Econometrica Supplementary Material

SUPPLEMENT TO "IDENTIFYING EQUILIBRIUM MODELS OF LABOR MARKET SORTING"

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The supplement contains most formal proofs and derivations (Section A), detailed description of implementation strategy and computational algorithms (Sections B and C), complete specification of the model with on-the-job search and the identification proof for this model (Section D), details of empirical analysis using large German matched employer–employee data (Section E), and figures describing Monte Carlo results for the benchmark model and various alternative specifications (Section F).

A. PROOFS AND DERIVATIONS

A.1. *Derivation of Value Functions*

WE DERIVE WORKERS' VALUE FUNCTIONS only since the functions for firms follow by symmetry.

An unemployed worker becomes employed only if he meets a firm in his acceptance set, and does not experience immediate match destruction. Otherwise, the worker remains unemployed in the next period:

$$\begin{aligned} V_u(x) = {} & \underbrace{\beta(1-\delta)\mathbb{M}_u \int_{B^w(x)} \frac{d_v(\tilde{y})}{V} V_e(x,\tilde{y})\,\mathrm{d}\tilde{y}}_{\text{successful matching}} \\ & + \underbrace{\beta\delta V_u(x)}_{\text{destruction}} + \underbrace{\beta(1-\delta)(1-\mathbb{M}_u)V_u(x)}_{\text{no meeting}} \\ & + \underbrace{\beta(1-\delta)\mathbb{M}_u V_u(x) \int_{\overline{B^w(x)}} \frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y}}_{\text{meet unacceptable firm}}. \end{aligned}$$

To express the continuation value from successful matching in terms of surplus, subtract $V_u(x)$ from the first integrand and add it back to rebalance the equation. Then, use (1) to obtain

$$\begin{aligned} V_u(x) = {} & \beta\alpha(1-\delta)\mathbb{M}_u \int_{B^w(x)} \frac{d_v(\tilde{y})}{V} S(x,\tilde{y})\,\mathrm{d}\tilde{y} \\ & + \beta\delta V_u(x) + \beta(1-\delta)(1-\mathbb{M}_u)V_u(x) \\ & + \beta(1-\delta)\mathbb{M}_u V_u(x)\left[\int_{\overline{B^w(x)}} \frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y} + \int_{B^w(x)} \frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y}\right], \end{aligned}$$

where terms cancel to give (3).

An employed worker receives $w(x, y)$, and remains employed next period with probability $(1-\delta)$ or becomes unemployed with complementary probability. Minor rearranging

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and (1) yield (5):

$$\begin{aligned}V_e(x,y) &= w(x,y) + \beta\delta V_u(x) + \beta(1-\delta)V_e(x,y)\\ &= w(x,y) + \beta\delta V_u(x) + \beta\alpha(1-\delta)S(x,y) + \beta(1-\delta)V_u(x)\\ &= w(x,y) + \beta V_u(x) + \beta\alpha(1-\delta)S(x,y).\end{aligned}$$

A.2. *Proofs of Results in Section 3.1*

PROOF OF RESULT 1(i): Adding (5) and (6) yields

$$V_e(x,y) + V_p(x,y) = f(x,y) + \beta V_v(y) + \beta V_u(x) + \beta(1-\delta)S(x,y),$$

and, equivalently,

$$\begin{aligned}&V_e(x,y) - V_u(x) + V_p(x,y) - V_v(y)\\ &\quad = f(x,y) + (\beta-1)V_v(y) + (\beta-1)V_u(x)\\ &\qquad + \beta(1-\delta)S(x,y),\end{aligned}$$

so that, using (1), gives

$$S(x,y)\big(1-\beta(1-\delta)\big) = f(x,y) + (\beta-1)V_v(y) + (\beta-1)V_u(x),$$

and thus surplus equals

$$S(x,y) = \frac{f(x,y) + (\beta-1)V_v(y) + (\beta-1)V_u(x)}{1-\beta(1-\delta)}. \tag{A.1}$$

Using (5) again gives us wages[1]

$$\begin{aligned}w(x,y) &= S(x,y)\alpha\big(1-\beta(1-\delta)\big) + (1-\beta)V_u(x)\\ &= \alpha f(x,y) + \alpha(\beta-1)V_v(y) + (1-\alpha)(1-\beta)V_u(x).\end{aligned} \tag{A.2}$$

We now establish that $V_u(x)$ is increasing in x. From (3),

$$V_u(x)(1-\beta) = \beta\alpha(1-\delta)\mathbb{M}_u \int_{B^w(x)} \frac{d_v(\tilde{y})}{V} S(x,\tilde{y})\,\mathrm{d}\tilde{y},$$

so that

$$\frac{\partial V_u(x)}{\partial x}(1-\beta) = \beta\alpha(1-\delta)\mathbb{M}_u \int_{B^w(x)} \frac{d_v(\tilde{y})}{V} \frac{\partial S(x,\tilde{y})}{\partial x}\,\mathrm{d}\tilde{y},$$

[1]Wages can also be derived using (6):

$$\begin{aligned}w(x,y) &= f(x,y) - S(x,y)(1-\alpha)\big(1-\beta(1-\delta)\big) + (\beta-1)V_v(y)\\ &= f(x,y) - (1-\alpha)f(x,y) - (1-\alpha)(\beta-1)V_u(x) + \alpha(\beta-1)V_v(y)\\ &= \alpha f(x,y) + (1-\alpha)(1-\beta)V_u(x) + \alpha(\beta-1)V_v(y).\end{aligned}$$

keeping in mind that either $S(x, y) = 0$ at the interior boundaries of the matching set or the non-interior boundaries do not change with x. More precisely, consider for simplicity $B^w(x) = [\underline{\varphi}(x), \overline{\varphi}(x)]$. If $\underline{\varphi}(x) \neq 0$, then $S(x, \underline{\varphi}(x)) = 0$. If $\underline{\varphi}(x) = 0$, then $\frac{\partial \underline{\varphi}(x)}{\partial x} = 0$. Analogously, if $\overline{\varphi}(x) \neq 1$, then $S(x, \overline{\varphi}(x)) = 0$. If $\overline{\varphi}(x) = 1$, then $\frac{\partial \overline{\varphi}(x)}{\partial x} = 0$.

As a result, we have, using (A.1), that

$$\begin{aligned}&\frac{\partial V_u(x)}{\partial x}(1-\beta)\\&\quad = \frac{\beta\alpha(1-\delta)\mathbb{M}_u}{1-\beta(1-\delta)} \int_{B^w(x)} \frac{d_v(\tilde{y})}{V} \frac{\partial f(x, \tilde{y}) + (\beta - 1)V_u(x)}{\partial x} \,\mathrm{d}\tilde{y}.\end{aligned}$$

Solving for $\frac{\partial V_u(x)}{\partial x}$ yields

$$\begin{aligned}&\frac{\partial V_u(x)}{\partial x}\left(1-\beta + \frac{(1-\beta)\beta\alpha(1-\delta)\mathbb{M}_u}{1-\beta(1-\delta)} \int_{B^w(x)} \frac{d_v(\tilde{y})}{V} \,\mathrm{d}\tilde{y}\right)\\&\quad = \frac{\beta\alpha(1-\delta)\mathbb{M}_u}{1-\beta(1-\delta)} \int_{B^w(x)} \frac{d_v(\tilde{y})}{V} \frac{\partial f(x, \tilde{y})}{\partial x} \,\mathrm{d}\tilde{y}\end{aligned}$$

and thus $\frac{\partial V_u(x)}{\partial x} > 0$ since $\frac{\partial f(x,y)}{\partial x} > 0$.

To show that $w(x, y)$ is increasing in x, we differentiate (A.2):

$$\frac{\partial w(x, y)}{\partial x} = \alpha \frac{\partial f(x, y)}{\partial x} + (1-\alpha)(1-\beta)\frac{\partial V_u(x)}{\partial x},$$

which is positive because $\frac{\partial f(x,y)}{\partial x} > 0$ and $\frac{\partial V_u(x)}{\partial x} > 0$.

Finally, we show that $V_e(x, y)$ is increasing in x as well. We have

$$V_e(x, y) = w(x, y) + \beta\delta V_u(x) + \beta(1-\delta)V_e(x, y),$$

and thus that

$$V_e(x, y)\big(1 - \beta(1-\delta)\big) = w(x, y) + \beta\delta V_u(x),$$

which is increasing in x since $\frac{\partial w(x,y)}{\partial x} > 0$ and $\frac{\partial V_u(x)}{\partial x} > 0$. *Q.E.D.*

PROOF OF RESULT 1(ii): Let $y^{\min}(x)$ be a firm type such that worker x is indifferent between matching with this firm and staying unemployed,

$$V_e\big(x, y^{\min}(x)\big) = V_u(x).$$

$y^{\min}(x)$ is the firm that pays the reservation wage to a worker of type x. Then (5) can be written as

$$V_e\big(x, y^{\min}(x)\big) = w\big(x, y^{\min}(x)\big) + \beta V_u(x),$$

so that

$$w\big(x, y^{\min}(x)\big) = V_e\big(x, y^{\min}(x)\big) - \beta V_u(x) = (1-\beta)V_u(x)$$

which from Result 1(i) is increasing in x. *Q.E.D.*

PROOF OF RESULT 1(iii): The maximum wage is given by $w(x, y^{\max}(x))$. Taking derivatives with respect to x yields

$$\begin{aligned}\frac{\partial w\big(x, y^{\max}(x)\big)}{\partial x} &= w_x\big(x, y^{\max}(x)\big) + w_y\big(x, y^{\max}(x)\big) y_x^{\max}(x) \\ &= w_x\big(x, y^{\max}(x)\big) > 0. \end{aligned}$$

Q.E.D.

PROOF OF RESULT 1(iv): Assume that the matching sets are unions of intervals. For the ease of exposition, we assume that there is just one interval:

$$B^w(x) = \big[\underline{\varphi}(x), \overline{\varphi}(x)\big].$$

First rewrite the adjusted average wage as

$$\begin{aligned} w^{av}(x) &= w\big(x, y^{\min}(x)\big) \\ &\quad + \mathbb{M}_u(1-\delta)\int_{B^w(x)} \frac{d_v(\tilde{y})}{V}\big[w(x, \tilde{y}) - w\big(x, y^{\min}(x)\big)\big]\,\mathrm{d}\tilde{y}. \end{aligned}$$

Take derivatives with respect to x:

$$\begin{aligned} \frac{\partial w^{av}(x)}{\partial x} &= \frac{\partial w\big(x, y^{\min}(x)\big)}{\partial x} \\ &\quad + \mathbb{M}_u(1-\delta)\int_{B^w(x)} \frac{\partial w(x, \tilde{y}) - w\big(x, y^{\min}(x)\big)}{\partial x}\frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y} \\ &\quad + \mathbb{M}_u(1-\delta)\overline{\varphi}'(x)\frac{d_v\big(\overline{\varphi}(x)\big)}{V}\big[w\big(x, \overline{\varphi}(x)\big) - w\big(x, y^{\min}(x)\big)\big] \\ &\quad - \mathbb{M}_u(1-\delta)\underline{\varphi}'(x)\frac{d_v\big(\underline{\varphi}(x)\big)}{V}\big[w\big(x, \underline{\varphi}(x)\big) - w\big(x, y^{\min}(x)\big)\big]. \end{aligned}$$

The last two terms are zero since either $w(x, \overline{\varphi}(x)) = w(x, y^{\min}(x))$ or $\overline{\varphi}'(x) = 0$ and either $w(x, \underline{\varphi}(x)) = w(x, y^{\min}(x))$ or $\underline{\varphi}'(x) = 0$. Now simply rewrite

$$\begin{aligned} &\frac{\partial w^{av}(x)}{\partial x} \\ &\quad = \frac{\partial w\big(x, y^{\min}(x)\big)}{\partial x}\left[1 - \mathbb{M}_u + \delta\mathbb{M}_u + \mathbb{M}_u(1-\delta)\int_{\overline{B^w(x)}} \frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y}\right] \\ &\qquad + \mathbb{M}_u(1-\delta)\int_{B^w(x)} \frac{\partial w(x, \tilde{y})}{\partial x}\frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y} \end{aligned}$$

to see that $\frac{\partial w^{av}(x)}{\partial x} > 0$. *Q.E.D.*

A.3. *Proofs of Results in Section 3.2*

PROOF OF RESULT 2: For the value of a vacancy we have that

$$V_v(y)(1-\beta) = \beta(1-\alpha)(1-\delta)\mathbb{M}_v \int_{B^f(y)} \frac{d_u(\tilde{x})}{U} S(\tilde{x}, y)\,\mathrm{d}\tilde{x},$$

so that (using again as in the Proof of Result 1(i) that the terms involving the derivatives of the boundaries are zero)

$$\begin{aligned}
&\frac{\partial V_v(y)}{\partial y}(1-\beta) \\
&\quad = \beta(1-\alpha)(1-\delta)\mathbb{M}_v \int_{B^f(y)} \frac{d_u(\tilde{x})}{U} \frac{\dfrac{\partial f(\tilde{x}, y) + (\beta-1)V_v(y)}{\partial y}}{1-\beta(1-\delta)}\,\mathrm{d}\tilde{x},
\end{aligned}$$

and thus that

$$\begin{aligned}
&\frac{\partial V_v(y)}{\partial y}\left(1-\beta + \frac{(1-\beta)\beta(1-\alpha)(1-\delta)\mathbb{M}_v}{1-\beta(1-\delta)} \int_{B^f(y)} \frac{d_u(\tilde{x})}{U}\,\mathrm{d}\tilde{x}\right) \\
&\quad = \beta(1-\alpha)(1-\delta)\mathbb{M}_v \int_{B^f(y)} \frac{d_u(\tilde{x})}{U} \frac{\dfrac{\partial f(\tilde{x}, y)}{\partial y}}{1-\beta(1-\delta)}\,\mathrm{d}\tilde{x} > 0,
\end{aligned}$$

so that $\frac{\partial V_v(y)}{\partial y} > 0$ since the coefficient multiplying it is positive. Finally, we show that the value of a filled job for a firm is increasing in y. We have that

$$\begin{aligned}
V_p(x, y) &= f(x, y) - w(x, y) + \beta V_v(y) + \beta(1-\alpha)(1-\delta)S(x, y) \\
&= f(x, y)(1-\alpha) - (1-\alpha)(1-\beta)V_u(x) \\
&\quad + \alpha(1-\beta)V_v(y) + \beta V_v(y) \\
&\quad + \beta(1-\delta)\big(V_p(x, y) - V_v(y)\big),
\end{aligned}$$

so that

$$\begin{aligned}
V_p(x, y)\big(1-\beta(1-\delta)\big) &= f(x, y)(1-\alpha) - (1-\alpha)(1-\beta)V_u(x) \\
&\quad + V_v(y)\big(\beta\delta + \alpha(1-\beta)\big),
\end{aligned}$$

and

$$\begin{aligned}
&\frac{\partial V_p(x, y)}{\partial y}\big(1-\beta(1-\delta)\big) \\
&\quad = \frac{\partial f(x, y)}{\partial y}(1-\alpha) + \frac{\partial V_v(y)}{\partial y}\big(\beta\delta + \alpha(1-\beta)\big) > 0.
\end{aligned}$$

Q.E.D.

PROOF OF RESULT 4: The main part of the proof is in the main text. Here we only show that we can use Ω to rank firms even if some or all workers match with all firm types or if some or all firms match with all worker types. In particular, we show that it does not matter that if a worker of type x is accepted by all firms, the lowest wage $w(x, y^{\min}(x))$ is not equal to the reservation wage and thus not equal to the return of being unemployed.

The derivative of Ω with respect to y equals

$$\begin{aligned}\frac{\partial\Omega(y)}{\partial y} &= (1-\delta)\mathbb{M}_v \int_{B^f(y)} \frac{d_u(\tilde{x})}{U}\frac{\partial w(\tilde{x}, y)}{\partial y}\,\mathrm{d}\tilde{x} \\ &\quad + (1-\delta)\mathbb{M}_v\bigg\{\big(w\big(\overline{\varphi}(y), y\big) - w\big(\overline{\varphi}(y), y^{\min}\big(\overline{\varphi}(y)\big)\big)\big) \\ &\quad \times \frac{d_u\big(\overline{\varphi}(y)\big)}{U}\frac{\partial\overline{\varphi}(y)}{\partial y}\bigg\} \\ &\quad - (1-\delta)\mathbb{M}_v\bigg\{\big(w\big(\underline{\varphi}(y), y\big) - w\big(\underline{\varphi}(y), y^{\min}\big(\underline{\varphi}(y)\big)\big)\big) \\ &\quad \times \frac{d_u\big(\underline{\varphi}(y)\big)}{U}\frac{\partial\underline{\varphi}(y)}{\partial y}\bigg\},\end{aligned}$$

where for simplicity $B^f(y) = [\underline{\varphi}(y), \overline{\varphi}(y)]$. The terms $(w(\overline{\varphi}(y), y) - w(\overline{\varphi}(y), y^{\min}(\overline{\varphi}(y))))\frac{\partial\overline{\varphi}(y)}{\partial y}$ and $(w(\underline{\varphi}(y), y) - w(\underline{\varphi}(y), y^{\min}(\underline{\varphi}(y))))\frac{\partial\underline{\varphi}(y)}{\partial y}$ are both zero, because one of the two factors is zero.

Without loss of generality consider the first term with $\overline{\varphi}(y)$. The argument for the second term with $\underline{\varphi}(y)$ is identical. If the matching set is interior, that is, $\overline{\varphi}(y) \in (0, 1)$, the lowest wage of worker type $\overline{\varphi}(y)$, $w(\overline{\varphi}(y), y^{\min}(\overline{\varphi}(y)))$, is equal to the reservation wage and also equal to $w(\overline{\varphi}(y), y)$. If the matching is not interior, that is, $\overline{\varphi}(y) \in \{0, 1\}$, $\frac{\partial\overline{\varphi}(y)}{\partial y} = 0$.

Thus, independent of whether the matching set is interior or not, the term is zero and the derivative of Ω with respect to y equals

$$\frac{\partial\Omega(y)}{\partial y} = (1-\delta)\mathbb{M}_v \int_{B^f(y)} \frac{d_u(\tilde{x})}{U}\frac{\partial w(\tilde{x}, y)}{\partial y}\,\mathrm{d}\tilde{x}.$$

By the same logic, it holds that

$$\frac{\partial \int_{B^f(y)} \frac{d_u(x)}{U}\big(V_e(x, y) - V_u(x)\big)\,\mathrm{d}x}{\partial y} = \int_{B^f(y)} \frac{d_u(\tilde{x})}{U}\frac{\partial V_e(\tilde{x}, y)}{\partial y}\,\mathrm{d}\tilde{x},$$

which is proportional to $\frac{\partial\Omega(y)}{\partial y}$ since $\frac{\partial V_e(x,y)}{\partial y}(1-\beta(1-\delta)) = \frac{\partial w(x,y)}{\partial y}$. As a result, the statistic $\Omega(y)$ is increasing in y independent of the properties of the matching set. *Q.E.D.*

A.4. *Measuring α in the Data*

In the model of Shimer and Smith (2000), the value of α is fixed at $\frac{1}{2}$. More generally, one may consider leaving the value of α unrestricted in the (0, 1) interval and recovering it from the data. Note that α governs the responsiveness of wages to changes in match surplus (if $\alpha \to 0$, workers receive b regardless of the movements in the match surplus, while if $\alpha \to 1$, workers' wages fully co-move with surplus). While this provides a natural source of variation for the identification of this parameter, the fluctuations of surplus are absent from the simple baseline version of the model considered in the main text. In this appendix, we extend the model to incorporate two sources of fluctuation in match surplus, an idiosyncratic and an aggregate productivity shock, and show how each of these stochastic components allows to identify the bargaining powers.[2]

A.4.1. *Measure α From Fluctuation in Firm Output*

To measure the bargaining power α in the data, we first consider an extended version of the model with i.i.d. shocks to the firm's technology, ε_j, which changes output from $f(x, y)$ to $f(x, y) + \varepsilon_j$ for all worker types x employed at firm j of type y. In response to such a shock to the firm's technology, Nash bargaining with worker bargaining power α implies that profits increase by $(1 - \alpha)$ and wages increase by α. To measure α using this experiment, we can use any data where the response of wages is observable. This approach for identifying the bargaining powers was pursued in a number of papers in the literature reviewed in Hagedorn and Manovskii (2008).

Adding these shocks to our model is simple and does not change any of our other results and conclusions as we verified in simulated data. The reason is that these shocks are unanticipated and their impact is only to make statistics slightly noisier in the same way as measurement error does (and we have established in the main text that adding a large amount of measurement error does not have a significant impact on our inference). The ranking of workers within a firm is not affected at all since all wages within a firm are shifted by the same amount, $\alpha\varepsilon_j$, and thus the ranking of workers is preserved. The ranking of firms is based on the statistic Ω which is proportional to the value of a vacancy. Since the technology shocks are unanticipated, this statistic is not affected either. Neither is the estimation of the production function f. The only object that is affected is the estimation of the matching set, as now workers may become acceptable only because of a large positive ε_j whereas they were not acceptable in the absence of any shocks. This makes the model computationally much more burdensome. Given that adding these idiosyncratic shocks to the model obviously allows to identify α but has no material impact on any of our results, we adopted a simpler model as a benchmark in the main text.

A.4.2. *Using Business Cycles to Measure α*

We now show how the bargaining power α can be measured in the data by considering an extended version of the model with business cycles, that is, exogenous changes in aggregate productivity z. The output of a pair (x, y) is then $zf(x, y)$. Consider two worker types x and x' (have to be different types), working at firm y when productivity is z and when it is $\hat{z}$. The wages of worker x in the two business cycle states are $w(x, y, z)$ and $w(x, y, \hat{z})$, respectively. For worker x', the corresponding wages are $w(x', y, z)$ and $w(x', y, \hat{z})$. These wages are observed. The equation for wages with business cycles is straightforward and follows the same arguments as the one without business cycles. For the value of a job, it holds with the obvious notation that

$$\begin{aligned} V_e(x, y, z) = {} & w(x, y, z) + \beta E\bigl(V_u\bigl(x, z'\bigr) \mid z\bigr) \\ & + \beta\alpha(1 - \delta)E\bigl(S\bigl(x, y, z'\bigr) \mid z\bigr), \end{aligned}$$

[2] Eeckhout and Kircher (2011) have argued that the bargaining power can also be identified in the baseline deterministic version of the model. Unfortunately, their proof appears to contain a mistake (Eq. (28) in their paper does not follow from Eq. (26) since the term $w_x^*(x)$ is missing in Eq. (28)).

and for the value of a filled vacancy that

$$V_p(x,y,z) = zf(x,y) - w(x,y,z) + \beta E\big(V_v(y,z') \mid z\big) \\ + \beta(1-\alpha)(1-\delta)E\big(S(x,y,z') \mid z\big).$$

Adding up these two Bellman equations yields

$$V_e(x,y,z) + V_p(x,y,z) \\ = zf(x,y) + \beta E\big(V_v(y,z') \mid z\big) + \beta E\big(V_u(x,z') \mid z\big) \\ + \beta(1-\delta)E\big(S(x,y,z') \mid z\big),$$

and equivalently,

$$S(x,y,z) = V_e(x,y,z) - V_u(x,z) + V_p(x,y,z) - V_v(y,z) \\ = zf(x,y) - V_v(y,z) - V_u(x,z) + \beta E\big(V_v(y,z') \mid z\big) \\ + \beta E\big(V_u(x,z') \mid z\big) + \beta(1-\delta)E\big(S(x,y,z') \mid z\big).$$

Motivated by the observation that productivity basically follows a random walk, we now make the approximation that

$$E\big(S(x,y,z') \mid z\big) = S(x,y,z) + \text{expectational error},$$

so that the surplus equals

$$S(x,y,z)\big(1-\beta(1-\delta)\big) = zf(x,y) - V_v(y,z) - V_u(x,z) \\ + \beta E\big(V_v(y,z') \mid z\big) + \beta E\big(V_u(x,z') \mid z\big).$$

Using the Bellman equation for V_e and the approximation, we can solve for wages:

$$w(x,y,z) = \alpha S(x,y,p)\big(1-\beta(1-\delta)\big) + V_e(x,z) \\ - \beta E\big(V_u(x,z') \mid z\big).$$

Making the same approximation for V_u,

$$E\big(V_u(x,z') \mid z\big) = V_u(x,z) + \text{expectational error},$$

and using the equation for the surplus S, we obtain

$$w(x,y,z) = \alpha\big(zf(x,y) - V_v(y,z) - (1-\beta)V_u(x,z) \\ + \beta E\big(V_v(y,z') \mid z\big)\big) + V_u(x,z)(1-\beta) \\ = \alpha zf(x,y) + \alpha\big(\beta E\big(V_v(y,z') \mid z\big) - V_v(y,z)\big) \\ + (1-\alpha)(1-\beta)V_u(x,z).$$

The differences in wages for types x and x' equals

$$w(x',y,z) - w(x,y,z) = \alpha z\big(f(x',y) - f(x,y)\big) \\ + (1-\alpha)(1-\beta)\big(V_u(x',z) - V_u(x,z)\big).$$

To figure out α, we have to measure $V_u(x,z)$ and $V_u(x',z)$ in the data. For this, we use the Bellman equation for V_e and the approximation for the expected surplus

$$\begin{aligned}
V_u(x,z) &= V_e\big(x, \underline{y}(x,z), z\big) \\
&= w\big(x, \underline{y}(x,z), z\big) + \beta E\big(V_u(x,z') \mid z\big) \\
&\quad + \beta\alpha(1-\delta)E\big(S\big(x, \underline{y}(x,z), z'\big) \mid z\big) \\
&= w\big(x, \underline{y}(x,z), z\big) + \beta V_u(x,z) + \beta\alpha(1-\delta)S\big(x, \underline{y}(x,z), z\big) \\
&= w\big(x, \underline{y}(x,z), z\big) + \beta V_u(x,z),
\end{aligned}$$

so that

$$V_u(x,z)(1-\beta) = w\big(x, \underline{y}(x,z), z\big),$$

that is, we measure the value of employment at the lowest firm at productivity level z through the lowest wage accepted by type x at level z. Using this expression for the reservation wage in the wage equation to substitute for the value of unemployment yields

$$\begin{aligned}
&w\big(x', y, z\big) - w(x,y,z) \\
&\quad = \alpha z\big(f\big(x',y\big) - f(x,y)\big) + (1-\alpha)(1-\beta)\big(V_u\big(x',z\big) - V_u(x,z)\big) \\
&\quad = \alpha z\big(f\big(x',y\big) - f(x,y)\big) \\
&\qquad + (1-\alpha)\big(w\big(x', \underline{y}\big(x',z\big), z\big) - w\big(x, \underline{y}(x,z), z\big)\big).
\end{aligned}$$

For the empirical implementation, define then dummies $\delta_{x,y}$, which are 1 if worker type x works at firm type y and zero otherwise. We then regress

$$\begin{aligned}
w_t\big(x'\big) - w_t(x) &= z_t(\delta_{x',y} - \delta_{x,y}) \\
&\qquad + \kappa\big(w\big(x', \underline{y}\big(x',z\big), z\big) - w\big(x, \underline{y}(x,z), z\big)\big).
\end{aligned}$$

The estimated value of κ is then our estimate of $1-\alpha$ so that $\hat{\alpha} = (1-\kappa)$.

B. COMPUTATION AND IMPLEMENTATION

In this section, we describe how we compute the model, and construct and measure the variables mentioned in the text. We first discretize the continuous type space for both workers and firms with 50 evenly distributed grid points on the type space $[0,1]$. To compute the model, we use an iterative procedure on the match density, $d_m(x,y)$, and the surplus, $S(x,y)$. Let $d_{m,k}(x,y)$ and $S_k(x,y)$ be the values in the kth iteration. To initialize the iteration, we set, $\forall(x,y)$, the initial match distribution, $d_{m,0}(x,y) = 0.5$, and the initial surplus, $S_0(x,y) = f(x,y)$. We obtain a solution by alternatively updating exactly once on either the match density (8) or the flow equation for the surplus (which we get by summing (3)–(6)). When $d_{m,k}(x,y) < 10^{-6}$, we set $d_{m,k}(x,y) = 0$. A solution is found if the maximum absolute difference between iterations of both surplus and match density is less than 10^{-12}.

If no solution admitting a pure acceptance strategy is found (due to discretization), we solve for a mixed strategy; that is, unemployed agents accept matches with a probability (between 0 and 1) such that the surplus of the match is positive, but very close to zero. Denote iteration k of the acceptance strategy of workers with $A_k^w(x, y)$. $A_k^w(x, y)$ is the probability worker x accepts a job at firm y. We then update the acceptance strategy in the following way:

Mixed strategy
if $S_k(x, y) > 5 \times 10^{-7}$ and $A_k^w(x, y) < 1$
$A_{k+1}^w(x, y) = A_k^w(x, y) + 0.001 \cdot \text{rand}() \cdot (1 - A_k^w(x, y))$
elseif $S_k(x, y) < 5 \times 10^{-7}$ and $A_k^w(x, y) > 0$
$A_{k+1}^w(x, y) = A_k^w(x, y) - 0.001 \cdot \text{rand}() \cdot (1 - A_k^w(x, y))$
end

where rand() is a pseudo-random value drawn from the standard uniform distribution on the open interval $(0, 1)$. A mixed solution is found if the maximum absolute change between iterations of both the surplus and the match density is less than 2.5×10^{-6}. We find a mixed strategy solution in all parameterizations that we use.

With the computed solution, we simulate 600 workers and 600 jobs for each grid point, giving 60,000 agents (30,000 workers and 30,000 jobs) over a period of 240 months with an initial burn-in of 100 months. This corresponds to 20 years of monthly data.

Where order is meaningful (e.g., ranks, types, or bins), higher numbers correspond to higher productivity; for example, a worker with rank 10 is better than a worker with rank 2, a firm in bin 7 is better than a firm in bin 3.

Here, we define quantities that we will use to sketch the procedures we use:

(i) #workers = #jobs = $N = 30{,}000$.
(ii) #worker types = X = #firm types = Y = 50.
(iii) Worker ID, $i = 1, \dots, N$.
(iv) Rank of worker i, $\hat{i} = 1, \dots, N$.
For example, if $i = 4$ has rank 10, $\hat{i}(4) = 10$.
(v) True worker type $x = 1, \dots, X$. Each x has N/X individual workers.
E.g., if $i = 6$ has type 3, $x(6) = 3$. For convenience, $x(i) = 1$ if $i \in \{1, \dots, N/X\}$, $x(i) = 2$ if $i \in \{1+N/X, \dots, 2N/X\}$, and so on. In our estimation of the assignment of individual workers to worker types, $\hat{x}$, we use no information on the true assignment x.
(vi) Estimated worker type (worker bin) $\hat{x} = 1, \dots, X$. Each $\hat{x}$ has N/X workers.
For example, if $i = 5$ is in bin 45, $\hat{x}(5) = 45$.
For our simulations, $\hat{x}(i) = 1$ if $\hat{i} \in \{1, \dots, N/X\}$, $\hat{x}(i) = 2$ if $\hat{i} \in \{1+N/X, \dots, 2N/X\}$, and so on.
(vii) Firm ID, $j = 1, \dots, J$. $J = N/100$. Jobs and vacancies sum to 100 at all j.
(viii) Rank of firm j, $\hat{j} = 1, \dots, J$.
For example, if $j = 4$ has rank 10, $\hat{j}(4) = 10$.
(ix) True firm type, $y = 1, \dots, Y$. Each y has $N/(100 \cdot Y)$ unique j's. Denote $J_Y \equiv J/Y = N/(100 \cdot Y)$.
For example, if $j = 4$ has type 10, $y(4) = 10$.
(x) Estimated firm type (firm bin) $\hat{y} = 1, \dots, Y$. Each $\hat{y}$ has J_Y unique j's.
For example, if $j = 4$ is in bin 10, $\hat{y}(4) = 10$.

First, we take simulated matched employer–employee data sets and rank workers using the algorithm described in Appendix C. The algorithm delivers the ranking of workers $\hat{i}(i)$ and the estimated worker type $\hat{x}(i)$. At each firm j, we observe all workers i matching with this firm and we have their estimated type $\hat{x}$. This gives us an estimate of the set of worker types matching with this firm j, that is, we obtain an indicator function $\hat{\mathcal{B}}(\hat{x}, j)$, which is 1 if firm j hires a worker of type $\hat{x}$ and is zero otherwise. We now want to refine this estimate of which types match with firm j. The reason is that whereas we observe whether a worker i works at a firm j in the data, his type $\hat{x}(i)$ is just estimated, potentially with error due to large measurement error in wages. To take this into account, we now provide an algorithm to detect misranked workers. We then exclude the wage histories of these misranked workers. Using IDNoise, we locate matches that are likely caused by very noisy wage histories. Note that this algorithm does not apply to noise generated by match-specific productivity, the presence and magnitude of which is measured following Hagedorn and Manovskii (2012). We include all these workers in the set $\hat{\mathcal{N}}$.[3] This algorithm also updates the estimate of the set of worker types matching with firm j, $\hat{\mathcal{B}}(\hat{x}, j)$, by excluding those estimated types of workers who are included in $\hat{\mathcal{N}}$.

ALGORITHM 1—IDNoise$[\hat{x}(i)] \implies [\hat{\mathcal{B}}(\hat{x}, j), \hat{\mathcal{N}}]$:
Construct $p(\hat{x}, j)$, $\pi(\hat{x}, j)$ and $N(j)$.[4]
for each firm j
    Compute $F(p(\hat{x}, j); N(j), \pi(\hat{x}, j))$.[5]
    $\forall \hat{x}$, Initialize $\hat{\mathcal{B}}(\hat{x}, j) = 1$ iff $p(\hat{x}, j) > 0$.
    ***for** $\hat{x}$ with $\hat{\mathcal{B}}(\hat{x}, j) = 1$
        if $\hat{x} \in \{1, X\}$ and $F(p(\hat{x}, j); N(j), \pi(\hat{x}, j)) < 0.1\chi$[6]
            Set $\hat{\mathcal{B}}(\hat{x}, j) = 0$.
            Return to $*$.
        else
            if $(\hat{\mathcal{B}}(\hat{x}+1, j) = 0 \quad | \quad \hat{\mathcal{B}}(\hat{x}-1, j) = 0)$
                if $F(p(\hat{x}, j); N(j), \pi(\hat{x}, j)) < 0.1\chi$
                    Set $\hat{\mathcal{B}}(\hat{x}, j) = 0$.
                    Return to $*$.
                end
            end
        end
    end
end
$i \in \hat{\mathcal{N}}$ if a firm j, which matches with i, exists such that $\hat{\mathcal{B}}(\hat{x}(i), j) = 0$.
return $[\hat{\mathcal{B}}(\hat{x}, j), \hat{\mathcal{N}}]$

[3]The fraction of workers excluded is small (less than 5%) for most parameterizations.

[4]$p(\hat{x}, j)$ is the number of workers of estimated type $\hat{x}$ hired by firm j. $N(j) = \sum_{\hat{x}} p(\hat{x}, j)$ is the total number of workers actually hired by firm j which sums over all types from the matching set of firm j. The theoretical fraction of workers of type $\hat{x}$ hired by firm j over all workers hired by j is

$$\pi(\hat{x}, j) = \frac{u(\hat{x})\mathbb{1}\{p(\hat{x}, j) > 0\}}{\sum_{\hat{x}} u(\hat{x}) \cdot \mathbb{1}\{p(\hat{x}, j) > 0\}}.$$

[5]The probability of observing at most $p(\hat{x}, j)$ given the hiring probability $\pi(\hat{x}, j)$ from $N(j)$ trials is

$$F\big(p(\hat{x}, j); N(j), \pi(\hat{x}, j)\big) = \sum_{i=0}^{p(\hat{x},j)} \binom{N(j)}{i} \pi(\hat{x}, j)^i \big(1 - \pi(\hat{x}, j)\big)^{N(j)-i}.$$

[6]Where $\chi = 0$ in the presence of match-specific productivity.

The next crucial statistic to estimate is reservation wages for each worker $\hat{w}_{\text{res}}(i)$. To this end, we implement ResWage.

ALGORITHM 2—ResWage$[w(i,j), \hat{x}(i), \hat{\mathcal{N}}] \implies \hat{w}_{\text{res}}(i)$:
Consider wages histories of $i \notin \hat{\mathcal{N}}$.
for $\hat{x}$
  Construct $J(\hat{x}) = \{j \text{ s.t. } j \text{ hires any } i \in \hat{x}\}$.
  foreach $j \in J(\hat{x})$, compute $\bar{w}(\hat{x}, j) =$ average wage paid by j to workers $i \in \hat{x}$.
  $w_{\text{res}}(\hat{x}) =$ lowest average of $\bar{w}(\hat{x}, j)$ possible from pooling J_Y firms in J.[7]
end
return $\hat{w}_{\text{res}}(i) = w_{\text{res}}(\hat{x}(i))$

Then, for each firm j, compute the average wage premium as in (15). We next estimate job filling rates $\hat{q}(j)$ using information from all workers (whether or not they belong to $\hat{\mathcal{N}}$) over the acceptance set $\hat{\mathcal{B}}$ of firm j, which includes all types $\hat{x}$ for which $\hat{\mathcal{B}}(\hat{x}, j) = 1$. Our estimate of $\hat{q}(j)$ is $\hat{\mathbb{M}}_v \sum_{\hat{x} \in \hat{\mathcal{B}}} \frac{u(\hat{x})}{u}$. Multiplying the average wage premium and the acceptance rate gives the statistic $\hat{\Omega}$ which allows us to rank firms.

We now assign individual firms to firm types $\hat{y}$. Using our ranking of firms, we can assign the first J_Y firms to firm bin 1, the next J_Y firms to firm bin 2, and so on. The assignment of firms to types allows us to compute statistics for firm types only. For example, statistics for all firms belonging to firm type $\hat{y} = 1$ will be the firm size (measured by average employment) weighted average of firms with $\hat{j}(j) \in \{1, \dots, J_Y\}$. This step only serves to aggregate information across firms and yields smoother statistics and more precise estimates. We could have also proceeded by assigning each individual firm to its own type, that is, $\hat{y}(j) = \hat{j}(j)$.

Taking present values of estimated minimum wages for each bin yields $V_u(\hat{x})$. Compute the average wages each bin $\hat{x}$ receives with all firms of bin $\hat{y}$. This is $w_{av}(\hat{x}, \hat{y})$. Compute the corresponding value of employment, $V_e(\hat{x}, \hat{y})$ and $V_v(\hat{y})$ from $\hat{\Omega}(\hat{y})$. The estimate of the production function $\hat{f}(\hat{x}, \hat{y})$ follows.

Using unemployment rates at the $\hat{x}$ level and estimated firm size at the $\hat{j}$ level, we can estimate frictional output with the estimated production function.

To measure output losses due to frictions we optimally assign a subsample (5000 workers and 5000 jobs) from the pool of employed workers.[8] The subsample reflects the estimated type distributions of employed workers and producing firms. To evaluate the accuracy of our method, our estimated gains from eliminating search frictions are compared with those obtained when repeating the same procedure using true model generated distributions and production functions.

C. RANK AGGREGATION

Our goal is to rank workers according to their productivity. We know that wages within a firm are increasing in worker productivity x. Thus, in the absence of measurement error, considering the workers within one specific firm gives us a correct ranking among these

[7]For $\hat{x} > 1$, we additionally impose $w_{\text{res}}(\hat{x}) > w_{\text{res}}(\hat{x} - 1)$ which is consistent with theory.
[8]See Section 4.6 for references to the algorithms used.

workers. Repeating this ranking for every firm yields a globally consistent and, if workers are sufficiently mobile between firms, a complete ranking of workers since worker rankings are transitive across firms.

However, wage data might contain measurement error. Consequently, within one firm, a less productive worker could be ranked above a truly more productive worker because of measurement error. Furthermore, the ranking between these two workers may not be transitive across firms where they happen to be coworkers. Thus, the rankings from all firms are not consistent and thus do not yield an aggregate ranking. To solve this problem, we build on the insights from social choice theory, which considers a equivalent problem in the context of voting.

Imagine that voters were asked to rank candidates from the most to the least preferred one. Voters will rank candidates according to their own preferences, but when the need to have a single ranking of candidates comes up, a disagreement is likely to arise. Unless every voter ranks all candidates identically, there will not be an aggregate ranking that all voters agree with completely. This requires then the specification of how to aggregate disagreements between voters and a method how to find this aggregate ranking.

C.1. *Kemeny–Young Rank Aggregation*

Given many (perhaps) inconsistent rankings of candidates, how does one aggregate the ranks to determine who the best candidate is? This problem is ancient, and was first studied by de Borda (1781) and Condorcet (1785). One natural starting point to use as a metric for evaluating the posited aggregate ranking is the number of disagreements generated in the voter submitted ranks as done in the Kemeny–Young formulation of this classic problem.[9] The goal then is to find an aggregate ranking which generates the minimum number of disagreements with the data. Drissi-Bakhkhat and Truchon (2004) argued in a context of a social choice model that the disagreements in the ranking of two alternatives should be weighted by the probability that the voters compare them correctly. Similarly, in our labor market application, weighting means that the disagreements are weighted by the probabilities that a worker is ranked higher than another worker (which are computed from wage data). Fortunately, the computer science literature provides algorithms to handle these weighted ranking problems as well since they can be cast as a special case of a weighted feedback arc set problem on tournaments (see, e.g., Ailon, Charikar, and Newman (2008)).

For a candidate ranking Π, $\Pi(i,j)=1$ if i is ranked higher than j and $\Pi(i,j)=0$ otherwise. There are no ties. The objective is to find ranking Π which maximizes

$$\sum_{i>j} c(i,j)\Pi(i,j)+c(j,i)\Pi(j,i), \tag{A.3}$$

where the weighting $c(i,j)$ is the probability (computed from wage observations) that i is ranked above j.

We now construct $c(i,j)$. First, we use head-to-head wage information at all firms to calculate the probability that worker i is ranked higher than worker j. Note that we can only use this ranking when we observe worker i and worker j at the same firm. We first discuss the simple case where we only observe i and j at one firm.

[9]This was first described in Kemeny (1959) and Kemeny and Snell (1963).

Suppose we observe $n_{i,k}$ wage observations and $n_{j,k}$ from workers i and j, respectively, at firm k.[10] We know that observed wages follows

$$\hat{w}_{i,k,t} = w_{i,k} + \varepsilon_t,$$

which contains noise ε with variance σ^2. We can compute the average wages $\bar{w}_{i,k}$ and $\bar{w}_{j,k}$, which can be written as

$$\begin{aligned}\bar{w}_{i,k} - \bar{w}_{j,k} &= \frac{1}{n_{i,k}} \sum_{t=1}^{n_{i,k}} \hat{w}_{i,k,t} - \frac{1}{n_{j,k}} \sum_{t=1}^{n_{j,k}} \hat{w}_{j,k,t} \\ &= w_{i,k} - w_{j,k} + \frac{1}{n_{i,k}} \sum_{t=1}^{n_{i,k}} \varepsilon_{i,k,t} - \frac{1}{n_{j,k}} \sum_{t=1}^{n_{j,k}} \varepsilon_{j,k,t},\end{aligned}$$

where all of the ε's are independent.

We are interested in computing the probability that $w_{i,k} > w_{j,k}$ given the observations on $\bar{w}_{i,k}$ and $\bar{w}_{j,k}$. A Bayesian approach seems a natural one to follow to accomplish this. Suppose that we had a normal prior distribution over wages, that is, we assume that

$$w_{i,k} \sim \mathcal{N}\big(\mu_0, \tau_0^2\big).$$

The posterior density over $w_{i,k}$ conditional on knowing σ^2 (we explain below how to measure it in the data) is given by

$$p(w_{i,k} | \hat{w}_{i,k,1}, \dots, \hat{w}_{i,k,n_{i,k}}) = p(w_{i,k} | \bar{w}_{i,k}) = \mathcal{N}\big(\mu_n, \tau_n^2\big),$$

where

$$\mu_n = \frac{\dfrac{1}{\tau_0^2}\mu_0 + \dfrac{n_{i,k}}{\sigma^2}\bar{w}_{i,k}}{\dfrac{1}{\tau_0^2} + \dfrac{n_{i,k}}{\sigma^2}}$$

and

$$\frac{1}{\tau_n^2} = \frac{1}{\tau_0^2} + \frac{n_{i,k}}{\sigma^2}.$$

If in the baseline case we assume an uninformative prior, that is, we take $\tau_0^2 \to \infty$, this simplifies to

$$\mu_n = \bar{w}_{i,k}$$

and

$$\frac{1}{\tau_n^2} = \frac{n_{i,k}}{\sigma^2}.$$

[10] $n_{i,k}$ periods (months, in our case) of observations need not be in one employment spell. Moreover, i and j do not need to be employed at the same time.

Then the posterior densities for $w_{i,k}, w_{j,k}$ given the data would just be given by

$$p(w_{i,k}|\bar{w}_{i,k}) = \mathcal{N}\left(\bar{w}_{i,k}, \frac{\sigma^2}{n_{i,k}}\right),$$

$$p(w_{j,k}|\bar{w}_{j,k}) = \mathcal{N}\left(\bar{w}_{j,k}, \frac{\sigma^2}{n_{j,k}}\right).$$

Since these posteriors are independent normals, we know that the distribution over the difference $p(w_{i,k} - w_{j,k}|\bar{w}_{i,k}, \bar{w}_{j,k})$ is simply

$$p(w_{i,k} - w_{j,k}|\bar{w}_{i,k}, \bar{w}_{j,k}) = \mathcal{N}\left(\bar{w}_{i,k} - \bar{w}_{j,k}, \frac{\sigma^2}{n_{i,k}} + \frac{\sigma^2}{n_{j,k}}\right).$$

Thus, the posterior probability that $w_{i,k} > w_{j,k}$ can simply be computed as

$$\mathbb{P}(w_{i,k} > w_{j,k}) = 1 - \Phi\left(\frac{0 - (\bar{w}_{i,k} - \bar{w}_{j,k})}{\dfrac{\sigma^2}{n_{i,k}} + \dfrac{\sigma^2}{n_{j,k}}}\right) = \Phi\left(\frac{\bar{w}_{i,k} - \bar{w}_{j,k}}{\dfrac{\sigma^2}{n_{i,k}} + \dfrac{\sigma^2}{n_{j,k}}}\right).$$

If i and j are on the same payroll at only one firm, $\mathbb{P}(w_{i,k} > w_{j,k}) = c(i, j)$. If more than one firm hires i and j, we compute $\mathbb{P}(w_{i,k} > w_{j,k})$ for all those firms and assign the product of these probabilities to $c(i, j)$, that is, we consider observations in different firms as independent.

The variance of noise is computed from the variance of wages for all workers within jobs since all variation in wages within a specific job arises from measurement error only.

The solution to the problem of finding the best ranking is then conceptually trivial: (1) Enumerate all possible rankings. (2) Evaluate (A.3) for all of them. (3) Select the rank which maximizes the objective function. Unfortunately, the *Kemeny–Young rank aggregation* problem is NP-hard.[11] We therefore approximate the solution to the problem and use the following algorithm:

ALGORITHM 3—Single Worker Moves:
Initialize $\Pi(i, j)$ that maximizes (A.3). Choose ranking from:
(a) lowest wage,
(b) highest wage,
(c) adjusted average wage,
While some rearrangement of $\Pi(i, j)$ improves (A.3)
Choose worker x and rank j.
Rearrange $\Pi(i, j)$ so that x has rank j, leaving all other relative rankings intact.[12]
Return $\Pi(i, j)$

This algorithm is a simplified version of the algorithm in Kenyon-Mathieu and Schudy (2007) which is capable of approximating the solution arbitrarily well. We choose this algorithm as it provides for us the best compromise between accuracy and computational

[11]See Bartholdi, Tovey, and Trick (1989). Consider a simple case of 100 candidates and at least 4 submitted rankings. There are $100 \times 99 \times \cdots \times 2$ combinations to consider!

[12]Suppose there are workers, A, B, C, and D ranked alphabetically, $\{A, B, C, D\}$. Moving C to rank 2 would mean rearranging them so that the ranking is now $\{A, C, B, D\}$.

complexity. Indeed, we show that this simplified algorithm provides a very accurate ranking of workers in our model. It is straightforward to implement the complete algorithms in Kenyon-Mathieu and Schudy (2007) if more precision is required for a particular application.

D. MODEL WITH ON-THE-JOB SEARCH

We first describe the details of the model with on-the-job search used in the data analysis. We then prove the results, mentioned in the main text, to obtain identification, and we finally evaluate its performance in Monte Carlo simulations.

D.1. *The Model*

In this section, we describe the model which introduces on-the-job search into the environment of Shimer and Smith (2000) analyzed in the main text. The basic features of the two models are the same and we describe here only the necessary modifications.

All workers and all unmatched firms engage in random search. The exogenous search intensity of employed (relative to unemployed) workers is $\phi \in [0, 1]$. The total search effort is the weighted sum $s = U + \phi E$. A function $m : [0, 1] \times [0, 1] \to [0, \min(s, V)]$ takes the total mass of searchers s and vacant firms V as its inputs and generates meetings. The probabilities that an unemployed or an employed worker meets a potential employer are given by $\mathbb{M}_u = \frac{m(s,V)}{s}$ and $\mathbb{M}_e = \phi\frac{m(s,V)}{s}$, while the probability of a vacant firm meeting a potential hire is $\mathbb{M}_v = \frac{m(s,V)}{V}$. These probabilities are time-invariant in the steady-state equilibrium we will consider. The probability to meet a firm $y \in Y \subset [0, 1]$ equals $\mathbb{M}_u\frac{\int_Y d_v(y)\,dy}{V}$ for unemployed and $\mathbb{M}_e\frac{\int_Y d_v(y)\,dy}{V}$ for employed workers. For firms, these probabilities depend on the employment status of the worker, since unemployed and employed workers not only search with different intensities but also endogenously exhibit different distributions. Conditional on meeting a worker, we therefore define the probabilities that the worker is unemployed or employed by $\mathbb{C}_u = \frac{U}{U+\phi E}$ and $\mathbb{C}_e = \frac{\phi E}{U+\phi E}$, respectively. The probability for a firm to meet an unemployed worker $x \in X \subset [0, 1]$ then equals $\mathbb{M}_v\mathbb{C}_u\frac{\int_X d_u(x)\,dx}{U}$ and the probability to meet an employed worker $x \in X \subset [0, 1]$ equals $\mathbb{M}_v\mathbb{C}_e\frac{\int_X d_e(x)\,dx}{E}$. Not all meetings necessarily result in matches. Some meetings are between an unemployed worker and a vacant firm who are unwilling to consummate a match and who prefer to continue the search process. Some other meetings are between an employed worker and a vacant firm where the worker prefers to stay with the current firm so that the meeting does not result in a new match.

An unemployed worker makes a take-it-or-leave-it offer to a firm and thus extracts the full surplus. As in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006), when a worker employed at some firm $\tilde{y}$ meets a firm y which generates higher surplus, the two firms engage in Bertrand competition such that the worker moves to firm y and obtains the full surplus generated with firm $\tilde{y}$. Small costs of writing an offer prevent firms y which generate lower surplus than the current firm from engaging in Bertrand competition. Let $V_u(x)$ denote the value of unemployment for a worker of type x; $V_e(x, y, S^o)$ is the value of employment for a worker of type x at a firm of type y, $V_v(y)$ the value of a vacancy for firm y, and $V_p(x, y, S^o)$ the value of firm y employing a worker of type x. The value functions V_e and V_p depend on S^o, which is the current surplus for a worker hired out of unemployment or the surplus from the previous job when a worker is poached from another firm. The surplus of a match between worker x and firm y and for some S^o is[13]

$$S(x, y) := V_p(x, y, S^o) - V_v(y) + V_e(x, y, S^o) - V_u(x). \tag{A.4}$$

Matching takes place when both the worker and the firm find it mutually acceptable. For a worker of type x, the *matching set* $B^w(x)$ consists of firms which match with worker type x. Symmetrically, for a firm of type y, $B^f(y)$ consists of workers who are matching with firm type y. For a worker of type x employed at a firm of type y, the set $B^e(x, y)$ are firms which match with worker type x and are preferred by worker x to his current firm of type y. Finally, the set $B^p(y)$, with corresponding density $d_y(\tilde{x}, \tilde{y})$, are worker-firm pairs $(\tilde{x}, \tilde{y})$ where worker type $\tilde{x}$ and firm type $\tilde{y}$ accept each other ($\tilde{x} \in B^f(\tilde{y})$, $\tilde{x} \in B^u(\tilde{y})$) and the worker prefers firm type y to firm type $\tilde{y}$. In this case, a worker of type $\tilde{x}$ currently employed at firm $\tilde{y}$ moves to a firm of type y. We denote by $\overline{X}$ the complement of a set X (in the obvious universe). The matching sets for unemployed workers and vacant firms can be characterized through the surplus function:

$$\begin{aligned} B^w(x) &= \{y : S(x, y) \geq 0\}, \\ B^f(y) &= \{x : S(x, y) \geq 0\}, \\ B^e(x, y) &= \{\tilde{y} : S(x, \tilde{y}) \geq S(x, y) \geq 0\}, \\ B^p(y) &= \{(\tilde{x}, \tilde{y}) : S(\tilde{x}, y) \geq S(\tilde{x}, \tilde{y}) \geq 0\}. \end{aligned} \tag{A.5}$$

The steady-state value functions are

$$V_u(x) = \beta V_u(x) + \beta(1-\delta)\mathbb{M}_u \int_{B^u(x)} \frac{d_v(\tilde{y})}{V} S(x, \tilde{y})\, \mathrm{d}\tilde{y}, \tag{A.6}$$

$$\begin{aligned} V_v(y) &= \beta V_v(y) \\ &\quad + \beta(1-\delta)\mathbb{M}_v\mathbb{C}_e \int_{B^p(y)} \frac{d_y(\tilde{x}, \tilde{y})}{E} \big(S(\tilde{x}, y) - S(\tilde{x}, \tilde{y})\big)\, \mathrm{d}\tilde{x}\, \mathrm{d}\tilde{y}, \end{aligned} \tag{A.7}$$

$$\begin{aligned} V_e(x, y, S^o) &= w(x, y, S^o) + \beta V_u(x) \\ &\quad + \beta(1-\delta)\left[1 - \mathbb{M}_e + \mathbb{M}_e \int_{\overline{B^e(x,y)}} \frac{d_v(\tilde{y})}{V}\, \mathrm{d}\tilde{y}\right] S^0 \\ &\quad + \beta(1-\delta)\mathbb{M}_e \int_{B^e(x,y)} \frac{d_v(\tilde{y})}{V} S(x, y)\, \mathrm{d}\tilde{y}, \end{aligned} \tag{A.8}$$

$$\begin{aligned} &V_p(x, y, S^o) \\ &\quad = f(x, y) - w(x, y, S^o) + \beta V_v(y) \\ &\qquad + \beta(1-\delta)\left[1 - \mathbb{M}_e + \mathbb{M}_e \int_{\overline{B^e(x,y)}} \frac{d_v(\tilde{y})}{V}\, \mathrm{d}\tilde{y}\right]\big(S(x, y) - S^o\big), \end{aligned} \tag{A.9}$$

[13]As in Lise and Robin (2013), the surplus S^o does not affect the surplus in the current match, $S(x, y)$, but only the sharing of the surplus between the worker and the firm.

and free entry implies

$$c = \int V_v(\tilde{y})\,\mathrm{d}\tilde{y}. \tag{A.10}$$

The worker's value of being employed then equals

$$V_e\big(x, y, S^o\big) = V_u(x) + S^o \tag{A.11}$$

and the firm's value of producing equals

$$V_p\big(x, y, S^o\big) = V_v(y) + \big(S(x, y) - S^o\big). \tag{A.12}$$

In a *steady-state search equilibrium* (SE), all workers and firms maximize expected payoff, taking the strategies of all other agents as given. The economy is in steady-state. A SE is then characterized by the density $d_u(x)$ of unemployed workers, the density $d_v(y)$ of vacant firms, the density of formed matches $d_m(x, y)$, and wages $w(x, y, S^o)$. The density $d_m(x, y)$ implicitly defines the matching sets, as it is zero if no match is formed and is strictly positive if a match is consummated. Wages are set as described above and match formation is optimal given wages w, that is, a match is formed whenever the surplus (weakly) increases (see (A.5)). The densities $d_u(x)$ and $d_v(x)$ ensure that, for all worker and firm type combinations in the matching set, the numbers of destroyed matches (into unemployment and to other jobs) and created matches (hires from unemployment and from other jobs) are the same.

D.2. *Identification*

D.2.1. *Some Useful Results*

Before we turn to the specific identification results, we derive results for the surplus and wages. We first derive the Bellman equation of surplus:

$$\begin{aligned}
S(x, y) &= V_p\big(x, y, S^o\big) - V_v(y) + V_e\big(x, y, S^o\big) - V_u(x) \\
&= f(x, y) - (1-\beta)\big(V_v(y) + V_u(x)\big) \\
&\quad + \beta(1-\delta)\left[1 - \mathbb{M}_e + \mathbb{M}_e \int_{\overline{B^e(x,y)}} \frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y}\right] S(x, y) \\
&\quad + \beta(1-\delta)\mathbb{M}_e \int_{B^e(x,y)} \frac{d_v(\tilde{y})}{V} S(x, y)\,\mathrm{d}\tilde{y} \\
&= f(x, y) - (1-\beta)\big(V_v(y) + V_u(x)\big) + \beta(1-\delta)S(x, y)
\end{aligned}$$

so that

$$S(x, y)\big[1 - \beta(1-\delta)\big] = f(x, y) - (1-\beta)\big(V_v(y) + V_u(x)\big). \tag{A.13}$$

We can also compute how the surplus changes with worker type x:

$$\frac{\partial S(x, y)}{\partial x} = \frac{\dfrac{\partial f(x, y)}{\partial x} - (1-\beta)\dfrac{\partial V_u(x)}{\partial x}}{1 - \beta(1-\delta)}. \tag{A.14}$$

Similarly, the derivative with respect to y equals

$$\frac{\partial S(x,y)}{\partial y} = \frac{\dfrac{\partial f(x,y)}{\partial y} - (1-\beta)\dfrac{\partial V_v(y)}{\partial y}}{1-\beta(1-\delta)}. \tag{A.15}$$

From the Bellman equation for an employed worker, we get that the wage equals

$$\begin{aligned} w(x,y,S^o) &= S^o + (1-\beta)V_u(x) \\ &\quad - \beta(1-\delta)\left[1 - \mathbb{M}_e + \mathbb{M}_e \int_{\overline{B^e(x,y)}} \frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y}\right] S^0 \\ &\quad - \beta(1-\delta)\mathbb{M}_e \int_{B^e(x,y)} \frac{d_v(\tilde{y})}{V} S(x,y)\,\mathrm{d}\tilde{y}. \end{aligned} \tag{A.16}$$

For a worker coming out of unemployment, this equals

$$w(x,y,S^o) = S(x,y) + (1-\beta)V_u(x) - \beta(1-\delta)S(x,y). \tag{A.17}$$

And thus using the surplus equation,

$$\begin{aligned} w(x,y,S^o) &= f(x,y) - (1-\beta)\big(V_v(y) + V_u(x)\big) + (1-\beta)V_u(x) \\ &= f(x,y) - (1-\beta)V_v(y). \end{aligned} \tag{A.18}$$

Finally, we can also show that workers can be ranked by their lowest as well as by their highest wage. Let $y^{\min}(x)$ be a firm type such that worker x is indifferent between matching with this firm and staying unemployed,

$$V_e\big(x, y^{\min}(x)\big) = V_u(x).$$

$y^{\min}(x)$ is the firm that pays the lowest wage to a worker of type x hired from unemployment. Of course, this equation can be satisfied for more than one firm. In this case, simply pick the lowest firm type. It then holds that, from the Bellman equation for V_e,

$$\begin{aligned} V_e\big(x, y^{\min}(x)\big) &= w\big(x, y^{\min}(x)\big) + \beta V_u(x) \\ &\quad + \beta(1-\delta)\mathbb{M}_e \int_{B^e(x,y^{\min}(x))} \frac{d_v(y)}{V} S(x,y)\,\mathrm{d}y \end{aligned}$$

and from the Bellman equation for V_u,

$$V_u(x)(1-\beta) = \beta(1-\delta)\mathbb{M}_u \int_{B^w(x)} \frac{d_v(y)}{V} S(x,y)\,\mathrm{d}y,$$

and thus that

$$\frac{\partial}{\partial x}\beta(1-\delta)\mathbb{M}_u \int_{B^w(x)} \frac{d_v(y)}{V} S(x,y)\,\mathrm{d}y = \frac{\partial}{\partial x} V_u(x)(1-\beta) > 0.$$

Since $B_v(x, \tilde{y}(x)) = B^w(x)$, it follows that

$$\begin{aligned}\frac{\partial w\big(x, y^{\min}(x)\big)}{\partial x} &= (1-\beta)\frac{\partial}{\partial x}V_u(x) - (1-\beta)\frac{\mathbb{M}_e}{\mathbb{M}_u}\frac{\partial}{\partial x}V_u(x) \\ &= (1-\beta)\frac{\partial}{\partial x}V_u(x)\frac{\mathbb{M}_u - \mathbb{M}_e}{\mathbb{M}_u},\end{aligned}$$

which is greater than zero if $\mathbb{M}_u > \mathbb{M}_e$.

Similarly for the highest wage, let $y^{\max}(x)$ be the firm type that generates the highest surplus with worker x. The highest wage of worker type x equals $w(x, y^{\max}(x), S(x, y^{\max}(x)))$. For the wage, it holds that

$$\begin{aligned}w\big(x, y, S^o\big) = S^o &+ (1-\beta)V_u(x) \\ &- \beta(1-\delta)\bigg[1 - \mathbb{M}_e + \mathbb{M}_e\int_{\overline{B^e(x,y)}}\frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y}\bigg]S^0 \\ &- \beta(1-\delta)\mathbb{M}_e\int_{B^e(x,y)}\frac{d_v(\tilde{y})}{V}S(x,y)\,\mathrm{d}\tilde{y}.\end{aligned} \tag{A.19}$$

For the highest wage, we therefore get

$$\begin{aligned}&w\big(x, y^{\max}(x), S\big(x, y^{\max}(x)\big)\big) \\ &\quad= S\big(x, y^{\max}(x)\big) + (1-\beta)V_u(x) \\ &\qquad - \beta(1-\delta)\bigg[1 - \mathbb{M}_e + \mathbb{M}_e\int_{\overline{B^e(x,y^{\max}(x))}}\frac{d_v(\tilde{y})}{V}\,\mathrm{d}\tilde{y}\bigg]S\big(x, y^{\max}(x)\big) \\ &\qquad - \beta(1-\delta)\mathbb{M}_e\int_{B^e(x,y^{\max}(x))}\frac{d_v(\tilde{y})}{V}S(x,y)\,\mathrm{d}\tilde{y} \\ &\quad= S\big(x, y^{\max}(x)\big)\big(1-\beta(1-\delta)\big) + (1-\beta)V_u(x) \\ &\quad= f\big(x, y^{\max}(x)\big) - (1-\beta)\big(V_v\big(y^{\max}(x)\big) + V_u(x)\big) + (1-\beta)V_u(x) \\ &\quad= f\big(x, y^{\max}(x)\big) - (1-\beta)V_v\big(y^{\max}(x)\big).\end{aligned} \tag{A.20}$$

The Envelope Theorem then implies that the highest wage, given by $w(x, y^{\max}(x), S(x, y^{\max}(x)))$, is increasing in x as the production function f is increasing in x.

D.2.2. *Ranking Workers*

The wage of a worker in the first job following an unemployment spell is equal to

$$w\big(x, y, S^o\big) = f(x,y) - (1-\beta)V_v(y). \tag{A.21}$$

Thus, within a firm, wages of workers hired from unemployment are increasing in worker type: $\partial w(x, y, S^o)/\partial x = \partial f(x,y)/\partial x > 0$. The same methodology applied to workers hired out of unemployment as in the benchmark model can therefore be used to rank workers.

D.2.3. *Ranking Firms*

To rank firms, we establish that the value of a vacancy is increasing in y. We then show that the value of the vacancy can be expressed as a function of wages observed in the data.

RESULT A-1: *$V_v(y)$ is increasing in y.*

Differentiating (A.7), we have

$$\frac{\partial V_v(y)}{\partial y} = \beta \frac{\partial V_v(y)}{\partial y} + \beta(1-\delta)\mathbb{M}_v \mathbb{C}_e \int_{B^p(y)} \frac{d_y(\tilde{x}, \tilde{y})}{E} \frac{\partial S(\tilde{x}, y)}{\partial y} \, \mathrm{d}\tilde{x} \, \mathrm{d}\tilde{y}. \tag{A.22}$$

Plugging in

$$\frac{\partial S(\tilde{x}, y)}{\partial y} = \frac{\dfrac{\partial f(\tilde{x}, y)}{\partial y} - (1-\beta)\dfrac{\partial V_v(y)}{\partial y}}{1-\beta(1-\delta)}, \tag{A.23}$$

and solving for $\frac{\partial V_v(y)}{\partial y}$ yields the desired result. Using (A.7), this result immediately implies the following:

RESULT A-2: *The expected surplus from newly hired workers poached from other firms given by*

$$(1-\delta)\mathbb{M}_v \mathbb{C}_e \int_{B^p(y)} \frac{d_y(\tilde{x}, \tilde{y})}{E} \big(S(\tilde{x}, y) - S(\tilde{x}, \tilde{y})\big) \, \mathrm{d}\tilde{x} \, \mathrm{d}\tilde{y}$$

is increasing in y.

The next step is to relate these monotone statistics to workers' value functions.

RESULT A-3: *The expected surplus premium given by*

$$(1-\delta)\mathbb{M}_v \mathbb{C}_e \int_{B^p(y)} \frac{d_y(\tilde{x}, \tilde{y})}{E} \Big(V_e\big(\tilde{x}, y, S(\tilde{x}, y)\big) - V_e\big(\tilde{x}, \tilde{y}, S(\tilde{x}, \tilde{y})\big)\Big) \, \mathrm{d}\tilde{x} \, \mathrm{d}\tilde{y}$$

is increasing in y.

This result is immediately implied by (A.11) as

$$S(\tilde{x}, y) - S(\tilde{x}, \tilde{y}) = V_e\big(\tilde{x}, y, S(\tilde{x}, y)\big) - V_e\big(\tilde{x}, \tilde{y}, S(\tilde{x}, \tilde{y})\big). \tag{A.24}$$

We now relate this statistic to wages which are observable in the data. Since

$$S(\tilde{x}, y) = \frac{w\big(\tilde{x}, y, S(\tilde{x}, y)\big) - (1-\beta)V_u(\tilde{x})}{1-\beta(1-\delta)}, \tag{A.25}$$

$$S(\tilde{x}, y) - S(\tilde{x}, \tilde{y}) = \frac{w\big(\tilde{x}, y, S(\tilde{x}, y)\big) - w\big(\tilde{x}, \tilde{y}, S(\tilde{x}, \tilde{y})\big)}{1-\beta(1-\delta)}, \tag{A.26}$$

we finally obtain the key result that allows to rank firms:

RESULT A-4: *The expected wage premium given by*

$$\Omega(y) = (1-\delta)\mathbb{M}_v\mathbb{C}_e \int_{B^p(y)} \frac{d_y(\tilde{x}, \tilde{y})}{E} \big(w\big(\tilde{x}, y, S(\tilde{x}, y)\big) - w\big(\tilde{x}, \tilde{y}, S(\tilde{x}, \tilde{y})\big)\big)\,\mathrm{d}\tilde{x}\,\mathrm{d}\tilde{y} \tag{A.27}$$

is increasing in y.

Once again it is useful to decompose $\Omega(y)$ into the average wage difference, $\Omega^e(y)$, and the probability to fill a vacancy with an employed worker, $q^e(y)$. The average wage difference equals

$$\Omega^e(y) = \int_{B^p(y)} \frac{\frac{d_y(\tilde{x}, \tilde{y})}{E}\big(w\big(\tilde{x}, y, S(\tilde{x}, y)\big) - w\big(\tilde{x}, \tilde{y}, S(\tilde{x}, \tilde{y})\big)\big)}{\int_{B^p(y)} \frac{d_y(\tilde{x}, \tilde{y})}{E}\,\mathrm{d}\tilde{x}\,\mathrm{d}\tilde{y}}\,\mathrm{d}\tilde{x}\,\mathrm{d}\tilde{y}. \tag{A.28}$$

The probability that a vacancy of type y is filled with an employed equals

$$q^e(y) = (1-\delta)\mathbb{M}_v\mathbb{C}_e \int_{B^p(y)} \frac{d_y(\tilde{x}, \tilde{y})}{E}\,\mathrm{d}\tilde{x}\,\mathrm{d}\tilde{y}. \tag{A.29}$$

It then holds that

$$\Omega(y) = q^e(y)\Omega^e(y). \tag{A.30}$$

Measuring $q^e(y)$. The probability for a type y firm to fill a vacancy with an employed worker is

$$q^e_y = (1-\delta)\mathbb{M}_v\mathbb{C}_e \int_{B^p(y)} \frac{d_y(\tilde{x}, \tilde{y})}{E}\,\mathrm{d}\tilde{x}\,\mathrm{d}\tilde{y} = (1-\delta)\mathbb{M}_v\mathbb{C}_e\tilde{q}^e_y. \tag{A.31}$$

It can be directly measured with vacancy data at the firm level. If such data are not available, q^e_y can still be easily estimated using, for example, only the aggregate number of vacancies, as we now show.

For hires out of unemployment, let

$$q^u_y = (1-\delta)\mathbb{M}_v\mathbb{C}_u \int_{B^f(y)} \frac{d_u(\tilde{x})}{U}\,\mathrm{d}\tilde{x} = (1-\delta)\mathbb{M}_v\mathbb{C}_u\tilde{q}^u_y \tag{A.32}$$

be the probability a firm fills a vacancy with an unemployed worker. $\tilde{q}^u_y$ is simply the share of unemployed workers that firm j is willing to hire out of unemployment and can be measured in the data.

For hiring out of unemployment, denote by $H^u(y)$ the observed number of new hires (out of unemployment) for a firm of type y, and by $V(y)$ the unobserved number of vacancies posted by such a firm. For a single firm, we get

$$H^u(y) = q^u_y V(y). \tag{A.33}$$

Aggregating and rearranging yields

$$\mathbb{M}_v\mathbb{C}_u = \frac{1}{1-\delta}\frac{\displaystyle\int_{[0,1]}\frac{H^u(\tilde{y})}{\tilde{q}^u_{\tilde{y}}}\,\mathrm{d}\tilde{y}}{\displaystyle\int_{[0,1]}V(\tilde{y})\,\mathrm{d}\tilde{y}}. \tag{A.34}$$

Denote by $H^e(y)$ the observed number of new hires (from other firms) of a firm of type y. For a single firm, we get

$$H^e(y) = q^e_y V(y). \tag{A.35}$$

Aggregating and rearranging yields

$$\mathbb{M}_v\mathbb{C}_e = \frac{1}{1-\delta}\frac{\displaystyle\int_{[0,1]}\frac{H^e(\tilde{y})}{\tilde{q}^e_{\tilde{y}}}\,\mathrm{d}\tilde{y}}{\displaystyle\int_{[0,1]}V(\tilde{y})\,\mathrm{d}\tilde{y}}. \tag{A.36}$$

The total number of vacancies, $\int_{[0,1]}V(\tilde{y})\,\mathrm{d}\tilde{y}$, if unobserved, can be inferred by matching the wage share in output.

What remains to be obtained is an estimate of $\tilde{q}^e(y)$ which requires an estimate of $B^p(y)$. To better estimate $B^p(y)$ in short panels, we can augment mobility information with wage data by utilizing (A.25): Conditional on worker type x, a worker moves job-to-job to firms which pay higher wages out of unemployment, as the surplus in these firms is higher. Summing the number of worker-firm matches over the estimated matching set $B^p(y)$ for each firm gives $\tilde{q}^e(y) = \int_{B^p(y)}\frac{d_y(\tilde{x},\tilde{y})}{E}\,\mathrm{d}\tilde{x}\,\mathrm{d}\tilde{y}$.

We can therefore compute $\mathbb{M}_v\mathbb{C}_e$ and $\mathbb{M}_v\mathbb{C}_u$ and therefore also $\frac{\mathbb{C}_e}{\mathbb{C}_u}$, which delivers an estimate of ϕ, and thus both $\mathbb{C}_e$ and $\mathbb{C}_u$ are available which allows us to obtain $\mathbb{M}_v$. This then yields, using the estimates of δ, $\mathbb{M}_v$, and $\mathbb{C}_e$, $q^e_y = (1-\delta)\mathbb{M}_v\mathbb{C}_e\tilde{q}^e(y)$.

D.2.4. *Measuring Output $f(x, y)$*

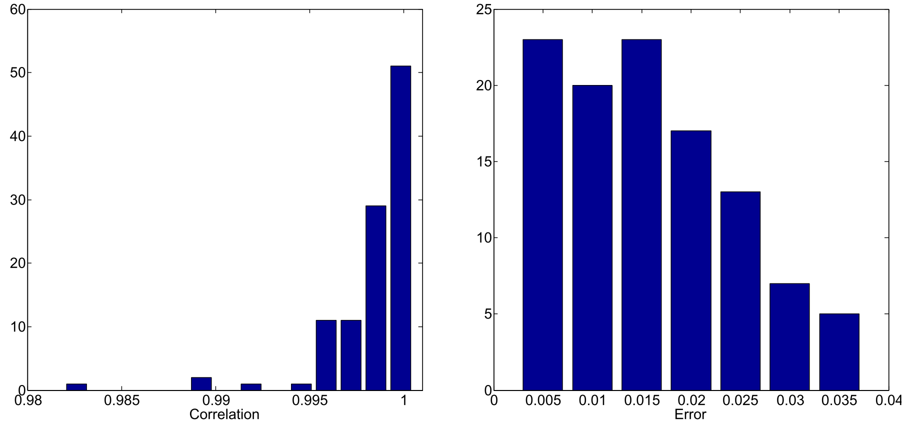
Inverting the wage equation (A.21) for workers hired from unemployment, we obtain

$$f(x, y) = w\big(x, y, S^o\big) + (1-\beta)V_v(y). \tag{A.37}$$

The output of a match is determined by inverting the wage equation, expressing the output $f(x, y)$ as a function of the observed wage $w(x, y, S^o)$ and the outside option $V_v(y)$ measured above.

D.3. *Quantitative Evaluation*

The objective of this section is to evaluate the performance of the proposed measurement approach over a wide range of parameter values that are likely to be encountered in empirical work for the model with on-the-job search. The approach and the parameterization are the same as in the benchmark model. In addition, the on-the-job search efficiency parameter ϕ is set to 0.2 to generate the monthly probability of a job-to-job move ranging from 1% to 2% across parameterizations.



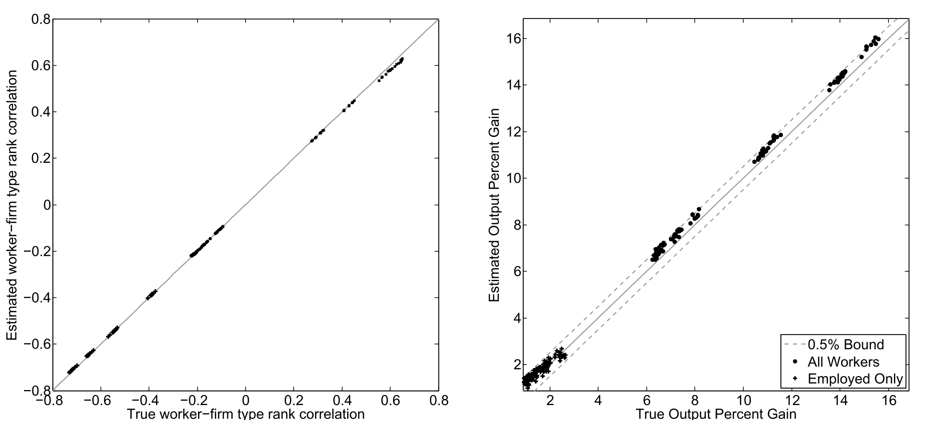
(a) Distribution of correlation between true and estimated production functions across parameterizations.

(b) Distribution of the difference between true and estimated production functions across parameterizations: $\frac{\sum_{x,y}[|f(x,y)-\hat{f}(x,y)|d_m(x,y)]}{\sum_{x,y}[f(x,y)d_m(x,y)]}$, $d_m(x,y)$ normalized to integrate to 1.

FIGURE A-1.—Model with on-the-job search: recovering the production function.

All combinations of parameters result in 108 distinct parameterizations. Across the parameterizations, all the key variables of interest fall within empirically plausible ranges. Figures A-1(a) and A-1(b) plot the distribution of the correlation between the true and the estimated production functions and the corresponding distribution of the differences between them. The lowest correlation is 0.9812 and the median is above 0.995, indicating that the proposed identification and estimation strategy recovers the underlying production function very precisely.

Figure A-2(a) plots the correlation between identified worker and firm ranks against the true correlation for all parameterizations. Clearly, the proposed identification strategy identifies the sign of sorting and measures the strength of sorting very well. It also



(a) Correlation between identified worker and firm ranks against true correlation.

(b) Estimated gains from eliminating frictions.

FIGURE A-2.—Model with on-the-job search: frictions and sorting.

allows to accurately estimate gains from optimal worker reallocation as illustrated in Figure A-2(b).

E. DETAILS OF EMPIRICAL ANALYSIS

As explained in the main text, our empirical work closely follows Card, Heining, and Kline (2013). Here, we provide basic details and explain all the differences.

Our raw LIAB data contain employment histories of 2,087,683 German males aged 16 and above observed working in 1,168,301 unique establishments. The worker data are continuous (up to a day), and are based on notifications submitted by employers to various social insurance agencies upon a change in the conditions of employment. The data include 34,263,798 spells from the Employment History (Beschäftigten-Historik—BeH), which cannot be longer than a year since an annual notification is required for all jobs in progress on December 31; and 6,488,810 spells from the Benefit Recipient History (Leistungsempfänger-Historik—LeH), which can span multiple years.

Our analysis is based on daily wages in the main job of West German male workers aged 20–60. While the data are continuous, we aggregate it to monthly frequency. In case of several concurrent jobs in a given calendar month, we define the main one to be the job in which the worker earns the most in that month. We drop all spells from the Benefit Recipient History, spells with real (2005 base) daily earnings below 10 Euro, as well as spells that correspond to individuals in training or working from home. We also drop several individuals with over 150 employment spells. After this initial data preparation, we are left with 698,374 establishments, 1,973,679 workers, and 22,675,589 employment spells.

Wages are censored at the social security maximum. We follow Dustman, Ludsteck, and Schönberg (2009) and impute censored wages by multiplying the censoring threshold by 1.2.[14] Our identification strategy is based on wages of workers who start new employment cycles, that is, individuals (1) who start their first ever job, (2) whose start of a new job is preceded by compensated unemployment, or (3) who have an uncompensated gap between two jobs longer than one month. Only 2.41% of spells in this sample are censored.

To construct residual wages, we follow Card, Heining, and Kline (2013). In particular, we regress individual log real daily wage y_{it} of individual i in month t on a worker fixed effect α_i and an index of time-varying observable characteristics $x'_{it}\beta$ which include an unrestricted set of year dummies as well as a quadratic and cubic terms in age fully interacted with educational attainment:

$$y_{it} = \alpha_i + x'_{it}\beta + r_{it},$$

where r_{it} is an error component. The residual wage which serves as input into the analysis is then defined as $w_{it} = \exp(y_{it} - x'_{it}\hat{\beta})$.[15]

[14] Card, Heining, and Kline (2013) used a different algorithm in Dustman, Ludsteck, and Schönberg (2009) and stochastically imputed censored wages using a series of Tobit models. One argument of their Tobit model is the censoring rate of an individual's coworkers. It is not possible to reliably construct this variable in LIAB data for establishments outside of the Establishment Panel because not all workers employed in those establishments are observed.

[15] Card, Heining, and Kline (2013) also included establishment fixed effects in the regression. This difference is inconsequential for our purposes, as the inclusion of establishment fixed effects has virtually no impact on $\hat{\beta}$. In particular, $\operatorname{corr}(x'_{it}\hat{\beta}, x'_{it}\hat{\beta}_{\text{CHK}}) = 0.9925$ and $\operatorname{corr}(\log(w_{it}), \log(w_{it,\text{CHK}})) = 0.9993$, where $w_{it,\text{CHK}} = \exp(y_{it} - x'_{it}\hat{\beta}_{\text{CHK}})$.

Having constructed w_{it}, we rank workers. The ranking algorithm uses all available pairwise wage comparisons of workers who start employment cycles within an establishment and does not require that all workers in the establishment are observed. Thus, we include the comparisons in all establishments available in the LIAB data regardless of whether they belong to the Establishment Panel.[16] On this sample, we also measure the labor market transition rates.

After the ranking of workers has been constructed, we drop all establishments for which we do not observe all workers (those that are not in the IAB Establishment Panel). We also drop the establishments that employ fewer than 20 workers on average during the sample period. This leaves us with a sample of 1,328,402 workers and 5,349 establishments. This generates 13,381,974 employment-year spells of which 2,857,275 are out of unemployment. Establishments in this sample are ranked following the procedure in Appendix D.2.3. Following this, the production function is recovered.

F. APPENDIX FIGURES

F.1. *Figures: Benchmark Model*

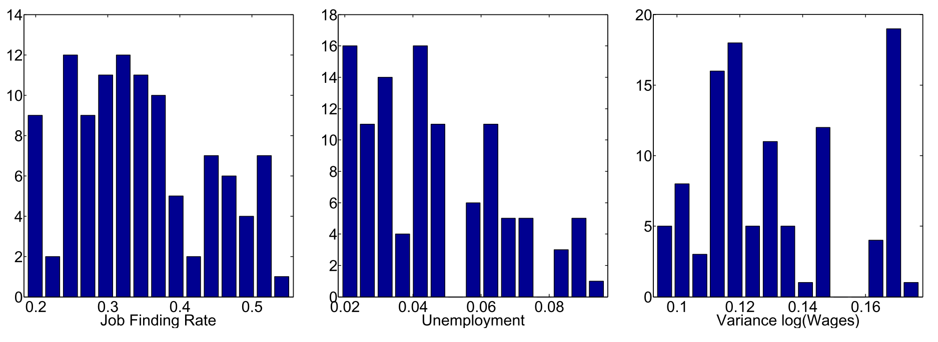


FIGURE A-3.—Distributions of selected variables of interest across all parameterizations.

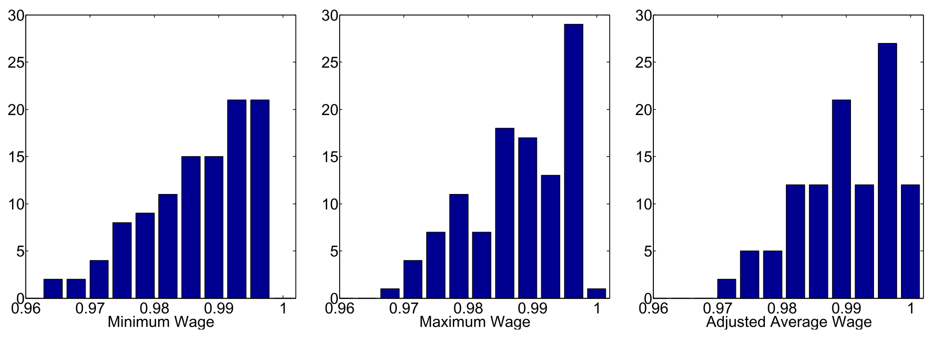


FIGURE A-4.—Distribution of the correlation between the true and estimated, using indicated alternative ranking procedures, worker ranks across all parameterizations.

[16]Even restricting the pairwise connections to workers hired out of unemployment into the same firms implies a highly connected set of workers. In particular, the largest connected set on this sample contains 98.75% of workers. This is only a small reduction in connectedness relative to the full sample where the largest connected set contains 99.81% of workers.

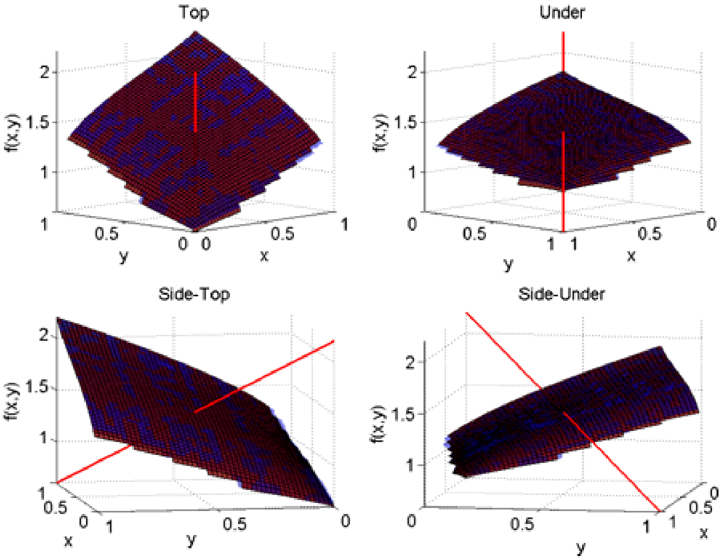


FIGURE A-5.—True and estimated PAM production function.

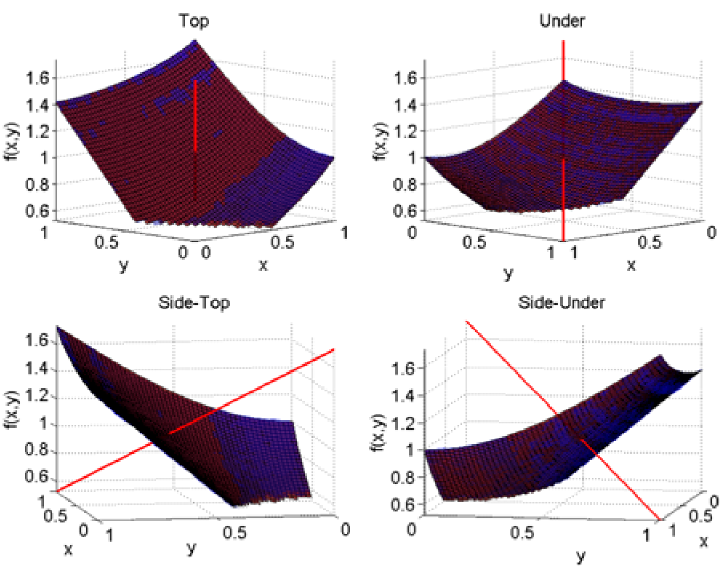


FIGURE A-6.—True and estimated NAM production function.

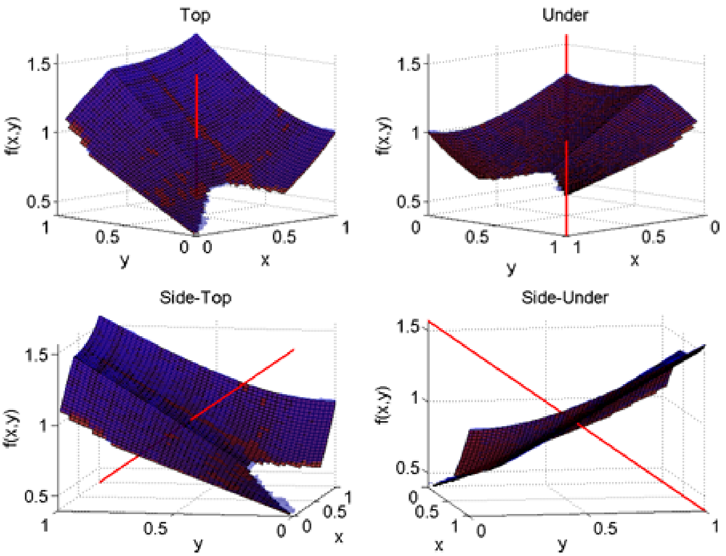
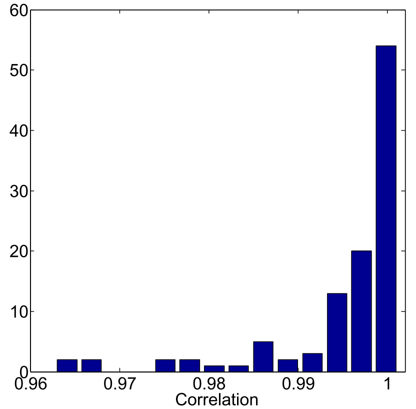
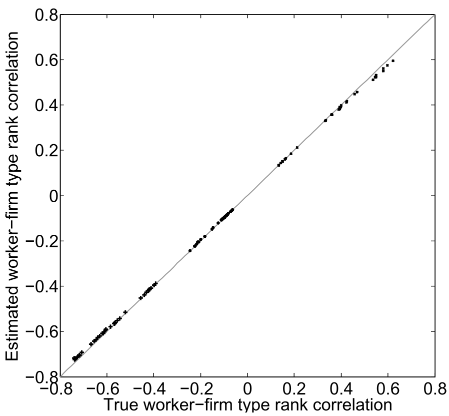


FIGURE A-7.—True and estimated Neither NAM nor PAM production function.

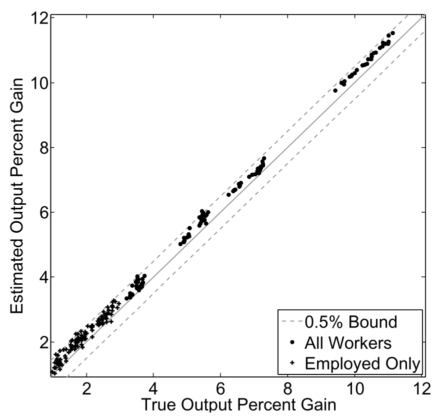
F.2. *Robustness: Shorter Time Horizon of 10 Years*



(a) Distribution of correlation between true and estimated production functions across parameterizations.



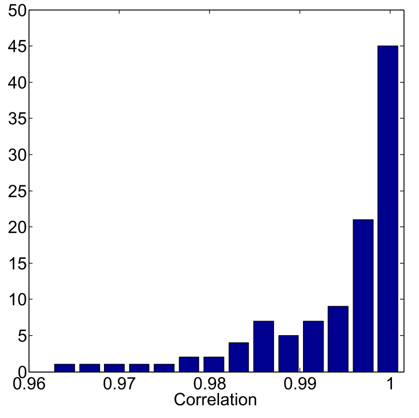
(b) Correlation between identified worker and firm ranks against true correlation.



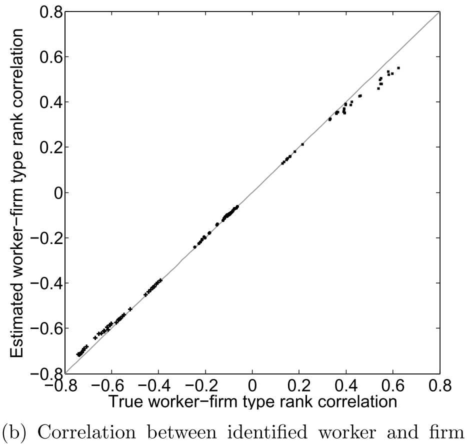
(c) Estimated gains from eliminating frictions.

FIGURE A-8.—Monte Carlo results with a 10-year panel.

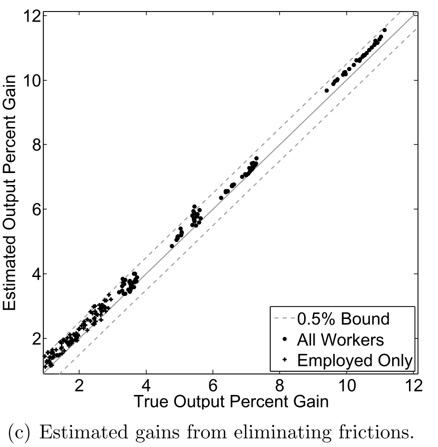
F.3. *Robustness: Small Firms*



(a) Distribution of correlation between true and estimated production functions across parameterizations.



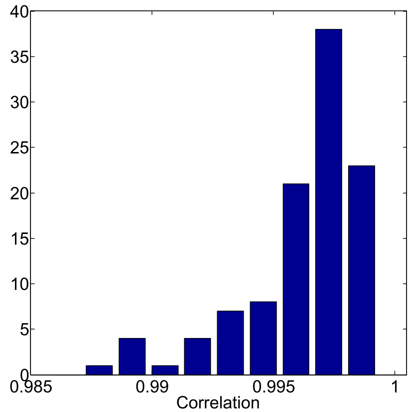
(b) Correlation between identified worker and firm ranks against true correlation.



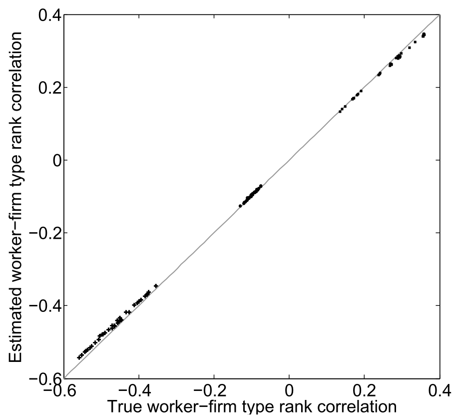
(c) Estimated gains from eliminating frictions.

FIGURE A-9.—Monte Carlo results with maximum firm size of 20 workers.

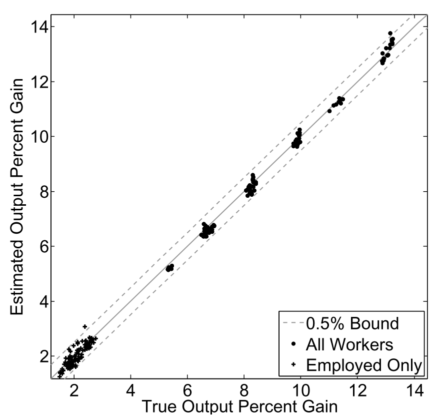
F.4. *Robustness: Stochastic Match Quality*



(a) Distribution of correlation between true and estimated production functions across parameterizations.



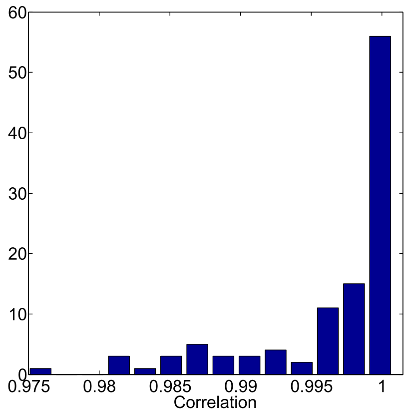
(b) Correlation between identified worker and firm ranks against true correlation.



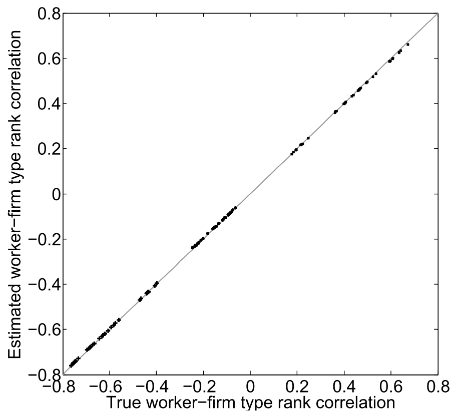
(c) Estimated gains from eliminating frictions.

FIGURE A-10.—Monte Carlo results on a model with stochastic match quality.

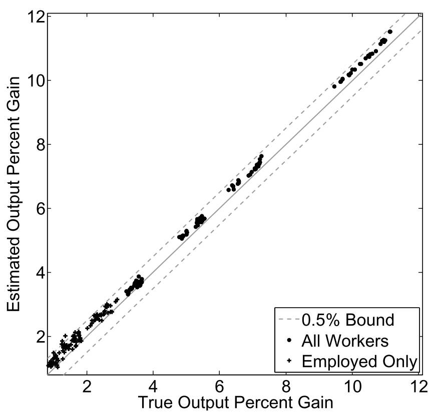
F.5. *Robustness: Discount Factor Close to 1*



(a) Distribution of correlation between true and estimated production functions across parameterizations.



(b) Correlation between identified worker and firm ranks against true correlation.



(c) Estimated gains from eliminating frictions.

FIGURE A-11.—Monte Carlo results with monthly discount factor of 0.999.

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