Econometrica Supplementary Material

SUPPLEMENT TO "TIME PREFERENCES AND BARGAINING"
(*Econometrica*, Vol. 86, No. 1, January 2018, 173–217)

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APPENDIX B: SUPPLEMENTAL MATERIAL

B.1. *Multiplicity and Delay Under Weak Present Bias*

SUPPLEMENTING SECTION 5.2.2, I here present two examples of how violations of immediacy result in multiplicity and, possibly, also delay. The first is one of dynamically consistent preferences (ED) and was presented already by Rubinstein (1982, concl. I). To the best of my knowledge, its set of equilibria has not yet been explicitly characterized, however.

EXAMPLE 4: Let the two parties' preferences be given by $U_i(q, t) = q - ct$, for $c \in (0, 1)$. Due to preference symmetry, player indices are omitted in what follows. The preferences are covered by Assumption 1 once $U(0, \infty) \equiv -\infty$ is specified; in particular, impatience property (3)(c) is satisfied: $U(1, t)$ tends to minus infinity, whereas $u(0) = 0$.[1] In the assumed absence of uncertainty, they actually satisfy ED, albeit with "strongly" convex instantaneous utility: $U(q, t) = \ln(\delta^t u(q))$ for $\delta \equiv \exp(-c)$ and $u(q) \equiv \exp(q)$. Hence they exhibit a weak present bias but violate immediacy (increasing shares by the same amount leaves indifferent).[2]

This results in a multiplicity of stationary equilibrium: any $q \in [c, 1]$ is a proposer's equilibrium share in some stationary equilibrium (with immediate agreement, of course). Applying the characterization of Theorem 1, $v^* = c$ and $w^* = 0$, where both of these minimal proposer and rejection values correspond to a player's least preferred stationary equilibrium. Using these two least preferred stationary equilibria as optimal punishments, non-stationary delay equilibria can be constructed, and equation (6) offers a formula to compute the maximal such delay for any $c \in (0, 1)$:

$$\begin{aligned}
\kappa(t, c, c) &= \min\{c + ct, 1\} + \min\{ct, 1\} \\
&= \begin{cases} (2t + 1)c, & t \leq \dfrac{1 - c}{c}, \\ 1 + ct, & \dfrac{1 - c}{c} \leq t \leq \dfrac{1}{c}, \\ 2, & \dfrac{1}{c} \leq t, \end{cases} \\
\Rightarrow \quad t^* &= \sup\big\{t \in T \,|\, \kappa(t, c, c) \leq 1\big\} \\
&= \max\left\{t \in T \,\middle|\, t \leq \frac{1}{2} \cdot \frac{1 - c}{c}\right\}
\end{aligned}$$

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[1] U violates the requirement of Assumption 1 that $U(0, \infty) \in \mathbb{R}$, but the positive monotonic transformation $\exp(U)$ represents the same preferences and satisfies also this property.

[2] One may interpret such preferences as there being a cost to bargaining. To justify the non-negativity of each player's share in any proposal, assume then that players have an "outside option" of leaving the bargaining table forever, which is equivalent to obtaining a zero share immediately.

 https://doi.org/10.3982/ECTA14396

$$= \left\lfloor \frac{1}{2} \cdot \frac{1-c}{c} \right\rfloor.$$

For instance, if $c = \frac{1}{100}$, so that the cost per bargaining round equals one percent of the surplus per player, then the maximal equilibrium delay is 49 periods, with an associated efficiency loss of 98 percent of the surplus. To determine the values of c for which delayed agreement is an equilibrium outcome, simply solve $\kappa(1, c, c) \le 1$ for c, yielding $c \le \frac{1}{3}$. The set of equilibrium divisions with a given delay $t \le t^*$ in game G_1 equals $\{x \in X | c + ct \le x_1 \le 1 - ct\}$ and is monotonically shrinking in t.

The second example is one of dynamically inconsistent preferences (with an actual present bias) that are non-separable, following the magnitude-effects model of Noor (2011).

EXAMPLE 5: Let the two parties' preferences be symmetrically given by $U_i(q, t) = \delta(q)^t \cdot u(q)$ with $\delta(q) = 0.5 + 0.49 \cdot q^{0.5}$ and $u(q) = q^{0.5}$.

While both $U_i(q, 0) = q^{0.5}$ and $U_i(q, 1) = 0.5 \cdot q^{0.5} + 0.49 \cdot q$ are concave, these preferences violate immediacy; for example, once delayed share $q' = 0.75$ is indifferent to immediate share $q \approx 0.64$, but upon increasing both by $\varepsilon = 0.05$ the delayed one is preferred. Equations (4) and (5) for $t_i = 0$ have here three solutions, all of which correspond to a (symmetric) stationary equilibrium, with respective proposer shares 0.04, 0.57 and 0.98. (All numbers are rounded.) These different stationary equilibria can be used as (non-stationary) threats to support further equilibrium outcomes.

Indeed, given weak present bias (see footnote 36), the extreme *stationary* equilibria deliver the extreme equilibrium values; hence, they constitute optimal punishments supporting *all* equilibrium outcomes. Here the smallest stationary-equilibrium proposer share equals 0.04, and any *immediate* division with the initial proposer's share between this smallest amount and the largest stationary share of 0.98 can be supported. For any such division x, it can easily be verified that the following is an equilibrium: player 1 initially proposes division x, player 2 accepts with threshold x_2, and in case of a rejection, (i) if the initial offer was less than x_2, the players continue with the stationary equilibrium in which player 2, as the proposer of round 2, receives the largest credible share of 0.98, and (ii) if the initial offer was at least x_2, the players continue with the stationary equilibrium in which player 2, as the proposer of round 2, receives the smallest credible share of 0.04.

Computing all other equilibrium outcomes is straightforward using the indifference property (due to preference symmetry, player indices are omitted in what follows): for a single period of delay, the delayed share indifferent to the smallest immediate share of 0.04 equals 0.10, and the surplus cost κ of this delay therefore equals $0.04 + 0.10 = 0.14$, which is feasible. Hence, any once-delayed division with the initial proposer's share between 0.10 and $1 - 0.04 = 0.96$ can be supported. Let player 2 be the initial proposer and take any such division x; it can easily be verified that the following is an equilibrium: player 2 initially demands the entire surplus (offers zero), player 1 accepts with threshold 0.96, and in case of a rejection, (i) if the initial offer was zero, then the players continue with the immediate-agreement equilibrium described above for division x, and (ii) if the initial offer was positive, then the players continue with the stationary equilibrium in which player 1, as the proposer of round 2, receives the largest credible share of 0.98.

Continuing this way until the surplus cost of the delay becomes infeasible—that is, $\kappa > 1$—we can describe the set of equilibrium divisions for any feasible delay. The maximal delay t^* equals seven rounds, and the set of equilibrium divisions with this delay equals that of all divisions with the initial proposer's share between 0.48 and $1 - 0.43 = 0.57$.

B.2. *Unbounded Equilibrium Delay*

The following example slightly modifies Example 3 to exhibit unbounded equilibrium delay.

EXAMPLE 6: Let the two players' preferences be symmetrically given by $U_i(q, t) = d(t) \cdot q$ with

$$d(t) = \begin{cases} \delta^t, & t \le \tau, \\ \gamma\delta^{\tau+1}, & t > \tau, \end{cases} \quad (\delta, \gamma) \in (0, 1)^2 \text{ and } \tau > 0.$$

Due to preference symmetry, the player subscript is again omitted in what follows.

The difference to Example 3 is that delays beyond horizon $\tau + 1$ are not discounted. Observe, however, that $\Delta(t)$ equals δ for all $t \le \tau$ and $\gamma\delta$ for all $t > \tau$, exactly as in Example 3. Hence, whenever there is an equilibrium in which agreement is delayed by τ periods, $v^* = \frac{1-\delta}{1-\gamma\delta^2}$ and $w^* = \gamma\delta v^*$, as was found there.

The absence of discounting beyond a delay of $\tau + 1$ periods implies that equilibrium delay is unbounded if and only if $1 \ge \kappa(\tau + 2, v^*, v^*) = 2\frac{v^*}{\gamma\delta^{\tau+1}}$, which reduces to

$$\delta^\tau \ge \frac{2}{\gamma\delta} \cdot \frac{1-\delta}{1-\gamma\delta^2} \tag{23}$$

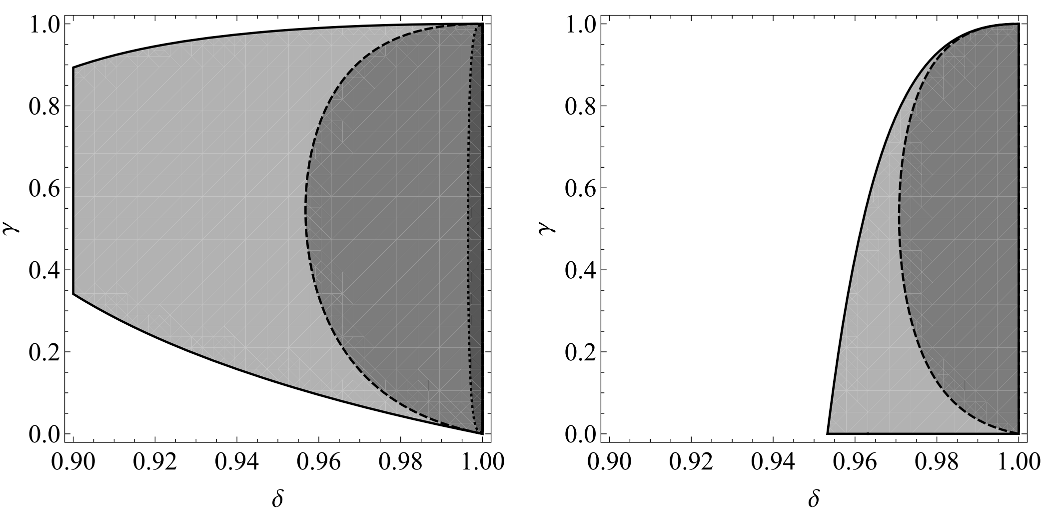

FIGURE 3.—Graphs regarding unbounded equilibrium delay in Example 3. The panel on the left shows the parametric regions (δ, γ) such that equilibrium delay is unbounded for three given values of τ, which are 1 (bounded by solid line), 25 (bounded by dashed line), and 1000 (bounded by dotted line). The panel on the right illustrates how the respective parametric regions for existence of delay equilibria (bounded by solid line) and unbounded equilibrium delay (bounded by dashed line) are related for the case of $\tau = 50$.

after substituting for v^*. Notice that this inequality is more stringent than Example 3's inequality (8), which shows when delay equilibria exist; in particular, $\gamma > 0$ is here required. Indeed, γ might be too low: despite existence of an equilibrium with delay τ, which fully determines the optimal punishments, proposing players would then require too large a compensation for longer delays, as those would involve additional discounting through γ. Nonetheless, for any given $\tau > 0$ and $\gamma < 1$, there again exist large enough values of δ such that also inequality (23) is satisfied, with the set of parameters γ and τ such that equilibrium delay is unbounded expanding as δ increases. Figure 3 illustrates this.

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Co-editor Itzhak Gilboa handled this manuscript.

Manuscript received 12 May, 2016; final version accepted 10 August, 2017; available online 16 August, 2017.