

SUPPLEMENT TO “COMPETING ON SPEED”
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APPENDIX C: PROOF OF LEMMAS

C.1. *Proof of Lemma 3*

THE PROOF is contained in Appendix D.

C.2. *Proof of Lemma 4*

The welfare formula in equation (9) reflects the joint trading surplus of investors. Transfers from investors to venue owners do not represent net social gains.

Consider the segmented case first. The welfare of type σ joining venue i is

$$W_i(\sigma) - W_{\text{out}} = \frac{s_i \sigma^\dagger(p_i, \rho_i)}{r} \bar{a} + \frac{s_i}{2r} \max(0; \sigma - \sigma^\dagger(p_i, \rho_i)),$$

where $\sigma^\dagger(p, \rho) \equiv \frac{r+\rho+\gamma}{r+\rho}(rp - \mu)$. We write $\sigma_i^\dagger \equiv \sigma^\dagger(p_i, \rho_i)$ for brevity. The net value of participation, $W - W_{\text{out}}$, is composed of two parts. The first part is independent of σ and represents the option to sell the asset on the exchange: $\frac{s\bar{a}\sigma^\dagger}{r} = \frac{\rho}{r+\rho}(p - \frac{\mu}{r})\bar{a}$. It is the value that can be achieved by types $\sigma < \sigma^\dagger$ that trade only once. The second part, $\frac{s}{2r} \max(0; \sigma - \sigma^\dagger)$, is the value of trading repeatedly and is super-modular in (s, σ) . The mass of light traders in venues 1 and 2 is $(\frac{1}{2\bar{a}} - 1)(G(\sigma_2) - G(\sigma_1))$ and $(\frac{1}{2\bar{a}} - 1)(1 - G(\sigma_2))$, respectively. Thus, total social gains for this group are given by

$$\frac{1}{r} \left(\frac{1}{2} - \bar{a} \right) \left((G(\sigma_2) - G(\sigma_1)) s_1 \sigma_1^\dagger + (1 - G(\sigma_2)) s_2 \sigma_2^\dagger \right). \quad (29)$$

The welfare gains of heavy traders in venues 1 and 2 are

$$\begin{aligned} & s_1 \int_{\sigma_1}^{\sigma_2} \left(\frac{\bar{a}\sigma_1^\dagger}{r} + \frac{\sigma - \sigma_1^\dagger}{2r} \right) dG(\sigma) \\ & = \int_{\sigma_1}^{\sigma_2} \frac{s_1 \sigma}{2r} dG(\sigma) - (G(\sigma_2) - G(\sigma_1)) \frac{s_1 \sigma_1^\dagger}{r} \left(\frac{1}{2} - \bar{a} \right), \end{aligned} \quad (30)$$

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$$\begin{aligned}
& s_2 \int_{\sigma_2}^{\bar{\sigma}} \left(\frac{\bar{a}\sigma_2^\dagger}{r} - \frac{\sigma - \sigma_2^\dagger}{2r} \right) dG(\sigma) \\
&= \int_{\sigma_2}^{\bar{\sigma}} \frac{s_2\sigma}{2r} dG(\sigma) - (1 - G(\sigma_2)) \frac{s_2\sigma_2^\dagger}{r} \left(\frac{1}{2} - \bar{a} \right).
\end{aligned} \tag{31}$$

Adding (29), (30), and (31), and subtracting speed investment and entry costs, yields the total net gains from trade:

$$\mathcal{W}(2) = \frac{s_1}{2r} \int_{\sigma_1}^{\sigma_2} \sigma dG(\sigma) + \frac{s_2}{2r} \int_{\sigma_2}^{\bar{\sigma}} \sigma dG(\sigma) - \sum_{i=1,2} C(s_i) - 2\kappa.$$

Consider now the case of price integration. There is only one price so

$$W_i(\sigma) - W_{\text{out}} = \frac{s_i\sigma^\dagger(p, \rho_i)}{r} \bar{a} + \frac{s_i}{2r} \max(0; \sigma - \sigma^\dagger(p, \rho_i)).$$

Light traders join venue 1. Therefore, their utility is $(\frac{1}{2} - \bar{a})(1 - G(\sigma_{1,\text{int}})) \frac{s_1\sigma_1^\dagger}{r}$. Welfare for heavy traders is given by expressions analogous to equations (30) and (31). Thus,

$$\begin{aligned}
\mathcal{W}^{\text{int}}(2) &= \int_{\sigma_{1,\text{int}}}^{\sigma_{2,\text{int}}} \frac{s_1\sigma}{2r} dG(\sigma) + \int_{\sigma_{2,\text{int}}}^{\bar{\sigma}} \frac{s_2\sigma}{2r} dG(\sigma) \\
&\quad - \left(\frac{1}{2} - \bar{a} \right) (1 - G(\sigma_{2,\text{int}})) \left(\frac{s_2\sigma_{2,\text{int}}^\dagger}{r} - \frac{s_1\sigma_{1,\text{int}}^\dagger}{r} \right). \quad \text{Q.E.D.}
\end{aligned}$$

C.3. Proof of Lemma 5

The proof is a generalization of the result in Lemma 2 and is thus omitted.

APPENDIX D: TRANSITION DYNAMICS UNDER INTEGRATION

In this section, we compute the system dynamics when the price is not constant. This happens when the market clearing in stocks is not enough to ensure market clearing in flows. To understand the issue, consider a duopoly with n_1 agents in venue 1 and n_2 in venue 2. That is, $n_1 = \ell_1 + G(\sigma_2) - G(\sigma_1)$ and $n_2 = \ell_2 + 1 - G(\sigma_2)$. The total quantity of tradable assets is $(n_1 + n_2)\bar{a}$. Let $\bar{a}_{i,t}$ be the average holding of agents in venue i . All assets must be held; therefore we must have, at any point in time, $n_1\bar{a}_{1,t} + n_2\bar{a}_{2,t} = (n_1 + n_2)\bar{a}$.

D.1. Price Segmentation

Let us show that market clearing in flows is always satisfied under segmentation. Let $\bar{a}_{i,t}^*$ be the average optimal demand in venue $i \in \{1, 2\}$. The flow market clearing condition is $n_i\rho_i\bar{a} = n_i\rho_i\bar{a}_{i,t}^*$, which implies $\bar{a} = \bar{a}_{i,t}^*$, a time-invariant demand. The average demand of light traders is zero. The average demand of heavy traders is $\frac{1}{2}$, so

$$\bar{a}_1^* = \frac{\frac{1}{2}(G(\sigma_2) - G(\sigma_1))}{\ell_1 + G(\sigma_2) - G(\sigma_1)}; \quad \bar{a}_2^* = \frac{\frac{1}{2}(1 - G(\sigma_2))}{\ell_2 + 1 - G(\sigma_2)}.$$

The flow market clearing conditions in venues 1 and 2 are therefore $(\ell_1 + G(\sigma_2) - G(\sigma_1))\bar{a} = \frac{1}{2}(G(\sigma_2) - G(\sigma_1))$ and $(\ell_2 + 1 - G(\sigma_2))\bar{a} = \frac{1}{2}(1 - G(\sigma_2))$, which are the conditions derived in Section 5.2.

D.2. Price Integration

The single flow market clearing condition is now $n_1\rho_1\bar{a}_{1,t} + n_2\rho_2\bar{a}_{2,t} = n_1\rho_1\bar{a}_{1,t}^* + n_2\rho_2\bar{a}_{2,t}^*$. Let us show that there is excess demand in the short term. Consider time 0^+ when all agents still hold \bar{a} , so the supply is $(n_1\rho_1 + n_2\rho_2)\bar{a}$. Suppose that the demand is given by the long-term time-invariant functions. The demand is then $\frac{1}{2}\rho_1(n_1 - \ell_1) + \frac{1}{2}\rho_2n_2$ and the flow market clearing condition in the short run is

$$\frac{1}{2}(\rho_1(n_1 - \ell_1) + \rho_2n_2) = (n_1\rho_1 + n_2\rho_2)\bar{a}. \quad (32)$$

In the long run, we have the same dynamics of α as in the one-venue case. In venue 1, we have a fraction $\alpha_-^1(1)$ of agents who want to sell and a fraction $\alpha_+^1(0)$ who want to buy of equal value: $\alpha_-^1(1) = \alpha_+^1(0) = \frac{1}{4}\frac{\gamma}{\gamma + \rho_1}$. So, in fact, we have clearing in both venues, and we can imagine in the long run that slow buyers buy only from slow sellers and fast buyers from fast sellers. This is another way of saying that, in the long run, assets no longer migrate from the slow to the fast venue. The assets held in the fast venue are

$$n_2(\alpha_+^2(1) + \alpha_-^2(1)) = \frac{n_2}{4}\left(\frac{\gamma}{\gamma + \rho_2} + \frac{2\rho_2 + \gamma}{\gamma + \rho_2}\right) = \frac{n_2}{2},$$

and assets held in the slow venue are $n_1(\alpha_+^1(1) + \alpha_-^1(1)) = \frac{n_1 - \ell_1}{2}$. The market clearing in stock in the long run is

$$\frac{1}{2}(n_1 - \ell_1 + n_2) = (n_1 + n_2)\bar{a}, \quad (33)$$

which is identical to the condition used in the paper: $\frac{1 - G(\sigma_1)}{2} = (\ell_1 + 1 - G(\sigma_1))\bar{a}$.

One can see that expressions (32) and (33) are inconsistent with each other. More precisely, suppose that (33) holds. Then, excess short run demand is $\Delta = \frac{1}{2}\rho_1(n_1 - \ell_1) + \frac{1}{2}\rho_2n_2 - (n_1\rho_1 + n_2\rho_2)\bar{a}$. Using equation (33), we get $\Delta = (\frac{1}{2} - \bar{a})(\rho_2 - \rho_1)n_2$ as a measure of short run excess demand conditional on long run market clearing. We can see why there is a gap simply by rewriting (32) and (33) as $\frac{1}{2} - \frac{1}{n_1 + \frac{\rho_2}{\rho_1}n_2}\frac{\ell_1}{2} = \bar{a}$ and $\frac{1}{2} - \frac{1}{n_1 + n_2}\frac{\ell_1}{2} = \bar{a}$. When $\frac{\rho_2}{\rho_1} > 1$, we over-sample the fast venue where demand is high and supply is low. Let us now compute the equilibrium transition dynamics.

D.3. Computing the Transition Dynamics Under Integration

The trader distribution consists of $n_2 = 1 - G(\sigma_2)$ heavy traders in the fast venue, with long run flow demand $\frac{\rho_2 n_2}{2}$, and $n_1 = \ell_1 + G(\sigma_2) - G(\sigma_1)$ light and heavy traders in the slow venue, with long run flow demand $\frac{\rho_1(n_1 - \ell_1)}{2}$. We know that there is excess demand at time 0, so some of the heavy traders in the slow venue do not buy at time 0. Let $\sigma_{1,t}^1 \geq \sigma_1$ be the time-varying marginal trading type. We assume here that there is an interior solution for $\sigma_{1,t}^1$ (the analysis easily extends to the case of a corner solution). The flow demand from investors in the slow venue is $\frac{\rho_1 \tilde{n}_{1,t}}{2}$ where $\tilde{n}_{1,t} \equiv G(\sigma_2) - G(\sigma_{1,t}^1)$. Moreover, we know that

assets migrate over time from the slow to the fast venue. Let m_t measure the net stock of migrated assets, so that total assets held in venues 2 and 1 at time t are $n_2\bar{a} + m_t$ and $n_1\bar{a} - m_t$. Market clearing requires that the flow demand equals the flow supply:

$$\rho_2 \frac{n_2}{2} + \rho_1 \frac{\tilde{n}_{1,t}}{2} = \rho_1(n_1\bar{a} - m_t) + \rho_2(n_2\bar{a} + m_t). \quad (34)$$

Next, we need to find the law of motion for m_t . Consider the flows from and to venue 2. In gross terms, venue 2 traders sell an amount $\rho_2(n_2\bar{a} + m_t)$ and buy $\frac{\rho_2 n_2}{2}$, so the net asset migration is

$$\frac{dm_t}{dt} = \rho_2 \frac{n_2}{2} - \rho_2(n_2\bar{a} + m_t). \quad (35)$$

Given the initial condition $m_0 = 0$, the solution of the ODE (35) is $m(t) = n_2(\frac{1}{2} - \bar{a})(1 - e^{-\rho_2 t})$. When we consider an interior solution, the transition dynamics of m do not depend on ρ_1 because $\sigma_{1,t}^\dagger$ adjusts. One can use equation (34) to compute $\sigma_{1,t}^\dagger$.

We can then use the market clearing condition to compute $\tilde{n}_{1,t}$. Using $\frac{n_1 - \ell_1 + n_2}{2} = (n_1 + n_2)\bar{a}$, we can write $\tilde{n}_{1,t} = n_1 - \ell_1 - n_2(1 - 2\bar{a})\frac{\rho_2 - \rho_1}{\rho_1}e^{-\rho_2 t}$. So in the long run, we have $\tilde{n}_{1,\infty} = n_1 - \ell_1 = G(\sigma_2) - G(\sigma_1)$, or equivalently, $\sigma_{1,\infty}^\dagger = \sigma_1^\dagger$. In terms of the time-varying marginal trading type $\sigma_{1,t}^\dagger$, we have

$$G(\sigma_{1,t}^\dagger) = G(\sigma_1) + n_2(1 - 2\bar{a})\frac{\rho_2 - \rho_1}{\rho_1}e^{-\rho_2 t},$$

which gives us the path of convergence of $\sigma_{1,t}^\dagger$ to σ_1 . One can then back out the price from the indifference condition in equation (2): $\bar{u}(a; \sigma_{1,t}^\dagger, 1) = r p_t a$. Thus, $p_t = \frac{\mu}{r} + \frac{\sigma_{1,t}^\dagger}{r} \frac{r + \rho_1}{r + \gamma + \rho_1}$.

Quantifying the Value Approximation

Assuming a uniform distribution of types, we have that the time-varying marginal trading type and price satisfy

$$\sigma_{1,t}^\dagger p_t = \sigma_1 + \bar{\sigma} n_2 (1 - 2\bar{a}) \frac{\rho_2 - \rho_1}{\rho_1} e^{-\rho_2 t}; \quad p_t = \bar{p} + k e^{-\rho_2 t},$$

where $\bar{p} = \frac{\mu}{r} + \frac{\sigma_1}{r} \frac{r + \rho_1}{r + \gamma + \rho_1}$ and $k = \bar{\sigma} n_2 (1 - 2\bar{a}) \frac{(\rho_2 - \rho_1)}{\rho_1 r} \frac{r + \rho_1}{r + \gamma + \rho_1}$.

Consider now the value functions for the light traders given price p_t : $rW_t = \mu a + \rho_1(p_t a - W_t) + \frac{\partial W_t}{\partial t}$. The value function is of the form $W_t = A + B e^{-\rho_2 t}$, so we have

$$r(A + B e^{-\rho_2 t}) = \mu a + \rho_1(\bar{p} a + k e^{-\rho_2 t} - A - B e^{-\rho_2 t}) - \rho_2 B e^{-\rho_2 t},$$

where $A = \frac{\mu + \rho_1 \bar{p}}{r + \rho_1} \bar{a}$ and $B = \frac{\rho_1}{r + \rho_1 + \rho_2} k \bar{a}$. We can see that A is just like \tilde{W} in the formulation of the proof of Proposition 1. The time-varying part $B e^{-\rho_2 t}$ is new.

We can compare the exact value of W at time 0 with its steady-state approximation as follows:

$$\frac{(W_0 - W_{\text{out}}) - (W_\infty - W_{\text{out}})}{(W_\infty - W_{\text{out}})} = \frac{B}{A - \frac{\mu}{r}} = \frac{\bar{\sigma}}{\sigma_1} n_2 (1 - 2\bar{a}) \frac{r + \rho_1}{\rho_1} \frac{\rho_2 - \rho_1}{r + \rho_1 + \rho_2}.$$

Take the case where $\frac{1}{2}\rho_2 = \rho_1 \gg r$ (note that we have assumed an interior solution for $\sigma_{1,l}^\dagger$, so we cannot take ρ_2/ρ_1 to be too large, or we would need to use the formula for the corner solution, which is less interesting, and in any case does not change the main point). Then we make an approximation of the order $\frac{1}{3} \frac{n_2(1-2\bar{a})}{n_1-\ell_1}$. So with long run participation of 60% in the fast venue, 30% in the slow one, and $\bar{a} = 0.45$, we get an approximation of $2 * 0.1/3 = 6.66\%$ in our value functions.

APPENDIX E: SUPPLEMENT TO SECTIONS 7 AND 8

E.1. *Example of Excess Entry With Three Venues*

This appendix analyzes the possibility of excess entry in a market with three venues. Following the notation of Section 7.2, l and h denote the slow and fast incumbents, respectively, and e denotes the entrant. We use the baseline calibration for equities as in Section 8. We report $\frac{\sigma_i}{\bar{\sigma}}$ instead of σ_i because it is easier to interpret. We also normalize $\Pi_l(\rho_l, \rho_h) = 100$ in the duopoly equilibrium and normalize welfare relative to the Walrasian outcome.

With two incumbents, the optimal speeds are $\rho_l = 239.13$ and $\rho_h = 23,758.2$, as displayed in Panel II of Table IV. The marginal types are $\frac{\sigma_l}{\bar{\sigma}} = 0.125$ and $\frac{\sigma_h}{\bar{\sigma}} = 0.417$. Profits are $\Pi_l(\rho_l, \rho_h) = 100$ and $\Pi_h(\rho_l, \rho_h) = 690.69$ and aggregate welfare is $\mathcal{W} = 90.51$.

Consider now a slow entrant $\rho_e \leq \rho_l$. The entrant optimally chooses $\rho_e = 127.70$ and the marginal types are $\frac{\sigma_e}{\bar{\sigma}} = 0.031$, $\frac{\sigma_l}{\bar{\sigma}} = 0.146$, and $\frac{\sigma_h}{\bar{\sigma}} = 0.458$. There is a significant decrease in participation in the fast venue, as predicted by Lemma 1. The profits are $\Pi_e(\rho_e, \rho_l, \rho_h) = 7.09$, $\Pi_l(\rho_e, \rho_l, \rho_h) = 53.71$, and $\Pi_h(\rho_e, \rho_l, \rho_h) = 594.50$. The profits of the slow incumbent are almost halved by competition from the entrant. Aggregate welfare decreases to $\mathcal{W} = 89.48$ mostly because welfare generated by the fast venue decreases from 81.56 to $\mathcal{W}_h = 77.96$.

The outcome is very different if the entrant has a high speed $\rho_e \geq \rho_l$. In that case, the entrant would optimally choose $\rho_e = 29,319$. The marginal types become $\frac{\sigma_e}{\bar{\sigma}} = 0.334$, $\frac{\sigma_l}{\bar{\sigma}} = 0.241 \times 10^{-3}$, and $\frac{\sigma_h}{\bar{\sigma}} = 0.802 \times 10^{-3}$. The venues generate welfare $\mathcal{W}_l \approx 0$, $\mathcal{W}_h = 10.63$, and $\mathcal{W}_e = 87.75$. Aggregate welfare increases to $\mathcal{W} = 98.38$. We also find (in untabulated results) that welfare increases when the entrant has an intermediate speed $\rho_l \leq \rho_e \leq \rho_h$.

To summarize, we find that entry can reduce welfare when the entrant has a low speed relative to that of the incumbent. The reason is that increased participation by low- σ types is not enough to compensate for the misallocation of high- σ types. On the other hand, when entry takes place at the high end of the speed ladder, we find that it improves welfare.

E.2. *Comparing Calibration Approaches*

Let us compare our calibration approach, PP, to that of DGP (2005, 2007). First and most obvious, there are several parameters that are unique to our model. PP considered heterogeneous agents, adding a distribution for investor types, and endogenized the market structure, thus considering speed cost, entry costs, and other parameters related to the modeling of competing venues and regulations.

One parameter calibration that is common to both papers is the rate of preference shocks γ . Volume depends on the number of transactions and on the average transaction size. Trade size is normalized to 1 both in our paper and in DGP, so it is natural to focus on

the *number of trades* per unit of time (a day in our calibration) as opposed to total volume.¹ Consider a standard trading day and a single trading venue. Let ρ denote the daily market contact rate, which can also be interpreted as the bilateral contact rate in DGP. Let \mathcal{V} denote the total number of trades and v the per capita number of trades. Assume there are n traders in the venue. A (steady-state) fraction F of traders are willing to trade given their holdings, preferences, and the prevailing market price. We can then write $\mathcal{V} = \rho \times n \times F(\gamma, \cdot)$ and $v = \frac{\mathcal{V}}{n} = \rho \times F(\gamma, \cdot)$. The fraction F structurally depends on the rate of preference change γ . Let \bar{a} denote the per capita asset supply, so the total supply is $n\bar{a}$. We can express traders' turnover as $\mathcal{T} = \frac{\mathcal{V}}{n\bar{a}} = \frac{v}{\bar{a}} = \frac{\rho F(\gamma)}{\bar{a}}$. The calibration strategies can be then summarized as follows: DGP started from \mathcal{T} and ρ and then computed $\gamma_{\text{DGP}} = F_{\text{DGP}}^{-1}(\frac{\bar{a}\mathcal{T}}{\rho})$; PP started from \mathcal{V} and ρ and then computed $\gamma_{\text{PP}} = F_{\text{PP}}^{-1}(\frac{\mathcal{V}}{\rho n})$. In words, the main empirical difference is that DGP considered a stylized trader turnover figure and fixed an arbitrary per capita asset supply $\bar{a} \in [0, 1]$ to derive γ . Instead, we looked at the aggregate number of trades for a particular asset and calibrated the number of active institutions to recover a representative institution's γ . We used volume mainly because we found that statistic more readily available across asset classes and trading instruments. A second advantage of using volume is de-emphasizing the role of the ad hoc parameter \bar{a} .

The calibration results in $\gamma_{\text{PP}} > \gamma_{\text{DGP}}$. Let us use the corporate bond market data points to do a numerical comparison. DGP chose asymmetric preference shock rates. Using their notation, in terms of yearly rates, $\lambda_u = 5$ and $\lambda_d = 0.5$, implying that an investor spends an average of 2 years as a high type and 0.2 years as a low type. Since, on average, each investor spends nearly 91% of the time as a high type, the weighted average yearly rate of preference change is $0.91 \times 5 + 0.09 \times 0.5 \approx 4.55$. The equivalent daily rate is $\gamma_{\text{DGP}} = \frac{4.55}{252} = 0.0181$. PP's comparable rate of preference change is $\frac{\mathcal{V}}{2} = 0.417$ ($\frac{1}{2}$ is the conditional probability of type change given the arrival of a shock). Roughly speaking, there are two orders of magnitude difference between the models. This difference chiefly stems from the difference in volume figure. Our TRACE data sample shows that active corporate bonds trade $1.97 \times 252 = 496.44$ times a year. Based on DGP's figures, instead, annual volume is turnover \times asset supply $= 0.5 \times 0.8 = 0.4$ trades a year. Actively traded corporate bonds then trade 1,000 times more than what is predicted in DGP calibration. Despite the fact that a different trading volume will naturally map into a different γ , and different potential gains from trade, the qualitative conclusions of the calibration exercise remain the same. In particular, the main economic interpretations that arise from comparing market outcomes to the solution of the constrained efficient problem are not affected by the specific value of γ .

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¹See the discussion in Section 8 about order splitting. Needless to say, this simplification will better represent the data when trade size is rather homogeneous.