Econometrica Supplementary Material

SUPPLEMENT TO "STOCHASTIC CHOICE AND CONSIDERATION SETS": SUPPLEMENTAL APPENDICES

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This supplement contains Appendixes C and D.

KEYWORDS: Discrete choice, random utility, logit model, Luce model, consideration sets, bounded rationality, revealed preferences.

APPENDIX C: NECESSITY OF THE AXIOMS

I-ASYMMETRY: $\frac{p(a,A\setminus\{b\})}{p(a,A)} \neq 1 \Rightarrow \frac{p(b,A\setminus\{a\})}{p(b,A)} = 1$.

Let $p_{\succ,\gamma}$ be a random consideration set rule. For all $A \in \mathcal{D}$, all $a, b \in A$ with $b \neq a$:

$$\frac{p_{\succ,\gamma}(a, A\setminus\{b\})}{p_{\succ,\gamma}(a, A)} = \frac{\gamma(a)\displaystyle\prod_{c\in A\setminus\{b\}:c\succ a}(1-\gamma(c))}{\gamma(a)\displaystyle\prod_{c\in A:c\succ a}(1-\gamma(c))} = \begin{cases} \dfrac{1}{1-\gamma(b)}, & \text{if } b \succ a, \\ 1, & \text{if } a \succ b, \end{cases}$$

so that $\frac{p(a,A\setminus\{b\})}{p(a,A)} = 1 \Leftrightarrow a \succ b$. Therefore, if $\frac{p(a,A\setminus\{b\})}{p(a,A)} \neq 1$, we have $b \succ a$ and, thus, $\frac{p(b,A\setminus\{a\})}{p(b,A)} = 1$.

I-INDEPENDENCE: $\frac{p(a,A\setminus\{b\})}{p(a,A)} = \frac{p(a,B\setminus\{b\})}{p(a,B)}$ and $\frac{p(a^*,A\setminus\{b\})}{p(a^*,A)} = \frac{p(a^*,B\setminus\{b\})}{p(a^*,B)}$.

Let $p_{\succ,\gamma}$ be a random consideration set rule. Then, for all $A, B \in \mathcal{D}$ and for all $a, b \in A \cap B$, $a \neq b$:

$$\begin{aligned} \frac{p_{\succ,\gamma}(a, A\setminus\{b\})}{p_{\succ,\gamma}(a, A)} &= \frac{\gamma(a)\displaystyle\prod_{c\in A\setminus\{b\}:c\succ a}(1-\gamma(c))}{\gamma(a)\displaystyle\prod_{c\in A:c\succ a}(1-\gamma(c))} \\ &= \begin{cases} \dfrac{1}{1-\gamma(b)} & \text{if } b \succ a \\ 1 & \text{if } a \succ b \end{cases} \\ &= \frac{\gamma(a)\displaystyle\prod_{c\in B\setminus\{b\}:c\succ a}(1-\gamma(c))}{\gamma(a)\displaystyle\prod_{c\in B:c\succ a}(1-\gamma(c))} = \frac{p_{\succ,\gamma}(a, B\setminus\{b\})}{p_{\succ,\gamma}(a, B)}, \end{aligned}$$

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as desired. Similarly, we have

$$\frac{p_{\succ,\gamma}(a^*, A\setminus\{b\})}{p_{\succ,\gamma}(a^*, A)} = \frac{\prod_{c\in A\setminus\{b\}}(1-\gamma(c))}{\prod_{c\in A}(1-\gamma(c))} = \frac{1}{1-\gamma(b)}$$

$$= \frac{\prod_{c\in B\setminus\{b\}}(1-\gamma(c))}{\prod_{c\in B}(1-\gamma(c))} = \frac{p_{\succ,\gamma}(a, B\setminus\{b\})}{p_{\succ,\gamma}(a, B)}.$$

In Manzini and Mariotti (2014), we reported that the random consideration set rule satisfies i-Neutrality; below, we show that i-Neutrality is indeed necessary:

I-NEUTRALITY: $\frac{p(a,A\setminus\{c\})}{p(a,A)}, \frac{p(b,A\setminus\{c\})}{p(b,A)} > 1 \Rightarrow \frac{p(a,A\setminus\{c\})}{p(a,A)} = \frac{p(b,A\setminus\{c\})}{p(b,A)}$ for all $A \in \mathcal{D}$, $a \in A^*$, and $b, c \in A$.

Let $p_{\succ,\gamma}$ be a random consideration set rule. For all $A \in \mathcal{D}$, $a \in A^*$, and $b, c \in A$ with $c \neq a$:

$$\frac{p_{\succ,\gamma}(a, A\setminus\{c\})}{p_{\succ,\gamma}(a, A)} = \frac{\gamma(a)\prod_{d\in A\setminus\{c\}:d\succ a}(1-\gamma(d))}{\gamma(a)\prod_{d\in A:d\succ a}(1-\gamma(d))} = \begin{cases} \frac{1}{1-\gamma(c)}, & \text{if } c \succ a, \\ 1, & \text{if } a \succ c, \end{cases}$$

so that $\frac{p(a,A\setminus\{c\})}{p(a,A)} > 1 \Leftrightarrow \frac{p(a,A\setminus\{c\})}{p(a,A)} = \frac{1}{1-\gamma(c)}$ for all $a \in A$ such that $a \neq a^*$, while if $a = a^*$, then

$$a^* A b = \frac{p_{\succ,\gamma}(a^*, A\setminus\{c\})}{p_{\succ,\gamma}(a^*, A)} = \frac{\prod_{d\in A\setminus\{c\}:d\succ a}(1-\gamma(d))}{\prod_{d\in A:d\succ a}(1-\gamma(d))} = \frac{1}{1-\gamma(c)}.$$

Therefore, $\frac{p(a,A\setminus\{c\})}{p(a,A)} > 1$ and $\frac{p(b,A\setminus\{c\})}{p(b,A)} > 1$ imply $\frac{p(a,A\setminus\{c\})}{p(a,A)} = \frac{1}{1-\gamma(c)} = \frac{p(b,A\setminus\{c\})}{p(b,A)}$.

APPENDIX D: INDEPENDENCE OF THE AXIOMS

First, we recall the axioms; next, we provide examples of random choice rules that fail to satisfy only one of the axioms, and show that they are not random consideration set rules.

I-ASYMMETRY: $\frac{p(a,A\setminus\{b\})}{p(a,A)} \neq 1 \Rightarrow \frac{p(b,A\setminus\{a\})}{p(b,A)} = 1$.

I-INDEPENDENCE: $\frac{p(a,A\setminus\{b\})}{p(a,A)} = \frac{p(a,B\setminus\{b\})}{p(a,B)}$ and $\frac{p(a^*,A\setminus\{b\})}{p(a^*,A)} = \frac{p(a^*,B\setminus\{b\})}{p(a^*,B)}$.

FAILS ONLY I-ASYMMETRY: Let $X = \{a, b\}$, and assume choice probabilities are as in Table D.I, where $x, y > 0$, $x + y < 1$, and $\alpha \in (\frac{1}{1-x}, \frac{x+y}{y})$. We have that $\frac{x+y}{y} < \frac{1}{y}$, ensuring that $p(a^*, \{b\}) = 1 - \alpha y > 0$, while the upper bound on α also ensures that $p(a^*, \{a\}) < 1$. This random choice rule fails i-Asymmetry, since $\frac{p(a,S_1\setminus\{b\})}{p(a,S_1)} = \frac{x+(1-\alpha)y}{x(1-\alpha y)} < 1$ (since $\alpha > \frac{1}{1-x}$) and $\frac{p(b,S_1\setminus\{a\})}{p(b,S_1)} = \alpha > 1$. i-Independence holds trivially for a and b. For the default alternative:

$$\frac{p(a^*, S_1 \setminus \{a\})}{p(a^*, S_1)} = \frac{p(a^*, \{b\})}{p(a^*, \{a, b\})}$$

$$= \frac{1 - \alpha y}{1 - x - y} = \frac{p(a^*, \emptyset)}{p(a^*, \{a\})} = \frac{p(a^*, S_2 \setminus \{a\})}{p(a^*, S_2)},$$

$$\frac{p(a^*, S_1 \setminus \{b\})}{p(a^*, S_1)} = \frac{p(a^*, \{a\})}{p(a^*, \{a, b\})} = \frac{\frac{1 - x - y}{1 - \alpha y}}{1 - x - y} = \frac{1}{1 - \alpha y}$$

$$= \frac{p(a^*, \emptyset)}{p(a^*, \{b\})} = \frac{p(a^*, S_3 \setminus \{b\})}{p(a^*, S_3)},$$

so that i-Independence holds. Finally, we show directly that there is no $p_{\succ,\gamma}$ that returns the above probabilities. First of all, by the arguments in the proof of Theorem 1, it must be that $\gamma(a) = \frac{x+(1-\alpha)y}{1-\alpha y}$ and $\gamma(b) = \alpha y$. If $a \succ b$, then

$$p_{\succ,\gamma}\big(b, \{a, b\}\big) = \gamma(b)\big(1 - \gamma(a)\big) = \frac{(1 - x - y)\alpha y}{1 - \alpha y} \neq y = p\big(b, \{a, b\}\big),$$

TABLE D.I

A RANDOM CHOICE RULE THAT FAILS I-ASYMMETRY

		$p(a, S_i)$	$p(b, S_i)$	$p(a^*, S_i)$
S_1	$\{a, b\}$	x	y	$1 - x - y$
S_2	$\{a\}$	$\frac{x+(1-\alpha)y}{1-\alpha y}$	–	$1 - \frac{x+(1-\alpha)y}{1-\alpha y} = \frac{1-x-y}{1-\alpha y}$
S_3	$\{b\}$	–	αy	$1 - \alpha y$
	$\emptyset$	–	–	1

where the inequality follows from our assumption that $\alpha > \frac{1}{1-x}$. Next, if $b \succ a$, then

$$p_{\succ,\gamma}\big(a, \{a, b\}\big) = \gamma(a)\big(1-\gamma(b)\big) = \frac{x+(1-\alpha)y}{1-\alpha y}(1-\alpha y)$$
$$= x + (1-\alpha)y \neq x = p\big(a, \{a, b\}\big),$$

where the inequality follows from $\alpha > \frac{1}{1-x} > 1$ and $y > 0$.

FAILS ONLY I-INDEPENDENCE: Let $X = \{a, b\}$, and assume choice probabilities are as in Table D.II, where $x, y > 0$, $x + y < 1$, and $\alpha \in (1, \frac{1}{y})$ with $\alpha \neq \frac{1}{1-x}$. While i-Asymmetry holds (since $\frac{p(a,S_1\setminus\{b\})}{p(a,S_1)} = 1$ and $\frac{p(b,S_1\setminus\{a\})}{p(b,S_1)} = \alpha \neq 1$), i-Independence does not, since

$$\frac{p(a^*, S_1 \setminus \{a\})}{p(a^*, S_1)} = \frac{p(a^*, \{b\})}{p(a^*, \{a, b\})} = \frac{1-\alpha y}{1-x-y}$$
$$\neq \frac{1}{1-x} = \frac{p(a^*, \emptyset)}{p(a^*, \{a\})} = \frac{p(a^*, S_2 \setminus \{a\})}{p(a^*, S_2)},$$

where the inequality follows from our assumption $\alpha \neq \frac{1}{1-x}$.

To see that there is no $p_{\succ,\gamma}$ that returns the above probabilities, observe that, by the usual arguments, it must be that $\gamma(a) = x$ and $\gamma(b) = \alpha y$. Now if $a \succ b$, we have

$$p_{\succ,\gamma}\big(b, \{a, b\}\big) = \gamma(b)\big(1-\gamma(a)\big) = \alpha y(1-x) \neq y = p\big(b, \{a, b\}\big),$$

where the inequality follows from $\alpha \neq \frac{1}{1-x}$. If instead $b \succ a$, then

$$p_{\succ,\gamma}\big(a, \{a, b\}\big) = \gamma(a)\big(1-\gamma(b)\big) = x(1-\alpha y) \neq x = p\big(a, \{a, b\}\big),$$

where the inequality follows from $\alpha, y > 0$.

TABLE D.II

A RANDOM CHOICE RULE THAT FAILS I-INDEPENDENCE

		$p(a, S_i)$	$p(b, S_i)$	$p(a^*, S_i)$
S_1	$\{a, b\}$	x	y	$1-x-y$
S_2	$\{a\}$	x	–	$1-x$
S_3	$\{b\}$	–	αy	$1-\alpha y$
	$\emptyset$	–	–	1

REFERENCE

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