SUPPLEMENT TO "TASKS, AUTOMATION, AND THE RISE IN U.S. WAGE INEQUALITY"

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DARON ACEMOGLU
Department of Economics, MIT

PASCUAL RESTREPO
Department of Economics, Boston University

APPENDIX A

A-1. *Glossary of Variables Used in Section 4*

- (Direct) Task displacement$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot (-d\ln s_i^{L,\text{auto}})$, where $-d\ln s_i^{L,\text{auto}}$ is the observed percent decline of the labor share in industry i or the automation-driven component thereof.
- Industry shifters$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot d\ln s_i^Y$, where s_i^Y denotes the value added share of industry i.
- Exposure to industry labor share declines$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot (-d\ln s_i^{L,\text{auto}})$.
- Relative specialization in routine jobs$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R}$.
- Exposure to industry shock$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \text{Shock}_i$.
- Exposure of routine jobs to industry shock$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot \text{Shock}_i$.
- Exposure to change in markups$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot$ Percent change in markups$_i$.
- Exposure of routine jobs to change in markups$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot$ Percent change in markups$_i$.
- Exposure to change in concentration$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot$ Change in concentration$_i$.
- Exposure of routine jobs to change in concentration$_g = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot$ Change in concentration$_i$.

A-2. *Proofs of the Results in the Main Text*

We first provide conditions for the single-sector and multi-sector economies to produce finite output. Let $H(y_1, \dots, y_I)$ denote the production function for the final good, taking sectoral outputs as its inputs. Define the derived aggregate production function of the economy, depending on the total amount of capital used in production, k, and the vector of labor supplies, $\boldsymbol{\ell}$ as

$$F(k, \boldsymbol{\ell}) = \max H(y_1, \dots, y_I) \tag{A-1}$$

$$\text{subject to: } y_i = \left(\frac{1}{M}\int_{\mathcal{T}} \big(M\cdot y(x)\big)^{\frac{\lambda-1}{\lambda}} \cdot dx\right)^{\frac{\lambda}{\lambda-1}} \quad \forall i \in \mathcal{I},$$

Daron Acemoglu: daron@mit.edu
Pascual Restrepo: pascual@bu.edu

$$y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_{g \in \mathcal{G}} A_g \cdot \psi_g(x) \cdot \ell_g(x) \quad \forall x \in \mathcal{T},$$

$$\ell_g = \int_{\mathcal{T}} \ell_g(x) \cdot dx \quad \forall g \in \mathcal{G},$$

$$k = \int_{\mathcal{T}} \big(k(x)/q(x)\big) \cdot dx.$$

PROPOSITION A-1—Finite output: *The economy produces finite output if and only if the following Inada condition holds*:

$$\lim_{k \to \infty} F_k(k, \boldsymbol{\ell}) < 1. \tag{A-2}$$

Moreover, in any equilibrium with positive and finite consumption, we have $s^K \in [0, 1)$, *and in any equilibrium with infinite output, we have* $s^K = 1$.

PROOF: A competitive equilibrium maximizes the strictly concave function $c(k) = F(k, \boldsymbol{\ell}) - k$. When the Inada condition (A-2) holds, the function $c(k)$ reaches a unique maximum at some $k^* \geq 0$. Since $c(k^*) = (1 - s^K)F(k^*, \boldsymbol{\ell})$, we also have $s^K \in [0, 1)$.

Because F is concave, $\lim_{k\to\infty} F_k(k, \boldsymbol{\ell})$ exists. Suppose now that the Inada condition (A-2) fails, so that $\lim_{k\to\infty} F_k(k, \boldsymbol{\ell}) \geq 1$. Then, $c(k)$ is an increasing function on $\mathbb{R}_+$, and thus has no well-defined maximizer and the economy reaches infinite output. Since in this case $\lim_{k\to\infty} F_k(k, \boldsymbol{\ell}) \geq 1$ and F exhibits constant returns to scale, $F_k(k, \boldsymbol{\ell})$ is a decreasing function that converges to some limit $m > 1$ as $k \to \infty$. Therefore,

$$s^K = \lim_{k\to\infty} \frac{F_k(k, \boldsymbol{\ell}) \cdot k}{F(k, \boldsymbol{\ell})} \geq m \cdot \lim_{k\to\infty} \frac{k}{F(k, \boldsymbol{\ell})} = m \cdot \lim_{k\to\infty} \frac{1}{F_k(k, \boldsymbol{\ell})} = 1,$$

where we used l'Hôpital's rule in the third step. This implies that $s^K = 1$ as wanted.

We also note that in the single-sector case, the Inada condition (A-2) is equivalent to $A_k^{\lambda-1}\Gamma_k < 1$, as noted in the text. *Q.E.D.*

PROOF OF PROPOSITION 1: We first show that an equilibrium exists and is unique. The equilibrium of this economy solves the following optimization problem:

$$\max_{\{k(x), \ell_1(x), \ldots, \ell_G(x)\}_{x\in\mathcal{T}}} y - \int_{\mathcal{T}} \big(k(x)/q(x)\big) \cdot dx$$

$$\text{subject to: } y = \left(\frac{1}{M} \int_{\mathcal{T}} \big(M \cdot y(x)\big)^{\frac{\lambda-1}{\lambda}} \cdot dx\right)^{\frac{\lambda}{\lambda-1}},$$

$$y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_{g \in \mathcal{G}} A_g \cdot \psi_g(x) \cdot \ell_g(x) \quad \forall x \in \mathcal{T},$$

$$\ell_g = \int_{\mathcal{T}} \ell_g(x) \cdot dx \quad \forall g \in \mathcal{G}.$$

This is related to (A-1), except that it is for the single-sector case and maximizes over the production of capital inputs as well. The objective function is concave, while the constraint set is convex. Hence, this optimization problem either reaches a unique maximal value (though it might have non-unique maximizers) or has no solution (meaning that it reaches infinite output). Proposition A-1 rules out the latter case under (A-2), which we have imposed. Hence, we focus on the former case. Let w_g be the Lagrange multiplier associated with the constraint for labor of type g. Then the solution can be expressed by the following allocation of tasks to factors:

$$\mathcal{T}_g \subseteq \left\{x : \frac{w_g}{A_g \cdot \psi_g(x)} \leq \frac{w_{g'}}{A_{g'} \cdot \psi_{g'}(x)}, \frac{1}{\psi_k(x) \cdot q(x) \cdot A_k} \text{ for all } g'\right\},$$

$$\mathcal{T}_k \subseteq \left\{x : \frac{1}{\psi_k(x) \cdot q(x) \cdot A_k} \leq \frac{w_g}{A_g \cdot \psi_g(x)}, \text{ for all } g\right\}.$$

The tie-breaking rule described in footnote 7 then selects a unique equilibrium allocation. This argument shows that, when the maximization problem is bounded, there is a unique equilibrium, where the task allocation is as described in the main text. In what follows, we characterize the equilibrium as a function of this unique task allocation.

The demand for task x is

$$y(x) = \frac{1}{M} \cdot y \cdot p(x)^{-\lambda}, \tag{A-3}$$

where $p(x)$ is this task's price. Given the allocation of tasks $\{\mathcal{T}_k, \mathcal{T}_1, \dots, \mathcal{T}_G\}$, this price is

$$p(x) = \begin{cases} \dfrac{1}{A_k \cdot q(x) \cdot \psi_k(x)} & \text{if } x \in \mathcal{T}_k, \\ \dfrac{w_g}{A_g \cdot \psi_g(x)} & \text{if } x \in \mathcal{T}_g. \end{cases} \tag{A-4}$$

This implies that the demand for capital and labor at the task level is given by

$$\frac{k(x)}{q(x)} = \begin{cases} \dfrac{1}{M} \cdot y \cdot \left(A_k \cdot q(x) \cdot \psi_k(x)\right)^{\lambda-1} & \text{if } x \in \mathcal{T}_k, \\ 0 & \text{if } x \notin \mathcal{T}_k, \end{cases}$$

$$\ell_g(x) = \begin{cases} \dfrac{1}{M} \cdot y \cdot \left(A_g \cdot \psi_g(x)\right)^{\lambda-1} \cdot w_g^{-\lambda} & \text{if } x \in \mathcal{T}_g, \\ 0 & \text{if } x \notin \mathcal{T}_g. \end{cases}$$

To derive equation (2), we integrate over the demand for labor across tasks in the previous expression and rearrange to obtain

$$\ell_g = \int_{\mathcal{T}_g} \frac{1}{M} \cdot y \cdot \left(A_g \cdot \psi_g(x)\right)^{\lambda-1} \cdot w_g^{-\lambda} \cdot dx$$

$$\Rightarrow \quad w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\frac{1}{M} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1}\, dx\right)^{\frac{1}{\lambda}}.$$

Equation (1) follows by noting that, by definition, gross output y is $y = \int_{\mathcal{T}} y(x) p(x)\, dx$. Substituting for $y(x)$ from equation (A-3), we obtain the ideal price condition:

$$1 = \frac{1}{M} \int_{\mathcal{T}} p(x)^{1-\lambda}\, dx. \tag{A-5}$$

Substituting for the equilibrium task prices from equation (A-4) yields

$$1 = A_k^{\lambda-1} \cdot \left(\frac{1}{M}\int_{\mathcal{T}_k} \big(q(x)\cdot\psi_k(x)\big)^{\lambda-1}\,dx\right) + \sum_{g\in\mathcal{G}} \left(\frac{w_g}{A_g}\right)^{1-\lambda} \cdot \left(\frac{1}{M}\int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1}\,dx\right).$$

Next substituting for w_g from equation (2), we rewrite this equation in terms of task shares:

$$1 = A_k^{\lambda-1}\cdot\Gamma_k + \sum_{g\in\mathcal{G}} \Gamma_g^{\frac{1}{\lambda}} \cdot \left(\frac{y}{A_g\cdot\ell_g}\right)^{\frac{1-\lambda}{\lambda}}.$$

Rearranging and using the fact that $A_k^{\lambda-1}\Gamma_k < 1$ establishes (1).

Finally, we can compute factor shares as

$$s^K = \frac{\frac{1}{M}\int_{\mathcal{T}_k} y\cdot p(x)^{1-\lambda}\,dx}{y} = A_k^{\lambda-1}\cdot\Gamma_k.$$

Because of constant returns to scale, we have $s^L = 1 - s^K$. *Q.E.D.*

PROOF OF PROPOSITION 2: We now characterize the effects of a small change in technology. As in the text, we use $\mathcal{D}_g \subset \mathcal{T}_g$ to denote the set of tasks that used to be performed by group g and, after the technological change, will switch to capital.

To characterize the effects of technology on wages, we first log-differentiate equation (2):

$$d\ln w_g = \frac{1}{\lambda} d\ln y + \frac{\lambda-1}{\lambda} d\ln A_g + \frac{1}{\lambda} d\ln\Gamma_g.$$

The definitions of $d\ln\Gamma_g^{\text{deep}}$ and $d\ln\Gamma_g^{\text{auto}}$ in the main text, together with Assumption 1, imply

$$d\ln\Gamma_g = (\lambda-1)d\ln\Gamma_g^{\text{deep}} - d\ln\Gamma_g^{\text{auto}},$$

which yields the expression for wage changes in (6) in the text.

Let us next define changes in TFP as

$$d\ln \text{tfp} = d\ln y - s^K\cdot d\ln k|_q,$$

where $k = \int_{\mathcal{T}_k} k(x)/q(x)\,dx$ denotes the total capital stock and $d\ln k|_q$ denotes changes in the capital stock coming from capital quantities and not prices. For a small change in technology, this can be computed as

$$s^K\cdot d\ln k|_q = \frac{1}{y}\,dk = \int_{\mathcal{T}_k} s^K(x)\,d\ln k(x)\,dx + \frac{1}{y}\sum_{g\in\mathcal{G}}\int_{\mathcal{D}_g} \big(k^{\text{new}}(x)/q^{\text{new}}(x)\big)\,dx,$$

where the $k^{\text{new}}(x)$ and $q^{\text{new}}(x)$ denote capital usage and prices in the newly-automated tasks.

We now show that changes in TFP also satisfy the dual representation:

$$d\ln \text{tfp} = \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln w_g - \int_{\mathcal{T}_k} s^K(x)\, d\ln q(x)\, dx, \tag{A-6}$$

where $s^K(x)$ denotes the share of capital $k(x)$ in gross output and s_g^L denotes the share of labor of type g in gross output.

Equation (A-6) follows from the fact that we have competitive markets and constant returns to scale. In particular, Euler's theorem implies $y = \sum_{g\in\mathcal{G}} w_g \ell_g + \int_{\mathcal{T}_k} k(x)/q(x)\, dx$. For any small change in technology, we therefore have

$$\begin{aligned} d\ln y = {} & \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln w_g + \int_{\mathcal{T}_k} s^K(x)\, d\ln k(x)\, dx - \int_{\mathcal{T}_k} s^K(x)\, d\ln q(x)\, dx \\ & + \frac{1}{y} \sum_{g\in\mathcal{G}} \int_{\mathcal{D}_g} \big(k^{\text{new}}(x)/q^{\text{new}}(x)\big)\, dx. \end{aligned}$$

We can rearrange this as

$$\begin{aligned} & d\ln y - \left(\int_{\mathcal{T}_k} s^K(x)\, d\ln k(x)\, dx + \frac{1}{y} \sum_{g\in\mathcal{G}} \int_{\mathcal{D}_g} \big(k^{\text{new}}(x)/q^{\text{new}}(x)\big)\, dx \right) \\ & \quad = \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln w_g - \int_{\mathcal{T}_k} s^K(x)\, d\ln q(x)\, dx, \end{aligned}$$

which is equivalent to (A-6).

We now return to the contributions of different types of technologies to TFP. For this, we use the ideal price index condition in equation (A-5), which we can rewrite as

$$1 = A_k^{\lambda-1} \cdot \left(\frac{1}{M} \int_{\mathcal{T}_k} \big(q(x) \cdot \psi_k(x)\big)^{\lambda-1} dx \right) + \sum_{g\in\mathcal{G}} \left(\frac{w_g}{A_g} \right)^{1-\lambda} \cdot \left(\frac{1}{M} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1}\, dx \right).$$

Log-differentiating this equation following a change in technology and capital prices, we obtain

$$\begin{aligned} & \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln w_g - \int_{\mathcal{T}_k} s^K(x)\, d\ln q(x)\, dx \\ & \quad = s^K \cdot \big(d\ln A_k + d\ln \Gamma_k^{\text{deep}}\big) + \sum_{g\in\mathcal{G}} s_g^L \cdot \big(d\ln A_g + d\ln \Gamma_g^{\text{deep}}\big) + \Delta, \end{aligned} \tag{A-7}$$

where

$$\Delta = \frac{1}{\lambda-1} \left[s^K \cdot d\ln \Gamma_k^{\text{auto}} - \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln \Gamma_g^{\text{auto}} \right]$$

represents the reallocation of tasks from labor to capital. Using the definitions of $\pi_g(x)$ and π_g in the main text, Δ can be rewritten as

$$\begin{aligned}
\Delta &= \sum_{g\in\mathcal{G}} \frac{1}{\lambda-1}\left[A_k^{\lambda-1}\cdot\frac{1}{M}\int_{\mathcal{D}_g}\big(q(x)\cdot\psi_k(x)\big)^{\lambda-1}\,dx - \left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\frac{1}{M}\int_{\mathcal{D}_g}\psi_g(x)^{\lambda-1}\,dx\right] \\
&= \sum_{g\in\mathcal{G}} \frac{1}{M}\int_{\mathcal{D}_g}\frac{1}{\lambda-1}\left[\big(A_k\cdot q(x)\cdot\psi_k(x)\big)^{\lambda-1} - \left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\psi_g(x)^{\lambda-1}\right]dx \\
&= \sum_{g\in\mathcal{G}} \frac{1}{M}\int_{\mathcal{D}_g}\left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\psi_g(x)^{\lambda-1}\cdot\pi_g(x)\,dx \\
&= \sum_{g\in\mathcal{G}}\left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\left(\frac{1}{M}\int_{\mathcal{D}_g}\psi_g(x)^{\lambda-1}\,dx\right)\cdot\pi_g.
\end{aligned}$$

Next, using the fact that $s_g^L = (\frac{w_g}{A_g})^{1-\lambda}\cdot(\frac{1}{M}\int_{\mathcal{T}_g}\psi_g(x)^{\lambda-1}\,dx)$, we get

$$\Delta = \sum_{g\in\mathcal{G}} s_g^L\cdot\frac{\frac{1}{M}\int_{\mathcal{D}_g}\psi_g(x)^{\lambda-1}\,dx}{\frac{1}{M}\int_{\mathcal{T}_g}\psi_g(x)^{\lambda-1}\,dx}\cdot\pi_g = \sum_{g\in\mathcal{G}} s_g^L\cdot d\ln\Gamma_g^{\text{auto}}\cdot\pi_g.$$

Substituting this expression for Δ into equation (A-7) and using the dual representation of TFP in equation (A-6), we obtain (8).

Equation (7) can be obtained from (8) by using the fact that $d\ln y = d\ln \text{tfp} + s^K\cdot d\ln k$. Moreover, $k = s^K\cdot y$, which implies $d\ln k = d\ln s^K + d\ln y$. Combining this expression with the equation for $d\ln y$, we obtain

$$d\ln y = \frac{1}{1-s^K}\big(d\ln\text{tfp} + s^K\cdot d\ln s^K\big) \quad\text{and}\quad d\ln k = \frac{1}{1-s^K}\big(d\ln\text{tfp} + d\ln s^K\big).$$

To derive the factor share changes, note that

$$d\ln s^K = (\lambda-1)\cdot\big(d\ln A_k + d\ln\Gamma_k^{\text{deep}}\big) + d\ln\Gamma_k^{\text{auto}},$$

which follows from the fact that $s^K = A_k^{\lambda-1}\cdot\Gamma_k$. We can rewrite this expression as

$$\begin{aligned}
d\ln s^K &= (\lambda-1)\cdot\big(d\ln A_k + d\ln\Gamma_k^{\text{deep}}\big) + \frac{1}{s^K}\cdot\left((\lambda-1)\cdot\Delta + \sum_{g\in\mathcal{G}} s_g^L\cdot d\ln\Gamma_g^{\text{auto}}\right) \\
&= (\lambda-1)\cdot\big(d\ln A_k + d\ln\Gamma_k^{\text{deep}}\big) + \frac{1}{s^K}\cdot\sum_{g\in\mathcal{G}} s_g^L\cdot d\ln\Gamma_g^{\text{auto}}\cdot\big(1+(\lambda-1)\cdot\pi_g\big),
\end{aligned}$$

which yields equation (9), completing the proof. *Q.E.D.*

PROOF OF PROPOSITION 3: The equilibrium of the multi-sector economy is a solution to the following optimization problem:

$$\max_{\{k(x),\ell_1(x),\ldots,\ell_G(x)\}_{x\in\mathcal{T}_i,i\in\mathcal{I}}} H(y_1,\ldots,y_I)-\int_{\mathcal{T}}\big(k(x)/q(x)\big)\cdot dx$$

$$\text{subject to: } y_i=\left(\frac{1}{M}\int_{\mathcal{T}}\big(M\cdot y(x)\big)^{\frac{\lambda-1}{\lambda}}\cdot dx\right)^{\frac{\lambda}{\lambda-1}}\quad\forall i\in\mathcal{I},$$

$$y(x)=A_k\cdot\psi_k(x)\cdot k(x)+\sum_{g\in\mathcal{G}}A_g\cdot\psi_g(x)\cdot\ell_g(x)\quad\forall x\in\mathcal{T},$$

$$\ell_g=\int_{\mathcal{T}}\ell_g(x)\cdot dx\quad\forall g\in\mathcal{G}.$$

As in the proof of Proposition 1, this is a concave problem, and under the conditions of Proposition A-1, it has a solution and reaches a unique maximal value. As before, let w_g be the Lagrange multiplier for the constraint for labor of type g. The allocation of tasks to factors is unique (under our tie-breaking rule from footnote 7) and given by:

$$\mathcal{T}_{gi}\subseteq\left\{x:\frac{w_g}{A_{gi}\cdot\psi_g(x)}\le\frac{w_{g'}}{A_{g'i}\cdot\psi_{g'}(x)},\frac{1}{\psi_k(x)\cdot q(x)\cdot A_{ki}}\text{ for all }g'\right\},$$

$$\mathcal{T}_{ki}\subseteq\left\{x:\frac{1}{\psi_k(x)\cdot q(x)\cdot A_{ki}}\le\frac{w_g}{A_{gi}\cdot\psi_g(x)},\text{ for all }g\right\}.$$

As in the proof of Proposition 1, the demand for task x in sector i is

$$y(x)=\frac{1}{M_i}\cdot y\cdot s_i^Y(\boldsymbol{p})\cdot p(x)^{-\lambda}\cdot(A_ip_i)^{\lambda-1},$$

the price of task x is

$$p(x)=\begin{cases}\dfrac{1}{A_k\cdot q(x)\cdot\psi_k(x)} & \text{if } x\in\mathcal{T}_{ki},\\[2mm] \dfrac{w_g}{A_g\cdot\psi_k(x)} & \text{if } x\in\mathcal{T}_{gi},\end{cases}$$

and the demand for capital and labor at task x is

$$\frac{k(x)}{q(x)}=\begin{cases}\dfrac{1}{M_i}\cdot y\cdot s_i^Y(\boldsymbol{p})\cdot(A_ip_i)^{\lambda-1}\cdot\big(A_k\cdot q(x)\cdot\psi_k(x)\big)^{\lambda-1} & \text{if } x\in\mathcal{T}_{ki},\\ 0 & \text{if } x\notin\mathcal{T}_k,\end{cases}$$

$$\ell_g(x)=\begin{cases}\dfrac{1}{M_i}\cdot y\cdot s_i^Y(\boldsymbol{p})\cdot(A_ip_i)^{\lambda-1}\cdot\big(A_g\cdot\psi_g(x)\big)^{\lambda-1}\cdot w_g^{-\lambda} & \text{if } x\in\mathcal{T}_g,\\ 0 & \text{if } x\notin\mathcal{T}_g.\end{cases}$$

Following the same steps as in the proof of Proposition 1, we have

$$\ell_g=\sum_{i\in\mathcal{I}}\int_{\mathcal{T}_{gi}}\frac{1}{M_i}\cdot y\cdot s_i^Y(\boldsymbol{p})\cdot(A_ip_i)^{\lambda-1}\cdot\big(A_g\cdot\psi_g(x)\big)^{\lambda-1}\cdot w_g^{-\lambda}\cdot dx$$

$$\Rightarrow \quad w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p}) \cdot (A_i p_i)^{\lambda-1} \cdot \frac{1}{M_i}\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}\,dx\right)^{\frac{1}{\lambda}},$$

which establishes equation (10).

To derive the industry price index in equation (11), we observe that

$$p_i \cdot y_i = \int_{\mathcal{T}_i} p(x)\cdot y(x)\,dx \quad \Rightarrow \quad p_i = \frac{1}{A_i}\left(\frac{1}{M_i}\int_{\mathcal{T}_i} p(x)^{1-\lambda}\,dx\right)^{\frac{1}{1-\lambda}}.$$

Equation (12) then follows by substituting for the equilibrium task prices to obtain

$$\begin{aligned} p_i &= \frac{1}{A_i}\left(\frac{1}{M_i}\int_{\mathcal{T}_i} p(x)^{1-\lambda}\,dx\right)^{\frac{1}{1-\lambda}} \\ &= \frac{1}{A_i}\Bigg(A_k \cdot \left(\frac{1}{M_i}\int_{\mathcal{T}_{ki}} \big(q(x)\cdot\psi_k(x)\big)^{\lambda-1}\,dx\right) \\ &\quad + \sum_{g\in\mathcal{G}} w_g^{1-\lambda}\cdot A_g^{\lambda-1}\cdot\left(\frac{1}{M_i}\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}\,dx\right)\Bigg)^{\frac{1}{1-\lambda}}. \end{aligned}$$

Because industry shares must add up to 1, equation (12) holds, completing the proof.

Although not included in the proposition, factor shares can be computed as

$$\begin{aligned} s^K &= A_k^{\lambda-1}\cdot\sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p})\cdot(A_i p_i)^{\lambda-1}\cdot\Gamma_{ki} \quad \text{and} \\ s^L &= 1 - A_k^{\lambda-1}\cdot\sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p})\cdot(A_i p_i)^{\lambda-1}\cdot\Gamma_{ki}. \end{aligned}$$

Q.E.D.

PROOF OF PROPOSITION 4: We first provide a proof for the existence and the properties of the propagation matrix Θ.

Define the matrix

$$\Sigma = \mathbb{1} - \frac{1}{\lambda}\frac{\partial \ln \Gamma(\boldsymbol{w}, \boldsymbol{\zeta}, \Psi)}{\partial \ln \boldsymbol{w}}.$$

We now establish several properties of this matrix. First, because $\partial\Gamma_g/\partial w_{g'} \geq 0$, all of its off-diagonal entries are negative. This implies that Σ is a Z-matrix.

Second, Σ has a positive dominant diagonal. This follows from the fact that

$$\Sigma_{gg} = 1 - \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln w_g} > 0,$$

and

$$\Sigma_{gg} - \sum_{g'\neq g}|\Sigma_{gg'}| = 1 - \sum_{g'}\frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} > 1.$$

This last inequality follows because $\sum_{g'}\frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} \leq 0$, which is true since when all wages rise by the same amount, workers lose tasks to capital but do not experience task reallocation among them.

Third, all eigenvalues of Σ have a real part that exceeds 1. This follows from the Gershgorin circle theorem, which states that for each eigenvalue ε of Σ, we can find a dimension g such that $\|\varepsilon - \Sigma_{gg}\| < \sum_{g'\neq g} |\Sigma_{gg'}|$. This inequality implies

$$\Re(\varepsilon) \in \left[\Sigma_{gg} - \sum_{g'\neq g} |\Sigma_{gg'}|, \Sigma_{gg} + \sum_{g'\neq g} |\Sigma_{gg'}|\right].$$

Because $\Sigma_{gg} - \sum_{g'\neq g} |\Sigma_{gg'}| > 1$ for all g, as shown above, all eigenvalues of Σ have a real part that is greater than 1.

Fourth, since Σ has negative off-diagonal elements and all of its eigenvalues have a positive real part, we can conclude that it is an M-matrix. Because Σ is an M-matrix, its inverse Θ exists and has positive and real entries, $\theta_{gg'} \geq 0$, as desired. Moreover, each eigenvalue of Θ has a real part that is positive and less than 1. Finally, the row and column sums of Θ are also less than 1. In particular, let us denote by θ^r_g the sum of the elements of row g of Θ. Then,

$$\Theta \cdot (1, 1, \dots, 1)'_1 = \left(\theta^r_1, \theta^r_2, \dots, \theta^r_G\right)' \Rightarrow \Sigma \cdot \left(\theta^r_1, \theta^r_2, \dots, \theta^r_G\right)' = (1, 1, \dots, 1)'.$$

This equality requires that

$$\Sigma_{gg} \cdot \theta^r_g + \sum_{g'\neq g} \Sigma_{gg'} \cdot \theta^r_{g'} = 1. \tag{A-8}$$

Now, suppose without loss of generality that $\theta^r_1 > \theta^r_2 > \cdots > \theta^r_G > 0$ (all rows must have strictly positive sums, since $\theta_{gg'} = 0$ for all g' would imply that Θ is singular, contradicting the fact that all its eigenavlues have real parts in $(0, 1)$). Equation (A-8) for $g = 1$ gives

$$\Sigma_{11} \cdot \theta^r_1 + \sum_{g'\neq 1} \Sigma_{1g'} \cdot \theta^r_{g'} = 1,$$

and thus

$$\left(1 - \frac{1}{\lambda}\frac{\partial \ln \Gamma_1}{d \ln w_1}\right) \cdot \theta^r_1 = 1 + \frac{1}{\lambda}\sum_{g'\neq 1} \frac{\partial \ln \Gamma_1}{d \ln w_{g'}} \cdot \theta^r_{g'} \leq 1 + \frac{1}{\lambda}\sum_{g'\neq 1} \frac{\partial \ln \Gamma_1}{d \ln w_{g'}} \cdot \theta^r_1.$$

Because $\sum_{g'} \frac{\partial \ln \Gamma_1}{d \ln w_{g'}} \leq 0$, we can rewrite this inequality as

$$\theta^r_1 < 1 + \frac{1}{\lambda}\sum_{g'} \frac{\partial \ln \Gamma_1}{d \ln w_{g'}} \cdot \theta^r_1 \leq 1.$$

An identical argument establishes that column sums of Θ are between 0 and 1.

We next derive the formulas characterizing the effects of technology on wages, industry prices, and TFP. First, define $w^e_g = w_g/A_g$ as the wage per efficiency unit of labor of g workers. Equation (10) then implies

$$w^e_g = \left(\frac{y}{A_g \cdot \ell_g}\right)^{\frac{1}{\lambda}} \cdot \Gamma_g(\boldsymbol{w}, \boldsymbol{\zeta}, \Psi)^{\frac{1}{\lambda}}.$$

Log-differentiating this equation with respect to an automation technology, we obtain

$$d\ln w_g^e = \frac{1}{\lambda} d\ln y - \frac{1}{\lambda} d\ln \Gamma_g^{\text{auto}} + \frac{1}{\lambda}\sum_{i\in\mathcal{I}} \omega_{gi}\cdot d\ln\zeta_i + \frac{1}{\lambda}\frac{\partial \ln\Gamma_g}{\partial \ln \boldsymbol{w}}\cdot d\ln \boldsymbol{w}.$$

Stacking these equations for all groups, we have

$$\begin{pmatrix} d\ln w_1^e \\ d\ln w_1^e \\ \cdots \\ d\ln w_G^e \end{pmatrix} = \frac{1}{\lambda}\begin{pmatrix} d\ln y \\ d\ln y \\ \cdots \\ d\ln y \end{pmatrix} + \frac{1}{\lambda}\begin{pmatrix} \sum_{i\in\mathcal{I}} \omega_{1i}\cdot d\ln\zeta_i \\ \sum_{i\in\mathcal{I}} \omega_{2i}\cdot d\ln\zeta_i \\ \cdots \\ \sum_{i\in\mathcal{I}} \omega_{Gi}\cdot d\ln\zeta_i \end{pmatrix} - \frac{1}{\lambda}\begin{pmatrix} d\ln \Gamma_1^{\text{auto}} \\ d\ln \Gamma_2^{\text{auto}} \\ \cdots \\ d\ln \Gamma_G^{\text{auto}} \end{pmatrix}$$

$$+ \frac{1}{\lambda}\frac{\partial \ln \Gamma}{\partial \ln \boldsymbol{w}}\cdot \begin{pmatrix} d\ln w_1^e \\ d\ln w_1^e \\ \cdots \\ d\ln w_G^e \end{pmatrix},$$

which yields

$$\begin{pmatrix} d\ln w_1^e \\ d\ln w_1^e \\ \cdots \\ d\ln w_G^e \end{pmatrix} = \frac{1}{\lambda}\Theta\cdot\begin{pmatrix} d\ln y \\ d\ln y \\ \cdots \\ d\ln y \end{pmatrix} + \frac{1}{\lambda}\Theta\cdot\begin{pmatrix} \sum_{i\in\mathcal{I}} \omega_{1i}\cdot d\ln\zeta_i \\ \sum_{i\in\mathcal{I}} \omega_{2i}\cdot d\ln\zeta_i \\ \cdots \\ \sum_{i\in\mathcal{I}} \omega_{Gi}\cdot d\ln\zeta_i \end{pmatrix} - \frac{1}{\lambda}\Theta\cdot\begin{pmatrix} d\ln \Gamma_1^{\text{auto}} \\ d\ln \Gamma_2^{\text{auto}} \\ \cdots \\ d\ln \Gamma_G^{\text{auto}} \end{pmatrix},$$

and thus

$$d\ln w_g = \frac{\varepsilon_g}{\lambda} d\ln y + \frac{1}{\lambda}\Theta_g\cdot d\ln\zeta - \frac{1}{\lambda}\Theta_g\cdot d\ln\Gamma^{\text{disp}},$$

where

$$d\ln\zeta_g = \sum_{i\in\mathcal{I}} \omega_{gi}\cdot d\ln\zeta_i = \sum_{i\in\mathcal{I}}\omega_{gi}\cdot\left(\frac{\partial \ln s_i^Y(\boldsymbol{p})}{\partial \ln \boldsymbol{p}}\cdot d\ln\boldsymbol{p} + (\lambda-1)\cdot d\ln p_i\right).$$

Turning to industry prices, note that these are given by equation (12). By definition, the equilibrium task allocation $\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \ldots, \mathcal{T}_{Gi}\}$ solves the cost-minimization problem:

$$p_i = \min_{\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \ldots, \mathcal{T}_{Gi}\}} \frac{1}{A_i}\left(A_k^{\lambda-1}\cdot\Gamma_{ki} + \sum_{g\in\mathcal{G}} w_g^{1-\lambda}\cdot A_g^{\lambda-1}\cdot\Gamma_{gi}\right)^{\frac{1}{1-\lambda}}.$$

The envelope theorem then implies that

$$d\ln p_i = \sum_{g\in\mathcal{G}} s_{gi}^L\cdot d\ln w_g - \Delta_i,$$

where

$$\Delta_i = (A_i p_i)^{\lambda-1}\frac{1}{\lambda-1}\left[A_k^{\lambda-1}\cdot d\Gamma_{ki}^{\text{auto}} - \sum_{g\in\mathcal{G}} w_g^{1-\lambda}\cdot A_g^{\lambda-1}\cdot d\Gamma_{gi}^{\text{auto}}\right]$$

is cost savings from the reallocation of tasks from labor to capital and industry i, and is thus a generalization of the term Δ in the proof of Proposition 2.

Average cost savings from automating tasks in the set $\mathcal{D}_{gi}$ in industry i are now

$$\pi_{gi} = \frac{\frac{1}{M_i}\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}\cdot\pi_{gi}(x)\,dx}{\frac{1}{M_i}\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}\,dx},$$

where

$$\pi_{gi}(x) = \frac{1}{\lambda-1}\left[\left(w_g\frac{A_k\cdot q(x)\cdot\psi_k(x)}{A_g\cdot\psi_g(x)}\right)^{\lambda-1} - 1\right] > 0.$$

Using these definitions, and following the same steps as in the proof of Proposition 2, we can write Δ_i as

$$\Delta_i = (A_i p_i)^{\lambda-1}\sum_{g\in\mathcal{G}}\left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\left(\frac{1}{M_i}\int_{\mathcal{D}_{gi}}\psi_g(x)^{\lambda-1}\,dx\right)\cdot\pi_{gi}.$$

Again as in the proof of Proposition 2, we use $s_{gi}^L = (A_i p_i)^{\lambda-1}(\frac{w_g}{A_g})^{1-\lambda}\cdot(\frac{1}{M_i}\int_{\mathcal{T}_g}\psi_g(x)^{\lambda-1}\,dx)$ to get

$$\Delta_i = \sum_{g\in\mathcal{G}} s_{gi}^L\cdot\frac{\frac{1}{M_i}\int_{\mathcal{A}_{gi}}\psi_g(x)^{\lambda-1}\,dx}{\frac{1}{M_i}\int_{\mathcal{T}_g}\psi_g(x)^{\lambda-1}\,dx}\cdot\pi_{gi} = \sum_{g\in\mathcal{G}} s_{gi}^L\cdot d\ln\Gamma_{gi}^{\text{auto}}\cdot\pi_{gi},$$

which yields the desired formula for $d\ln p_i$ in the proposition.

To derive a formula for TFP, first note that given a price vector $\boldsymbol{p}$, we can define the cost of producing the final good as $c^h(\boldsymbol{p})$. Moreover, Shephard's lemma implies that

$$\frac{\partial c^h(\boldsymbol{p})}{\partial p_i}\frac{p_i}{c^h} = s_i^Y(\boldsymbol{p}).$$

Our choice of numeraire, which implies that the final good has a price of 1, then implies that $1 = c^h(\boldsymbol{p})$. Log-differentiating this expression yields

$$\begin{aligned} 0 &= \sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p})\cdot d\ln p_i \\ &= \sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p})\cdot\left(\sum_{g\in\mathcal{G}} s_{gi}^L\cdot\left(d\ln w_g - d\ln\Gamma_{gi}^{\text{auto}}\cdot\pi_{gi}\right)\right)\end{aligned}$$

$$= \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln w_g - \sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p}) \sum_{g\in\mathcal{G}} \cdot s_{gi}^L \cdot \pi_{gi}.$$

Rearranging this expression, and using the dual representation of TFP (which in this case is given by $d\ln \text{tfp} = \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln w_g$) yields the formula for the contribution of automation to TFP in the proposition.

For aggregate output, we again have $d\ln y = d\ln \text{tfp} + s^K \cdot d\ln k$ (from the primal definition of TFP) and $d\ln k = d\ln s^K + d\ln y$ (from $k = s^K \cdot y$). Combining these equations, we obtain

$$d\ln y = \frac{1}{1-s^K}\left(d\ln \text{tfp} + s^K \cdot d\ln s^K\right) \quad \text{and} \quad d\ln k = \frac{1}{1-s^K}\left(d\ln \text{tfp} + d\ln s^K\right).$$

Finally, the change in the capital share is given by

$$d\ln s^K = -\frac{1-s^K}{s^K} d\ln s^L = -\frac{1}{s^K}\sum_{g\in\mathcal{G}} s_g^L \cdot (d\ln w_g - d\ln y).$$

Q.E.D.

A-3. *Measuring Task Displacement*

This section derives our measures of task displacement. To derive the adjustments we perform in our empirical work, we also allow for markups and differences in the user cost of capital across industries. In the presence of these generalizations, the labor share of industry i can be written as

$$s_i^L = \frac{1}{\mu_i} \cdot \frac{\sum_g \Gamma_{gi} \cdot w_g^{1-\lambda}}{\sum_g \Gamma_{gi} \cdot w_g^{1-\lambda} + \Gamma_{ki} \cdot R_i^{1-\lambda}}, \tag{A-9}$$

where μ_i and R_i are, respectively, the markup and user cost of capital in industry i.

We assume that tasks can be partitioned into routine tasks $\mathcal{R}_i$ and non-routine tasks $\mathcal{N}_i$, whose union equals $\mathcal{T}_i$. Moreover, let $\mathcal{R}_{gi}$ and $\mathcal{N}_{gi}$ denote the (disjoint) sets of routine and non-routine tasks allocated to workers of type g.

Assumption 2 implies that only routine tasks can be automated, that is, $\mathcal{D}_{gi} \subset \mathcal{R}_{gi}$, and also that routine tasks in a given industry will be automated at the same rate for all workers. Therefore,

$$\frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}\,dx}{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}\,dx} = \vartheta_i \geq 0 \quad \text{for all } g.$$

Before continuing with our derivations, we introduce some notation that we will use in the rest of the appendix. Define by ω_X^Y the share of wages in some cell X earned within another sub-cell Y. For example, define ω_g^i as the share of wages earned by members of group g in industry i as a fraction of their total wage income:

$$\omega_g^i = \frac{s_i^Y(\boldsymbol{p}) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{gi}}{\sum_{i'\in\mathcal{I}} s_{i'}^Y(\boldsymbol{p}) \cdot (A_{i'} p_{i'})^{\lambda-1} \cdot \Gamma_{gi'}}.$$

Define ω_{gi}^R as the share of wages earned by members of group g in industry i in routine jobs as a fraction of the total wage income earned by workers of group g in industry i:

$$\omega_{gi}^R = \frac{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}\,dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}\,dx}.$$

And define ω_i^R as the share of wages earned by workers in industry i in routine jobs as a fraction of the total wage income earned by workers in industry i:

$$\omega_i^R = \frac{\sum_{g\in\mathcal{G}} w_g^{1-\lambda} \int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}\,dx}{\sum_{g\in\mathcal{G}} w_g^{1-\lambda} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}\,dx}.$$

Average cost savings from automation in industry i are

$$\pi_i = \sum_{g\in\mathcal{G}} \frac{\omega_i^{Rg}}{\omega_i^R} \cdot \pi_{gi},$$

where ω_i^{Rg} is the share of wages in industry i paid to g workers in routine jobs, and ω_i^R is the share of wages in industry i paid to workers in routine jobs.

The next proposition characterizes the change in the labor share in response to automation.

PROPOSITION A-2—Task displacement and industry labor shares: *Suppose that Assumption 2 holds and routine tasks in industry i are automated at the rate ϑ_i. The resulting change in the labor share of industry i holding wages, markups, and other technologies constant is given by*

$$d\ln s_i^{L,\text{auto}} = -\big(1 + (\lambda - 1)\cdot s_i^L \cdot \pi_i\big)\cdot \omega_i^R \cdot \vartheta_i.$$

This implies that the task displacement due to automation for group g in industry i is

$$d\ln \Gamma_{gi}^{\text{auto}} = \frac{\omega_{gi}^R}{\omega_i^R} \cdot \frac{-d\ln s_i^{L,\text{auto}}}{1 + (\lambda - 1)\cdot s_i^L \cdot \pi_i}.$$

PROOF: The denominator in equation (A-9) is also equal to

$$(A_i p_i)^{1-\lambda} = A_k^{\lambda-1}\cdot \Gamma_{ki} + \sum_{g\in\mathcal{G}} w_{gi}^{e\;1-\lambda}\cdot \Gamma_{gi}.$$

The effect of automation on s_i^L (holding prices and other technologies constant) is

$$d\ln s_i^{L,\text{auto}} = -\sum_{g\in\mathcal{G}} \omega_i^{Rg}\cdot \vartheta_i + (1-\lambda)\cdot s_i^L \cdot \sum_{g\in\mathcal{G}} \omega_i^{Rg}\cdot \vartheta_i \cdot \pi_{gi},$$

where the first term captures the effect of automation on the numerator of (A-9) and the second term the effect on the denominator of (A-9). Using the definition of π_i, this can be written as

$$d\ln s_i^{L,\text{auto}} = -\big(1 + (\lambda - 1)\cdot s_i^L \cdot \pi_i\big)\cdot \omega_i^R \cdot \vartheta_i.$$

Turning to the second part of the proposition, by definition we have

$$d\ln \Gamma_{gi}^{\text{auto}} = \frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}\,dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}\,dx} = \frac{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}\,dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}\,dx}\cdot \frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}\,dx}{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}\,dx} = \omega_{gi}^R \cdot \vartheta_i$$
$$= \frac{\omega_{gi}^R}{\omega_i^R}\cdot \frac{-d\ln s_i^{L,\text{auto}}}{1 + (\lambda-1)\cdot s_i^L \cdot \pi_i}. \qquad \textit{Q.E.D.}$$

We now provide the details for our measurement of the (percent) decline in the labor share of industry i driven by automation, $d\ln s_i^{L,\text{auto}}$. Differentiating (A-9), we have

$$-d\ln s_i^L = -d\ln s_i^{L,\text{auto}} - s_i^K \cdot \big[\big(1-\sigma_i^L\big)\cdot d\ln w_i - \big(1-\sigma_i^K\big)\cdot d\ln R_i\big] + d\ln \mu_i + \varepsilon_i. \qquad \text{(A-10)}$$

Here, $d\ln \mu_i$ is the (percent) increase in industry i markups and ε_i is a residual term that captures the role of other technologies (factor-augmenting and productivity-deepening technologies on the labor share). The term $s_i^K \cdot [(1-\sigma_i^L)\cdot d\ln w_i - (1-\sigma_i^K)\cdot d\ln R_i]$ adjusts for the effect of changing factor prices on the labor share. In particular, $d\ln w_i = \sum_{g\in\mathcal{G}} \omega_i^g \cdot d\ln w_g$ denotes the average wage increase experienced by industry i, and the elasticities σ_i^L and σ_i^K give the effect of changing factor prices on the labor share. In a world with a single labor aggregate, we would have $\sigma_i^L = \sigma_i^K = \sigma_i$, where σ_i is the elasticity of substitution between capital and this labor aggregate. However, with multiple types of workers, σ_i^L varies depending on whether groups experiencing a wage increase are more or less substitutable for capital at marginal tasks. Finally, ε_i denotes the influence of other technologies on the labor share. Appendix B-4 provides a full derivation of equation (A-10) and shows that the contribution of ε_i to changes in the labor share between 1987 and 2016 has been small.

Our two measures of task displacement are based on different ways of estimating $-d\ln s_i^{L,\text{auto}}$. In both cases, we approximate the discrete changes between 1987 and 2016 with our theory-based differential changes.

- Our first measure of task displacement, exploiting the observed changes in industry labor shares, is based on setting $\lambda = 1$, $d\ln\mu_i = 0$, $\varepsilon_i = 0$, and using Assumption 1 to rule out ripple effects. Under these assumptions, $\sigma_i^L = \sigma_i^K = 1$, and equation (A-10) implies

$$-d\ln s_i^{L,\text{auto}} = -d\ln s_i^L.$$

- Our second measure of task displacement, exploiting the automation-driven component of changes in industry labor share, proceeds as follows. We again set $\lambda = 1$, $d\ln\mu_i = 0$, $\varepsilon_i = 0$ and use Assumption 1 to rule out ripple effects, so that $\sigma_i^L = \sigma_i^K = 1$. However, instead of using the full observed change in industry labor shares, we use its component that is (linearly) predicted by our three proxies for automation technologies:

$$-d\ln s_i^{L,\text{auto}} = \mathbb{E}\big[-d\ln s_i^L | Z_i\big],$$

where Z_i denotes the vector of the three measures of automation technologies for industry i. This strategy also works when there are markup differences and other influences on labor shares, and in this case, relies on the formal identifying assumption: $Z_i \perp\!\!\!\perp \mu_i, \varepsilon_i$, meaning that these differences are orthogonal to our instruments (as noted in the text).

- In Section 5 and Table A.IV in this supplement, we generalize these measures and allow for $\sigma_i^L = \sigma_i^K = \sigma_i = 0.8$ and $\sigma_i^L = \sigma_i^K = \sigma_i = 1.2$. In this case, our first measure is computed simply as

$$-d\ln s_i^{L,\text{auto}} = -d\ln s_i^L + s_i^K \cdot (1-\sigma_i)\cdot (d\ln w_i - d\ln R_i).$$

- Similarly, for our second measure, we compute $-d\ln s_i^{L,\text{auto}} = \mathbb{E}[-d\ln s_i^L + s_i^K \cdot (1 - \sigma_i)\cdot (d\ln w_i - d\ln R_i)|Z_i]$ as the predicted component of a linear regression of the adjusted decline in the labor share across industries on our proxies of automation. This is again valid when there are markup differences and other influences on labor shares under the assumption that $Z_i \perp\!\!\!\perp \mu_i, \varepsilon_i$.
- In Table 6, we allow for the effects of changes in markups. In this case, we continue to set $\lambda = 1$ and abstract from ripple effects, but now adjust for the estimated change in markups. Equation (A-10) now gives

$$-d\ln s_i^{L,\text{auto}} = -d\ln s_i^L - d\ln \mu_i$$

for our first measure and

$$-d\ln s_i^{L,\text{auto}} = \mathbb{E}\big[-d\ln s_i^L - d\ln \mu_i | Z_i\big]$$

for our second measure. This is again valid when there are other influences on labor shares under the assumption that $Z_i \perp\!\!\!\perp \varepsilon_i$.

A-4. *Additional Tables*

This appendix includes additional tables discussed in the main text:

- Table A.I: Determinants of industry-level labor share changes, 1987–2016.
- Table A.II: Summary statistics for demographic groups.
- Table A.III: Task displacement versus SBTC, 1980–2016—controlling for changes in relative supply.
- Table A.IV: Task displacement based on adjusted labor share declines and changes in real hourly wages—measures of task displacement based on adjusted labor share decline.
- Table A.V: Task displacement and changes in real hourly wages, 1980–2016—alternative measures of jobs that can be automated.
- Table A.VI: Task displacement and changes in real hourly wages, 1980–2016—controlling for differential effects on low-paying jobs.
- Table A.VII: Task displacement and changes in real hourly wages for men, women, and native-born workers, 1980–2016.

- Table A.VIII: Task displacement and changes in real hourly wages, stacked-differences models for 1980–2000 and 2000–2016.
- Table A.IX: Task displacement and changes in real hourly wages, 1980–2016—alternative labor share measures.
- Table A.X: GMM estimates of the propagation matrix.
- Table A.XI: Robustness checks for estimates of the general equilibrium effects.

TABLE A.I

DETERMINANTS OF INDUSTRY-LEVEL LABOR SHARE CHANGES, 1987–2016.

	Dependent Variable: Percent Labor Share Changes, 1987–2016								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Adjusted penetration of robots	−1.27		−0.92	−0.90	−0.93	−0.94	−1.00	−0.94	−0.73
	(0.35)		(0.42)	(0.40)	(0.42)	(0.43)	(0.42)	(0.43)	(0.53)
Change in share of dedicated machinery services		−3.32	−2.42	−2.37	−2.43	−2.40	−2.18	−2.43	−2.13
		(0.59)	(0.72)	(0.75)	(0.71)	(0.73)	(0.77)	(0.73)	(0.80)
Change in share of specialized software services	−6.33	−5.81	−6.95	−6.67	−6.99	−6.87	−6.97	−7.02	−6.84
	(1.71)	(1.87)	(1.57)	(1.71)	(1.69)	(1.75)	(1.78)	(1.64)	(1.50)
Change in share of imported intermediates				−0.56					
				(0.55)					
Change in K/Y ratio					−0.01				
					(0.03)				
Change tail index of revenue concentration						−0.06			
						(0.23)			
Change in accounting markups (%)							−0.24		
							(0.36)		
Change Chinese import competition								0.12	
								(0.25)	
De-unionization rate									−0.25
									(0.26)
F-stat technology variables	11.02	18.02	16.04	13.65	14.82	14.83	11.53	15.04	9.37
Share variance explained by technology	0.33	0.33	0.45	0.44	0.45	0.45	0.45	0.46	0.40
R-squared	0.33	0.33	0.45	0.46	0.45	0.45	0.47	0.45	0.47
Observations	49	49	49	49	49	49	49	49	49

Note: This table presents estimates of the relationship between labor share changes (in %) between 1987 and 2016 at the industry level and automation technologies, offshoring, capital deepening, changes in market structure (proxied by markups or rising sales concentration), and changes in Chinese import competition for the 49 industries in our analysis. All regressions are weighted by industry value added in 1987. Standard errors robust to heteroskedasticity are reported in parentheses.

TABLE A.II

SUMMARY STATISTICS FOR DEMOGRAPHIC GROUPS.

			Labor-Market Outcomes			Educational Levels and Gender				
Quintile	N	Task Displacement	Change in Hourly Wages, 1980–2016	Employment to Population Ratio Change 1980–2016	Hourly Wage 1980	Completed High School	Some College	Completed College	Post-Graduate	Share Male
PANEL A. QUINTILES OF TASK DISPLACEMENT BASED ON LABOR SHARE DECLINES										
1—Lowest	191	4.77%	26.51%	0.00 p.p.	$26.9	0.05%	12.21%	42.10%	44.84%	80.00%
2	141	15.53%	5.91%	−0.80 p.p.	$18.3	17.54%	69.15%	1.81%	0.13%	61.79%
3	63	21.01%	3.07%	−3.71 p.p.	$17.3	72.96%	13.17%	0.18%	0.00%	55.49%
4	69	24.95%	−5.06%	−8.72 p.p.	$15.1	36.93%	19.39%	0.01%	0.00%	66.33%
5—Highest	36	28.87%	−11.95%	−16.23 p.p.	$15.7	61.19%	1.19%	0.01%	0.00%	99.34%
PANEL B. QUINTILES OF TASK DISPLACEMENT BASED ON AUTOMATION-DRIVEN LABOR SHARE DECLINES										
1—Lowest	182	3.80%	26.52%	0.01 p.p.	$27.0	0.02%	12.04%	42.71%	45.18%	80.60%
2	119	13.95%	3.21%	−2.21 p.p.	$19.3	32.43%	64.46%	1.10%	0.12%	69.30%
3	93	19.22%	5.69%	−0.84 p.p.	$15.6	77.87%	8.99%	0.26%	0.00%	38.39%
4	68	23.11%	−6.07%	−10.41 p.p.	$15.9	43.04%	13.88%	0.01%	0.00%	77.66%
5—Highest	38	27.72%	−11.98%	−18.46 p.p.	$14.5	40.26%	1.62%	0.01%	0.00%	99.86%
PANEL C. ALL WORKERS										
All	500	16.84%	7.18%	−4.80	$19.9	32.83%	22.28%	13.38%	13.87%	72.98%

Note: This table presents summary statistics for the 500 demographic groups used in our analysis. These groups are defined by gender, education, age, race, and native/immigrant status. The table breaks down these groups by quintiles of exposure to task displacement (measured using the percent labor share decline in Panel A and the automation-driven labor share decline in Panel B) and reports summary statistics for all groups in Panel C. See the main text and Appendix B-3 for definitions and data sources.

TABLE A.III

TASK DISPLACEMENT VERSUS SBTC, 1980-2016—CONTROLLING FOR CHANGES IN RELATIVE SUPPLY.

	Dependent Variable: Change in Hourly Wages 1980–2016					
	SBTC by Education Level			Allowing for SBTC by Wage Level		
Task Displacement Measure	(1)	Labor Share Declines (2)	Automation-Driven Declines (3)	(4)	Labor Share Declines (5)	Automation-Driven Declines (6)
Gender: women	0.19	0.09	0.13	0.25	0.16	0.18
	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
Education: no high school	−0.10	−0.04	−0.01	0.04	0.01	0.03
	(0.08)	(0.05)	(0.05)	(0.03)	(0.03)	(0.03)
Education: some college	0.13	−0.07	−0.04	0.03	−0.05	−0.03
	(0.06)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)
Education: full college	0.37	−0.03	0.04	0.19	0.01	0.06
	(0.08)	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)
Education: more than college	0.50	0.03	0.14	0.29	0.07	0.16
	(0.07)	(0.08)	(0.06)	(0.05)	(0.07)	(0.06)
Log of hourly wage in 1980				0.25	0.14	0.13
				(0.06)	(0.05)	(0.06)
Change in supply	−0.10	−0.06	−0.04	−0.01	−0.03	−0.02
	(0.06)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)
Task displacement		−1.72	−1.60		−1.15	−1.06
		(0.31)	(0.28)		(0.21)	(0.25)
Share variance explained by:						
—educational dummies	0.75	0.04	0.18	0.39	0.08	0.19
—baseline wage				0.16	0.09	0.08
—supply changes	−0.28	−0.16	−0.11	−0.04	−0.07	−0.05
—task displacement		0.72	0.63		0.48	0.41
R-squared	0.43	0.75	0.77	0.80	0.83	0.82
Observations	493.00	493.00	493.00	493.00	493.00	493.00
Other covariates:						
Industry shifters and manufacturing share	✓	✓	✓	✓	✓	✓

Note: This table presents estimates of the relationship between SBTC proxies, task displacement, and the change in hourly wages across 500 demographic groups. These groups are defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in hourly wages for each group between 1980 and 2016. Columns 2 and 5 report results using our measure of task displacement based on observed labor share declines. Columns 3 and 6 report results using our measure of task displacement based on automation-driven labor share declines. In all specifications, we measure changes in labor supply by the change in hours worked between 1980 and 2016, and instrument it using the predetermined trend in hours for 1970–1980. In addition to the covariates reported in the table, all specifications control for industry shifters and baseline wage shares in manufacturing. The bottom rows of the table report the share of variance explained by task displacement and the different proxies of skill-biased technical change. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

TABLE A.IV

TASK DISPLACEMENT BASED ON ADJUSTED LABOR SHARE DECLINES AND CHANGES IN REAL HOURLY WAGES—MEASURES OF TASK DISPLACEMENT BASED ON ADJUSTED LABOR SHARE DECLINE.

	Dependent Variables: Change in Wages and Wage Declines, 1980–2016							
	Task Displacement Measured From Observed Labor Share declines				Task Displacement Measured From Automation-Driven Labor Share Declines			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Panel A. Task displacement for $\lambda = 1$ and $\sigma_i = 0.8$							
Task displacement	−1.35	−1.02	−1.19	−2.05	−1.45	−1.13	−1.24	−2.70
	(0.12)	(0.15)	(0.17)	(0.38)	(0.11)	(0.17)	(0.20)	(0.47)
Share variance explained by task displacement	0.57	0.43	0.51	0.87	0.60	0.47	0.51	1.11
R-squared	0.57	0.65	0.84	0.84	0.60	0.63	0.83	0.84
Observations	500	500	500	500	500	500	500	500
	Panel B. Task displacement for $\lambda = 1$ and $\sigma_i = 1.2$							
Task displacement	−1.73	−1.53	−1.26	−0.73	−1.83	−1.71	−1.37	−0.99
	(0.09)	(0.15)	(0.17)	(0.54)	(0.10)	(0.22)	(0.21)	(0.58)
Share variance explained by task displacement	0.71	0.63	0.52	0.30	0.67	0.63	0.50	0.36
R-squared	0.71	0.73	0.83	0.83	0.67	0.67	0.82	0.82
Observations	500	500	500	500	500	500	500	500
	Panel C. Task displacement for $\lambda = 0.5$ and $\sigma_i = 0.8$							
Task displacement	−1.22	−0.92	−1.07	−1.86	−1.30	−1.02	−1.11	−2.40
	(0.10)	(0.14)	(0.16)	(0.35)	(0.10)	(0.15)	(0.18)	(0.42)
Share variance explained by task displacement	0.58	0.44	0.51	0.88	0.60	0.47	0.51	1.10
R-squared	0.58	0.65	0.84	0.84	0.60	0.63	0.83	0.84
Observations	500	500	500	500	500	500	500	500
	Panel D. Task displacement for $\lambda = 0.5$ and $\sigma_i = 1$							
Task displacement	−1.44	−1.19	−1.17	−1.47	−1.48	−1.27	−1.21	−1.64
	(0.08)	(0.14)	(0.17)	(0.40)	(0.09)	(0.18)	(0.19)	(0.43)
Share variance explained by task displacement	0.67	0.56	0.55	0.69	0.64	0.55	0.53	0.71
R-squared	0.67	0.70	0.84	0.84	0.64	0.66	0.83	0.83
Observations	500	500	500	500	500	500	500	500
	Panel E. Task displacement for $\lambda = 0.5$ and $\sigma_i = 1.2$							
Task displacement	−1.54	−1.36	−1.12	−0.63	−1.64	−1.53	−1.23	−0.87
	(0.08)	(0.14)	(0.16)	(0.49)	(0.09)	(0.20)	(0.19)	(0.52)
Share variance explained by task displacement	0.71	0.63	0.52	0.29	0.67	0.63	0.50	0.36
R-squared	0.71	0.73	0.83	0.83	0.67	0.67	0.82	0.82
Observations	500	500	500	500	500	500	500	500
Covariates:								
Industry shifters		✓	✓	✓		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓				✓

Note: This table presents estimates of the relationship between task displacement and the change in hourly wages across demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in hourly wages for each group between 1980 and 2016. We measure task displacement using the general formula in equation (17). Our baseline measure sets $\lambda = \sigma_i = 1$. The panels in this table use different combinations of λ and σ_i to measure the adjusted labor share decline. Columns 1–4 report results for our measure of task displacement based on observed (and adjusted) labor share declines. Columns 5–8 report results for our measure of task displacement based on automation-driven labor share declines. In addition to the covariates reported in the table, columns 3–4 and 7–8 control for baseline wage shares in manufacturing and dummies for education (for no high-school degree, completed high school, some college, college degree and post-graduate degree) and gender, and columns 4 and 8 control for groups' exposure to industry labor share declines and groups' relative specialization in routine jobs. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

TABLE A.V

TASK DISPLACEMENT AND CHANGES IN REAL HOURLY WAGES, 1980–2016—ALTERNATIVE MEASURES OF JOBS THAT CAN BE AUTOMATED.

	Dependent Variables: Change in Wages and Wage Declines, 1980–2016							
	Task Displacement Measured From Observed Labor Share declines				Task Displacement Measured From Automation-Driven Labor Share Declines			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	PANEL A. TOP 40							
Task displacement	−1.39	−1.02	−1.10	−2.50	−1.48	−1.04	−1.15	−2.70
	(0.15)	(0.16)	(0.19)	(0.53)	(0.15)	(0.19)	(0.22)	(0.54)
Share variance explained by task displacement	0.52	0.38	0.41	0.93	0.51	0.36	0.39	0.92
R-squared	0.52	0.64	0.82	0.84	0.51	0.60	0.81	0.83
Observations	500	500	500	500	500	500	500	500
	PANEL B. ALTERNATIVE DEFINITIONS							
Task displacement	−1.88	−1.67	−1.67	−1.79	−1.99	−1.98	−1.87	−1.54
	(0.08)	(0.15)	(0.20)	(0.47)	(0.09)	(0.20)	(0.24)	(0.51)
Share variance explained by task displacement	0.76	0.67	0.67	0.72	0.75	0.74	0.70	0.58
R-squared	0.76	0.77	0.85	0.85	0.75	0.75	0.84	0.84
Observations	500	500	500	500	500	500	500	500
	PANEL C. OCCUPATIONS SUITABLE TO AUTOMATION VIA ROBOTS							
Task displacement	−1.18	−1.16	−0.85	−0.66	−1.26	−1.30	−0.89	−1.41
	(0.08)	(0.11)	(0.16)	(0.29)	(0.09)	(0.15)	(0.18)	(0.34)
Share variance explained by task displacement	0.69	0.68	0.49	0.38	0.65	0.67	0.46	0.72
R-squared	0.69	0.69	0.81	0.82	0.65	0.65	0.80	0.82
Observations	500	500	500	500	500	500	500	500
	PANEL D. OCCUPATIONS SUITABLE TO AUTOMATION VIA SOFTWARE							
Task displacement	−1.76	−1.71	−1.46	−1.55	−1.89	−1.96	−1.50	−2.86
	(0.13)	(0.15)	(0.22)	(0.51)	(0.16)	(0.21)	(0.25)	(0.63)
Share variance explained by task displacement	0.68	0.66	0.56	0.59	0.64	0.67	0.51	0.97
R-squared	0.68	0.68	0.81	0.82	0.64	0.64	0.81	0.82
Observations	500	500	500	500	500	500	500	500
	PANEL E. OCCUPATIONS SUITABLE TO AUTOMATION VIA ROBOTS OR SOFTWARE							
Task displacement	−1.46	−1.42	−1.03	−0.87	−1.50	−1.50	−1.02	−1.38
	(0.09)	(0.12)	(0.17)	(0.32)	(0.11)	(0.15)	(0.20)	(0.38)
Share variance explained by task displacement	0.71	0.69	0.50	0.42	0.66	0.66	0.45	0.61
R-squared	0.71	0.71	0.81	0.82	0.66	0.66	0.80	0.81
Observations	500	500	500	500	500	500	500	500
Covariates:								
Industry shifters		✓	✓	✓		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓				✓

Note: This table presents estimates of the relationship between task displacement and the change in hourly wages across demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in hourly wages for each group between 1980 and 2016. In Panel A, we define routine occupations as the top 40% in the routine index distribution (as opposed to the top 30%). In Panel B, we use an alternative construction of the routine index described in Appendix B-3. In Panel C, we use a measure of occupational suitability to automation via robots from Webb (2020). In Panel D, we use a measure of occupational suitability to automation via software from Webb (2020). In Panel E, we combine these two indices in a single one. Columns 1–4 report results for our measure of task displacement based on observed labor share declines. Columns 5–8 report results for our measure of task displacement based on automation-driven labor share declines. In addition to the covariates reported in the table, columns 3–4 and 7–8 control for baseline wage shares in manufacturing and dummies for education (for no high-school degree, completed high school, some college, college degree and post-graduate degree) and gender, and columns 4 and 8 control for groups' exposure to industry labor share declines and groups' relative specialization in routine jobs. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

TABLE A.VI

TASK DISPLACEMENT AND CHANGES IN REAL HOURLY WAGES, 1980–2016—CONTROLLING FOR DIFFERENTIAL EFFECTS ON LOW-PAYING JOBS.

	Dependent Variables: Change in Hourly Wages, 1980–2016			
	(1)	(2)	(3)	(4)
	PANEL A. TASK DISPLACEMENT BASED ON LABOR SHARE DECLINES			
Task displacement	−1.78	−1.74	−1.51	−1.89
	(0.09)	(0.17)	(0.20)	(0.47)
Relative specialization in low-pay jobs	0.03	0.03	−0.03	−0.12
	(0.03)	(0.03)	(0.03)	(0.05)
Effect mediated through low-pay jobs	0.16	0.13	0.43	0.87
	(0.17)	(0.19)	(0.17)	(0.25)
Industry shifters		0.02	0.06	0.22
		(0.09)	(0.14)	(0.15)
Exposure to industry labor share decline				−0.95
				(0.62)
Relative specialization in routine jobs				0.08
				(0.08)
Share variance explained by task displacement	0.74	0.73	0.63	0.79
R-squared	0.76	0.76	0.85	0.86
Observations	500	500	500	500
	PANEL B. TASK DISPLACEMENT BASED ON AUTOMATION-DRIVEN LABOR SHARE DECLINES			
Task displacement	−1.79	−1.88	−1.45	−1.69
	(0.10)	(0.20)	(0.20)	(0.58)
Relative specialization in low-pay jobs	0.05	0.05	−0.01	−0.08
	(0.04)	(0.04)	(0.03)	(0.05)
Effect mediated through low-pay jobs	−0.04	−0.04	0.31	0.75
	(0.29)	(0.29)	(0.26)	(0.34)
Industry shifters		−0.05	−0.05	0.12
		(0.11)	(0.14)	(0.16)
Exposure to industry labor share decline				−1.96
				(0.74)
Relative specialization in routine jobs				0.05
				(0.10)
Share variance explained by task displacement	0.70	0.73	0.56	0.66
R-squared	0.71	0.71	0.84	0.84
Observations	500	500	500	500
Other covariates:				
Manufacturing share, education and gender dummies			✓	✓

Note: This table presents estimates of the relationship between task displacement and the change in hourly wages across 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in hourly wages for each group between 1980 and 2016. Panel A reports results for our measure of task displacement based on observed labor share declines. Panel B reports results for our measure of task displacement based on automation-driven labor share declines. In addition to the covariates reported in the table, columns 3 and 4 control for baseline wage shares in manufacturing and dummies for education (for no high-school degree, completed high school, some college, college degree and post-graduate degree) and gender. All specifications control for groups' relative specialization in low-pay jobs (defined as occupations in the bottom tercile of the overall wage distribution in 1980) and differential effects of industry labor share declines (observed or automation-driven) on workers in low-pay jobs. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

TABLE A.VII

TASK DISPLACEMENT AND CHANGES IN REAL HOURLY WAGES FOR MEN, WOMEN, AND NATIVE-BORN WORKERS, 1980–2016.

	Dependent Variables: Change in Wages and Wage Declines, 1980–2016							
	Task Displacement Measured From Observed Labor Share Declines				Task Displacement Measured From Automation-Driven Labor Share Declines			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PANEL A. CHANGE IN HOURLY WAGES FOR NATIVE-BORN WORKERS 1980–2016								
Task displacement	−1.57	−1.29	−1.48	−1.66	−1.63	−1.35	−1.56	−1.93
	(0.10)	(0.19)	(0.23)	(0.53)	(0.11)	(0.22)	(0.26)	(0.52)
Share variance explained by task displacement	0.68	0.56	0.64	0.72	0.65	0.54	0.63	0.77
R-squared	0.68	0.71	0.85	0.85	0.65	0.67	0.84	0.84
Observations	250	250	250	250	250	250	250	250
PANEL B. CHANGE IN HOURLY WAGES FOR MEN 1980–2016								
Task displacement	−1.52	−1.08	−0.83	−1.57	−1.56	−1.04	−0.79	−1.70
	(0.11)	(0.19)	(0.08)	(0.30)	(0.12)	(0.21)	(0.09)	(0.35)
Share variance explained by task displacement	0.84	0.60	0.46	0.87	0.81	0.54	0.41	0.89
R-squared	0.84	0.86	0.96	0.96	0.81	0.85	0.96	0.96
Observations	250	250	250	250	250	250	250	250
PANEL C. CHANGE IN HOURLY WAGES FOR WOMEN 1980–2016								
Task displacement	−1.57	−1.68	−2.66	−2.80	−1.62	−2.49	−3.86	−4.49
	(0.18)	(0.23)	(0.37)	(0.79)	(0.21)	(0.29)	(0.46)	(1.06)
Share variance explained by task displacement	0.53	0.57	0.90	0.95	0.47	0.72	1.12	1.30
R-squared	0.53	0.54	0.66	0.68	0.47	0.58	0.70	0.70
Observations	250	250	250	250	250	250	250	250
Covariates:								
Industry shifters		✓	✓	✓		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓				✓

Note: This table presents estimates of the relationship between task displacement and the change in hourly wages across demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in hourly wages for each group between 1980 and 2016. Panel A provides results for native-born groups of workers. Panel B provides results for men. Panel C provides results for women. Columns 1–4 report results for our measure of task displacement based on observed labor share declines. Columns 5–8 report results for our measure of task displacement based on automation-driven labor share declines. In addition to the covariates reported in the table, columns 3–4 and 7–8 control for baseline wage shares in manufacturing and dummies for education (for no high-school degree, completed high school, some college, college degree and post-graduate degree) and gender, and columns 4 and 8 control for groups' exposure to industry labor share declines and groups' relative specialization in routine jobs. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

TABLE A.VIII

TASK DISPLACEMENT AND CHANGES IN REAL HOURLY WAGES, STACKED-DIFFERENCES MODELS FOR 1980–2000 AND 2000–2016.

	Dependent Variable Change in Hourly Wages 1980–2000, 2000–2016			:
	(1)	(2)	(3)	(4)
	PANEL A. COMMON COEFFICIENTS ACROSS PERIODS			
Task displacement	−1.31	−1.04	−0.94	−0.61
	(0.10)	(0.13)	(0.20)	(0.35)
Industry shifters		0.25	−0.44	−0.44
		(0.06)	(0.11)	(0.13)
Exposure to industry labor share decline				0.19
				(0.40)
Exposure to routine occupations				−0.06
				(0.04)
Share variance explained by				
—task displacement	0.46	0.36	0.33	0.21
—task displacement in 80s				
—task displacement in 00s				
R-squared	0.42	0.46	0.56	0.57
Observations	1000	1000	1000	1000
	PANEL B. ALLOW COVARIATES TO HAVE PERIOD-SPECIFIC COEFFICIENTS			
Task displacement	−1.31	−1.21	−1.27	−1.42
	(0.10)	(0.13)	(0.14)	(0.27)
Share variance explained by				
—task displacement	0.46	0.42	0.44	0.50
—task displacement in 80s				
—task displacement in 00s				
R-squared	0.42	0.58	0.74	0.74
Observations	1000	1000	1000	1000
	PANEL C. PERIOD-SPECIFIC ESTIMATES OF TASK DISPLACEMENT			
Task displacement 80–00	−2.08	−1.33	−1.36	−2.11
	(0.28)	(0.25)	(0.25)	(0.73)
Task displacement 00–16	−1.10	−1.16	−1.22	−1.08
	(0.11)	(0.14)	(0.17)	(0.39)
Share variance explained by				
—task displacement	0.45	0.42	0.44	0.45
—task displacement in 80s	0.42	0.27	0.27	0.42
—task displacement in 00s	0.49	0.52	0.55	0.48
R-squared	0.46	0.58	0.74	0.74
Observations	1000	1000	1000	1000
Covariates:				
Industry shifters		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓

Note: This table presents estimates of the relationship between task displacement and the change in hourly wages across 500 demographic groups using a stacked-differences specification for 1980–2000 and 2000–2016. The table uses our measure of task displacement based on observed labor share declines. These groups are defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in hourly wages for 1980–2000 and 2000–2016. Panel A provides estimates assuming common coefficients across periods. Panel B allows covariates to have period-specific coefficients. Panel C provides period-specific estimates of task displacement. In addition to the covariates reported in the table, column 2 controls for industry shifters, column 3 controls for groups' baseline wage share in manufacturing at the beginning of each period and for education and gender dummies, and column 4 controls for relative specialization in routine jobs and groups' exposure to industry labor share declines. Observations are weighted by total hours worked by each group at the beginning of each period. Standard errors robust to heteroskedasticity are reported in parentheses.

TABLE A.IX

TASK DISPLACEMENT AND CHANGES IN REAL HOURLY WAGES, 1980–2016—ALTERNATIVE LABOR SHARE MEASURES.

	Dependent Variable: Change in Hourly Wages 1980–2016			
	(1)	(2)	(3)	(4)
	PANEL A. EXCLUDING COMMODITIES			
Task displacement	−1.67	−1.32	−1.39	−2.14
	(0.12)	(0.17)	(0.20)	(0.46)
Share variance explained by task displacement	0.63	0.50	0.52	0.80
R-squared	0.63	0.67	0.83	0.84
Observations	500	500	500	500
	PANEL B. WINSORIZED LABOR SHARE CHANGES			
Task displacement	−1.59	−1.31	−1.34	−1.89
	(0.10)	(0.16)	(0.20)	(0.44)
Share variance explained by task displacement	0.66	0.54	0.56	0.78
R-squared	0.66	0.69	0.84	0.84
Observations	500	500	500	500
	PANEL C. EXCLUDING INDUSTRIES WITH RISING LABOR SHARES			
Task displacement	−1.49	−1.25	−1.32	−1.96
	(0.09)	(0.16)	(0.20)	(0.42)
Share variance explained by task displacement	0.66	0.55	0.58	0.86
R-squared	0.66	0.68	0.84	0.84
Observations	500	500	500	500
	PANEL D. GROSS LABOR SHARE CHANGES			
Task displacement	−1.39	−1.11	−0.91	−1.19
	(0.08)	(0.11)	(0.13)	(0.31)
Share variance explained by task displacement	0.66	0.53	0.43	0.57
R-squared	0.66	0.74	0.83	0.83
Observations	500	500	500	500
Covariates:				
Industry shifters		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓

Note: This table presents estimates of the relationship between task displacement and the change in hourly wages across 500 demographic groups using different measures of the labor share decline. These groups are defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in hourly wages between 1980 and 2016. The table uses our measure of task displacement based on observed labor share declines. In Panel A, we exclude sectors producing commodities. In Panel B, we winsorized the observed labor share changes at the 5th and 95th percentiles when constructing the task displacement measure. In Panel C, we exclude industries with rising labor shares. In Panel D, we use the percent decline in the labor share of gross output to construct our measure, which also accounts for substitution of labor for intermediates. In addition to the covariates reported in the table, column 2 controls for industry shifters, column 3 controls for each group's baseline wage share in manufacturing and dummies for education level and gender, and column 4 controls for relative specialization in routine jobs and groups' exposure to industry labor share declines. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

TABLE A.X

GMM ESTIMATES OF THE PROPAGATION MATRIX.

	Dependent Variable: Change in Wages 1980–2016					
	Task Displacement Measured From Observed Labor Share Declines			Task Displacement Measured From Automation-Driven Labor Share Declines		
	(1)	(2)	(3)	(4)	(5)	(6)
	PANEL A. BASELINE ESTIMATES COMPUTING THE ADJUSTED LABOR SHARE DECLINE WITH $\sigma_i = 1$					
Own effect, θ/λ	0.88	0.87	0.81	0.88	0.95	0.88
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
Contribution of ripple effects via occupational similarity	0.65	0.63	0.50	0.74	0.88	0.74
	(0.17)	(0.18)	(0.18)	(0.20)	(0.20)	(0.21)
Contribution of ripple effects via industry similarity	0.24	0.24	0.55	0.44	0.45	0.74
	(0.19)	(0.19)	(0.21)	(0.22)	(0.22)	(0.24)
Contribution of ripple effects via education–age groups	0.19	0.19	0.19	0.19	0.19	0.18
	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
	PANEL B. ESTIMATES COMPUTING THE ADJUSTED LABOR SHARE DECLINE WITH $\sigma_i = 0.8$					
Own effect, θ/λ	0.68	0.67	0.62	0.74	0.77	0.72
	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)
Contribution of ripple effects via occupational similarity	0.51	0.48	0.43	0.50	0.55	0.51
	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)
Contribution of ripple effects via industry similarity	0.08	0.08	0.22	0.22	0.23	0.32
	(0.10)	(0.10)	(0.10)	(0.11)	(0.10)	(0.11)
Contribution of ripple effects via education–age groups	0.20	0.20	0.19	0.18	0.17	0.17
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
	PANEL C. ESTIMATES COMPUTING THE ADJUSTED LABOR SHARE DECLINE WITH $\sigma_i = 1.2$					
Own effect, θ/λ	1.05	1.04	0.95	1.04	1.18	1.08
	(0.06)	(0.06)	(0.06)	(0.06)	(0.07)	(0.08)
Contribution of ripple effects via occupational similarity	0.20	0.18	0.13	0.32	0.40	0.34
	(0.10)	(0.11)	(0.11)	(0.12)	(0.12)	(0.12)
Contribution of ripple effects via industry similarity	0.39	0.39	0.59	0.50	0.57	0.75
	(0.12)	(0.12)	(0.13)	(0.14)	(0.14)	(0.15)
Contribution of ripple effects via education–age groups	0.15	0.15	0.14	0.16	0.15	0.14
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
	PANEL D. SETTING $\kappa = 1$ IN THE SIGMOID FUNCTION					
Own effect, θ/λ	0.88	0.87	0.81	0.88	0.95	0.88
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
Contribution of ripple effects via occupational similarity	0.65	0.63	0.50	0.74	0.88	0.74
	(0.17)	(0.18)	(0.18)	(0.20)	(0.20)	(0.21)

(*Continues*)

TABLE A.X

Continued.

	Dependent Variable: Change in Wages 1980–2016					
	Task Displacement Measured From Observed Labor Share Declines			Task Displacement Measured From Automation-Driven Labor Share Declines		
	(1)	(2)	(3)	(4)	(5)	(6)
Contribution of ripple effects via industry similarity	0.24	0.24	0.55	0.44	0.45	0.74
	(0.19)	(0.19)	(0.21)	(0.22)	(0.22)	(0.24)
Contribution of ripple effects via education–age groups	0.19	0.19	0.19	0.19	0.19	0.18
	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
PANEL E. SETTING $\kappa = 5$ IN THE SIGMOID FUNCTION						
Own effect, θ/λ	0.91	0.90	0.85	0.92	1.00	0.95
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
Contribution of ripple effects via occupational similarity	0.25	0.24	0.23	0.31	0.34	0.33
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
Contribution of ripple effects via industry similarity	0.18	0.18	0.24	0.26	0.29	0.33
	(0.06)	(0.06)	(0.06)	(0.07)	(0.07)	(0.07)
Contribution of ripple effects via education–age groups	0.16	0.16	0.15	0.15	0.14	0.13
	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
Covariates:						
Industry shifters		✓	✓		✓	✓
Manufacturing share			✓			✓

Note: This table presents estimates of the propagation matrix. Ripple effects are parameterized as functions of the similarity of groups in terms of their 1980 occupational distribution, industry distribution, and education×age groups. The table reports our estimates of the common diagonal term θ and a summary measure of the strength of ripple effects operating through each of these dimensions, defined by

$$\text{Contributionofrippleeffects}_n = \frac{\beta_n}{\lambda} \cdot \left(\frac{1}{s^L} \sum_g \sum_{g' \neq g} f\left(d^n_{gg'}\right) \cdot s^L_g \cdot s^L_{g'} \right),$$

which equals the average sum of the off-diagonal terms of the propagation matrix explained by each dimension of similarity. Estimates and standard errors are obtained via GMM. Columns 1–3 provide GMM estimates using our measure of task displacement based on observed labor share declines. Columns 4–6 provide GMM estimates using our measure of task displacement based on automation-driven labor share declines. The panels report results for different measures of the adjusted labor share decline and using different values of κ in the sigmoid function. All models are weighted by total hours worked by each group in 1980.

TABLE A.XI

ROBUSTNESS CHECKS FOR ESTIMATES OF THE GENERAL EQUILIBRIUM EFFECTS.

	Baseline (1)	Labor Supply Response With Hicksian of 0.3 (2)	Setting $\lambda = 0.3$ (3)	Setting $\lambda = 0.7$ (4)	Setting $\pi = 50\%$ (5)	Setting $\kappa = 1$ in Sigmoid Function (6)	Setting $\kappa = 5$ in Sigmoid Function (7)	Computing Adjusted Labor Share Decline With $\sigma_i = 0.8$ (8)	Computing Adjusted Labor Share Decline With $\sigma_i = 1.2$ (9)
WAGE STRUCTURE:									
Share wage changes explained:									
—due to industry shifts	6.33%	3.96%	3.31%	8.06%	7.38%	6.38%	6.45%	5.10%	7.50%
—adding direct displacement effects	93.34%	58.34%	142.92%	70.21%	94.38%	93.38%	93.45%	97.04%	89.57%
—accounting for ripple effects	46.88%	46.88%	43.74%	50.49%	45.04%	47.16%	47.77%	39.04%	53.68%
Rise in college premium	21.02%	21.02%	20.19%	21.96%	20.60%	20.77%	21.76%	19.68%	21.62%
—part due to direct displacement effect	37.71%	23.57%	62.86%	26.94%	37.71%	37.71%	37.71%	42.00%	33.43%
Rise in post-college premium	22.42%	22.42%	21.08%	23.96%	21.56%	22.40%	23.33%	20.51%	23.46%
—part due to direct displacement effect	43.57%	27.23%	72.62%	31.12%	43.57%	43.57%	43.57%	48.90%	38.25%
Change in gender gap	1.90%	1.90%	0.97%	2.97%	1.23%	2.25%	1.73%	−2.15%	6.43%
—part due to direct displacement effect	5.94%	3.71%	9.90%	4.24%	5.94%	5.94%	5.94%	2.19%	9.69%
Share with declining wages	42.26%	42.26%	45.82%	46.34%	35.10%	42.25%	49.04%	48.86%	42.39%
—part due to direct displacement effects	51.52%	51.52%	51.58%	48.76%	39.20%	55.30%	48.65%	55.60%	43.54%
Wages for men with no high school	−8.41%	−8.41%	−7.71%	−9.23%	−4.81%	−8.46%	−8.40%	−5.60%	−11.10%
—part due to direct displacement effects	−15.11%	−9.44%	−25.75%	−10.53%	−11.27%	−16.41%	−14.35%	−13.76%	−16.18%
Wages for women with no high school	−3.40%	−3.40%	−3.53%	−3.23%	−0.51%	−2.99%	−3.77%	−4.84%	−1.36%
—part due to direct displacement effects	−2.82%	−1.76%	−5.27%	−1.76%	1.01%	−4.13%	−2.06%	−4.94%	−0.43%
AGGREGATES:									
Change in average wages, $d \ln w$	5.18%	5.18%	5.18%	5.18%	8.63%	5.18%	5.18%	5.88%	4.48%
Change in GDP per capita, $d \ln y$	20.95%	20.95%	20.78%	21.13%	22.87%	20.30%	21.33%	24.13%	17.91%
Change in TFP, $d \ln \text{tfp}$	3.42%	3.42%	3.42%	3.42%	5.70%	3.42%	3.42%	3.88%	2.96%
Change in labor share, ds^L	−10.41 p.p.	−9.38 p.p.	−10.30 p.p.	−10.53 p.p.	−9.40 p.p.	−9.98 p.p.	−10.66 p.p.	−12.04 p.p.	−8.87 p.p.
Change in K/Y ratio	38.10%	34.93%	37.77%	38.47%	34.98%	36.78%	38.87%	42.97%	33.29%
SECTORAL PATTERNS:									
Share manufacturing in GDP	−0.52 p.p.	−0.52 p.p.	−0.50 p.p.	−0.53 p.p.	−0.76 p.p.	−0.52 p.p.	−0.52 p.p.	−0.47 p.p.	−0.56 p.p.
Change in manufacturing wage bill	−12.85%	−12.85%	−12.93%	−12.76%	−12.30%	−13.51%	−12.49%	−11.68%	−13.88%

Note: The table summarizes the general equilibrium effects of automation on the wage distribution, wage levels, aggregates, and industry outcomes, computed using the formulas in Proposition 4 and the parameterization and estimates for the industry demand system and the propagation matrix described in the column headers. In all cases, we use our measure of task displacement based on automation-driven labor share declines.

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