Econometrica Supplementary Material

SUPPLEMENT TO "PARETO-IMPROVING TAX REFORMS AND THE EARNED INCOME TAX CREDIT"

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APPENDIX A: PROOFS

A.1. *Proof of Theorem 1 and Proposition 1*

TO PROVE Theorem 1, we proceed in two steps. We first show that the non-existence of a Pareto-improving one-bracket reform implies that $\mathcal{R}(y) = R^s_{\tau\ell}(0, 0, y) \in [0, 1]$ for all y. We then show that the non-existence of a Pareto-improving two-bracket reform implies that $y \mapsto \mathcal{R}(y)$ is non-increasing.

Reforms With One Bracket. Adapting inequality (4) to the case of a one-bracket reform, we find that a small reform with $\tau > 0$ that increases marginal tax rates from the status quo is Pareto-improving if, for some $\ell > 0$,

$$R^s_\tau(0, \ell, \hat{y}) - \ell > 0, \tag{A.1}$$

that is, if marginal revenue gains are so large that even those agents are made better off whose tax bill increases by the maximal amount of $\max_y h^s(y) = \ell$. For a one-bracket reform with $\tau < 0$, we have to compare -1 times the derivative $R^s_\tau(0, \ell, \hat{y})$ with $\max_y[-h^s(y)] = 0$. Consequently, a small one-bracket reform that reduces marginal tax rates is Pareto-improving if

$$R^s_\tau(0, \ell, \hat{y}) < 0, \tag{A.2}$$

so that a tax cut leads to larger tax revenues, a logic familiar from analyses of the Laffer curve.

Below, we exploit the fact that a one-bracket reform on a bracket of length zero does not affect tax revenue, that is, $R^s_\tau(0, 0, \hat{y}) = 0$ for any $\hat{y}$. To see this, recall that the new tax schedule satisfies $T_1(y) = T_0(y)$ for any $y \le \hat{y}$, and $T_1(y) = T_0(y) + \tau\ell$ for any $y \ge \hat{y} + \ell$. For a one-bracket reform on a bracket of length $\ell = 0$, the new tax schedule is thus identical to the status quo tax schedule, $T_1(y) = T_0(y)$ for all y, independent of the step size τ. Hence, a variation in τ affects neither the budget set C_1 nor individual behavior y^*, nor tax revenue $R^s(\tau, 0, \hat{y})$.

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LEMMA A.1: (i) *If* $R^s_{\tau\ell}(0,0,\hat{y})-1>0$ *for some* $\hat{y}\in\mathcal{Y}$, *there exists a Pareto-improving one-bracket reform with* $\tau>0$ *and* $\ell>0$.
(ii) *If* $R^s_{\tau\ell}(0,0,\hat{y})<0$ *for some* $\hat{y}\in\mathcal{Y}$, *there exists a Pareto-improving one-bracket reform with* $\tau<0$ *and* $\ell>0$.
(iii) *If there is no Pareto-improving reform, then* $R^s_{\tau\ell}(0,0,\hat{y})\in[0,1]$ *for all* $\hat{y}$.

PROOF: As explained above, we have $R^s_\tau(0,0,\hat{y})=0$. If $R^s_{\tau\ell}(0,0,\hat{y})>1$, this implies that $R^s_\tau(0,\ell,\hat{y})-\ell$ turns positive if, starting from $\ell=0$, the length of the interval is slightly increased. This proves (i). Analogously, if $R^s_{\tau\ell}(0,0,\hat{y})<0$, this implies that $R^s_\tau(0,\ell,\hat{y})$ turns negative if, starting from $\ell=0$, the length of the interval is slightly increased. This proves (ii). Thus, necessary conditions for the non-existence of a Pareto-improving one-bracket reform are $R^s_{\tau\ell}(0,0,\hat{y})\le 1$ and $R^s_{\tau\ell}(0,0,\hat{y})\ge 0$. This proves (iii). *Q.E.D.*

Reforms With Two Brackets. Lemma A.1 above gives sufficient conditions for the existence of Pareto-improving reforms with a single bracket. Proposition 1 gives the analogue for the case of two-bracket reforms. In particular, it shows that, if $y\mapsto\mathcal{R}(y)=R^s_{\tau\ell}(0,0,y)$ is increasing, the combination of two reforms—each of which would not be Pareto-improving on a stand-alone basis—yields a Pareto improvement. For this purpose, we denote by $R(\tau,h_2)$ the change in tax revenue due to a joint reform with two brackets, where $h_2=h^s_1+h^s_2$ is composed of two single-bracket reforms.

Proof of Proposition 1. Fix two income levels y_1 and y_2 such that $y_2>y_1$ and $R^s_{\tau\ell}(0,0,y_2)>R^s_{\tau\ell}(0,0,y_1)$. We now construct a Pareto-improving two-bracket reform with the parameters $\{(y_1,\tau_1,\ell_1,y_2,\tau_2,\ell_2,\tau,\ell)\}$. In particular, let $\tau_1<0$, $\tau_2>0$, and $\tau_1\ell_1+\tau_2\ell_2=0>\tau_1\ell_1$. This implies that $\max_y h_2(y)=0$. By the linearity of the Gateaux differential,[1] we moreover find that

$$\begin{aligned}R_\tau(0,h_2)&=\tau_1R^s_\tau(0,\ell\ell_1,y_1)+\tau_2R^s_\tau(0,\ell\ell_2,y_2),\quad\text{and}\\R_{\tau\ell}(0,h_2)&=\tau_1\ell_1R^s_{\tau\ell}(0,0,y_1)+\tau_2\ell_2R^s_{\tau\ell}(0,0,y_2)\\&=\tau_2\ell_2\big[R^s_{\tau\ell}(0,0,y_2)-R^s_{\tau\ell}(0,0,y_1)\big]>0.\end{aligned}$$

Hence, there exists $\hat{\ell}>0$ such that $R_\tau(0,h_2)-\max_y h(y)>0$ for all $\ell\in(0,\hat{\ell})$. Finally, by Equation (4), this implies that, for a reform as constructed above with $\ell\in(0,\hat{\ell})$, a small increase in τ is Pareto-improving.

Suppose that $R^s_{\tau\ell}(0,0,y_1)$ and $R^s_{\tau\ell}(0,0,y_2)$ are between 0 and 1. Then, there is no Pareto-improving one-bracket reform for incomes close to y_1 or close to y_2. If $R^s_{\tau\ell}(0,0,y_1)<R^s_{\tau\ell}(0,0,y_2)$, however, there is still scope for a Pareto improvement that involves two brackets.

[1]Gateaux differentials are not linear in general. To clarify the conditions under which they are, for $0<\bar{y}$, $a<\infty$, let $\tau\in[-a,a]$ and $h\in\mathcal{H}:=(\mathcal{C}[0,\bar{y}],\|\cdot\|_\infty)$, where $\|\cdot\|_\infty$ denotes the sup norm. We define the operator

$$\bar{R}:\mathcal{H}\to\mathcal{K}:h\mapsto\bar{R}(\tau,h),$$

where $\mathcal{K}:=(\mathcal{C}_b([-a,a]),\|\cdot\|_\infty)$ and $\mathcal{C}_b([-a,a])$ denotes the set of bounded continuous real functions defined on $[-a,a]$. Note that $\mathcal{H}$ and $\mathcal{K}$ are Banach spaces. In this setting, the Gateaux differential of $R_\tau(\tau,\cdot)$ is linear (Zorn (1945, Theorem 2.3)).

A.2. *Proof of Theorem 2*

A reform with an arbitrary number m of brackets can be characterized as a collection

$$\{(y_k, \tau\tau_k, \ell\ell_k)\}_{k=1}^m$$

of one-bracket reforms, where the marginal tax in the kth bracket is changed by $\tau\tau_k$ and the length of the kth bracket is given by $\ell\ell_k$. As before, the parameters (τ, ℓ) determine the size of the reform and the overall revenue is denoted by $R(\tau, h_m)$. The following lemma states sufficient conditions for the existence of a Pareto-improving reform with m brackets.

LEMMA A.2: *Consider a collection* $\{(y_k, \tau\tau_k, \ell\ell_k)\}_{k=1}^m$ *of simple reforms. Let* $\tau_0\ell_0 = 0$. *There is a reform* (τ, h_m) *with* $\tau > 0$ *and* $\ell > 0$ *that is Pareto-improving if*

$$\sum_{k=1}^m \tau_k\ell_k\mathcal{R}(y_k) - \max_{j\in\{0,1,\dots,m\}} \sum_{k=0}^j \tau_k\ell_k > 0.$$

PROOF: The linearity of the Gateaux differential implies that

$$R_\tau(0, h_m) = \sum_{k=1}^m \tau_k R_\tau^s(0, \ell\ell_k, y_k), \quad \text{and}$$

$$R_{\tau\ell}(0, h_m) = \sum_{k=1}^m \tau_k\ell_k\mathcal{R}(y_k).$$

Moreover,

$$\max_y h_m(y) = \ell \max_{j\in\{0,1,\dots,m\}} \sum_{k=0}^j \tau_k\ell_k.$$

As shown above, $R_\tau(0, h_m)$ equals zero for a reform with $\ell = 0$ such that all brackets have length zero. Hence, if the condition in the lemma is satisfied, there exists $\hat{\ell} > 0$ such that $R_\tau(0, h_m) - \max_y h_m(y) > 0$ for all $\ell \in (0, \hat{\ell})$. By Equation (4), this implies that, for such an m-bracket reform (τ, h_m) with $\ell \in (0, \hat{\ell})$, a small increase in the step size τ is Pareto-improving. *Q.E.D.*

Lemma A.2 states sufficient conditions for the existence of Pareto-improving reforms. If we limit attention to small reforms, these conditions are also necessary, that is, if they do not hold, there is no small reform that is Pareto-improving. The following lemma shows that, if the conditions in Theorem 2 hold, the condition in Lemma A.2 is violated for any collection of m single bracket reforms. Consequently, there is no small Pareto-improving m-bracket reform.

LEMMA A.3: *Suppose that the function* $y \mapsto \mathcal{R}(y)$ *is bounded from below by* 0, *bounded from above by* 1, *and non-increasing. Let* $\tau_0\ell_0 = 0$. *Then,*

$$\sum_{k=1}^m \tau_k\ell_k\mathcal{R}(y_k) - \max_{j\in\{0,1,\dots,m\}} \sum_{k=0}^j \tau_k\ell_k \le 0 \tag{A.3}$$

for any collection $\{(y_k, \tau_j\tau_k, \ell_j\ell_k)\}_{k=1}^m$, *and for any* $m \ge 1$.

PROOF: Let j^* be a bracket in which the function h_m achieves a maximum, $j^* := \text{argmax}_j \sum_{k=0}^{j} \tau_k \ell_k$. Note that this implies that $\sum_{k=z}^{j^*} \tau_k \ell_k \geq 0$ for any $z \in \{0, \ldots, j^*\}$ and $\sum_{k=j^*+1}^{z} \tau_k \ell_k \leq 0$ for any $z \in \{j^*+1, \ldots, m\}$; otherwise, j^* would not be a maximizer.

Step 1. We verify the following claim: Suppose that $j^* > 0$ and that

$$\sum_{k=z}^{j^*} \tau_k \ell_k \mathcal{R}(y_k) \leq \mathcal{R}(y_z) \sum_{k=z}^{j^*} \tau_k \ell_k \tag{A.4}$$

holds for some $z \in \{1, \ldots, j^*\}$. Then, if $z > 1$, we also have

$$\begin{aligned} \sum_{k=z-1}^{j^*} \tau_k \ell_k \mathcal{R}(y_k) &= \tau_{z-1} l_{z-1} \mathcal{R}(y_{z-1}) + \sum_{k=z}^{j^*} \tau_k \ell_k \mathcal{R}(y_k) \\ &\leq \tau_{z-1} l_{z-1} \mathcal{R}(y_{z-1}) + \underbrace{\mathcal{R}(y_z)}_{\leq \mathcal{R}(y_{z-1})} \underbrace{\sum_{k=z}^{j^*} \tau_k \ell_k}_{\geq 0} \\ &\leq \mathcal{R}(y_{z-1}) \sum_{k=z-1}^{j^*} \tau_k \ell_k. \end{aligned}$$

Condition (A.4) is obviously satisfied for $z = j^*$. Hence, a repeated application of the preceding argument yields

$$\sum_{k=1}^{j^*} \tau_k \ell_k \mathcal{R}(y_k) \leq \underbrace{\mathcal{R}(y_1)}_{\in[0,1]} \underbrace{\sum_{k=1}^{j^*} \tau_k \ell_k}_{\geq 0} \leq \sum_{k=1}^{j^*} \tau_k \ell_k = \sum_{k=0}^{j^*} \tau_k \ell_k. \tag{A.5}$$

Step 2. An analogous argument implies that

$$\sum_{k=j^*+1}^{m} \tau_k \ell_k \mathcal{R}(y_k) \leq \mathcal{R}(y_m) \underbrace{\sum_{k=j^*+1}^{m} \tau_k \ell_k}_{\leq 0} \leq 0. \tag{A.6}$$

Step 3. Together, (A.5) and (A.6) imply that, if $j^* \in \{1, \ldots, m-1\}$,

$$R_{\tau\ell}(0, h_m) = \sum_{k=1}^{j^*} \tau_k \ell_k \mathcal{R}(y_k) + \sum_{k=j^*+1}^{m} \tau_k \ell_k \mathcal{R}(y_k) \leq \sum_{k=0}^{j^*} \tau_k \ell_k, \tag{A.7}$$

which proves (A.3). Note that the cases $j^* = 0$ and $j^* = m$ are also covered. With $j^* = 0$, $\sum_{k=1}^{j^*} \tau_k \ell_k \mathcal{R}(y_k)$ does not enter the chain of inequalities and (A.6) directly implies (A.3). With $j^* = m$, $\sum_{k=j^*+1}^{m} \tau_k \ell_k \mathcal{R}(y_k)$ does not enter and (A.5) directly implies (A.3). *Q.E.D.*

While the previous arguments only refer to reforms with small brackets, $\ell \to 0$, they can easily be adjusted to cover reforms with large brackets as well. In particular, we note that the revenue implications of a reform with a bracket of length $\ell\ell_k > 0$, starting at income y_k, can be approximated arbitrarily well by the revenue implications of a reform with m evenly spaced small brackets in the interval $(y_k, y_k + \ell\ell_k)$, if m is chosen sufficiently large. The following proof of Corollary 1 makes this approximation argument explicit.

A.3. *Proof of Corollary 1 and Proposition 3*

Take any continuous reform direction h on $[0, \bar{y}]$. We approximate h with a piecewise linear reform direction h_m that involves m one-bracket reforms $(\tau_k, h_k^s)_{k=1}^m$, so that $h_m(y) = \sum_{k=1}^m \tau_k h_k^s(y)$. Throughout, we let $\ell = 1$ and divide the domain $[0, \bar{y}]$ into m brackets of equal length $\ell_k = \frac{1}{m}\bar{y}$, starting at incomes $y_1 = 0$, $y_2 = \frac{1}{m}\bar{y}, \dots$, and $y_m = \frac{m-1}{m}\bar{y}$. Thus, we have m adjacent brackets—a special case of our general formalism, which also allows for gaps between the brackets where marginal tax rates change. For any k, we then let

$$\tau_k = \frac{h(y_{k+1}) - h(y_k)}{\ell_k}, \quad \text{where we set } y_{m+1} = \bar{y}.$$

This yields an approximation of h by a piecewise linear function. The construction is illustrated in Figure A.1. By choosing m sufficiently large, the piecewise linear function h_m approximates h in the sense that, for any $\varepsilon > 0$, there exists $\hat{m}(\varepsilon)$ so that for any $m > \hat{m}(\varepsilon)$,

$$\sup_{y \in \mathcal{Y}} \big| h(y) - h_m(y) \big| < \varepsilon.$$

For later reference, we note that this implies in particular that, for any y^* that maximizes $h(y)$ over $\mathcal{Y}$, we have

$$h(y^*) - h_m(y^*) < \varepsilon. \tag{A.8}$$

The Gateaux differential is linear in the reform direction h and hence continuous. We therefore have

$$\lim_{m \to \infty} R_\tau(0, h_m) = R_\tau(0, h), \tag{A.9}$$

that is, the Gateaux differential for direction h_m converges to the Gateaux differential for direction h. We now provide a characterization of $\lim_{m\to\infty} R_\tau(0, h_m)$. By the linearity of the Gateaux differential, we have

$$R_\tau(0, h_m) = \sum_{k=1}^m \tau_k R_\tau(0, h_k^s) = \sum_{k=1}^m \tau_k R_\tau^s(0, \ell_k, y_k).$$

For m large and $\ell_k = \frac{\bar{y}}{m}$ close to zero, a first-order Taylor approximation moreover gives

$$R_\tau^s(0, \ell_k, y_k) \approx \ell_k R_{\tau\ell}^s(0, 0, y_k) = \ell_k \mathcal{R}(y_k).$$

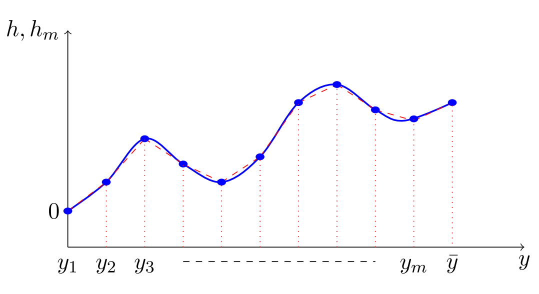


FIGURE A.1.—Approximation of function h (solid) by piecewise linear function h_m (dashed).

The approximation is perfect in the limit case $m \to \infty$ or, equivalently, $\ell_k = \frac{\bar{y}}{m} \to 0$. Therefore,

$$\begin{aligned}\lim_{m\to\infty} R_\tau(0, h_m) &= \lim_{m\to\infty} \sum_{k=1}^{m} \tau_k \ell_k \mathcal{R}(y_k) \\ &= \lim_{m\to\infty} \sum_{k=1}^{m} [y_{k+1} - y_k]\tau_k \mathcal{R}(y_k) \\ &= \int_{y\in\mathcal{Y}} h'(y)\mathcal{R}(y)\,dy, \end{aligned} \tag{A.10}$$

where the last term is the Riemann integral that gives the marginal revenue effect of a reform in direction h, and $h' : y \mapsto h'(y)$ is the change of the marginal tax rate at income y due to a unit increase in τ. To see this, note first that the term in the second line is the limit of a Riemann sum, the latter involving the step function $y \mapsto \tau_k \mathcal{R}(y_k)$ for $y \in [y_k, y_k + \ell_k]$. Second, note that both h and h_m are continuous functions on a compact interval. Hence, they have a bounded variation and are therefore differentiable almost everywhere. Moreover, we have $\lim_{m\to\infty} \ell_k = 0$ and therefore, for any y in the interior of bracket k,

$$\tau_k = h'_m(y) \longrightarrow_{m\to\infty} h'(y).$$

This completes the derivation of Equation (7) in Proposition 3.

We now show that the conclusion of Theorem 2 extends to all continuous reform directions. We first note that (A.9) implies that for any $\varepsilon > 0$, there is an $\tilde{m}(\varepsilon) \in \mathbb{R}$ such that, for $m > \tilde{m}(\varepsilon)$,

$$R_\tau(0, h) - R_\tau(0, h_m) < \varepsilon. \tag{A.11}$$

To complete the proof of Corollary 1, we proceed by contradiction. Suppose that the conditions of Theorem 2 apply and that there is a continuous reform direction h that is Pareto-improving, that is, that satisfies

$$R_\tau(0, h) - \max_{y\in\mathcal{Y}} h(y) = 2\varepsilon,$$

for some $\varepsilon > 0$. Then, by (A.8) and (A.11), for $m > \max\{\hat{m}(\varepsilon), \tilde{m}(\varepsilon)\}$, there is also an m-bracket reform such that $R_\tau(0, h_m) - \max_{y\in\mathcal{Y}} h_m(y) > 0$, that is, that is Pareto-improving. But this is impossible by Theorem 2. The assumption that there is a Pareto-improving direction h in the class of continuous functions on $[0, \bar{y}]$ has therefore led to a contradiction and must be false.

A.4. *Proof of Proposition 2*

The equivalence of statements A and C in Proposition 2 follows from Theorems 1 and 2 and from Corollary 1. To complete the proof, we establish the equivalence of A and B.

Preliminaries. First, as a preliminary step, we define a *solution to the inverse tax problem*. The inverse tax problem is based on the assumption that there is a social welfare function, $\mathcal{W}(\tau, h) := \mathbf{E}[G(v(\tau, h, \theta), \theta)]$, where $G : \mathbb{R} \times \Theta \to \mathbb{R}$ is increasing in its first argument. Upon using (3), the Gateaux differential of social welfare in direction h can be written as

$$\mathcal{W}_\tau(0,h)=\bar{g}\left\{R_\tau(0,h)-\frac{1}{\bar{g}}\mathbf{E}_y\big[g(y)h(y)\big]\right\}, \tag{A.12}$$

where the operator $\mathbf{E}_y$ indicates the computation of a population average using the status quo income distribution, which we take to be represented by the cdf F_y, function $y\mapsto g(y)$ is defined by $g(y):=\mathbf{E}[G'(v(0,h,\theta),\theta)u_c(\cdot)\mid y_0(\theta)=y]$, and $\bar{g}$ is the population average $\bar{g}:=\mathbf{E}_y[g(y)]$.

The interpretation is that $g(y)$ gives the marginal social benefit from increasing the consumption of people with earnings level y, whereas $\bar{g}$ gives the marginal social benefit from increasing everyone's consumption. The expression $\mathbf{E}_y[g(y)h(y)]$ interacts the marginal benefits with the way in which people's tax burdens change due to the reform. We say that the function $g: y\mapsto g(y)$ is a *solution to the inverse tax problem* if $\mathcal{W}_\tau(0,h)=0$, for any reform direction h.

Equivalence of A and B. We show the equivalence of statements A and B by considering three types of reforms: one-bracket reforms with a tax cut or tax hike, and two-bracket tax cuts. First, consider a small single-bracket reform with a bracket of length ℓ, starting at income y_1. To capture the welfare implications of this single-bracket reform, we write $\mathcal{W}^s(\tau,\ell,y_1)$ rather than $\mathcal{W}(\tau,h^s)$. Using this notation, the welfare implications of a single-bracket reform with τ and ℓ close to zero are given by the cross derivative

$$\mathcal{W}^s_{\tau\ell}(0,0,y_1)=\bar{g}\left\{\mathcal{R}(y_1)-\frac{1}{\bar{g}}\big[1-F_y(y_1)\big]\mathbf{E}_y\big[g(y)\mid y>y_1\big]\right\}.$$

If the first-order condition for welfare maximization is satisfied, $\mathcal{W}^s_{\tau\ell}(0,0,y_1)=0$, then we have

$$\mathbf{E}_y\big[g(y)\mid y>y_1\big]=\bar{g}\frac{\mathcal{R}(y_1)}{1-F_y(y_1)}.$$

Hence, the implicit welfare weights of people with incomes above y_1 are negative on average if and only if the revenue function is negative at income y_1. In this case, a small one-bracket tax cut for incomes close to y_1 is Pareto-improving by Lemma A.1. Note that the formula above generalizes Equation (7) in Lorenz and Sachs (2016), according to which a negative average welfare weight above some income implies an inefficiency in the tax-transfer system.

Analogously, for a small one-bracket tax increase at income level y_1, the first-order condition can be shown to imply that

$$\mathbf{E}_y\big[g(y)\mid y\le y_1\big]=\bar{g}\frac{1-\mathcal{R}(y_1)}{F_y(y_1)},$$

where we use that $\bar{g}=F_y(y_1)\mathbf{E}_y[g(y)\mid y\le y_1]+[1-F_y(y_1)]\mathbf{E}_y[g(y)\mid y>y_1]$. Hence, the implicit welfare weights of people with incomes below y_1 are negative on average if and only if the revenue function attains a value above 1 at income y_1. In this case, a small one-bracket tax increase for incomes close to y_1 is Pareto-improving by Lemma A.1.

Finally, consider a small two-bracket tax cut between incomes y_1 and $y_2>y_1$. For τ and ℓ close to zero, the welfare implications of such a two-bracket reform h_2 with $\tau_1<0$,

$\tau_2 > 0$, and $\tau_1 \ell_1 + \tau_2 \ell_2 = 0$ are given by $\mathcal{W}_{\tau\ell}(0, h_2)$, where

$$\begin{aligned}
\mathcal{W}_{\tau\ell}(0, h_2) &= \bar{g}\Big\{\tau_1 \ell_1 \mathcal{R}(y_1) + \tau_2 \ell_2 \mathcal{R}(y_2) \\
&\quad - \frac{1}{\bar{g}} \tau_1 \ell_1 \big[1 - F_y(y_1)\big] \mathbf{E}_y\big[g(y) \mid y > y_1\big] \\
&\quad - \frac{1}{\bar{g}} \tau_2 \ell_2 \big[1 - F_y(y_2)\big] \mathbf{E}_y\big[g(y) \mid y < y_2\big]\Big\} \\
&= \bar{g} \tau_2 \ell_2 \Big\{\mathcal{R}(y_2) - \mathcal{R}(y_1) \\
&\quad + \frac{1}{\bar{g}} \big[F_y(y_2) - F_y(y_1)\big] \mathbf{E}_y\big[g(y) \mid y \in (y_1, y_2)\big]\Big\}.
\end{aligned}$$

If such a reform satisfies the first-order condition, $\mathcal{W}_{\tau\ell}(0, h_2) = 0$, then the solution to the inverse tax problem is defined by

$$\mathbf{E}_y\big[g(y) \mid y \in (y_1, y_2)\big] = \bar{g} \frac{\mathcal{R}(y_1) - \mathcal{R}(y_2)}{F_y(y_2) - F_y(y_1)}.$$

Hence, the implicit welfare weights of people with incomes between y_1 and y_2 are negative on average if and only if the revenue function $\mathcal{R} : y \mapsto \mathcal{R}(y)$ is increasing so that $\mathcal{R}(y_2) > \mathcal{R}(y_1)$. By Proposition 1, this implies that a small two-bracket tax cut between incomes y_1 and $y_2 > y_1$ is Pareto-improving.

To sum up, if and only if one of the conditions listed in statement A is violated, the implicit welfare weights of people in some part of the income distribution are negative. If the average implicit weights above and below each income level, and in each subset of $[0, \bar{y}]$, are positive instead, then function $g(y)$ is bounded from below by 0.

Finally, note that, if the revenue function is continuously differentiable, the solution to the inverse tax problem can be characterized point-wise, with $g(y) = -\bar{g}\mathcal{R}'(y)/f_y(y)$ at each income level in the support of the pdf of the income distribution f_y. Differentiability of $\mathcal{R}$ cannot be taken for granted, however. For instance, our analysis of the EITC below gives rise to discontinuous revenue functions. Our formal arguments above accommodate this possibility.

Equivalence of B and C. By Theorem 1, a Pareto-improving reform exists if one of the conditions in statement A is violated ($C \Rightarrow A$). By Theorem 2 and Corollary 1, there is no Pareto-improving reform direction if these conditions are satisfied ($A \Rightarrow C$). In combination with the proof given above, this implies that B and C are equivalent as well: Positive implicit welfare weights are both necessary and sufficient for the non-existence of Pareto-improving reform directions.

APPENDIX B: THE OPTIMAL REFORM DIRECTION

Fix a status quo tax-transfer system (T_0, c_0) and a corresponding revenue function $\mathcal{R}(y)$. By Proposition 3, a reform direction h is Pareto-improving if it generates a strictly positive *free lunch*

$$\Pi(h) := \int_0^{\bar{y}} h'(y)\mathcal{R}(y)\,dy - \max_{y\in y_0(\Theta)} \int_0^y h'(z)\,dz. \tag{B.1}$$

Then, a small reform in direction h raises more tax revenue than what is needed to compensate the agents facing the largest tax increase. In the following, we solve for the reform direction h^* that maximizes the free lunch Π over the set of functions such that $h' : [0, \bar{y}] \to [-a, a]$ for some fixed $a > 0$. To simplify the exposition, we impose further assumptions that are satisfied in the context of our application in Section 3 (see Figure 7).

ASSUMPTION B.1: *Let* $y_0(\Theta) = [0, \bar{y}]$. *There is a unique triplet* (y_s, y_t, r) *with* $0 < y_s < y_t < \bar{y}$ *and* $r \in (0, 1)$ *with the following properties*:

1. $\mathcal{R}$ *is strictly decreasing on* $(0, y_s)$ *and on* $(y_t, \bar{y})$.
2. $\mathcal{R}(y_s) = \mathcal{R}(y_t) = r$.
3. $\mathcal{R}(y) \in (r, 1)$ *for each* $y \in (0, y_s)$ *and for each* $y \in (y_m, y_t)$, *for* $y_m := \frac{y_s+y_t}{2}$.
4. $\mathcal{R}(y) < r$ *for each* $y \in (y_s, y_m)$.
5. $\mathcal{R}(y) \in [0, r)$ *for each* $y \in (y_t, \bar{y})$.

PROPOSITION B.1: *Fix* $a > 0$. *Under Assumption* B.1, *the optimal reform direction* h^* *is, for any* $a > 0$, *given by a two-bracket tax cut with*

$$h^{*\prime}(y) = \begin{cases} 0 & \text{for } y \in [0, y_s), \\ -a & \text{for } y \in (y_s, y_m), \\ a & \text{for } y \in (y_m, y_t), \\ 0 & \text{for } y \in (y_t, \bar{y}]. \end{cases} \tag{B.2}$$

Moreover, $\Pi(h^*) = aI(T_0, c_0)$ *with*

$$I(T_0, c_0) = \int_{y_m}^{y_t} \mathcal{R}(y)\,dy - \int_{y_s}^{y_m} \mathcal{R}(y)\,dy. \tag{B.3}$$

PROOF: To solve for the optimal reform direction, we proceed in three steps. First, we show that $\max_{y\in y_0(\Theta)} h^*(y)$ equals zero. Second, we solve for the reform that maximizes $\Pi(h)$ subject to (i) $\max_{y\in y_0(\Theta)} h(y) = 0$, (ii) $h'(y) \in [-a, a]$ for all $y \in [y_s, y_t)$, and (iii) the additional restriction that $h'(y) = 0$ for all incomes below y_s and above y_t. Third, we show that the solution to this more restricted problem also solves the original maximization problem.

Step 1. The normalization that system $T(0) = 0$ for any tax system that we consider also implies $h(0) = 0$ for any reform direction that we consider. Therefore, $\max_{y\in y_0(\Theta)} h^*(y) \geq 0$. To show that $\max_{y\in y_0(\Theta)} h^*(y) = 0$, we provide a proof by contradiction. For this purpose, assume that there is some $\varphi > 0$ such that $\max_{y\in y_0(\Theta)} h^*(y) = \varphi$ and denote by y^* the lowest income level such that $h^*(y) = \varphi$. Then, $y^* > 0$ and there must be an income $y' \in (0, y^*)$ such that $h^*(y) > 0$ for all $y \in (y', y^*)$. Consider a perturbed reform h_ε such that $h_\varepsilon'(y) = h^{*\prime}(y) - \varepsilon$ for all incomes (y', y^*) and $h_\varepsilon'(y) = h^{*\prime}(y)$ for all other incomes. The free lunch from this perturbed reform is

$$\Pi(h_\varepsilon) = \int_0^{\bar{y}} h_\varepsilon'(y)\mathcal{R}(y)\,dy - \varphi + \varepsilon\big(y^* - y'\big).$$

The derivative of $\Pi(h_\varepsilon)$ with respect to ε is

$$\left.\frac{d\Pi(h_\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = -\int_{y'}^{y^*} \mathcal{R}(y)\,dy + \left(y^* - y'\right) > 0,$$

where the positive sign follows because $\mathcal{R}(y) < 1$ for any $y \in (0, \bar{y})$ by Assumption B.1. This contradicts the assumption that Π obtains a maximum at h^*.

Step 2. Consider the problem to maximize $\Pi(h)$ over the set of functions h such that (i) $h'(y) \in [-a, a]$ for all $y \in (y_s, y_t)$, (ii) $h'(y) = 0$ for all $y \leq y_s$ and all $y \geq y_t$, and (iii) $h(y) = \int_0^y h'(z)\,dz = \int_{y_s}^y h'(z)\,dz \leq 0$ for all $y \in (y_s, y_t)$. Note that the function given in (B.2) satisfies these constraints.

We now consider a Lagrangian for a more relaxed problem that takes only the constraint $h(y_t) \leq 0$ into account. We argue below that a solution to this relaxed problem satisfies (i)–(iii):

$$\mathcal{L}(t) = \int_{y_s}^{y_t} h'(y)\mathcal{R}(y)\,dy - \mu \int_{y_s}^{y_t} h'(y)\,dy,$$

where μ is a Lagrange multiplier. The solution to this restricted problem is given by a function $\tilde{h} : (y_s, y_t) \to [-a, a]$ and a value $\tilde{\mu}$ of the multiplier. For any $y \in (y_s, y_t)$, the derivative of $\mathcal{L}$ with respect to $h'(y)$ is given by

$$\frac{\partial \mathcal{L}}{\partial h'(y)} = \mathcal{R}(y) - \tilde{\mu}.$$

As the Lagrangian is linear in each $h'(y)$, the solution involves $\tilde{h}'(y)$ equal to the lower bound $-a$ for all y such that $\mathcal{R}(y) < \tilde{\mu}$, and $\tilde{h}'(y)$ equal to the upper bound a for all y such that $\mathcal{R}(y) > \tilde{\mu}$. Under Assumption B.1, this is only consistent with $\tilde{h}(y_t) = \int_{y_s}^{y_t} \tilde{h}'(y)\,dy = 0$ if $\tilde{\mu} = r = \mathcal{R}(y_s)$. Then, $\tilde{h}'(y) = -a$ for all $y \in (y_s, y_m)$ and $\tilde{h}'(y) = a$ for all $y \in (y_m, y_t)$. Hence, $\tilde{h}$ equals the function given in (B.2), so it satisfies (i)–(iii). Consequently,

$$\Pi(\tilde{h}) = \int_{y_s}^{y_m} -a\mathcal{R}(y)\,dy + \int_{y_m}^{y_t} a\mathcal{R}(y)\,dy = aI(T_0, c_0),$$

with $I(T_0, c_0)$ given in (B.3). We also note that, as $\tilde{\mu}$ is strictly positive, the constraint $\tilde{h}(y_t) \leq 0$ is binding.

Step 3. It remains to show that we cannot increase Π further by allowing $h'(y) \in \{-a, a\}$ for incomes below y_s and above y_t, while respecting the constraint $\int_0^y h(z)\,dz \leq 0$ for all $y \in [0, \bar{y}]$. A repeated application of the arguments in Step 2, once for incomes below y_s, and once for incomes above y_t, exploiting the monotonicity of $y \mapsto \mathcal{R}(y)$ over these income ranges, shows that any candidate solution to this problem will take the form

$$h^{*\prime}(y) = \begin{cases} 0 & \text{for } y < y_\alpha, \\ -a & \text{for } y \in (y_\alpha, y_m), \\ a & \text{for } y \in (y_m, y_\beta), \\ 0 & \text{for } y > y_\beta, \end{cases}$$

where $y_\alpha \leq y_s$ and $y_\beta \geq y_t$. The constraint $h(\bar{y}) = \int_0^{\bar{y}} h^*(y)\,dy \leq 0$ is only satisfied if $y_\beta - y_m \leq y_m - y_\alpha$. Finally, choosing y_α and y_β to maximize $\Pi(h)$ subject to $y_\alpha \leq y_s$ and $y_\beta \geq y_t$ and $y_\beta - y_m \leq y_m - y_\alpha$ shows that $y_\alpha = y_s$ and $y_\beta = y_t$ is an optimal choice. *Q.E.D.*

APPENDIX C: PARETO EFFICIENCY WHEN EARNINGS ARE BOUNDED AWAY FROM ZERO AND BOUNDED FROM ABOVE

Suppose that $y \mapsto \mathcal{R}(y)$ is non-increasing over $\mathcal{Y}$, so that there is no scope for a Pareto improvement by a two-bracket reform. Then, by Theorem 2 and Corollary 1, the status quo tax system is Pareto-efficient if $\mathcal{R}$ is bounded from above by 1 for the lowest incomes, and bounded from below by 0 for the highest incomes. Hence, our Pareto test only requires to verify one-bracket efficiency at the very top and the very bottom. Under more restrictive assumptions, even the conditions for one-bracket efficiency at the extremes become dispensable as we show in the following corollary. For this purpose, we denote by $y_{\min} := \inf y_0(\Theta)$ the infimum of the income levels, and by $y_{\max} := \sup y_0(\Theta)$ the supremum of the income levels chosen under the status quo tax policy.

COROLLARY C.1: *If $y_{\min} > 0$, $y_{\max} < \bar{y}$, and $y \mapsto \mathcal{R}(y)$ is non-increasing, then there is no Pareto-improving direction in the class of continuous functions.*

Under the conditions of Corollary C.1, the monotonicity of $y \mapsto \mathcal{R}(y)$ is sufficient for the non-existence of a Pareto-improving reform direction, that is, there is no need to invoke the requirements that this function must be bounded from below by 0 and from above by 1. The conditions are that all types choose their incomes in some interior subset of $\mathcal{Y}$. In this case, as we show formally below, we have $\mathcal{R}(y) = 1$ for all $y \in [0, y_{\min})$, and $\mathcal{R}(y) = 0$ for all $y \in (y_{\max}, \bar{y}]$. Consequently, all three sufficient conditions in Theorem 2 are satisfied if $y \mapsto \mathcal{R}(y)$ is non-increasing.[2]

The conditions in Corollary C.1 hold, for instance, in a Mirrleesian model of income taxation with only intensive-margin responses when Inada conditions ensure positive and bounded incomes for everybody.[3] By contrast, there is typically a positive mass of taxpayers with zero income in models with an intensive and an extensive margin. In this case, the requirement of boundedness does not follow from the requirement of monotonicity. Put differently, with a mass of non-working people, there can exist Pareto-improving reforms with one bracket even if there is no Pareto-improving reform with two brackets.

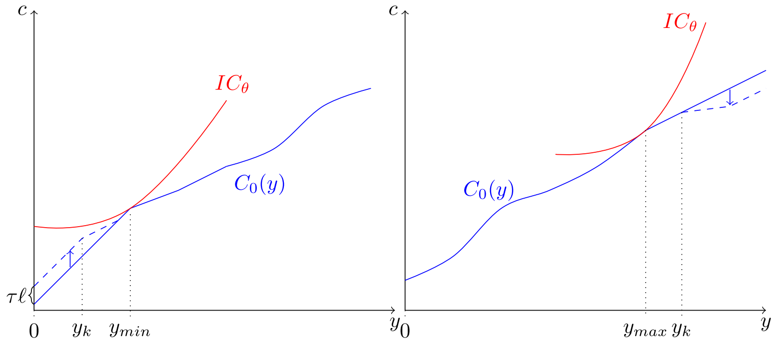
C.1. *Proof of Corollary C.1*

Recall that $y_0(\Theta)$ is the image function of y_0, that is, the set of income levels that are individually optimal for some type in Θ given the status quo tax system. The infimum of this set is denoted by $y_{\min}$ and the supremum by $y_{\max}$.

LEMMA C.1: *If $y_{\min} > 0$, then $\mathcal{R}(\hat{y}) = 1$ for any $\hat{y} \in [0, y_{\min})$.*

[2] Note that Corollary C.1 provides a more compact expression of our sufficient conditions, but it should not be interpreted as showing that two-bracket reforms can achieve strictly more than one-bracket reforms. In particular, a two-bracket reform with one bracket below $y_{\min}$ or above $y_{\max}$ is economically equivalent to a one-bracket reform.

[3] Technically, this also requires that the parameter $\bar{y}$ in $\mathcal{Y} = [0, \bar{y}]$ is chosen so large that this upper bound does not interfere with individual choices.



(a) A tax increase below y_{min}. (b) A tax increase above y_{max}.

FIGURE C.1.—Illustration of small one-bracket reforms below $y_{\min}$ and above $y_{\max}$.

PROOF: Fix a one-bracket reform $(\tau, \ell, \hat{y})$ such that $\hat{y} \geq 0$, $\hat{y} + \ell < y_{\min}$ and assume that $\tau\ell$ is close to zero. This implies that, at any income level $y \geq y_{\min}$, the tax burden increases by $\tau\ell$. We now argue that there are no behavioral responses to such a reform. More specifically, we show that "no behavioral responses" is consistent with both utility-maximizing behavior and the government budget constraint. When there are no behavioral responses and $y_0(\theta) > \hat{y} + \ell$ for all θ, this implies that the change of aggregate tax revenue equals $R^s(\tau, \ell, \hat{y}) = \tau\ell$. Since this additional tax revenue is rebated lump-sum, this also implies that all taxpayers receive additional transfers of $\tau\ell$. Hence, for any income in $y^*(\Theta)$, the additional tax payment and the additional transfer cancel each other out, implying that taxpayers face the same budget set before and after this reform, $C_1(y) = C_0(y)$. Moreover, for $\tau\ell$ sufficiently small, incomes smaller than $y_{\min}$ remain dominated by $y_0(\theta) \geq y_{\min}$ for each θ. From $R^s(\tau, \ell, \hat{y}) = \tau\ell$, we obtain $R^s_{\tau\ell}(0, 0, \hat{y}) = 1$, which completes the proof. *Q.E.D.*

Figure C.1(a) illustrates these arguments: The solid blue line depicts the status quo budget set $C_0(y)$; the dashed blue line shows the upward shift by $\tau\ell$ in the post-reform budget set $C_1(y)$ for incomes below y_k. The red line, finally, shows an indifference curve of the lowest-earning type θ such that $y_0(\theta) = y_{\min}$ before the reform. As is apparent from Figure C.1(a), the type continues to prefer $y_{\min}$ to any lower income and does not change her behavior as long as $\tau\ell$ is small enough.

LEMMA C.2: *If $y_{\max} < \bar{y}$, then $\mathcal{R}(y) = 0$ for any $y \in (y_{\max}, \bar{y}]$.*

PROOF: Fix a one-bracket reform $(\tau, \ell, \hat{y})$ such that $\hat{y} \in (y_{\max}, \bar{y})$ and $\tau\ell > 0$. This reform does not change the tax burden at any income $y \leq y_{\max}$. It increases the tax burden for incomes above $\hat{y}$ that were already dominated by some income below $y_{\max}$ for each type θ prior to the reform. We now argue that there are no behavioral responses to such a reform. More specifically, we show that "no behavioral responses" is consistent with both utility-maximizing behavior and the government budget constraint. When there are no behavioral responses and $y_0(\theta) < \hat{y}$ for each θ, this implies that the aggregate tax revenue does not change, $R^s(\tau, \ell, \hat{y}) = 0$. This also implies that the base transfer is not changed, $c_1 = c_0$. Hence, for any income $y \leq \hat{y}$, taxpayers face the same budget set before and after this reform, $C_1(y) = C_0(y)$. Moreover, because of the tax increase for incomes larger than $\hat{y}$, these incomes remain dominated by some income below $y_{\max}$ for each type θ. From $R^s(\tau, \ell, \hat{y}) = 0$, we obtain $\mathcal{R}(\hat{y}) = R^s_{\tau\ell}(0, 0, \hat{y}) = 0$, which completes the proof. *Q.E.D.*

Figure C.1(b) illustrates these arguments. Again, the solid blue line depicts the status quo budget set $C_0(y)$, and the dashed blue line depicts the downward shift in the budget set due to the reform. The red depicts an indifference curve of a type θ who chooses the highest income $y_{\max}$ before the reform. As is apparent from Figure C.1(b), the type continues to prefer $y_{\max}$ to any higher income and does not change her behavior after the reform.

Corollary C.1 follows from combining Theorem 2 with Lemmas C.1 and C.2. Thus, if $y_{\min} > 0$ and $y_{\max} < \bar{y}$, monotonicity of $\mathcal{R}$ is a sufficient condition for the non-existence of Pareto-improving tax reforms. The lower and upper bounds on $\mathcal{R}$ become dispensable. Put differently, whenever there is a Pareto-improving reform with one bracket, there is also a Pareto-improving two-bracket tax cut.

Now suppose that the conditions in Corollary C.1 are violated, so that either $y_{\min} = 0$ or $y_{\max} = \bar{y}$. In particular, the case with a mass of people with zero incomes is both empirically relevant and a typical outcome in the model with labor supply responses at the intensive and the extensive margin that we focus on in Section 3. In this case, it is possible that $\mathcal{R}(y)$ is monotonically decreasing over $(0, \bar{y})$ even though $\mathcal{R}(y) > 1$ for positive incomes close to 0. Then, there exists a Pareto-improving one-bracket reform, but no such reforms with two brackets. Hence, one-bracket efficiency at the bottom remains a substantive constraint.

By contrast, in the case $y_{\max} = \bar{y}$, we can simply raise the upper threshold $\bar{y}$ of the set of feasible incomes to make sure that all people choose incomes below $\bar{y}$. After this adjustment, $\mathcal{R}(y) = 0$ for all incomes between $y_{\max}$ and the adjusted level of $\bar{y}$ by Lemma C.2. Then, $\mathcal{R}(y) < 0$ for some income level close to $\bar{y}$ implies a violation of the condition that $y \mapsto \mathcal{R}(y)$ must be non-increasing over $[0, \bar{y}]$. Hence, whenever there is a Pareto-improving reform that reduces marginal taxes in one bracket of incomes below $y_{\max}$, there is also a Pareto-improving two-bracket reform. In particular, any reform that combines the previously mentioned one-bracket tax cut with a tax increase in some bracket of incomes above $y_{\max}$ will do the job. It should be noted, however, that such a two-bracket reform is economically equivalent to a one-bracket reform by the arguments given in the proof of Lemma C.2: The second bracket has no effect on behavior as no type will choose an income in or above the second bracket.

APPENDIX D: DATA DESCRIPTION AND BENCHMARK CALIBRATION

This section provides a description of the data we use in our empirical analysis and explains the choices for our benchmark calibration. We start with a description of the 1974 U.S. tax-transfer system. Subsequently, we describe how we obtained estimates of the relevant income distributions and our benchmark assumptions on behavioral responses to tax reforms.

Status Quo Tax Function: The U.S. Tax-Transfer System in 1974. We take account of the federal income tax and two large welfare programs, Aid for Families with Dependent Children (AFDC) and Supplementary Nutrition Assistance Programs (SNAP, also called Food Stamps). The two latter were considered the most important welfare programs at the time and contributed most to high effective tax rates; see Moffitt (1979) and Moffitt (2003). As shown in Table D.I based on CPS and PSID data, they also had many more recipients among, and provided larger payments to, low-income single parents than other programs. Moreover, they were eligible for all single parents, whereas other programs such as Social Security and the newly introduced Supplementary Security Income were

TABLE D.I

WELFARE TRANSFERS RECEIVED BY SINGLE PARENTS IN 1974.

Transfer program	Sample	CPS any income	CPS $y \leq 4000$	PSID any income	PSID $y \leq 4000$
Food stamps	Share	–	–	49.3%	71.0%
	Amount	–	–	\$369	\$573
AFDC	Share	37.0%	56.8%	35.0%	54.6%
	Amount	\$937	\$1501	\$912	\$1561
Supplementary Security Income (SSI)	Share	–	–	3.5%	5.6%
	Amount	–	–	\$58	\$104
Social Security	Share	12.4%	17.4%	14.6%	19.4%
	Amount	\$363	\$538	\$411	\$607
Unemployment or workmen's compensation	Share	–	–	4.0%	4.0%
	Amount	–	–	70	99
Other programs	Share	10.2%	11.3%	8.9%	14.3%
	Amount	\$155	\$196	\$210	\$385

Note: Table D.I reports information on different welfare transfers to single parents in 1974, based on CPS-ASEC 1975 data and PSID 1975 data. Sample restrictions: age 25–60, head of household, non-married, no partner or adult family member in household. Columns 2 and 4 show the shares of single parents receiving any positive transfers, and the average amounts of transfers received. Columns 3 and 5 show the shares of recipients and the average amounts received among single parents with earned income below 4000 USD. *Source*: Authors' calculations (see Table D.II for details).

restricted to aged, blind, and disabled individuals; see, for example, Daly and Burkhauser (2003). ADFC was available only for single parents and varied to some extent across U.S. states. We focus on the AFDC rules in California. SNAP was a federal program that was available both for single parents and childless singles, but was more generous for single parents. The transfer payments for single parents also depended on the number of children. In our benchmark analysis, we focus on the tax-transfer schedule for single parents with two children. In our data, the median number of children in single-parent households was two, and the arithmetic mean was about 2.2. Figure 4 in the main text depicts the effective tax rates for this subgroup of single parents and for childless singles. For the years 1975 and later, we also account for the Earned Income Tax Credit. Table D.II depicts the sources that we use for computing the U.S. tax-transfer systems for single parents and childless singles in the years 1974, 1975, and 1978.

Income Distribution. We estimate the 1974 income distributions based on Current Population Survey (CPS) data, using the Annual Social and Economic Supplement from the March 1975 wave (see Table D.III). We proceed in the same way for the years 1975 and 1978. For our benchmark estimates, we consider for each year the sample of non-married individuals aged 25 to 60 who do co-habit neither with an unmarried spouse nor with another adult family member. We estimate the earned income distributions for the subsamples of childless singles and single parents, respectively. For the benchmark analysis reported in the main text, we consider all singles with at least one qualifying child according to AFDC rules (i.e., either 15 years or younger, or 19 years and younger and in full-time education). In the subsequent sensitivity analysis, we also consider the subsets of single parents with one, two, and three children, and the somewhat larger set of singles with qualifying children according to EITC rules (i.e., either 19 years or younger, or 23 years and younger and in full-time education).

TABLE D.II

SOURCES FOR U.S. TAX-TRANSFER SYSTEM, 1974–1978.

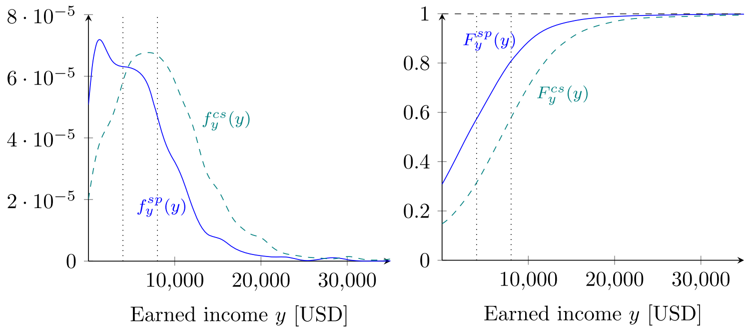
Information	Years	Sources
Income tax	1974–1975	Internal Revenue Service, "Instructions for Form 1040," years 1974, 1975. URLs: https://www.irs.gov/pub/irs-prior/i1040--1974.pdf; https://www.irs.gov/pub/irs-prior/i1040--1975.pdf.
	1974–1978	Internal Revenue Service, "Statistics of Income, Individual income tax returns," years 1974, 1975, 1978. URLs: https://www.irs.gov/pub/irs-soi/74inar.pdf; https://www.irs.gov/pub/irs-soi/75inar.pdf; https://www.irs.gov/pub/irs-soi/78inar.pdf.
EITC	1975, 1978	Tax Policy Center, "Earned Income Tax Credit Parameters, 1975–2021." URL: https://www.taxpolicycenter.org/file/190595/download?token=s_f3v98_.
AFDC	1974–1978	Office of the Assistant Secretary for Planning and Evaluation, "Aid to Families with Dependent Children. The Baseline," 1998. URL: https://aspe.hhs.gov/basic-report/aid-families-dependent-children-baseline
	1974	U.S. Department of Health, Education and Welfare, "Aid to Families with Dependent Children: Standards for Basic Needs, July 1974," 1974. URL: https://hdl.handle.net/2027/mdp.39015088906634.
	1975, 1978	TRIM3 project, "TRIM3 AFCD Rules." Accessible at trim3.urban.org.
SNAP	1974–1975	U.S. Bureau of the Census, "Characteristics of Households Purchasing Food Stamps. Current Population Reports," Series 9-23, No. 61, 1976. URL: https://www2.census.gov/library/publications/1976/demographics/p23-061.pdf.
	1978	Federal Register Vol. 43, No. 95, May 16, 1978. Accessible at https://www.govinfo.gov/app/collection/fr.
SNAP, Payroll tax	1974–1978	Social Security Administration, "Annual Statistical Supplement. Section 2: Program Provisions and SSA Administrative Data," 2010. URL: https://www.ssa.gov/policy/docs/statcomps/supplement/2010/2a1-2a7.html.

TABLE D.III

DESCRIPTIVE STATISTICS FOR DIFFERENT SAMPLES.

Sample	Sample size	Average age [yrs]	Share zero income [%]	Share EITC range [%]	Avg. earned income [USD]	Avg. capital income [USD]
1: Mothers	15,881	37.1	5.2	19.8	13,862	326
2: Single parents (BM)	1494	35.7	30.9	51.3	3968	224
3: Single parents (EITC)	1875	37.5	26.3	53.2	4432	248
4: Single parents (18-50)	1641	32.1	32.5	52.4	3582	152
5: Single parents (Cal)	187	35.7	25.8	52.2	4341	101
6: Single parents (PSID)	369	36.9	25.2	52.6	4568	100
7: Childless singles	3407	43.4	14.8	43.1	7355	425

Note: Table D.III shows descriptive statistics for different samples, based on CPS-ASEC 1975 data. Sample restrictions, unless otherwise stated: age 25–60, head of household, non-married, no partner or adult family member in household. Sample 1: any marital status, any household composition, at least one child qualifying for AFDC. Sample 2 (benchmark): at least one child qualifying for AFDC. Sample 3: at least one child qualifying for EITC. Sample 4: age 18–50. Sample 5: California resident, at least one child qualifying for AFDC. Sample 6: based on PSID 1975 data, at least one child aged 0–17. Sample 7: no dependent child. *Source*: Authors' calculations (see Table D.II for details).



(a) Probability density functions. (b) Cumulative distribution functions.

FIGURE D.1.—Income distributions of single parents and childless singles, US 1974. *Note*: Figure D.1 shows the kernel estimates of the U.S. income distributions among single parents (solid blue lines) and childless singles (dashed teal lines) in 1974. Panel (a) depicts the probability density functions; panel (b) depicts the cumulative distribution functions of both income distributions. The dotted vertical lines mark the endpoints of the phase-in range at 4000 USD and the phase-out range at 8000 USD of the 1975 EITC.*Source*: Authors' calculations (see Table D.II for details).

In line with the EITC rules, we consider as earned income the sum of (self-reported) wage income and self-employment income by all household members. In this sample, 30.9% of single parents and 14.8% of childless singles had zero or negative incomes, while 51.3% of single parents and 43% of childless singles had strictly positive incomes below 8000 USD, the eligibility threshold of the EITC. For our benchmark analysis, we estimate the distributions of earned income for both groups using a nonparametric kernel density estimation with a Gaussian kernel. In the benchmark, we use bandwidths of 997 USD for single parents and 1200 USD for childless singles, following Silverman's rule. Figure D.1 shows the estimated pdf and the cdf of both income distributions.

Behavioral Responses to Taxation. We draw on a rich literature providing estimates of labor supply responses at the intensive and the extensive margin; see the discussions in Saez, Slemrod, and Giertz (2012) or Chetty, Guren, Manoli, and Weber (2013). Based on a meta-study and focusing on population-wide averages, Chetty et al. (2013) suggested an intensive-margin elasticity of labor supply with respect to the net-of-tax rate of 0.33, and an extensive-margin elasticity with respect to net labor income of 0.25. Bargain, Dolls, Neumann, Peichl, and Siegloch (2014) provided similar estimates for a sample of childless singles. For single parents, various studies found larger responses at the extensive margin. Specifically, Bastian (2020) estimated labor supply responses of single mothers to the 1975 EITC introduction, and found an average participation elasticity of 0.58. Most earlier studies found similar or even larger participation responses by single mothers; for example, Meyer and Rosenbaum (2001). By contrast, Kleven (2021) recently estimated a participation elasticity close to zero based on EITC reforms in the 1990s. Besides, several studies found that persons with little formal education and low incomes respond more strongly at the extensive margin than persons with higher education and higher incomes—see, for example, Juhn, Murphy, and Topel (1991), Juhn, Murphy, and Topel (2002), Meghir and Phillips (2010). There is only limited empirical evidence on the relevance of income effects for labor supply. Recent evidence by Cesarini, Lindqvist, Notowidigdo, and Östling (2017) based on Swedish lottery winners suggests (i) a marginal propensity to earn out of unearned income (MPE) of −0.08, (ii) with about two-thirds of income effects arising at the intensive margin, one-third at the extensive margin, and (iii) with little heterogeneity in income effects along the income distribution. Imbens, Rubin, and Sacerdote (2001) reported similar estimates with an MPE of -0.11 for the United States; see also Holtz-Eakin, Joulfaian, and Rosen (1993).

In our benchmark calibration, we assume an average participation elasticity of 0.58 for single parents and 0.25 for childless singles, and an intensive-margin elasticity of 0.33 for both subgroups, following Bastian (2020) and Chetty et al. (2013). Moreover, we assume that participation elasticities are decreasing with income in both groups, according to the function $\pi_0(y) = \pi_a - \pi_b(y/\tilde{y})^{1/2}$, where $\tilde{y}$ equals 50,000 USD. Similar assumptions were employed by Jacquet, Lehmann, and Van der Linden (2013) and Hansen (2021). For single parents, we assume that the participation elasticity falls from 0.67 at very low incomes to 0.4 at incomes above 50,000 USD (i.e., $\pi_a = 0.67$, $\pi_b = 0.27$), giving rise to an average value of 0.58. For childless singles, we assume π to fall from 0.4 to 0.1 (i.e., $\pi_a = 0.4$, $\pi_b = 0.3$), giving rise to an average value around 0.25. In the benchmark calibration, we leave out income effects. In Bierbrauer, Boyer, and Hansen (2022), we consider a large range of alternative assumptions on labor supply elasticities.

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