SUPPLEMENT TO "A MODEL OF THE CONSUMPTION RESPONSE TO FISCAL STIMULUS PAYMENTS"

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THIS SUPPLEMENTARY MATERIAL IS ORGANIZED AS FOLLOWS. Appendix A illustrates another case of wealthy hand-to-mouth behavior which completes the discussion of Section 4.1 in the main text. Appendix B contains more details on our definitions of liquid and illiquid wealth from the SCF and on the measurement of hand-to-mouth households. Appendix C describes certain steps of the model's calibration omitted from the main text. Appendix D contains a robustness analysis on the baseline rebate experiment. The numerical computation of the model and the simulation of the policy experiments are delineated in Appendix E.

APPENDIX A: WEALTHY HAND-TO-MOUTH BEHAVIOR IN THE MODEL

Figure A.1 illustrates how the model can feature households with positive illiquid assets who, at the same time, use credit up to the limit. This is another type of wealthy hand-to-mouth (HtM) behavior, in addition to the one described in the main text (the latter being more prevalent in the data and in the model simulations). In Figure 2, the HtM behavior arises because the agent is at the zero kink for liquid wealth, whereas here she is at the borrowing limit.

After the first deposit into the illiquid account, households would like to increase their consumption to a target level that reflects the higher rate of return earned on their savings. In Figure 2, borrowing costs were prohibitive for the household, and after the deposit the household was immediately constrained. The key difference in the parameterization between the example in this Appendix and the example in the main text is that credit is much cheaper here. As a result, the household starts borrowing to finance consumption after its deposit (see panel (b) in Figure A.1), and it quickly reaches the credit limit. At that point, it stays at the limit for several periods, and consumes all of its earnings, net of the interest payment on debt. During this phase of the life-cycle, upon receiving the rebate check, it will consume a large part of the check, and, upon receiving the news of the rebate, it will not increase her expenditures.

As retirement gets closer, the life-cycle saving motive starts kicking in, and it begins repaying its debt and accumulating liquid wealth.

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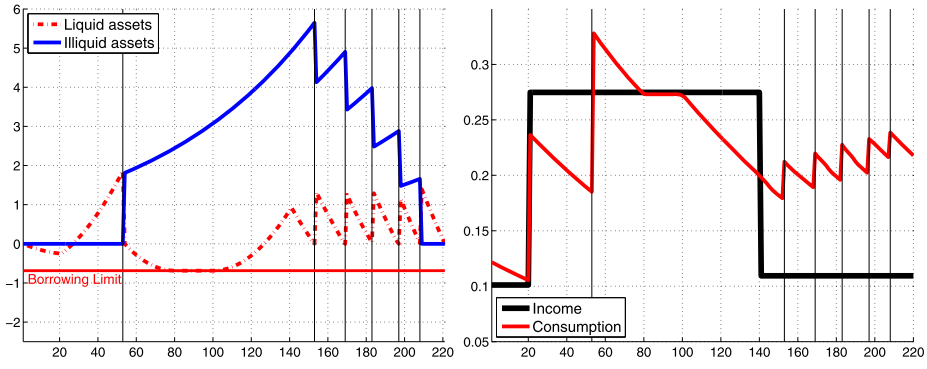

(a) Life-cycle asset accumulation (b) Life-cycle income and consumption path

FIGURE A.1.—Example of life-cycle of a "wealthy hand-to-mouth" agent in the two-asset model where hand-to-mouth behavior occurs when the agent hits the credit limit.

APPENDIX B: SCF DATA AND MEASUREMENT OF HAND-TO-MOUTH HOUSEHOLDS

B.1. *Estimation of Cash Holdings and Credit Card Debt*

Cash Imputation. The Survey of Consumer Finances (SCF) does not record cash holdings of households. To impute cash holdings to our measure of liquid assets, we make use of the Survey of Consumer Payment Choice, administered by the Federal Reserve Bank of Boston, for 2008 (the earliest survey year). This survey reports that median cash holdings on person and property was \$69 (Foster, Meijer, Schuh, and Zabek (2011, Table 9)). Median wealth in checking, saving, money market, and call accounts in the SCF 2001 is \$2,858. We therefore increase proportionately all individual household holdings of these assets by a factor of $1 + (69 \times 2)/2{,}858 = 1.05$, where the 2 multiplying the median individual holdings of cash accounts is for the fact that there are two adults in most households.

Unsecured Debt. As for the calculation of revolving credit card debt, the SCF asks the following questions about credit card balances: (i) "How often do you pay your credit card balance in full?" Possible answers are: (a) Always or almost always; (b) Sometimes; or (c) Almost never. (ii) "After the last payment, roughly what was the balance still owed on these accounts?" From the first question, we identify households with revolving debt as those who respond (b) Sometimes or (c) Almost never. We then use the answer to the second question, for these households only, to compute statistics about credit card debt. This strategy (common in the literature; e.g., see Telyukova (2013)) avoids including, as debt, purchases made through credit cards in between regular payments.

B.2. *Measurement of Hand-to-Mouth Households*

Based on the discussion of Section 4 in the paper, we use the following definitions of hand-to-mouth (HtM) households. Let m_i be the average balance of liquid assets over the past month for household i, and a_i be the stock of illiquid assets, as reported by the SCF. Let y_i be monthly labor income (annual labor income from the SCF divided by 12). Finally, let $\underline{m}_i$ be household's i reported credit limit in the survey.

Household i is HtM if either

$$0 \leq m_i \leq \frac{y_i}{2 \cdot f} \tag{B.1}$$

or

$$m_i < 0 \quad \text{and} \quad m_i \leq \frac{y_i}{2 \cdot f} - \underline{m}_i, \tag{B.2}$$

where f is the frequency of pay. For monthly frequency $f = 1$, for biweekly $f = 2$, and for weekly $f = 4$. Since the frequency of pay is not available from the SCF, we do all our calculations under three alternative assumptions: weekly, biweekly, and monthly frequency.

Household i is *wealthy* HtM if either

$$0 \leq m_i \leq \frac{y_i}{2 \cdot f} \quad \text{and} \quad a_i > 0 \tag{B.3}$$

or

$$m_i < 0 \quad \text{and} \quad m_i \leq \frac{y_i}{2 \cdot f} - \underline{m}_i \quad \text{and} \quad a_i > 0. \tag{B.4}$$

Poor HtM households are all the residual HtM households who are not wealthy HtM, that is, those who have $a_i = 0$.

Table B.I, row (i), reports the calculation with the baseline definition of liquid and illiquid wealth described in the main text.

We also offer a robustness analysis on these measures. First, we use a stricter definition of liquid wealth that only includes cash, checking, saving, money market, and call accounts (and therefore excludes directly held mutual funds, stocks, bonds, and T-Bills which are, arguably, less liquid). Second, we define as wealthy HtM only those with illiquid wealth above a positive threshold. As threshold, we choose \$3,000, which is roughly the median amount of liquid wealth held by the U.S. populations. Third, we use a broader definition of illiquid wealth that also includes vehicles (excluded from the baseline definition of illiquid assets). Note that over 80% of households in the SCF own a vehicle and, for many households, this is the major component of their non-liquid wealth. While the first modification increases the total number of HtM agents,

TABLE B.I

ESTIMATES OF HAND-TO-MOUTH (HTM) HOUSEHOLDS[a]

	Total HtM Households		
	Week	Bi-Week	Month
(i) Baseline	0.175	0.225	0.311
(ii) Strict liquid definition	0.188	0.245	0.350
(iii) Strict illiquid definition	0.175	0.225	0.311
(iv) Vehicles	0.175	0.225	0.311
	Wealthy HtM Households		
	Week	Bi-Week	Month
(i) Baseline	0.089	0.124	0.190
(ii) Strict liquid definition	0.100	0.142	0.226
(iii) Strict illiquid definition	0.071	0.102	0.165
(iv) Vehicles	0.134	0.179	0.260
	Total HtM in Net Worth		
	Week	Bi-Week	Month
(i) Baseline	0.094	0.113	0.139
(ii) Strict liquid definition	0.094	0.113	0.139
(iii) Strict illiquid definition	0.094	0.113	0.139
(iv) Vehicles	0.044	0.050	0.059

[a]Entries are fraction of the total population. The labels Week, Bi-Week, and Month refer to the assumptions on the frequency of pay. Row (i) reports the calculation with the baseline definition of liquid and illiquid wealth; row (ii) uses a stricter definition of liquid wealth which excludes directly held mutual funds, stocks, and bonds; row (iii) defines as wealthy HtM only those HtM households with at least $3,000 in illiquid assets; row (iv) adds to illiquid wealth the net value of vehicles.

the second and third ones only affect the split between "poor" and "wealthy" HtM, but not the total fraction of HtM households.

As reported in the main body of the paper, our analysis leads us to conclude that between 17.5% and 35% of U.S. households are HtM. This is a conservative estimate (for reasons explained in the main text). Moreover, we estimate that between 40% and 80% of these households are wealthy HtM, depending mainly on the pay frequency and on whether one expands the notion of illiquid wealth to include vehicles.

Finally, for comparison, we also compute the fraction of HtM households in terms of net worth. We apply the definition in (B.2) and (B.1), with the only difference that in those definitions we use net worth instead of liquid wealth. The bottom part of Table B.I shows that the fraction of agents HtM in terms of net worth never exceeds 14%, and is as low as 4–5% when including vehicles as wealth.

APPENDIX C: CALIBRATION

C.1. *Initial Asset Positions*

We divide households in the SCF into 21 groups based on their earnings and calculate (i) the fraction with zero holdings, and (ii) the median liquid and illiquid wealth in each group, conditional on positive holdings. When we simulate life-cycles in the model, we create the same groups based on the initial earnings draw. Within each group, we initialize a fraction of agents with zero assets, and the rest with the corresponding median holdings of liquid and illiquid wealth. For example, in the median initial earnings group, the fraction of households with zero liquid (illiquid) wealth is 14% (55%). For those with positive holdings, median liquid wealth is \$2,300, and median illiquid wealth is \$7,700.

C.2. *Service Flow From Housing*

To calculate the service flow from housing (the parameter ζ in the model), we start from the following relationship holding at any given date t:

$$\zeta_t = r_t^h - m_t^h - n_t^h - \left(1 - \tau_t^{\text{ded}}\right)\left(\tau_t^{\text{prop}} + i_t^{\text{mort}}\right), \tag{C.1}$$

where (as for the left hand side variable) every variable on the right hand side is expressed as a fraction of a unit of housing stock. Specifically, r_t^h is the rental value of a unit of housing, m_t^h are maintenance and repair expenditures, n_t^h are home-owner insurance expenditures, τ_t^{prop} are property taxes, and i_t^{mort} are mortgage interest payments. The formula accounts for the fact that these latter two items are tax deductible at the (average) marginal tax rate τ^{ded}. This formula reflects that owning housing wealth has both costs (maintenance, insurance, property taxes, and mortgage interest) and benefits (imputed rental value of the space and tax deductibility of mortgage interest and property taxes).

We omit from this calculation housing price appreciation net of physical depreciation because this component is included in the calculation of the financial return on total illiquid wealth described in Section C.3. We now explain how we measure all the ingredients in equation (C.1). Our final value for ζ is computed as an average of ζ_t over the period 1960–2009, the same period used to compute asset returns in Section C.3.

Our starting point is the total value of residential housing from the Flow of Funds (Table B100). Residential housing can be tenant-occupied or owner-occupied. NIPA Table 2.5.5 (line 20) reports rents from tenant-occupied housing. For owner-occupied housing, the National Income and Product Accounts (NIPA) use a "rental equivalence approach" stating that the housing services produced by an owner-occupied unit are deemed to be equal in value to the rentals that would be paid on the market for accommodations of the same size, quality, and type. NIPA Table 2.5.5 (line 21) reports these "imputed" rents.

Computing total rents over the total value of the residential housing stock over the sample period yields $r^h = 7.9\%$.

We set maintenance and repair expenditures m^h at 1 percent of the stock (an upper bound; see below). The Federal Reserve Board estimates the cost of home-owner insurance n^h at 0.35 percent per year. Poterba and Sinai (2008) reported an average annual property tax τ^{prop} of 1 percent.

To compute mortgage interest payments i^{mort} as a fraction of the value of the housing stock, we proceed as follows. As a measure of mortgage interest rates, we use the 30-year interest rate on conventional mortgages (series MORTG from the Federal Reserve Bank of St. Louis Federal Reserve Economic Data—"FRED"), which averages 8.3 percent over this period. To calculate the average loan-value ratio, we divide the total outstanding stock of home mortgages from the Flow of Funds (series HMLBSHNO from FRED) by the total value of residential housing from the Flow of Funds (the same series used above), which gives an average value of 0.36 over this period. By multiplying, year by year, the interest rate by the loan-value ratio, we obtain an estimate of mortgage interest payments per unit of housing owned of 2.9 percent.

Finally, Barro and Redlick (2011) reported that the average marginal Federal tax rate τ^{ded} over this period was 23.8 percent. Combining all these components into (C.1), and averaging over the sample period, we obtain an estimate of ζ of 4.2 percent per year. This estimate is a lower bound for various reasons.

First, if one repeats the calculation for r^h only on the stock of owner-occupied housing by using the value of residential housing wealth at current cost (i.e., market value) for owner-occupied housing from NIPA Table 5.1 (line 11) together with the imputed rents from owner-occupied housing from NIPA Table 2.5.5, one obtains a higher value for r^h, 8.6% instead of 7.9%, a result that confirms the conventional wisdom that the stock of owner-occupied housing is, on average, of better quality.

Second, the Census reports estimates of "maintenance and repair" expenditures both for owner-occupied housing and for all residential properties (http://www.census.gov/construction/c50/c50index.html). These estimates are considerably below our baseline of 1 percent per year. Using the Census estimates for m_t^h, we obtain values of ζ that are 0.8–0.9 percentage point higher.

Third, property taxes can be thought of as the price to pay to gain access to certain local services (notably, public schooling). As a result, they are not entirely a cost, as they imply a utility flow as well. Adding back 50 percent of property taxes in the calculation increases ζ by 0.9 percentage point.

Fourth, the service flow originates from the housing stock, whereas in the model a is the net value of illiquid assets. These two values differ because (1) housing is a leveraged claim, and (2) housing is only one asset class (albeit the largest) among illiquid wealth. From the SCF 2001, the median and mean gross housing wealth to net illiquid wealth ratios are, respectively, 1 and 1.6. By applying ζ to a, we implicitly use a ratio of 1.

To conclude, we choose a value of 1 percent per quarter for ζ, and the calculations reported in this section lead us to think that this may be a conservative estimate.

C.3. *Returns on Liquid and Illiquid Assets*

Risk-Adjustment. Since in the model we abstract from aggregate risk, we perform a "risk-adjustment" on the returns of all our asset classes.

In the data, assets have different returns because of the risk properties of their dividend stream and because of their liquidity value. In our model, the only source of return differentials is liquidity summarized (arguably, in reduced form) by the existence of transaction costs.

We outline two approaches to identify the portion of the return associated with the liquidity properties of the asset in question. The residual approach uses a minimum amount of asset pricing theory to filter out from the observed return the component due to aggregate risk and identifies the one due to liquidity residually. The direct approach uses existing estimates of liquidity premia from the literature.

C.3.1. *Residual Approach*

The Euler equation for an asset i at date t can be written as

$$1 = E_t\big[\mathrm{MRS}_{t+1}\big(1 + r^i_{t+1}\big)\big(1 - \ell^i_{t+1}\big)\big], \tag{C.2}$$

where MRS_{t+1} is the marginal rate of substitution of the asset holder, r^i_{t+1} is the return of the asset (price appreciation cum dividend), and $\ell^i_{t+1} \geq 0$ is an additional component of the return that captures the "liquidity value" of asset i (highest for $\ell^i_{t+1} = 0$). For example, Lagos (2010, Equation 1) derived the Euler equation (C.2) from a model with search frictions where some assets, beyond paying a stream of dividends, are better than others as a medium of exchange for the final consumption good in a decentralized frictional market. There, ℓ^i_{t+1} is a function of the model primitives (e.g., the lower the probability for the holder of asset i to meet a buyer in the frictional market, the higher is ℓ^i_{t+1}). For an asset which is safe, yields no dividends, and has perfect liquidity, the Euler equation (C.2) implies

$$1 = E_t[\mathrm{MRS}_{t+1}]. \tag{C.3}$$

Abstracting from second-order terms, $(1 + r^i_{t+1})(1 - \ell^i_{t+1}) \simeq 1 + r^i_{t+1} - \ell^i_{t+1}$, rearranging (C.2) and using (C.3), one can obtain the following reformulation for (the unconditional version of) that Euler equation:

$$E\big(r^i\big) = -\operatorname{cov}\big(\mathrm{MRS}, r^i\big) + \operatorname{cov}\big(\mathrm{MRS}, \ell^i\big) + E(\ell), \tag{C.4}$$

which yields an intuitive expression for the average return of the asset. The first term in the RHS of (C.4) encodes the classical risk premium due to the comovement between the return of the asset and the marginal rate of substitution of the asset holder. The second and third terms capture the additional components of the return associated with the liquidity value. An asset with low liquidity properties ($E(\ell)$ large) and liquidity value that is negatively correlated with the marginal rate of substitution (positive correlation between ℓ and MRS) must command a high financial return to be held by risk-averse households. See Lagos (2010, Equation 20) for a reinterpretation of the Euler equation (C.2) exactly along these lines. In this context, risk-adjusting the return r^i means eliminating the first covariance component $\text{cov}(\text{MRS}, r^i)$ from the return in (C.4). This covariance component, however, is model-specific since the MRS depends on preferences and market structure. Our model cannot be used for such calculation since it has no aggregate uncertainty. We therefore propose two empirical strategies to perform this risk-adjustment.

First, a plausible assumption, which allows making a risk-adjustment without taking a stand on the MRS, is

$$\text{(C.5)} \qquad \text{var}\left(r^i\right) > -\text{cov}\left(r^i, \text{MRS}\right).$$

Under this inequality, one can subtract from the expected return the observed variance of the return and obtain a lower bound for the component of the return which is associated to liquidity, that is, for the risk-adjusted return.

A second plausible upper bound for the term $-\text{cov}(r^a, \text{MRS})$ can be constructed using the insight that, empirically and theoretically, aggregate income volatility exceeds the volatility of the aggregate component of consumption. From NIPA Table 2.1 (series: Compensation of Employees plus $0.66\times$ series Proprietor's Income) and from the St. Louis FRED database (series: Civilian Employment), we compute labor income per worker and estimate a stochastic process for the residuals of this series around a deterministic linear trend. These residuals are well approximated as an AR(1) with autoregressive coefficient of 0.95 and annualized variance of the innovation equal to 0.003. Next, we use our Epstein–Zin–Weil preference specification parameterized as in our calibration (i.e., with risk-aversion equal to 4, IES equal to 1.5, and discount factor equal to 0.941) to compute the implied volatility of the MRS, when the consumption process equals the labor income process. See Chen, Favilukis, and Ludvigson (2013, Equation 5) for the analytical expression of the MRS with Epstein–Zin–Weil preferences.

Let $\overline{\text{MRS}}$ denote this alternative time-series proxy for the MRS. We find that $\text{std}(\overline{\text{MRS}}) = 0.044$. Since this is, arguably, an upper bound for the volatility of the MRS in the data, we can write the inequality

$$\text{(C.6)} \qquad -\text{cov}\left(r^a, \text{MRS}\right) < \text{std}\left(r^a\right) \cdot \text{std}(\text{MRS}) < \text{std}\left(r^a\right) \cdot \text{std}(\overline{\text{MRS}}),$$

and use the last (measurable from the data) term in this inequality for the risk-adjustment. In what follows, we refer to the first strategy based on inequality (C.5) as risk-adjustment strategy S1 and to the second strategy based on inequality (C.6) as strategy S2.

Nominal Returns. We apply this methodology to all individual asset classes we consider within the liquid and illiquid wealth groups. All our calculations refer to the period 1960–2006. We perform this calculation in nominal terms first, since we are interested in after-tax returns and taxes apply to nominal returns. Then, we make an adjustment for inflation. We set the annual inflation rate to 4% (the average over this period was 4.1%).

Recall that our definition of liquid assets comprises: cash, money market, checking, savings, and call accounts, plus directly held mutual funds, stocks, bonds, and T-Bills. Our baseline measure of illiquid assets includes net housing worth, retirement accounts, life insurance policies, CDs, and saving bonds.

We set the nominal return on cash and all non-interest bearing accounts to zero. We set the return on savings accounts, T-Bills, savings bonds, and life insurance (assuming actuarially fair contracts) to the interest rate on 3-month T-Bills (Federal Reserve Board, FRB hereafter, database). Over the period 1960–2006, we obtain an average nominal return on 3-month T-Bills of 5.33% (SD 2.76%) with an implied risk-adjusted return of 5.25% under strategy S1 and 5.21% under strategy S2.

For CDs (for which data are available only starting from 1964 in the FRB database), we compute a return of 6.29% (SD 3.13%) corresponding to a risk-adjusted return of 6.2% under both strategies.

For equities, we use Center for Research in Security Prices (CRSP) value-weighted returns, assuming dividends are reinvested, and obtain an annualized nominal return of 11.1% (SD 17.89%), with an implied risk-adjusted nominal return of 7.9% under strategy S1 and 10.3% under strategy S2. Note that our risk-adjustment S1 closes half of the gap between equity and bond returns. This is a generous adjustment, in light of the fact that Lagos (2010) concluded that 90% of the equity premium is liquidity driven (and hence the risk-adjustment would only account for 10 percent of the gap, similarly to what obtained from our risk-adjustment strategy S2).

The SCF reports the equity share for directly held mutual funds, stocks, and bonds, and for retirement accounts, which allows us to apply separate returns on the equity and safe components of each saving instrument. An important feature of retirement accounts is the employer's matching rate. Over 70% of households in our sample with positive balance on their retirement account have employer-run retirement plans. The literature on this topic finds that, typically, employers match 50% of employees' contributions up to 6% of earnings, but the vast majority of employees do not contribute above this threshold (e.g., Papke and Poterba (1995)). As a result, we raise the return on retirement accounts by a factor of 1.35.

To compute the rate of return on housing (appreciation net of physical depreciation), we follow two alternative methods. The first method replicates

the calculation in Favilukis, Ludvigson, and Van Nieuwerburgh (2010). We measure housing wealth for the household sector from the Flow of Funds (Table B100) and construct an index measuring the growth in residential housing wealth. We then subtract population growth in order to correct for the growth in housing quantity. We obtain an average annual nominal return of 6.6% (SD 7.3%) implying a risk-adjusted nominal return of 6% under the risk-adjustment strategy S1 and 6.2% under strategy S2.

Second, we use the calculations of Piazzesi and Schneider (2007), who listed different estimates for the real return on housing over the postwar period. Their Tables B1 and B2 report both means and standard deviation, and hence we can calculate risk-adjusted returns. We find that their estimates range between 1.7 and 2.7 percent per year in real terms under both risk-adjustment strategies, and hence in line with the 6% nominal obtained from the first approach, given our assumed inflation rate of 4%.

Finally, we note that both risk-adjustment strategies lead to very similar results, except for the case of stocks, where the first strategy S1 leads to much lower risk-adjusted returns.

C.3.2. *Direct Approach*

We take the view that the entire return on saving bonds, 3-month T-Bills, and on 3-month CDs is due to their imperfect liquidity (relative, say, to cash or bank accounts), and hence we do not perform any risk-adjustment. The calculations based on the residual approach outlined above suggest the adjustment would be rather trivial anyway.

The most widely cited recent paper on the measurement of liquidity risk for equities is Pastor and Stambaugh (2003; PS hereafter). PS studied whether liquidity (measured as the temporary effect of order flows on stock prices) is a relevant factor in explaining the cross-section of stock returns, over and above the standard Fama–French factors. Their answer is quite striking: the authors ranked stocks by decile of sensitivity to their measure of aggregate liquidity risk and showed that liquidity accounts for an excess return of 7.5% between the top and the bottom decile, and roughly 3.5% between the median and the bottom decile over the period 1966–1999.

If we assume that stocks in the bottom decile of the PS classification (the most liquid) are akin to T-Bills in their liquidity properties, and that the median stock is representative of the equity portfolio held by our agents, then we obtain a risk-adjusted nominal return for stocks of $5.33 + 3.5 = 8.83\%$ under this strategy (that we call S3).

Since we are not aware of an equivalent calculation in the literature for housing, we proceed as follows. Over the period 1966–1999, the illiquidity premium computed by PS represents $3.5/6.9 = 51\%$ of the excess return for stocks. It is reasonable to think, therefore, that, since housing is less liquid than the median stock, a larger portion of the excess return of housing (1.23%) stems from its

illiquid nature. If we assume that this portion is 2/3, we obtain a risk-adjusted nominal return for housing of $5.33 + 1.23 \times 0.66 = 6.14\%$.

Overall, strategy S3 yields a return differential between total illiquid and liquid wealth in between that obtained with strategy S1 and that obtained with strategy S2.

C.3.3. *Calculation of Real After-Tax Returns on Liquid and Illiquid Assets*

In light of these results, we proceed with our calculations using the first, more conservative, strategy for risk-adjustment, S1. To complete our calculations, we need estimates for (i) tax rates and (ii) inflation.

Capital Income Tax Rates. Kiefer, Carroll, Holtzblatt, Lerman, McCubbin, Richardson, and Tempalski (2002, Table 5) reported the effective tax schedule on interests and dividends and on long-term capital gains by ten income brackets in 2000. We apply the interests and dividend tax rates on all asset returns with two exceptions. First, we apply the capital gain tax rate on the return to retirement accounts. Second, we follow Poterba and Sinai (2008) and set the effective tax rate on housing returns to zero. They wrote that "since 1997, married (single) homeowners have been able to realize \$500,000 (\$250,000) of capital gains tax-free after a holding period of two years. Relatively few accruing housing capital gains are likely to face taxation under this regime."

Real After-Tax Returns. We apply these nominal returns (by asset type) and these tax rates (by asset type and household income bracket) to each household portfolio in the SCF and compute average risk-adjusted after-tax nominal returns in the population for liquid wealth, illiquid wealth, and net worth. Finally, we subtract 4% inflation to each rate of return, and obtain risk-adjusted after-tax real returns of -1.48% for liquid wealth, 2.29% for illiquid wealth, and 1.67% for net worth. Table C.I summarizes these calculations.

C.4. *Dynamics of Liquid Wealth Around Retirement*

Figure C.1 zooms on the age range 50–65 to display the hump in median liquid wealth around retirement in the model and in the SCF data. In the model, households accumulate liquid wealth in anticipation of retirement to smooth the drop in income. The micro data do display a similar pattern. Unsurprisingly, in the data, the hump is smoother since not every individual retires at the same age.

APPENDIX D: ROBUSTNESS

Table D.I summarizes our sensitivity analysis with respect to preference parameters (risk aversion and IES), access to credit (borrowing costs and limits), desirability of the illiquid asset (financial return and consumption flow), and

TABLE C.I

SUMMARY OF CALCULATIONS FOR RETURNS OF VARIOUS ASSET CLASSES (1960–2009)[a]

	Nominal		
	Mean	SD	Risk-Adjusted
Cash, checking accounts	0.00	0.00	0.00
3-month T-bills	5.33	2.76	5.25
Saving acc./bonds, life ins.	5.33	2.76	5.25
3-month CDs (1964–2009)	6.29	3.13	6.20
Stocks	11.06	17.89	7.86
Housing	6.56	7.30	6.03
	Nominal Risk-Adjusted	Mean Tax Rate	Real After-Tax Risk-Adjusted
Liquid wealth	3.30	23.19	−1.48
Illiquid wealth	6.84	7.86	2.29
Net worth	6.30	10.37	1.67

[a]Risk adjustment based on strategy S1.

size of the idiosyncratic risk.[47] The analysis is done for all three information structures, and for both the one-asset and the two-asset models. For every pa-

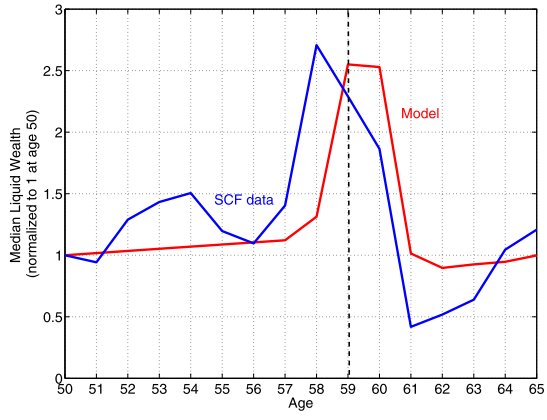

FIGURE C.1.—Pattern of median liquid wealth around retirement in the model (where retirement age is 59 for all households) and in the SCF data. SCF data are 3-year moving averages. Model is yearly averages of quarterly values.

[47]The table does not report sensitivity with respect to the transaction cost κ because it can be easily inferred from the figures in the paper.

TABLE D.I

ROBUSTNESS ANALYSIS[a]

		Rebate Coefficient					
Information Structure:		Baseline		Surprise for All		Anticipated by All	
Assets in Model:		One	Two	One	Two	One	Two
Borrowing rate	5%	<1%	19%	<1%	19%	<1%	19%
	10%	<1%	15%	3%	25%	<1%	11%
	15%	3%	25%	3%	30%	<1%	17%
Credit limit	0	<1%	28%	3%	30%	<1%	27%
	0.74	<1%	15%	3%	25%	<1%	11%
	1.48	<1%	14%	3%	24%	<1%	11%
Risk aversion	2	<1%	13%	3%	23%	<1%	10%
	4	<1%	15%	3%	25%	<1%	11%
	6	<1%	17%	4%	28%	3%	23%
IES	1.05	<1%	9%	3%	17%	<1%	7%
	1.5	<1%	15%	3%	25%	<1%	11%
	2	<1%	20%	3%	33%	<1%	16%
Return wedge	2.54	<1%	15%	3%	25%	<1%	11%
	3.54	<1%	15%	3%	25%	<1%	11%
	4.54	<1%	14%	3%	25%	<1%	11%
Housing service flow	0.02	<1%	14%	3%	22%	<1%	11%
	0.04	<1%	15%	3%	25%	<1%	11%
	0.06	<1%	18%	3%	25%	<1%	15%
Variance of shocks	0.002	<1%	14%	3%	23%	<1%	11%
	0.003	<1%	15%	3%	25%	<1%	11%
	0.004	<1%	16%	4%	27%	<1%	11%

[a]The borrowing rate is the nominal annual rate on unsecured credit. The credit limit is expressed as a fraction of quarterly income, as in the model. The return wedge is the differential after-tax return between illiquid and liquid assets. In all sensitivity analyses, the middle row is the value of the baseline calibration. For every parameterization, we recalibrate the discount factor β to match median illiquid wealth (as a fraction of average income).

rameterization, we recalibrate the discount factor β to match median illiquid wealth, or net worth (as a fraction of average income).

Preferences. Increasing the coefficient of relative risk aversion from 2 to 6 raises the rebate coefficient because households hold more illiquid wealth as a precautionary saving instrument in case they are hit by large shocks. As a result, the calibrated discount factor needed to match the median illiquid wealth-income ratio is lower. Higher impatience increases the MPC of all agents.

As we mention in the main text, the IES plays a powerful role. Households who are more willing to substitute consumption intertemporally are more likely to save heavily in the illiquid asset, and to be wealthy hand-to-mouth, during working-age to enjoy higher consumption at retirement. Moreover, those households who learn about the rebate in advance are less likely to use costly credit to start spending the check earlier, and would rather wait one extra

quarter to consume it. Indeed, with higher IES there are more hand-to-mouth agents and fewer agents using credit in the economy. Both forces push up the rebate coefficient.

Credit. Lowering and increasing the borrowing cost, relative to the baseline, increases the rebate coefficient. Cheap credit creates an arbitrage opportunity: many households borrow up to the limit to invest into the illiquid asset, and end up wealthy hand-to-mouth at the credit limit (recall the example in Appendix A). When credit is very expensive, few households ever borrow and there are many more hand-to-mouth households at the zero kink for liquid wealth.

Table D.I shows that our credit limit is not too binding. Doubling the limit has no impact on the rebate coefficient. Tightening the limit down to zero has similar effects to prohibitively increasing borrowing costs.

Desirability of the Illiquid Asset. Raising the return wedge and the housing-service flow makes the illiquid asset more desirable and induces more households to be wealthy hand-to-mouth, which, in turn, increases the rebate coefficient.

Idiosyncratic Earnings Risk. Making the individual earnings process more volatile has similar effects to raising risk aversion. It pushes households in the model to hold more illiquid wealth as a precautionary saving instrument. The discount factor required to replicate the median illiquid wealth-income ratio in the data is lower, and this lower degree of patience increases the MPC of all agents.

APPENDIX E: NUMERICAL SOLUTION OF THE MODEL

E.1. *Detailed Description of Model*

E.1.1. *Preliminaries*

An agent of age j can hold two assets in the model: an illiquid asset, a_j, that has an associated price q^a; and a liquid asset, m_j, that has an associated price $q^m(m_{j+1})$, where dependence on m_{j+1} reflects the possibility of a wedge between the borrowing cost and the interest rate on liquid saving.

In this Appendix, we make the following modifications relative to the main text:

1. For ease of notation, we let $\psi_j \equiv (\alpha, z_j)$, and write earnings at age j as $y_j(\psi_j)$. We denote by $F(\psi_j|\psi_{j-1})$ the conditional probability distribution of earnings and assume ψ_j can only take a finite number of values.

2. In the main text, we defined a tax function $\mathcal{T}(y_j, a_j, m_j)$. Since this tax function is separable between earnings and the two assets, in this appendix we express its earnings component as $T(y_j)$ to reflect the (nonlinear) tax on earnings, and interpret the prices (q^a, q^m) as after-tax prices.

3. We use e_j to denote total expenditures before tax. That is, $e_j \equiv c_j + h_j$, where c_j is non-durable expenditures and h_j is housing expenditures on the

rental market. Because of the assumption of a frictionless rental market for housing, the model can be solved in two stages. In the first stage, we solve for total expenditures, allowing for a flow of consumption services from the illiquid asset holdings in period j in the amount of ζa_{j+1}. In the second stage, we solve the within-period problem of allocating total spending on non-durables and rental housing services, conditional on the optimal total expenditure and holdings of illiquid assets. In Section E.2 below, we show that the solution to this second stage problem yields the indirect period utility function $e_{j+1} + \zeta a_{j+1}$, which we use in the first stage.

We define the following objects:

- x_j^N is total liquid funds available for consuming and saving, for an agent who is not adjusting:

$$x_j^N(m_j, a_j, y_j) \equiv m_j + y_j - T(y_j) + reb_j;$$

reb_j is equal to 0 unless a rebate is received in period j.

- x_j^A is total liquid funds available for consuming and saving, for an agent who is adjusting, before paying the adjustment cost:

$$\begin{aligned} x_j^A(m_j, a_j, y_j) &\equiv m_j + a_j + y_j - T(y_j) + reb_j \\ &= x_j^N(m_j, a_j, y_j) + a_j. \end{aligned}$$

- $V_j^A(x_j, \psi_j)$ is the value function if the agent accesses the illiquid asset. $e_j^A(x_j, \psi_j)$ is the associated consumption policy function.
- $V_j^N(x_j, a_j, \psi_j)$ is the value function if the agent does not access the illiquid asset. $e_j^N(x_j, a_j, \psi_j)$ is the associated consumption policy function.
- We define the expected value function, where the expectation is taken over the current period shocks, and so is a function of the current period holdings of the two types of assets (since these are chosen the period before) and the previous period's realization of the persistent component of earnings. Note that cash-on-hand is only realized when earnings are realized and so is not a state variable for the expected value function. Dependence of (x_j^A, x_j^N) on (m_j, a_j, y_j) is implicit in this function and those defined below:

$$\begin{aligned} &EV_j(m_j, a_j, \psi_{j-1}) \\ &\quad = \sum_{\psi_j \in \Psi_j} \max\left\{V_j^A\left(x_j^A, \psi_j\right), V_j^N\left(x_j^N, a_j, \psi_j\right)\right\} F(\psi_j | \psi_{j-1}). \end{aligned}$$

- We define a new operator, m̃ax. This operator chooses between two objects based on which of the corresponding value functions is higher. For example, $\tilde{\max}\{e^A, e^N\}$ selects consumption expenditures e^A when $V^A > V^N$ at the corresponding point in the state space.

- We define the risk-adjusted expected value function, RV_j, as

$$RV_j(m_j, a_j, \psi_{j-1})^{1-\gamma} = \sum_{\psi_j \in \Psi_j} \tilde{\max}\left\{V_j^A\left(x_j^A, \psi_j\right)^{1-\gamma}, V_j^N\left(x_j^N, a_j, \psi_j\right)^{1-\gamma}\right\} F(\psi_j|\psi_{j-1}).$$

- We define the functions $FV_{a,j}$ and $FV_{m,j}$ as

$$FV_{a,j}(m_j, a_j, \psi_{j-1}) = \sum_{\psi_j \in \Psi_j} \tilde{\max}\left\{V_j^A\left(x_j^A, \psi_j\right)^{-\gamma} \frac{\partial V_j^A}{\partial a_j}, V_j^N\left(x_j^N, a_j, \psi_j\right)^{-\gamma} \frac{\partial V_j^N}{\partial a_j}\right\} \times F(\psi_j|\psi_{j-1}),$$

$$FV_{m,j}(m_j, a_j, \psi_{j-1}) = \sum_{\psi_j \in \Psi_j} \tilde{\max}\left\{V_j^A\left(x_j^A, \psi_j\right)^{-\gamma} \frac{\partial V_j^A}{\partial m_j}, V_j^N\left(x_j^N, a_j, \psi_j\right)^{-\gamma} \frac{\partial V_j^N}{\partial m_j}\right\} \times F(\psi_j|\psi_{j-1}).$$

- We define $S_j = (m_j, a_j, \psi_{j-1})$.

E.1.2. *Decision Problems*

Problem if not Adjusting

$$V_j^N(x_j, a_j, \psi_j) = \max_{e_j, m_{j+1}} \left\{(1-\beta)(e_j + \zeta a_{j+1})^{1-\sigma} + \beta RV_{j+1}(S_{j+1})^{1-\sigma}\right\}^{1/(1-\sigma)}$$

subject to:

$$q^m(m_{j+1})m_{j+1} + \left(1+\tau^c\right)e_j \le x_j,$$
$$q^a a_{j+1} = a_j,$$
$$m_{j+1} \ge \underline{m}_{j+1}(y_j).$$

Problem if Adjusting

$$V_j^A(x_j, \psi_j) = \max_{e_j, m_{j+1}, a_{j+1}} \left\{(1-\beta)(e_j + \zeta a_{j+1})^{1-\sigma} + \beta RV_{j+1}(S_{j+1})^{1-\sigma}\right\}^{1/(1-\sigma)}$$

subject to:

$$q^m(m_{j+1})m_{j+1} + q^a a_{j+1} + \left(1+\tau^c\right)e_j \le x_j - \kappa,$$

$$m_{j+1} \geq \underline{m}_{j+1}(y_j),$$
$$a_{j+1} \geq 0.$$

E.1.3. *First-Order Necessary Conditions*

To solve the model, we derive the first-order conditions. Note that due to the non-convexity of the problem, these are not sufficient. Nonetheless, these conditions are necessary. Our computational approach is to look for all solutions to each set of FOCs, and then compare the associated value functions at each candidate solution.

No-Adjust Case. When agents do not adjust, there is one FOC, a standard Euler Equation (EE):

$$\frac{1-\beta}{1+\tau^c}(e_j+\zeta a_{j+1})^{-\sigma} = \begin{cases} \dfrac{\beta}{q^m} RV_{j+1}(S_{j+1})^{\gamma-\sigma} FV_{m,j+1}(S_{j+1}), & \text{if } m_{j+1} > 0, \\ \dfrac{\beta}{\bar{q}^m} RV_{j+1}(S_{j+1})^{\gamma-\sigma} FV_{m,j+1}(S_{j+1}), & \text{if } m_{j+1} < 0, \\ \in \left[\dfrac{1}{q^m}, \dfrac{1}{\bar{q}^m}\right] \cdot \beta RV_{j+1}(S_{j+1})^{\gamma-\sigma} FV_{m,j+1}(S_{j+1}), & \text{if } m_{j+1} = 0. \end{cases}$$

Adjust Case. For adjusting agents there are two FOCs. One is a standard Euler equation (intuitively, the liquid asset can be adjusted costlessly the following period so an EE holds), the other is a portfolio problem that equates the marginal value of investing in the two different assets:

$$\frac{1-\beta}{1+\tau^c}(e_j+\zeta a_{j+1})^{-\sigma} = \begin{cases} \dfrac{\beta}{q_j^m} RV_{j+1}(S_{j+1})^{\gamma-\sigma} FV_{m,j+1}(S_{j+1}), & \text{if } m_{j+1} > 0, \\ \dfrac{\beta}{\bar{q}^m} RV_{j+1}(S_{j+1})^{\gamma-\sigma} FV_{m,j+1}(S_{j+1}), & \text{if } m_{j+1} < 0, \\ \in \left[\dfrac{1}{q^m}, \dfrac{1}{\bar{q}^m}\right] \cdot \beta RV_{j+1}(S_{j+1})^{\gamma-\sigma} FV_{m,j+1}(S_{j+1}), & \text{if } m_{j+1} = 0, \end{cases}$$

$$\frac{1-\beta}{1+\tau^c}(e_j+\zeta a_{j+1})^{-\sigma} = \frac{1-\beta}{q^a}\zeta(e_j+\zeta a_{j+1})^{-\sigma} + \frac{\beta}{q^a} RV_{j+1}(S_{j+1})^{\gamma-\sigma} FV_{a,j+1}(S_{j+1}),$$

with an inequality for the second FOC when the nonnegativity constraint on illiquid assets ($a_{j+1} \geq 0$) binds.

Below, we transform these two equations into a Euler equation and a portfolio constraint, so that they can be solved by (i) guessing the solution to the intertemporal saving problem, and then (ii) solving the portfolio problem at each guessed value for savings.

E.1.4. *Envelope Conditions*

Here we derive the partial derivatives of value function that are required to evaluate $FV_{a,j}$ and $FV_{m,j}$. Our approach is to store these partial derivatives alongside the value function and policy functions, constructing them recursively. Of course, they may not be continuous, due to the discrete choice. However, (i) if there is enough uncertainty in the problem, the jumps tend to be smoothed away; and (ii) there are a finite number points of discontinuity.

Recall that

$$FV_{m,j}(S_j) = E\left[\tilde{\max}\left\{V_j^A\left(x_j^A, \psi_j\right)^{-\gamma}\frac{\partial V_j^A}{\partial m_j}, V_j^N\left(x_j^N, a_j, \psi_j\right)^{-\gamma}\frac{\partial V_j^N}{\partial m_j}\right\}\right],$$

$$FV_{a,j}(S_j) = E\left[\tilde{\max}\left\{V_j^A\left(x_j^A, \psi_j\right)^{-\gamma}\frac{\partial V_j^A}{\partial a_j}, V_j^N\left(x_j^N, a_j, \psi_j\right)^{-\gamma}\frac{\partial V_j^N}{\partial a_j}\right\}\right],$$

where the partial derivatives with respect to assets and cash on hand are related by

$$\frac{\partial V_j^A}{\partial m_j} = \frac{\partial V_j^A(x_j^A)}{\partial x_j} \equiv V_{x,j}^A,$$

$$\frac{\partial V_j^N}{\partial m_j} = \frac{\partial V_j^N(x_j^N)}{\partial x_j} \equiv V_{x,j}^N,$$

$$\frac{\partial V_j^A}{\partial a_j} = \frac{\partial V_j^A(x_j^A)}{\partial x_j} = V_{x,j}^A.$$

We denote the partial derivative with respect to illiquid assets when not adjusting by

$$\frac{\partial V_j^N}{\partial a_j} \equiv V_{a,j}^N.$$

Next, we compute these partial derivatives of the choice-specific value functions. For the adjust case, it is given by

$$V_{x,j}^A(x_j, \psi_j) = \frac{1-\beta}{1+\tau^c}(e_j + \zeta a_{j+1})^{-\sigma}\left(V_j^A\right)^{\sigma}.$$

For the no-adjust case, they are given by

$$V^N_{a,j}(x_j, a_j, \psi_j) = \frac{\zeta}{q^a}\frac{1-\beta}{1+\tau^c}\left(e_j + \zeta\frac{a_j}{q^a}\right)^{-\sigma}\left(V^N_j\right)^{\sigma} + \frac{\beta}{q^a}RV_{j+1}(S_{j+1})^{\gamma-\sigma}FV_{a,j+1}(S_{j+1})\left(V^N_j\right)^{\sigma},$$

$$V^N_{x,j}(x_j, a_j, \psi_j) = \frac{1-\beta}{1+\tau^c}(e_j + \zeta a_{j+1})^{-\sigma}\left(V^N_j\right)^{\sigma}.$$

In these expressions, e_j, m_{j+1}, and a_{j+1} on the RHS should be interpreted as the optimal decision rules at the point (x_j, a_j, ψ_j).

E.1.5. *Recursive Computation*

To make progress in constructing these objects recursively, it is useful to define some intermediate functions:

$$d_j(S_j) \equiv \left(\frac{1+\tau^c}{1-\beta}\right)FV_{a,j}(S_j)RV_j(S_j)^{\gamma-\sigma},$$

$$g_j(x_j, a_j, \psi_j) \equiv \left(\frac{1+\tau^c}{1-\beta}\right)\frac{V^N_{a,j}(x_j, a_j, \psi_j)}{(V^N_j)^{\sigma}},$$

$$\mu_j(S_j) \equiv \left(\frac{1+\tau^c}{1-\beta}\right)FV_{m,j}(S_j)RV_j(S_j)^{\gamma-\sigma}.$$

By substituting into the envelope conditions, we obtain the following recursions:

$$\mu_j(S_j) = RV_j(S_j)^{\gamma-\sigma}E\left[\tilde{\max}\left\{\left(V^A_j\right)^{\sigma-\gamma}\left(e^A_j + \zeta a^A_{j+1}\right)^{-\sigma}, \left(V^N_j\right)^{\sigma-\gamma}\left(e^N_j + \zeta\frac{a^N_j}{q^a}\right)^{-\sigma}\right\}\right],$$

$$g_j(x_j, a_j, \psi_j) = \frac{\zeta}{q^a}\left(e^N_j + \frac{\zeta}{q^a}a_j\right)^{-\sigma} + \frac{\beta}{q^a}d_{j+1}(S_{j+1}),$$

$$d_j(S_j) = RV_j(S_j)^{\gamma-\sigma}E\left[\tilde{\max}\left\{\left(V^A_j\right)^{\sigma-\gamma}\left(e^A_j + \zeta a^A_{j+1}\right)^{-\sigma}, \left(V^N_j\right)^{\sigma-\gamma}g_j(x_j, a_j, \psi_j)\right\}\right].$$

These recursions reflect the expected marginal values of illiquid assets (d_j) and total assets (μ_j).

E.1.6. *Euler Equations*

We can now finally substitute these into the first-order conditions and obtain the Euler equations that need to be solved.

For the no-adjust case, we have one Euler equation:

$$\left(e_j+\zeta\frac{a_j}{q^a}\right)^{-\sigma}=\begin{cases}\dfrac{\beta}{q^m}\mu_{j+1}(S_{j+1}), & \text{if } m_{j+1}>0,\\ \dfrac{\beta}{\bar{q}^m}\mu_{j+1}(S_{j+1}), & \text{if } m_{j+1}<0,\\ \in\left[\dfrac{1}{q^m},\dfrac{1}{\bar{q}^m}\right]\times\beta\mu_{j+1}(S_{j+1}), & \text{if } m_{j+1}=0.\end{cases}$$

For the adjusting agents, there are two Euler equations:

$$(e_j+\zeta a_{j+1})^{-\sigma}=\begin{cases}\dfrac{\beta}{q^m}\mu_{j+1}(S_{j+1}), & \text{if } m_{j+1}>0,\\ \dfrac{\beta}{\bar{q}^m}\mu_{j+1}(S_{j+1}), & \text{if } m_{j+1}<0,\\ \in\left[\dfrac{1}{q^m},\dfrac{1}{\bar{q}^m}\right]\times\beta\mu_{j+1}(S_{j+1}), & \text{if } m_{j+1}=0,\end{cases}$$

$$(e_j+\zeta a_{j+1})^{-\sigma}=\frac{\beta}{q^a}d_{j+1}(S_{j+1})+\left(1+\tau^c\right)\frac{\zeta}{q^a}(e_j+\zeta a_{j+1})^{-\sigma},\quad \text{if } a_{j+1}>0,$$

$$(e_j+\zeta a_{j+1})^{-\sigma}>\frac{\beta}{q^a}d_{j+1}(S_{j+1})+\left(1+\tau^c\right)\frac{\zeta}{q^a}(e_j+\zeta a_{j+1})^{-\sigma},\quad \text{if } a_{j+1}=0.$$

E.1.7. *Recursive Algorithm*

The model is computed by recursively solving these Euler equations backward from the last period of life $j=J$. At each point in the state space, we search for multiple solutions to the first-order conditions, compute the associated value functions, and choose the solution with the highest value. We explicitly allow for the possibility of solutions at each of the corners and compute the associated value function at these points.

E.2. *Subproblem for Housing and Non-Durable Consumption*

In this section, we outline the static subproblem at age j that yields the optimal choice of housing services h_j bought/sold on the rental market, and non-durable consumption c_j. In this problem, total expenditures e_j and the allocation of illiquid assets a_{j+1} are predetermined. Recall that total housing services s_j which yields utility to the agent also include the flow from the illiquid asset. The household faces the problem:

$$u(e_j, a_{j+1}) = \max_{c_j, s_j, h_j} c_j^{\phi} s_j^{1-\phi}$$

subject to:

$$c_j + h_j = e_j,$$
$$s_j = h_j + \zeta a_{j+1},$$
$$h_j \geq -\zeta a_{j+1},$$
$$c_j \geq 0.$$

The interior solution to this problem is

$$c_j = \phi(e_j + \zeta a_{j+1}),$$
$$s_j = (1-\phi)(e_j + \zeta a_{j+1}),$$
$$h_j = (1-\phi)e_j - \phi\zeta a_{j+1}.$$

The resulting indirect utility function (modulo a multiplicative constant) used in the first-stage problem is

$$u(e_j, a_{j+1}) = e_j + \zeta a_{j+1}.$$

E.3. *Bounds, Grids, and Interpolation*

We now describe the space for each of the state variables for the problem and our methods for interpolation.

E.3.1. (m_j, a_j) *Space*

The risk-adjusted expected value function RV_j and the expected marginal values of the two assets (μ_j, d_j) are defined over the space (m_j, a_j). We discretize this space as follows. Let the lower bound for liquid assets, m_j, be given by $\underline{m}_j$. Let M_j and A_j be an exogenous, age-dependent upper bound on liquid and illiquid assets, that will be chosen so that they never bind in the solution. Then the feasible set for (m_j, a_j) is

$$m_j \in [\underline{m}_j, M_j],$$
$$a_j \in [0, A_j],$$

that is, a rectangular space. We choose grid points in the a dimension to be polynomial spaced with more points closer to $a = 0$. We choose grids in the positive m dimension to be polynomial spaced between $m = 0$ and $m = M$, with an explicit point at $m = 0$. For the negative m dimension, the grid points are polynomial spaced between $\underline{m}$ and $\underline{m}/2$, and between $\underline{m}/2$ and 0, with more points closer to 0 and $\underline{m}$.

E.3.2. (x_j, a_j) *Space*

The value functions (V_j^A, V_j^N) and the decision rules are defined separately for the adjust and no-adjust cases.

When the agent is adjusting, these are defined over the space of cash on hand conditional on adjusting, x_j^A. This space is discretized as follows. The lowest possible value of x_j^A is

$$\underline{x}_j^A = \underline{m}_j + \min\{y_j - T(y_j)\}$$

and the highest possible value is

$$X_j^A = M_j + \max\{y_j - T(y_j)\}.$$

We choose grids in the positive dimension to be polynomial spaced between 0 and X_j^A, with an explicit point at $x_j^A = 0$. For the negative x_j^A dimension, the grid points are polynomial spaced between $\underline{x}_j^A$ and $\underline{x}_j^A/2$, and between $\underline{x}_j^A/2$ and 0, with more points closer to 0 and $\underline{x}_j^A$.

When the agent is not adjusting, these functions are defined over the space (x_j^N, a_j). We use the same space as defined above for a_j. The x_j^N space is discretized as follows. The lowest and highest possible values of x_j^N are

$$\underline{x}_j^N = \underline{m}_j + \min\{y_j - T(y_j)\},$$
$$X_j^N = M_j + \max\{y_j - T(y_j)\},$$

subject to these not violating the borrowing limit. The grid points are chosen in an analogous manner to the adjust case.

E.3.3. *Grid Sizes*

In the models without borrowing, we use 30 points each in the grids for a_j, m_j, and x_j^N, and 50 points in the grid for x_j^A. In the models with borrowing, we retain the same grid points as for the models without borrowing, but add 16 points in the negative regions for each of m_j, x_j^N, and x_j^A. We use 21 points in the grid for the realization of the permanent shock. Polynomial spaced grids with points concentrated at the lower bound are constructed by taking an equally spaced partition, z, of $[0, 1]$, then constructing a grid for x as $x_L + (x_H - x_L)z^{1/k}$. We use $k = 0.4$.

E.3.4. *Interpolation*

We use linear and bilinear interpolation. When using bilinear interpolation over the (m_{j+1}, a_{j+1}) space, we interpolate along the m_{j+1} dimension and a diagonal that holds total assets, $m_{j+1} + a_{j+1}$, constant. This provides much more accurate interpolations than standard bilinear interpolation since m_{j+1} is the relevant dimension if the agent does not adjust at $j+1$, while $m_{j+1} + a_{j+1}$ is the relevant dimension if the agent does adjust at $j+1$.

E.4. *Computation of Rebate Coefficients*

To compute the rebate coefficients implied by the model, we simulate two consumption paths for each of 200,000 individuals. Thus, the size of the simulated economy in the policy experiments is 400,000, two identical groups of size 200,000 each. In the first path, the timing of the arrival of the information and payment of the rebate check is as described for group A in the text. In the second path, the timing of the arrival of the information and payment of the rebate check is as described for group B in the text. These paths depend on the assumed information structure.

We compute the average rebate coefficient by regressing consumption growth of all individuals (combining both paths) on the amount of rebate received in that period (either \$500 or zero), a full set of quarter dummies, and a quadratic polynomial in age. We use only the quarters in which some individuals receive a check. This approach is equivalent to regression (1) in the main text. To mitigate the effects of outliers, we estimate this regression on a truncated sample of individuals whose individual-specific rebate coefficients are within 2 standard deviations either side of the mean, a procedure that results in dropping approximately the top and bottom 1% of individual-specific rebate coefficients. We compute the individual-specific rebate coefficients as the individual's average consumption growth in the periods when they receive the check, minus their average consumption growth in the periods when they do not receive the check, using only the periods where they receive the check in one of the paths. So, for example, in the baseline informational configuration, we use the average between consumption growth when the individual is in group A at Q2 and consumption growth when it is in group B at Q3, minus the average between consumption growth when it is in group A at Q3 and consumption growth when it is in group B at Q2.

To compute the aggregate consumption response to the policy, we simulate a third counterfactual consumption path for each of the 200,000 individuals in which they never receive a stimulus payment. We compute the aggregate consumption response as the average of the aggregate consumption for groups A and B minus the aggregate consumption along the counterfactual path.

E.5. *Other Computational Details*

Our model is very computationally intensive. However, by working with the first-order conditions directly, rather than using value function iteration, and by parallelizing the computation of decision rules and simulations, we are able to compute the model in a reasonable time on New York University's High Performance Computing Bowery cluster. Using 16 processors, it takes roughly 1–2 hours to solve one parameterization of the model. This involves iterating over the steady state of the model (to calibrate the discount factor, which is computationally equivalent to solving for the interest rate in a general equilibrium economy), iterating over the transition path induced by the policy change (to find the payroll tax that balances the government budget constraint), simulating the economy, and computing rebate coefficients.

The amount of memory (RAM) that is required to store the large number of decision rules—for each quarter along the transition at every quarter of the life-cycle over a very large state space—and the large number of simulations is significant. Our baseline model requires around 50 GB of RAM to run.

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