase the first term of (A.1) and decrease its second term.

Disjoint Ambiguity $\mathbf{D}_n^A$. The utility $U(\mathbf{D}_n^A)$ for a disjoint ambiguity lottery $\mathbf{D}_n^A$ under uniform prior is given by

$$\sum_{i=0}^{n} f\left(1 - \frac{i}{100}\right)\left[f\left(\frac{i+1}{2(n+1)}\right) - f\left(\frac{i}{2(n+1)}\right)\right]$$
$$+ \sum_{i=n+1}^{2n+1} f\left(\frac{2n+1-i}{100}\right)\left[f\left(\frac{i+1}{2(n+1)}\right) - f\left(\frac{i}{2(n+1)}\right)\right].$$

This can be approximated using a uniform random variable over [0, 1] with cumulative distribution function F as follows:

$$U = \int_0^{\widehat{n}} f(s)\,d\bigl(-f\bigl(1 - F(s)\bigr)\bigr) + \int_{1-\widehat{n}}^1 f(s)\,d\bigl(-f\bigl(1 - F(s)\bigr)\bigr),$$

where $\widehat{n}$ takes the values from 0.5 to 0 in disjoint ambiguity, and $F(s) = \frac{s}{2\widehat{n}}$ for $s \in [0, \widehat{n}]$ and $F(s) = \frac{s-(1-2\widehat{n})}{2\widehat{n}}$ for $s \in [1 - \widehat{n}, 1]$. Let x equal $\frac{s}{2\widehat{n}}$ in the first integral and equal $\frac{s-(1-2\widehat{n})}{2\widehat{n}}$ in the second integral. We have

$$U = \int_0^{0.5} -f(2\widehat{n}x)\,df(1-x) - \int_{0.5}^1 f\bigl(2\widehat{n}x + (1-2\widehat{n})\bigr)\,df(1-x).$$

Differentiating with respect to $\widehat{n}$ yields

$$U' = \int_0^{0.5} 2xf'(2\widehat{n}x)f'(1-x)\,dx + \int_{0.5}^1 (2x-2)f'\bigl(2\widehat{n}x + (1-2\widehat{n})\bigr)f'(1-x)\,dx.$$

Evaluating U' at $\widehat{n} = 0.5$ gives

$$U'|_{\widehat{n}=0} = \int_0^{0.5} 2xf'(x)f'(1-x)\,dx + \int_{0.5}^1 (2x-2)f'(x)f'(1-x)\,dx, \qquad (A.2)$$

which again equals 0 given the symmetry of the two terms.

Observe that $f(2\widehat{n}x) < f(x)$ when $x < 0.5$ and $f(2\widehat{n}x + (1-2\widehat{n})) > f(x)$ when $x > 0.5$. It follows that when f is convex, $U' < 0$, that is, aversion to increasing the number of possible compositions in disjoint ambiguity, since changing $f'(x)$ to $f'(2\widehat{n}x)$ will decrease the first term of (A.2) while changing $f'(x)$ to $f'(2\widehat{n}x + (1-2\widehat{n}))$ will increase the second term of (A.2).

Two-Point Ambiguity $\mathbf{T}_n^A$. The utility for a two-point ambiguity lottery $\mathbf{T}_n^A$ under uniform prior is given by

$$U_{\mathrm{RRDU}}(\mathbf{T}_n) = \bigl(1 - f(0.5)\bigr)f(0.5 - \widehat{n}) + f(0.5)f(0.5 + \widehat{n}).$$

Differentiating with respect to $\widehat{n}$ yields

$$U' = f(0.5)f'(0.5 + \widehat{n}) - \bigl(1 - f(0.5)\bigr)f'(0.5 - \widehat{n}).$$

Given f convex, we have $U'|_{\hat{n}=0} < 0$ and $U'|_{\hat{n}=0.5} > 0$, which in turn implies $U' < 0$ for $\hat{n}$ small, and $U' > 0$ as $\hat{n}$ approaches 0.5.

Stage-1 Spread. Given two ambiguous lotteries represented by stage-1 priors with cumulative distribution functions F and G such that G is a stage-1 spread of F, consider the difference:

$$\int f(x)\,d\bigl(-f\bigl(1-F(x)\bigr)\bigr) - \int f(x)\,d\bigl(-f\bigl(1-G(x)\bigr)\bigr).$$

This becomes $\int f'(x)[f(1-F(x)) - f(1-G(x))]\,dx$ after integrating by parts. We have that $f(1-F(x)) - f(1-G(x)) \geq 0$ for $x < 0.5$ and $f(1-F(x)) - f(1-G(x)) \leq 0$ for $x > 0.5$, since $F(x) \leq G(x)$ for $x < 0.5$ and $F(x) \geq G(x)$ for $x > 0.5$. Changing variables yields

$$\int_0^{0.5} f'(x)\bigl[f\bigl(1-F(x)\bigr) - f\bigl(1-G(x)\bigr)\bigr]\,dx$$
$$- \int_0^{0.5} f'(1-x)\bigl[f\bigl(1-G(1-x)\bigr) - f\bigl(1-F(1-x)\bigr)\bigr]\,dx.$$

We proceed to compare $\frac{f'(x)}{f'(1-x)}$ and $\frac{f(1-G(1-x))-f(1-F(1-x))}{f(1-F(x))-f(1-G(x))}$. Since f is convex, we have

$$f'\bigl(1-F(1-x)\bigr) \leq \frac{f\bigl(1-G(1-x)\bigr) - f\bigl(1-F(1-x)\bigr)}{F(1-x) - G(1-x)} \leq f'\bigl(1-G(1-x)\bigr),$$

and

$$f'\bigl(1-F(x)\bigr) \geq \frac{f\bigl(1-F(x)\bigr) - f\bigl(1-G(x)\bigr)}{G(x) - F(x)} \geq f'\bigl(1-G(x)\bigr).$$

Given symmetry of F and G, $F(1-x) - G(1-x) = G(x) - F(x)$. Dividing the above two inequalities yields

$$\frac{f'\bigl(1-F(1-x)\bigr)}{f'\bigl(1-F(x)\bigr)} \leq \frac{f\bigl(1-G(1-x)\bigr) - f\bigl(1-F(1-x)\bigr)}{f\bigl(1-F(x)\bigr) - f\bigl(1-G(x)\bigr)} \leq \frac{f'\bigl(1-G(1-x)\bigr)}{f'\bigl(1-G(x)\bigr)}.$$

Suppose F is a stage-1 spread of uniform prior; then $\frac{f'(1-F(1-x))}{f'(1-F(x))} \geq \frac{f'(x)}{f'(1-x)}$ for $x \leq 0.5$. It follows that the decision maker prefers G to F if both of them are spreads of uniform prior. Conversely, suppose uniform prior is a stage-1 spread of G; then $\frac{f'(1-G(1-x))}{f'(1-G(x))} \leq \frac{f'(x)}{f'(1-x)}$ at $x \leq 0.5$ and we have the decision maker preferring F to G if uniform prior is a stage-1 spread of both F and G.

A.3. *Source Preference of Chew and Sagi (2008)*

Chew and Sagi's (2008) model directly distinguishes among the even-chance bets from the three primitive sources of uncertainty in our experimental design: pure risk derived from the known composition of half red and half black in {50}, full ambiguity based on the unknown compositions of red and black in $[0, 100]^A$, and additionally the all-red or all-black ambiguity in $\{0, 100\}^A$. These three sources generate 50–50 probabilities that may be differentiated in terms of preference. In the following, we demonstrate how the source preference approach with built-in RCLA endogenously generates a two-stage representation for the various forms of partial ambiguity.

Interval Ambiguity $\mathbf{I}_n^A$. It comprises $100 - 2n$ cards with known composition—half red and half black—while the composition of the rest of the $2n$ cards is fully unknown. With RCLA for known probabilities, an interval ambiguous lottery induces a lottery on the overall "known" domain that delivers $(50 - n)/100$ chance of getting w, $(50 - n)/100$ chance of getting 0, and $2n/100$ chance of getting a fully ambiguous lottery, and it is represented as follows:

$$\left(\frac{50 - n}{100}, w; \frac{2n}{100}, c_{[0,100]^A}; \frac{50 - n}{100}, 0 \right),$$

where $c_{[0,100]^A}$ is the CE for a bet on the fully unknown deck $[0, 100]^A$.

Disjoint Ambiguity $\mathbf{D}_n^A$. It comprises $100 - n$ cards that are either all red or all black, and n cards with fully unknown composition. Similarly, the induced lottery on the "known" domain is

$$\left(\frac{100 - n}{100}, c_{\{0,100\}^A}; \frac{n}{100}, c_{[0,100]^A} \right),$$

where $c_{\{0,100\}^A}$ is the CE for a bet on the either-all-red-or-all-black deck $\{0, 100\}^A$. Notice that the expression above converges to $(0.5, c_{\{0,100\}^A}; 0.5, c_{[0,100]^A})$ rather than $c_{[0,100]^A}$ as n approaches 50. This behavior is related to an alternative description of full ambiguity $[0, 100]^A$. Besides its intended interpretation of being fully unknown, it can first be described as comprising 50 cards which are either all red or all black while the composition of the other 50 cards remains unknown. This process can be applied to the latter 50 cards to arrive at a further division into 25 cards which are either all red or all black while the composition of the remaining 25 cards remains unknown. Doing this ad infinitum gives rise to a dyadic decomposition of $[0, 100]^A$ into subintervals which are individually either all-red or all-black. For the source model to deliver the same CE for $[0, 100]^A$, we need to restrict its evaluation to undecomposed intervals of ambiguity, and the overall expression exhibits discontinuous behavior at $n = 50$.

Two-Point Ambiguity $\mathbf{T}_n^A$. It comprises $100 - 2n$ cards with known composition—half red and half black—while the composition of the rest of the $2n$ cards is either all red or all black, and the induced lottery on the "known" domain is given by

$$\left(\frac{50 - n}{100}, w; \frac{2n}{100}, c_{\{0,100\}^A}; \frac{50 - n}{100}, 0 \right).$$

Consider the baseline SEU model with a utility index u for objective risk on the "known" domain. The utility for an interval ambiguous lottery is then given by

$$\frac{50 - n}{100} u(w) + \frac{2n}{100} u(c_{[0,100]^A}) + \frac{50 - n}{100} u(0),$$

which is monotonically decreasing in n if we have $\frac{u(w) + u(0)}{2} > u(c_{[0,100]^A})$. This latter condition corresponds to standard ambiguity aversion $\{50\} \succ [0, 100]^A$. Behaviorally, a subject exhibits aversion to increasing the number of possible compositions under the same condition.

Similarly, the SEU for a two-point ambiguous lottery is

$$\frac{50 - n}{100} u(w) + \frac{2n}{100} u(c_{\{0,100\}^A}) + \frac{50 - n}{100} u(0),$$

and we have aversion to two-point spread, that is, the utility is monotonically decreasing in n, if $\frac{u(w)+u(0)}{2} > u(c_{\{0,100\}^A})$. This condition corresponds to a preference for betting on the known deck $\{50\}$ to betting on the deck $\{0, 100\}^A$ that comprises either all red or all black cards.

For disjoint ambiguity, we have aversion to increasing the number of possible compositions if $c_{\{0,100\}^A} > c_{[0,100]^A}$, which corresponds to a preference of $\{0, 100\}^A \succ [0, 100]^A$. Note that this implication does not depend on the assumption of SEU, but only on stochastic dominance. In contrast, it is possible for general non-expected utility preferences, for example, quadratic preference (Chew, Epstein, and Segal (1991)), to exhibit non-monotone behavior in interval and two-point ambiguity. See Chew, Miao, and Zhong (2013) for a discussion of Chew and Sagi's (2008) source preference model without reduction.

In line with Chew and Sagi's (2008) source preference approach, Abdellaoui, Baillon, Placido, and Wakker (2011) proposed a model with a source-dependent probability weighting function in conjunction with a cumulative prospect theory specification. As in the preceding exposition of source-dependent SEU, being silent on how compound lotteries may be evaluated in the absence of RCLA, this model exhibits considerable flexibility in modeling choice behavior in our setting and can distinguish among the three main sources of uncertainty should we take the view that each partial ambiguity lottery itself represents a possibly distinct source.

APPENDIX B: AXIOMS OF COMPOUND RISK AND THEIR IMPLICATIONS

Let $X_j = (p_j, x_j)_j$ denote a simple lottery paying x_j with probability p_j, and $\mathbf{X} = (q^k, X^k)_k$ a compound lottery paying simple lottery $X^k = (p_j^k, x_j^k)_j$ with probability q^k. In addition, denote a degenerate lottery paying x for sure by δ_x. Similarly, a degenerate compound lottery paying simple lottery X_j for sure is denoted by δ_{X_j} and $(q^j, \delta_{x_j})_j$ is another kind of degenerate compound lottery which pays degenerate simple lottery δ_{x_j} with probability q^j. We define two operations $+$ and $\oplus$ to combine risks at two different stages. We use $+$ to denote a mixture operation for simple risks and $\oplus$ to denote a mixture operation for stage-1 risks.

Given two simple lotteries $X_m = (p_1, x_1; p_2, x_2; \ldots; p_m, x_m)$ and $Y_n = (q_1, y_1; q_2, y_2; \ldots; q_n, y_n)$, a mixture with probability r, $rX_m + (1-r)Y_n$, is identified with a simple lottery given by

$$\left(rp_1, x_1; \ldots; rp_m, x_m; (1-r)q_1, y_1; \ldots; (1-r)q_n, y_n\right).$$

In other words, $+$ can be used as a mixture operation for stage-2 risks as follows:

$$\delta_{rX_m+(1-r)Y_n} = \delta_{(rp_1, x_1; \ldots; rp_m, x_m; (1-r)q_1, y_1; \ldots; (1-r)q_n, y_n)}.$$

In a similar vein, given two compound lotteries $\mathbf{X} = (p^1, X^1; p^2, X^2; \ldots; p^m, X^m)$ and $\mathbf{Y} = (q^1, Y^1; q^2, Y^2; \ldots; q^n, Y^n)$, a stage-1 mixture with probability r, $r\mathbf{X} \oplus (1-r)\mathbf{Y}$, is identified with the compound lottery

$$\left(rp^1, X^1; \ldots; rp^m, X^m; (1-r)q^1, Y^1; \ldots; (1-r)q^1, Y^n\right).$$

To illustrate the difference between $+$ and $\oplus$, consider two compound lotteries, $(1, (\frac{1}{2}, x; \frac{1}{2}, 0))$ and $(1, (\frac{1}{3}, y; \frac{2}{3}, 0))$. A mixture operation $+$ with probability $\frac{1}{2}$ delivers the following:

$$\left(1, \frac{1}{2}\left(\frac{1}{2}, x; \frac{1}{2}, 0\right) + \frac{1}{2}\left(\frac{1}{3}, y; \frac{2}{3}, 0\right)\right) = \left(1, \left(\frac{1}{4}, x; \frac{1}{6}, y; \frac{7}{12}, 0\right)\right),$$

while a mixture operation $\oplus$ with probability $\frac{1}{2}$ delivers the following:

$$\frac{1}{2}\left(1,\left(\frac{1}{2},x;\frac{1}{2},0\right)\right) \oplus \frac{1}{2}\left(1,\left(\frac{1}{3},y;\frac{2}{3},0\right)\right) = \left(\frac{1}{2},\left(\frac{1}{2},x;\frac{1}{2},0\right);\frac{1}{2},\left(\frac{1}{3},y;\frac{2}{3},0\right)\right).$$

We now introduce several commonly used axioms in the literature on decision making involving compound risk and discuss their implications for choice behavior in our setup. We begin with the following axiom which has largely been implicit in the literature on generalizing expected utility under different weakening of its independence axiom.

REDUCTION OF COMPOUND LOTTERY AXIOM (RCLA): $(q^k, X^k)_k \sim q^1 X^1 + q^2 X^2 + \cdots + q^k X^k$.

RCLA requires a compound lottery $(q^k, X^k)_k$ to be indifferent to a simple lottery $q^1 X^1 + q^2 X^2 + \cdots + q^k X^k$ whose outcomes are taken from the component lotteries X^k with the corresponding probabilities derived from the given compound lottery. This property is inherent to any utility model whose domain of choice comprises a set of probability measures with a convex combination of two lotteries interpreted as a compound lottery. In relaxing RCLA, people may exhibit distinct attitudes towards risks at different stages in a compound lottery. To accommodate this, we may limit the scope of a preference axiom to simple lotteries in a single-stage setting. Clearly, applying $+$ on within-stage risks does not imply that RCLA holds.

We adapt the independence axiom and the betweenness axiom to the domain of simple lotteries for both stage-1 and stage-2 risks.

STAGE-1 INDEPENDENCE: $(q^m, X^m)_m \succeq (q^n, X^n)_n$ implies that $\alpha(q^m, X^m)_m \oplus (1-\alpha)(q^l, X^l)_l \succeq \alpha(q^n, X^n)_n \oplus (1-\alpha)(q^l, X^l)_l$.

STAGE-2 INDEPENDENCE: $X_m \succeq X_n$ implies that $\alpha X_m + (1-\alpha)X_l \succeq \alpha X_n + (1-\alpha)X_l$.

Independence requires that the preference between two lotteries is preserved when each is mixed with a common third lottery at the same probability.

STAGE-1 BETWEENNESS: $(q^m, X^m)_m \succeq \alpha(q^m, X^m)_m \oplus (1-\alpha)(q^n, X^n)_n \succeq (q^n, X^n)_n$ if $(q^m, X^m)_m \succeq (q^n, X^n)_n$.

STAGE-2 BETWEENNESS: $X_m \succeq \alpha X_m + (1-\alpha)X_n \succeq X_n$ if $X_m \succeq X_n$.

Betweenness requires a mixture between two lotteries to be intermediate in preference between the preference for two respective lotteries.

The following axiom provides a link between stage-1 and stage-2 risk preference.

TIME NEUTRALITY: *Given* $X_j = (p_j, x_j)_j$, $\delta_{X_j} \sim (p^j, \delta_{x_j})_j$.

This axiom requires a decision maker to be indifferent between two degenerate compound lotteries if they reduce to the same simple lottery. Put differently, whether the resolution of risks occurs at stage 1 or stage 2 does not influence the preference for degenerate compound lotteries.

It is straightforward to see that RCLA implies time neutrality and that independence implies betweenness (see Segal (1990) for a detailed discussion). For completeness in exposition, we present the following axiom which together with RCLA implies independence.

COMPOUND INDEPENDENCE AXIOM: $X^k \succeq X^{k'}$ iff $(q^1, X^1; q^2, X^2; \ldots; q^k, X^{k'}; \ldots; q^n, X^n) \succeq (q^1, X^1; q^2, X^2; \ldots; q^k, X^{k'}; \ldots; q^n, X^n)$.

Compound independence requires a compound lottery $(q^k, X^k)_k$ to becomes less preferred if any of the component simple lotteries is replaced with a less preferred simple lottery.

We summarize below the implications of RCLA, stage-1 betweenness, and time neutrality for choice behavior in our experiments.

IMPLICATION R: RCLA implies that all compound lotteries are indifferent to {50}.

This follows from observing that the compound lotteries in our setup reduce to {50}.

IMPLICATION B: Stage-1 Betweenness implies that $[0, 100]^C$ is ranked between $[n, 100 - n]^C$ and $[0, n] \cup [100 - n, 100]^C$.

This follows from observing that $[0, 100]^C = \frac{2n}{100}[n, 100 - n]^C \oplus \frac{100-2n}{100}[0, n] \cup [100 - n, 100]^C$ in our setup, assuming the overlapping two points $\{n\}$ and $\{100 - n\}$ are negligible.

IMPLICATION T: Time Neutrality implies that $\{50\} \sim \{0, 100\}^C$.

This follows from observing that $\{50\}$ and $\{0, 100\}^C$ reduce to the same lottery.

APPENDIX C: SUPPLEMENTARY TABLES AND FIGURES

TABLE C.I

SUMMARY STATISTICS IN MAIN EXPERIMENT[a]

	N	Mean	SD	Seeking	Neutral	Averse
{50}	188	7.888	4.716	27	19	142
$[25, 75]^A$	188	6.346	4.314	30	65	93
$[0, 25] \cup [75, 100]^A$	188	5.840	4.297	19	65	104
$\{25, 75\}^A$	188	6.106	4.254	24	70	94
$[0, 100]^A$	188	5.686	4.483	22	69	97
$\{0, 100\}^A$	188	7.122	5.665	34	80	74
$[25, 75]^C$	188	6.686	4.256	29	78	81
$[0, 25] \cup [75, 100]^C$	188	6.128	4.199	39	54	95
$\{25, 75\}^C$	188	6.351	4.194	35	57	96
$[0, 100]^C$	188	5.830	4.259	25	63	100
$\{0, 100\}^C$	188	6.947	5.866	37	72	79

[a]Note. This table summarizes the mean and standard deviation of CEs, as well as the number of subjects who exhibit aversion, neutrality, and affinity towards each lottery.

TABLE C.II

TRENDS IN CHOICE PATTERNS ACROSS EXPERIMENTS[a]

	Ambiguity (M)	Compound Risk (M)	Ambiguity (S1)	Compound Risk (S2)
A: Interval Lotteries				
Increasing	12	15	0	12
Constant	49	47	20	21
Decreasing	79	85	25	30
B: Disjoint Lotteries				
Increasing	33	42	4	22
Constant	57	52	19	16
Decreasing	50	50	19	26
C: Two-point Lotteries				
Increasing	17	20	7	8
Constant	54	41	22	14
Decreasing 1	50	45	33	17
Decreasing 2	53	60	22	18

[a]Note. Panel A (B) displays the number of subjects whose CEs are increasing, constant, or decreasing as the number of possible compositions increases for the interval (disjoint) lotteries. Panel C displays the number of subjects whose CEs are increasing, constant, or decreasing in two ways (Decreasing 1: including the end-point; Decreasing 2: excluding the end-point) as the spread increases for the two-point lotteries.

TABLE C.III

END-POINT BEHAVIOR ACROSS EXPERIMENTS[a]

Experiment	$\{50\} > \{0, 100\}^A$	$\{50\} = \{0, 100\}^A$	$\{50\} < \{0, 100\}^A$
Main	74 (39.4%)	80 (42.6%)	34 (18.1%)
S1	33 (31.1%)	49 (46.2%)	24 (22.6%)
	$\{50\} > \{0, 100\}^C$	$\{50\} = \{0, 100\}^C$	$\{50\} < \{0, 100\}^C$
Main	79 (42.0%)	72 (38.3%)	37 (19.7%)
S2	43 (43.9%)	34 (34.7%)	21 (21.4%)

[a]Note. This table displays the number (percentage) of subjects in terms of their preference between $\{50\}$ and $\{0, 100\}^A$ in the main experiment and experiment S1, as well as between $\{50\}$ and $\{0, 100\}^C$ in the main experiment and experiment S2.

TABLE C.IV

SPEARMAN CORRELATION OF AMBIGUITY PREMIUM AND COMPOUND RISK PREMIUM

	$[25, 75]^C$	$[0, 25] \cup [75, 100]^C$	$\{25, 75\}^C$	$[0, 100]^C$	$\{0, 100\}^C$
$[25, 75]^A$	**0.614**	0.525	0.544	0.492	0.267
$[0, 25] \cup [75, 100]^A$	0.477	**0.613**	0.597	0.520	0.451
$\{25, 75\}^A$	0.564	0.556	**0.694**	0.600	0.382
$[0, 100]^A$	0.497	0.590	0.473	**0.580**	0.285
$\{0, 100\}^A$	0.170	0.271	0.378	0.199	**0.441**

TABLE C.V

CORRELATION BETWEEN AMBIGUITY AND COMPOUND RISK PREMIUMS ACROSS STUDIES

Study	Correlation	N	Remark
Halevy (2007)[a]	0.474	104	first round
Halevy (2007)[a]	0.810	38	robustness round
Dean and Ortoleva (2012)	0.730	190	–
Abdellaoui, Klibanoff, and Placido (2015)[a]	0.483	115	uniform compound risk
Abdellaoui, Klibanoff, and Placido (2015)[a]	0.557	115	hypergeometric compound risk
Gillen, Snowberg, and Yariv (2015)	0.440	786	no control for measurement error
Gillen, Snowberg, and Yariv (2015)	0.850	786	control for measurement error
Current study	0.580	188	[0, 100]
Current study	0.614	188	[25, 75]
Current study	0.613	188	[0, 25] ∪ [75, 100]
Current study	0.694	188	{25, 75}
Current study	0.441	188	{0, 100}
Chew, Miao, and Zhong (2017)	0.489	3146	[0, 100]

[a]The correlations are calculated based on data provided.

TABLE C.VI

COLOR PREFERENCE ACROSS STUDIES[a]

Study	Color Preference	N	Note
Abdellaoui et al. (2011)	1%	67	eight-color urn
Epstein and Halevy (2017)	5%	80	first experiment
Epstein and Halevy (2017)	23%	87	second experiment
Epstein and Halevy (2017)	14%	44	risk control with {50}
Epstein and Halevy (2017)	23%	39	single urn control with $[0, 100]^A$
Current study	13%	39	{50}
Current study	15%	39	$[0, 100]^A$
Current study	15%	39	$\{0, 100\}^A$

[a]Note. Abdellaoui et al. (2011) tested symmetry for one-color events, two-color events, and four-color events in an eight-color unknown urn. Epstein and Halevy (2017) tested symmetry with two unknown urns in two experiments, along with one risky urn and one fully ambiguous urn as controls. Our supplementary experiment follows Epstein and Halevy (2017) in which subjects make binary choices between betting on either color with slightly unequal prizes.

TABLE C.VII

ASSOCIATION BETWEEN AMBIGUITY ATTITUDE AND RCLA[a]

	Compound Risk		
	Reduction	Non-Reduction	Total
Panel A: Ambiguity first			
Ambiguity			
Neutral	18 (3.7)	2 (16.3)	20
Nonneutral	1 (15.3)	81 (66.7)	82
Total	19	83	102
Panel B: Compound risk first			
Ambiguity			
Neutral	12 (2.8)	8 (17.2)	20
Nonneutral	0 (9.2)	66 (56.8)	66
Total	12	74	86

[a]Note. The two-way table presents the number of subjects by whether RCLA holds and whether ambiguity neutrality holds. Each cell indicates the number of subjects with the expected number displayed in parentheses.

TABLE C.VIII

COMPARISON BETWEEN AMBIGUOUS AND COMPOUND TWO-POINT LOTTERIES[a]

	$\{0, 100\}$			
	$CE^A < CE^C$	$CE^A = CE^C$	$CE^A > CE^C$	Total
Panel A: Ambiguity first				
$\{25, 75\}$				
$CE^A < CE^C$	7 (3.8)	5 (5.8)	3 (5.4)	15
$CE^A = CE^C$	7 (11)	19 (17.1)	18 (15.9)	44
$CE^A > CE^C$	4 (3.3)	4 (5.1)	5 (4.7)	13
Total	18	28	26	72
Panel B: Compound risk first				
$\{25, 75\}$				
$CE^A < CE^C$	8 (5.6)	10 (9.2)	0 (3.2)	18
$CE^A = CE^C$	7 (8)	12 (13.6)	7 (4.7)	26
$CE^A > CE^C$	2 (3.4)	6 (5.6)	3 (2)	11
Total	17	28	10	55

[a]Note. The two-way table presents the number of subjects in each of the categories—$CE^A < CE^C$, $CE^A = CE^C$, and $CE^A > CE^C$—indicating that the CE of a two-point ambiguity lottery may be smaller than, equal to, or larger than that of the corresponding two-point compound lottery. Each cell indicates the number of subjects with the expected number displayed in parentheses.

TABLE C.IX

INDULGE TYPES WITH TWO-STAGE PERSPECTIVE[a]

	Compound Risk			
	REU	RRDU	Unclassified	Total
Panel A: Ambiguity first				
Ambiguity				
REU	11 (8.3)	8 (0.2)	4 (4.5)	23
RRDU	11 (14.1)	21 (17.3)	7 (7.6)	39
Unclassified	4 (3.6)	3 (4.4)	3 (1.9)	10
Total	26	32	14	72
Panel B: Compound risk first				
Ambiguity				
REU	16 (12.3)	5 (10.0)	4 (2.7)	25
RRDU	6 (9.3)	11 (7.6)	2 (2.1)	19
Unclassified	5 (5.4)	6 (4.4)	0 (1.2)	11
Total	27	22	6	55

[a]Note. The two-way table presents the number of subjects classified as REU or RRDU separately for ambiguity and compound risk. Each cell indicates the number of subjects with the expected number displayed in parentheses.

C.1. *Figures*
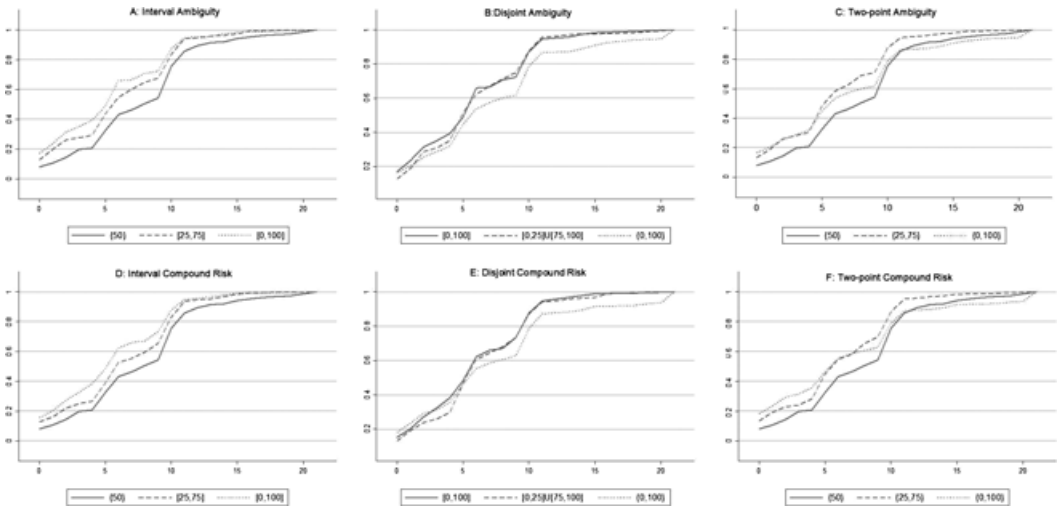
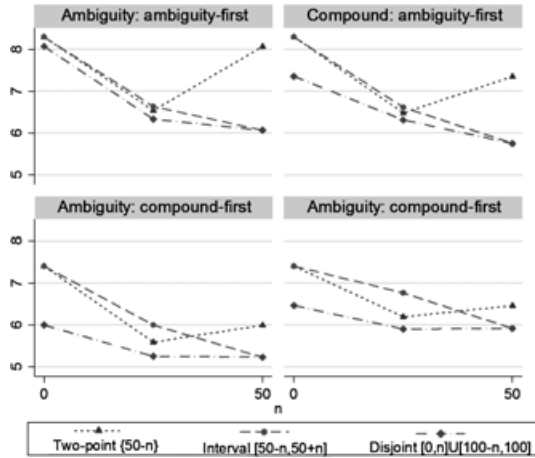FIGURE C.1.—Cumulative distribution of CEs in each kind of partial ambiguity/compound risk.

FIGURE C.2.—Aggregate choice patterns for the two treatments.

C.2. *Order Effect*

We assess the extent to which our results, especially those concerning the three key findings and individual type, are robust to the order of appearance of ambiguity and compound risk in our main experiment. The plots of the aggregate data displayed in Figure C.2 reveal that the observed choice patterns for interval, disjoint, and two-point ambiguity (compound risk) are generally similar across the two orders of appearance. Namely, there is a decreasing trend in the CEs for the interval and disjoint ambiguity (compound risk) as the number of possible compositions increases. For two-point ambiguity (compound risk), there is an initially decreasing trend in the CEs as the spread increases except for the end-point {0, 100}.

To check the robustness of the association between ambiguity attitude and RCLA for both orders, we plot the corresponding two-way tables separately (Table C.VII). For the 102 subjects under "ambiguity first," of the 19 subjects who are observed to reduce all compound lotteries, 18 are ambiguity neural. Of the 83 subjects who violate RCLA, 81 exhibit ambiguity non-neutrality. For the 86 subjects under "compound risk first," of the 12 subjects reducing all compound lotteries, all exhibit ambiguity neutrality. Of the 74 subjects who violate RCLA, 66 exhibit ambiguity non-neutrality. The null hypothesis of independence is strongly rejected in both order treatments (Pearson's chi-squared test, $p < 0.001$ for each treatment). This suggests that attitude towards partial ambiguity and attitude towards the corresponding compound risk are closely correlated within both treatments.

Table C.VIII further examines the robustness of the non-neutrality between two-point ambiguity and two-point compound risk. Under ambiguity first (compound risk first), 19 (12) subjects have similar CEs, 27 (16) subjects weakly prefer ambiguous two-point lotteries to compound two-point lotteries, and 19 (25) subjects exhibit the reverse behavior, with the rest of the 7 (2) subjects not revealing a consistent preference. The difference between the two treatments is significant for {0, 100} (multinomial logistic regression, $p < 0.023$), but not for {25, 75} (multinomial logistic regression, $p < 0.508$). Overall, while there are some differences between the two order treatments, the observation that substantial proportions of subjects value these two types of lotteries differently remains robust.

In an individual type analysis for REU and RRDU under the two treatments (see Table C.IX), 11 (16) are classified as REU type and 21 (11) are classified as RRDU type under ambiguity first (compound risk first) treatment. A proportion test reveals that the difference between these two treatments is statistically significant ($p < 0.039$). In sum, the three key findings appear robust to the two different orders. At the same time, we observe an order effect on the classification of individual type.

APPENDIX D: RESULTS OF SUPPLEMENTARY EXPERIMENTS

D.1. *Experiment S1 on Ambiguity*

This subsection presents the design and results of Experiment S1 on partial ambiguity. We first present the experimental design, and then analyze the choice patterns for the three kinds of partial ambiguity at both aggregate and individual levels.

D.1.1. *Design*

Experiment S1 comprises three kinds of six lotteries each with a total of 15 lotteries as follows.

Interval ambiguity. It comprises six lotteries with interval ambiguity: $\{50\}$, $\mathbf{I}_{10}^A = [40, 60]^A$, $\mathbf{I}_{20}^A = [30, 70]^A$, $\mathbf{I}_{30}^A = [20, 80]^A$, $\mathbf{I}_{40}^A = [10, 90]^A$, $[0, 100]^A$.

Disjoint ambiguity. It involves six lotteries with disjoint ambiguity: $[0, 100]^A$, $\mathbf{D}_{10}^A = [0, 10] \cup [90, 100]^A$, $\mathbf{D}_{20}^A = [0, 20] \cup [80, 100]^A$, $\mathbf{D}_{30}^A = [0, 30] \cup [70, 100]^A$, $\mathbf{D}_{40}^A = [0, 40] \cup [60, 100]^A$, $[0, 100]^A$.

Two-point ambiguity. It involves six lotteries with two-point ambiguity: $\{50\}$, $\mathbf{T}_{10}^A = \{40, 60\}^A$, $\mathbf{T}_{20}^A = \{30, 70\}^A$, $\mathbf{T}_{30}^A = \{20, 80\}^A$, $\mathbf{T}_{40}^A = \{10, 90\}^A$, $\{0, 100\}^A$.

In the experiment, subjects were shown the 15 decks of cards. For each lottery, betting correctly on the color of a drawn card would deliver SGD40 (about USD30) while betting incorrectly would deliver nothing. We use the price-list design and RIM in eliciting the CEs of various lotteries. The order of appearance of the 15 lotteries is randomized for the subjects who each make 150 choices in total (see Appendix E for detailed Experimental instructions). At the end of the experiment, in addition to a SGD5 show-up fee, each subject is paid based on his/her randomly selected decision in the experiment. One out of 150 choices is randomly chosen using dice. We recruited 112 undergraduate students from the National University of Singapore (NUS) by advertising on the university platform, the Integrated Virtual Learning Environment. The experiment consisted of four sessions with 20 to 30 subjects in each session.

D.1.2. *Observed Choice Behavior*

We present the observed choice behavior at both aggregate and individual levels for 106 subjects. Six subjects exhibit multiple switching in some of the tasks. Their data are excluded from our analysis. The choice data are similarly coded in terms of the switch point given by the number of times a subject chooses a given lottery over different increasingly ordered sure amounts before switching over to choosing the sure amounts.

We first examine the implication of ambiguity neutrality, that is, that subjects assign the same CE to the 15 lotteries. Using a Friedman test ($p < 0.001$), we reject the null hypothesis that the CEs of the 15 lotteries come from a single distribution. Besides replicating the standard finding on ambiguity aversion with CE of $\{50\}$ being significantly higher than that of $[0, 100]^A$ (paired Wilcoxon Signed-rank test, $p < 0.001$), our subjects have distinct attitudes towards different kinds of partial ambiguity. Specifically, for the comparison

between $\{50\}$ and $[0, 100]^A$, 62 subjects (58.5%) exhibit ambiguity aversion, 33 subjects (31.1%) exhibit ambiguity neutrality, and 11 subjects (10.4%) exhibit ambiguity affinity. Comparing $\{50\}$ with the other 14 ambiguous lotteries at the individual level, 16 out of 106 subjects (15.1%) have the same CEs for the 15 lotteries. Among the others, 48 subjects (45.3%) exhibit overall ambiguity aversion in having weakly larger CEs for $\{50\}$ than for the other 14 ambiguous lotteries. Thirteen subjects (12.3%) have weakly lower CEs for $\{50\}$ than that for the other 14 ambiguous lotteries, hence revealing some degree of ambiguity affinity. The remaining 29 subjects (24.3%) do not exhibit uniform attitude towards ambiguity. Using a similar analysis as in the main experiment, we replicate the observations for the three kinds of partial ambiguity.

For interval ambiguity, there is a statistically significant *decreasing* trend in the CEs as the number of possible compositions increases ($p < 0.001$). At the individual level, 20 subjects (18.9%) have the same CEs, 25 subjects (23.6%) have weakly decreasing CEs, while none of the subjects has weakly increasing CEs. For disjoint ambiguity, there is a statistically significant *decreasing* trend in the CEs as the number of possible compositions increases ($p < 0.001$). At the individual level, 19 subjects (17.9%) have the same CEs, 19 subjects (17.9%) have weakly decreasing CEs, and four subjects (3.8%) have weakly increasing CEs.

For two-point ambiguity, there is a significant *decreasing* trend in the CEs until the endpoint. Interestingly, the CE of $\{0, 100\}^A$ reverses this trend and is significantly higher than the CE of $\{10, 90\}^A$ (paired Wilcoxon Signed-rank test, $p < 0.001$). Moreover, the CE of $\{10, 90\}^A$ is not significantly different from that of $\{50\}$ (paired Wilcoxon Signed-rank test, $p > 0.225$). At the individual level, 22 subjects (20.8%) have the same CEs, 14 subjects (13.2%) have weakly decreasing CEs, 33 subjects (31.1%) have weakly decreasing CEs until $\{10, 90\}^A$ with an increase at $\{0, 100\}^A$, and seven subjects (6.6%) have weakly increasing CEs. Between $\{50\}$ and $\{0, 100\}^A$, 49 subjects (46.2%) have the same CEs, 33 subjects (31.1%) display a higher CE for $\{50\}$ than that for $\{0, 100\}^A$, and 24 subjects (22.6%) exhibit the reverse.

In summary, the observed patterns towards the three kinds of ambiguity replicate the observations in the main experiment regarding ambiguity lotteries.

D.2. *Experiment S2 on Compound Risk*

This subsection presents the design and results of Experiment S2 on compound lottery. We first present the experimental design, and then analyze the choice patterns for the three kinds of compound lottery at both aggregate and individual levels.

D.2.1. *Design*

Experiment S2 on uniform compound risk links naturally to the two-stage perspective of ambiguity under uniform priors. For each compound lottery, we implement objective uniform stage-1 prior as follows: one ticket is randomly drawn from a bag containing some tickets with different numbers written on them. The number drawn determines the number of red cards in the deck with the rest black. The resulting stage-1 risk is thus uniformly distributed among different numbers while the bet at stage 2 involves betting on the color of a card randomly drawn from the deck. There are three kinds of nine lotteries included in this experiment.

Interval compound risk. This involves four lotteries with symmetric interval stage-1 risk: $\{50\}$, $\mathbf{I}_{10}^C = [40, 60]^C$, $\mathbf{I}_{30}^C = [20, 80]^C$, $[0, 100]^C$.

Disjoint compound risk. This involves four lotteries with symmetric disjoint stage-1 risk: $\{0, 100\}^C$, $\mathbf{D}_{20}^C = [0, 20] \cup [80, 100]^C$, $\mathbf{D}_{40}^C = [0, 40] \cup [60, 100]^C$, $[0, 100]^C$.

Two-point compound risk. This involves four lotteries with symmetric two-point stage-1 risk: $\{50\}$, $\mathbf{T}_{10}^C = \{40, 60\}^C$, $\mathbf{T}_{30}^C = \{20, 80\}^C$, $\{0, 100\}^C$.

The elicitation mechanism and experimental procedure are similar to those in Experiment S1 (see Appendix E for detailed Experimental instructions). We have 109 subjects in Experiment S2.

D.2.2. *Observed Choice Behavior*

We report the observed choice patterns at both aggregate and individual levels. To test the implication of RCLA, that is, that the CEs of the nine lotteries are the same, we apply the Friedman test and reject the null hypothesis that their CEs come from the same distribution ($p < 0.001$). At the individual level, 12 of 98 subjects (12.2%) have the same CE for the nine lotteries. Besides these subjects, 39 subjects (40.0%) have weakly larger CEs for $\{50\}$ than that of the other eight compound lotteries, while 17 subjects (17.3%) exhibit the opposite pattern with weakly lower CEs for $\{50\}$ than the other eight lotteries. This suggests that subjects tend to weakly prefer receiving the reduced simple lottery than any of the eight other compound lotteries. To study the choice patterns across the three kinds of compound lotteries, we again apply a similar analysis as in the main experiment to examine whether there is a significant trend in each kind corresponding to attitudes towards different patterns of spread in stage-1 risks.

For interval compound risk, there is a statistically significant *decreasing* trend in the CEs as the stage-1 risks spread away from the mid-point ($p < 0.001$). At the individual level, 21 subjects (21.4%) have the same CEs, 30 subjects (30.6%) have weakly increasing CEs, and 12 subjects (12.2%) have weakly decreasing CEs, with the rest of the subjects not exhibiting monotonic preference in relation to uniform interval spread. For disjoint compound risk, there is a statistically significant *increasing* trend in the CEs as the stage-1 risks spread away from the mid-point ($p < 0.001$). At the individual level, 16 subjects (16.3%) have the same CEs, 26 subjects (26.5%) have weakly increasing CEs, and 22 subjects (22.4%) have weakly decreasing CEs, with the rest of the subjects not exhibiting monotonic preference in relation to uniform disjoint spread.

For two-point compound risk, there is a statistically significant *decreasing* trend in the CEs as the stage-1 risks spread away from the mid-point ($p < 0.042$). At the individual level, 18 subjects (18.4%) have weakly increasing CEs, 14 subjects (14.3%) have the same CEs, 8 subjects (8.2%) have weakly decreasing CEs, and 17 subjects (17.3%) have weakly decreasing CEs initially followed by an increase near the end-point, while the remaining 41 subjects (42.0%) do not exhibit any of these patterns. Focusing on $\{50\}$ and $\{0, 100\}^C$, 34 subjects (34.7%) have the same CEs, 43 subjects (43.9%) have a higher CE for $\{50\}$ than that of $\{0, 100\}^C$, while the other 21 subjects (21.4%) have the reverse preference. Paired Wilcoxon Signed-rank test shows that the CE of $\{50\}$ is significantly higher than that of $\{0, 100\}^C$ ($p < 0.022$).

In summary, the observed patterns towards the three kinds of compound lottery replicate the observations in the main experiment except for the end-point, which we elaborate in the manuscript.

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