

SUPPLEMENT TO “FROM POPULATION GROWTH TO FIRM  
DEMOGRAPHICS: IMPLICATIONS FOR CONCENTRATION,  
ENTREPRENEURSHIP AND THE LABOR SHARE”  
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APPENDIX A: PROOFS

A.1. *Entry and the Aggregate State*

THIS SECTION LAYS OUT THE STEPS to explicitly link entry and the aggregate state in the monopolistic competition economies with constant and variable markups.

*Monopolistic Competition With Constant Markups.* The optimal pricing rule is  $p(i) = 1/\eta s$ . Combining the pricing rule with the first-order condition for  $c(i)$  and the definition of  $U$  gives

$$\theta_t = \eta \left( \int s^{\frac{\eta}{1-\eta}} d\psi_t \right)^{\frac{1-\eta}{\eta}}.$$

Labor demand is given by  $n_t(s) = N_t c(s)/s$ . Substitute for  $c(s)$  using the first-order condition, and then use the optimal pricing rule and the expression for  $\theta_t$  to obtain

$$n_t(s) = N_t U_t \left( \int s^{\frac{\eta}{1-\eta}} d\psi_t \right)^{-\frac{1}{\eta}} s^{\frac{\eta}{1-\eta}}. \quad (\text{A.1})$$

Integrate (A.1) over the measure of firms, solve for  $U_t$  and substitute it back into the expression for  $z = N_t U_t \theta_t^{-\frac{1}{1-\eta}}$  to obtain the following:

$$z = \eta^{-\frac{1}{1-\eta}} \left( \int s^{\frac{\eta}{1-\eta}} d\psi_t^I + m_t \int s^{\frac{\eta}{1-\eta}} dG \right)^{-1} (N_t - c_e m_t), \quad (\text{A.2})$$

where the measure of productive firms  $\psi_t$  is given by the mixture of the measure of incumbents,  $\psi_t^I$ , and the measure of initial draws scaled by the mass of entrants,  $m_t G$ . The right-hand side of equation (A.2) is continuous and strictly decreasing in  $m_t$ . Assuming

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that  $z(\int s^{\frac{\eta}{1-\eta}} d\psi_t^I) < N_t$ , or equivalently, the necessary condition that labor demand for incumbent firms for the given value  $z$  does not exceed total labor force, there is a unique solution  $m_t^* > 0$  such that  $z$  remains constant.

*Monopolistic Competition With Variable Markups.* As in the constant markup case, we have  $z = N_t U_t \theta_t^{-\frac{1}{1-\eta}}$ . Following the same steps as above gives the relevant expression for  $z$ ,

$$z = \left( \int (s/p(s))^{\frac{1}{1-\eta}} d\psi_t^I + m_t \int (s/p(s))^{\frac{1}{1-\eta}} dG \right)^{-1} (N_t - c_e m_t),$$

where  $p(s)$  is given by limit pricing. Because the right-hand side of the equation is continuous and strictly decreasing in  $m_t$ , there is a unique value of  $m_t$  that will satisfy this condition provided that  $z \int (s/p(s))^{\frac{1}{1-\eta}} d\psi_t^I < N_t$ , exactly the necessary condition that labor demand of incumbents at these prices does not exceed total labor force.

## A.2. Stochastic Process for the Variable Markup Economy

In this section, we discuss how one might specify the employment process in the variable markup economy. The key elements are  $p(s)$ ,  $G(s)$  and the transition function  $F(s'|s)$ . Employment is given by

$$n(s, z) = z(s/p(s))^{\frac{1}{1-\eta}}, \quad (\text{A.3})$$

and profits

$$\pi(s, z) = z(s/p(s))^{\frac{1}{1-\eta}} (p(s) - 1) - c_f. \quad (\text{A.4})$$

For given  $\eta$  and  $z$ , there is a one-one correspondence between  $s/p(s)$  and the firm state in the baseline model. In particular,  $s^*/p(s^*)$  is pinned down by the employment threshold, the distribution of employment of entrants pins down an initial distribution for the variable  $s/p(s)$  on  $s \geq s^*$ , and the Markov process for employment pins down its conditional distribution. Likewise, we can map the fixed-cost function in the baseline model into a fixed-cost function  $c_f(s/p(s))$  in this variable markups setting.

We can write the value function:

$$v(s, z) = \pi(s, z) + \max(0, Ev(s', z)|s),$$

where upon substitution

$$\pi(s, z) = n(s, z)(p(s) - 1).$$

The value function has to meet two conditions:

$$v(s^*, z) = 0$$

and  $\int v(s, z) dG(s) = c_e$ , where the value  $c_e$  is the same as in the baseline model. Our previous analysis puts restrictions only on  $s/p(s)$ , and for fixed values of  $s/p(s)$  profits are increasing in  $p(s)$ . These two conditions can be easily met, given the flexibility in the choice of the function  $p(s)$ .

## APPENDIX B: ROBUSTNESS AND PROJECTIONS

## B.1. Sources and Alternative Measures of Labor Force Growth

What are the main drivers of labor force growth? Figure B.1(a) plots the labor force growth rate by decade, dividing the bars into the percentage contribution of growth in participation rates, birth rates 16 years prior, and other. The bulk of the changes in labor force growth are accounted for by birth rates 16 years prior.<sup>1</sup>

One potential source of concern when using the civilian labor force as a measure of labor supply is that it includes the unemployed, those employed by government, and the self-employed. Figure B.1(b) shows that total employment growth (excludes the unemployed) and private sector employment growth (excludes the self-employed and those working for government) follow a similar hump-shaped pattern as labor force growth. Another potential source of concern is the manufacturing sector, which has experienced negative overall employment growth since the 1980s (Fort, Pierce, and Schott (2018)). An exodus of workers from manufacturing into nonmanufacturing could reverse the trend of declining employment growth in nonmanufacturing sectors. Figure B.1(b) shows that this is not the case. Nonmanufacturing employment growth also shows a similar rise and fall pattern as labor force growth. The decline of manufacturing employment does not have a large effect on nonmanufacturing employment growth partly because the flow of workers out of manufacturing is small compared to the flow of workers entering the labor force. From 1977 to 2014, manufacturing employment shrank by 6 million workers while the labor force grew by 57 million workers.

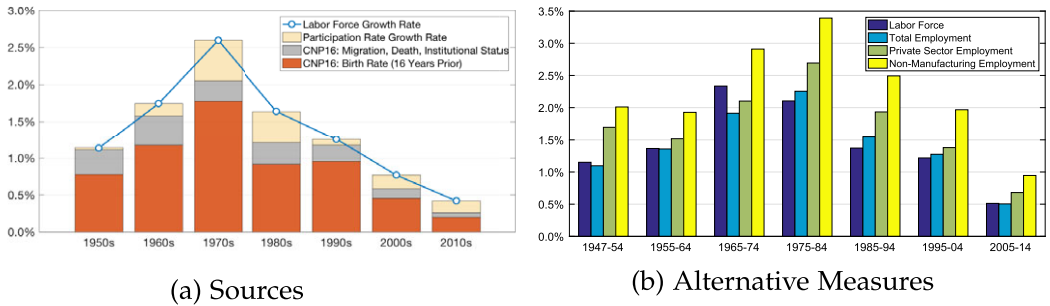


FIGURE B.1.—Labor force growth. *Sources:* Current Population Survey and Current Employment Statistics. *Note:* Panel A: The BLS' definition of labor force is  $LF_t = CNP16_t \times PR_t$  where  $LF_t$  is the civilian labor force at time  $t$ ,  $CNP16_t$  is the civilian noninstitutional population age 16 and over at time  $t$ , and  $PR_t$  is the participation rate at time  $t$ . It follows that labor force growth rate is the sum of the growth rate of each component  $LF \text{ Growth Rate}_t = CNP16 \text{ Growth Rate}_t + PR \text{ Growth Rate}_t$ . We further decompose  $CNP16$  growth rate at time  $t$  into the birth rate at time  $t - 16$  and a residual term  $Other_t$ ,  $CNP16 \text{ Growth Rate}_t = \text{Birth Rate}_{t-16} + Other_t$ , where the  $Other_t$  term includes death rates, net migration rates, and rates of entry and exit into institutional status. The size of the bars is calculated as the absolute value of the growth rate of a component divided by the sum of absolute growth rates of each component, multiplied by labor force growth. Panel B: Units are average annual growth rates. Data starts in 1947. Decade cutoffs are chosen so that full business cycles fall within the decade bin, effectively capturing the trend component in growth rates.

<sup>1</sup>On average, birth rates 16 years prior account for 64% of labor force growth across decades. The actual contribution of the birth rate to labor force growth is higher than 64% because the birth rate also has an effect on participation rates. For example, a portion of the decline of participation rates since the year 2000 is due to the baby boomer generation reaching the age of 55 and over, whose age group has low participation rates.

## B.2. Firm Age Regressions

TABLE B.1  
TIME TRENDS, 1978–2014.

Group	Average Size		Exit Rate		Concentration	
	Raw Data	Controlling for Sector	Raw Data	Controlling for Sector	Raw Data	Controlling for Sector
Aggregate	0.095 (0.007)	0.128 (0.015)	−0.042 (0.008)	−0.036 (0.004)	0.159 (0.013)	0.172 (0.017)
Age 0	0.001 (0.006)	0.003 (0.003)			0.080 (0.032)	0.074 (0.025)
Age 1	−0.026 (0.010)	−0.025 (0.005)	0.067 (0.037)	0.048 (0.016)	−0.005 (0.059)	−0.044 (0.034)
Age 2	−0.023 (0.012)	−0.024 (0.006)	−0.008 (0.021)	−0.007 (0.010)	0.017 (0.057)	−0.020 (0.030)
Age 3	−0.026 (0.015)	−0.027 (0.008)	−0.015 (0.017)	−0.010 (0.009)	0.002 (0.066)	−0.036 (0.032)
Age 4	−0.032 (0.015)	−0.032 (0.009)	−0.004 (0.015)	0.002 (0.008)	−0.032 (0.067)	−0.071 (0.033)
Age 5	−0.045 (0.019)	−0.045 (0.011)	0.010 (0.013)	0.015 (0.007)	−0.052 (0.082)	−0.098 (0.039)
Age 6 to 10	−0.081 (0.011)	−0.084 (0.011)	−0.001 (0.010)	−0.001 (0.006)	−0.154 (0.053)	−0.207 (0.030)
Age 11 to 15	−0.159 (0.032)	−0.166 (0.022)	0.021 (0.011)	0.016 (0.006)	−0.238 (0.113)	−0.314 (0.051)
Age 16 to 20	−0.399 (0.063)	−0.400 (0.041)	0.020 (0.014)	0.013 (0.008)	−0.418 (0.151)	−0.490 (0.070)
Age 21 to 25	−0.402 (0.112)	−0.464 (0.077)	0.020 (0.014)	0.009 (0.011)	0.163 (0.235)	−0.066 (0.108)
Age above 25	−1.598 (0.139)	−1.371 (0.141)	−0.007 (0.017)	−0.013 (0.009)	−0.125 (0.022)	−0.347 (0.038)

*Note:* The estimates for each variable indicate the annual trend for available years of data. Numbers in parentheses report standard errors. The number of observations for the raw data specification equals the number of years available in the sample for the corresponding group (e.g., 36 for the Aggregate group and 11 for the Age above 25 group.). The number of observations for the specifications Controlling for Sectors equals the number of years of data for the corresponding age group times the number of sectors (e.g.,  $36 \times 9$  for the Aggregate group and  $11 \times 9$  for the Age above 25 group). The Controlling by Sector regressions are also weighted by 1978 sectoral activity. For average size and exit rate, the sector activity weights correspond to share of firms. For concentration, the sector activity weights correspond to share of employment.

TABLE B.II  
TIME TRENDS FOR DIFFERENT MEASURES OF CONCENTRATION.

	Aggregate	Controlling for Sector	Controlling for Age	Controlling for Age and Sector	Controlling for Age, Sector, and Interaction
250+:	0.159 (0.013)	0.172 (0.017)	-0.068 (0.023)	-0.153 (0.018)	-0.149 (0.012)
500+:	0.146 (0.014)	0.153 (0.018)	-0.077 (0.022)	-0.138 (0.021)	-0.139 (0.014)
1000+:	0.133 (0.014)	0.132 (0.019)	-0.093 (0.022)	-0.148 (0.027)	-0.151 (0.016)
2500+:	0.116 (0.016)	0.106 (0.022)	-0.098 (0.021)	-0.159 (0.038)	-0.163 (0.021)
5000+:	0.091 (0.017)	0.070 (0.023)	-0.095 (0.021)	-0.064 (0.057)	-0.112 (0.026)
10,000+:	0.073 (0.017)	0.043 (0.024)	-0.060 (0.018)	-0.006 (0.061)	-0.049 (0.025)

*Note:* The estimates for each variable indicate the annual trend for years 1978–2014. Numbers in parentheses report standard errors. All regressions are weighted by sectoral activity.

### B.3. Entry Rate Projections

The main message from the decomposition exercises is that feedback effects from firm demographics and transitional dynamics are both quantitatively essential for the decline in firm entry. We next explore what projections of labor force growth imply for future entry rates. The Bureau of Labor Statistics (BLS) publishes projections of labor force growth up until the year 2060. We feed the BLS projections into the benchmark model and compute firm entry rates. Figure B.2 presents our findings.

The BLS projects that the labor force will slowly converge to a growth rate of about 0.25% by the year 2060. Through the lens of our model, these projections imply that the entry rate will rise from 8.75% in 2014 to 9.65% in 2060. The reason for the rebound is threefold. First, the exit rate along the transition is lower than the 2060 steady state,

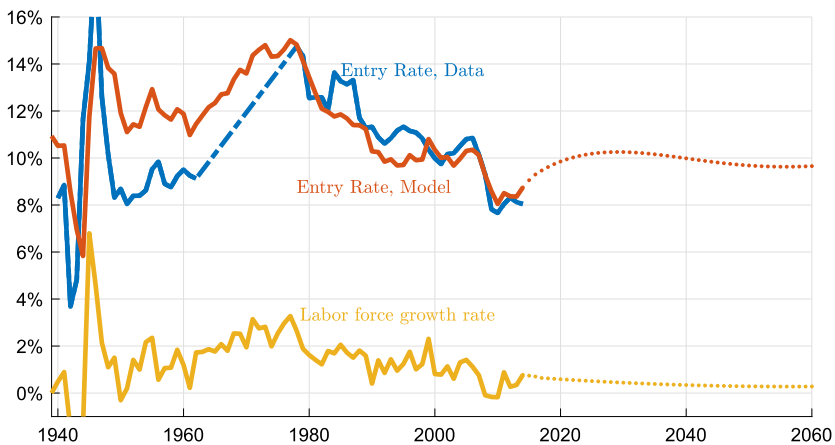


FIGURE B.2.—Projections.

9.19% versus 9.49%. This is because firms are older along the transition than in the 2060 steady state, and older firms exit at lower rates. Second, average firm size stops growing in the 2060 steady state, which adds an extra 0.95pp to the entry rate. Third, the residual goes from 0.26pp to zero as the economy converges to the 2060 steady state. Together these three forces more than offset the 0.52pp decline in the labor force growth rate from 2014 to 2060.

The projections also show that the convergence to the new steady state is non-monotonic. The entry rate rises above and then declines to its stationary level. This cycle in entry rates is due to the dynamic nature of entry. As past entrants age, they grow at slower rates and cannot absorb as much of the growth in labor supply. This creates room for new firms, raising the entry rate. As these new firms age and grow, they absorb a larger fraction of the growth in labor supply, lowering the entry rate and generating firm aging. This cycle repeats and dampens until convergence.

### APPENDIX C: DATA SOURCES

*Civilian Labor Force Growth Rate 1940–2014.* Civilian labor force data comes from the [Bureau of Labor Statistics’ Current Population Survey \(CPS\)](#) for the years 1947 to 2014, and from [Lebergott \(1964\)](#) from 1940 to 1946. The civilian labor force definition in CPS includes population 16 years of age and over while the definition in Lebergott includes population 14 years of age and over. We check the comparability of the two series from 1947 to 1960, the years where the two series overlap. Labor force growth rates of ages 14+ and 16+ are nearly identical for these years.

*Firm Data 1978–2014.* Firm data comes from the [U.S. Census Bureau’s Business Dynamics Statistics \(BDS\)](#). The BDS data set has near universal coverage of private sector firms with paid employees. BDS data starts in 1977, but common practice suggests dropping 1977 and 1978 due to suspected measurement error (e.g., [Moscarini and Postel-Vinay \(2012\)](#)). We drop entry rates for 1977, but keep 1978, as targeting 1978 or 1979 does not affect our quantitative results (the model matches the entry rate in both 1978 and 1979 almost exactly). We exclude the first 4 years of BDS data for age groups 6–10, 11–15, 15–20, and 20–25 in order to have consistent age-group definitions.

*Firm Entry Rates 1940–1962.* The firm entry rate is obtained from the now-discontinued [U.S. Bureau of Economic Analysis’ Survey of Current Business \(SCB\)](#). The entry rate is defined as “New Businesses” divided by “Operating Businesses.” The 1963 edition was the last one to report a “Business Population and Turnover” section. From 1963, the SCB reported “Business Incorporations” instead, which only include stock corporations. Detailed definitions can be found in the 1954 January issue and in the 1963 Supplement of the SCB. The main similarities and differences with the BDS data set are summarized below.

As in the BDS, the data on operating businesses and new businesses refer to number of firms as opposed to the number of establishments. A firm is defined as a business organization under one management with either an established place of business or at least one paid employee. The coverage in both data sets is similar: all nonfarm businesses are included. Also, as in the BDS, change of ownership does not count as entry or exit. New businesses include only firms which have been “newly established,” meaning that it excludes those undergoing a change in ownership (partnerships in which a member is added or dropped, corporations which are reorganized and reincorporated, as well as

businesses sold to or otherwise acquired by new owners, or firms which have undergone a change in legal form of organization such as from partnership to corporation). A change in ownership of an existing firm is listed separately under a category called “business transfers.” Discontinued businesses include closures of all kinds without reference to the reason for going out of business.

Unlike the BDS, the SCB includes some nonemployer businesses. Nonemployer business without an established place of business and those in “professional services,” such as dentist practices, are excluded. Although the fraction of 0-employee firms is not directly reported in the SCB publications, one way to get a sense of the size of this group is to compare the percentage of firms in comparable categories in both data sets. The 1950 May issue of the SCB reports that 95% of firms were of size 0 to 19. The BDS data reports that 90% of firms were of size 1 to 19 in 1978. Given the similarity between these two values, the 0-employee firms are not likely to have a large weight in the firm distribution in the SCB data.

*Birth Rates 1930–2000.* The birth rate series used in the calculations in Section B.1 of this Appendix is from the Centers for Disease Control and Prevention’s National Center for Health Statistics.

*Employment 1947–2014.* Total employment corresponds to the civilian employment in the Bureau of Labor Statistics’ Current Population Survey. Private sector employment and manufacturing employment are from the Bureau of Labor Statistics’ Current Employment Statistics. Nonmanufacturing employment is private sector employment minus manufacturing employment.

*Labor Force Projections 2014–2060.* Projections of labor force growth used in Section B.3 of this Appendix are from the Bureau of Labor Statistics; see Toossi (2016).

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