SUPPLEMENT TO "POLITICAL ECONOMY OF REDISTRIBUTION"
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A1. PROOFS

WE START WITH A FEW AUXILIARY LEMMAS that help us prove Proposition 1. In what follows, we let ξ_i^t denote transfers less transition costs, if any, obtained by player i in period t.

LEMMA A1: *Any protocol-free MPE σ is acyclic.*

PROOF: Let $\phi = \phi_\sigma$ be the equilibrium transition mapping generated by equilibrium σ. Suppose that there is a cycle starting from x: $\phi(x) \neq x$, but $\phi^l(x) = x$ for some $l > 1$. Without loss of generality, let l be the minimal such value, that is, the length of the cycle. Let us first show that for every $i \in V$, $[\phi^j(x)]_i = x_i$ for all j. Suppose not. Then without loss of generality we may assume to have chosen x such that $x_i \geq [\phi^j(x)]_i$ for all j (so i gets his maximum allocation along the cycle) and, moreover, that $[\phi(x)]_i < x_i$. Then, in the period that started with $x^{t-1} = x$ and where, in equilibrium, transition to $\phi_\sigma(x)$ is made, the continuation utility of player i satisfies (after taking the expectation over possible realizations of the protocols)

$$U_i^t \leq \left[\phi_\sigma(x)\right]_i + \xi + \beta\left(\left[\phi_\sigma^2(x)\right]_i + \xi\right) + \cdots + \beta^{l-1}\left(\left[\phi_\sigma^l(x)\right]_i + \xi\right) + \beta^l U_i^t,$$

where $\xi \in [0, (b+1)\varepsilon]$ is the maximum possible value of ξ_i^t over different periods. We thus have

$$\begin{aligned} U_i^t &\leq \frac{\left[\phi_\sigma(x)\right]_i + \xi + \beta\left(\left[\phi_\sigma^2(x)\right]_i + \xi\right) + \cdots + \beta^{l-1}\left(\left[\phi_\sigma^l(x)\right]_i + \xi\right)}{1-\beta^l} \\ &\leq \frac{(x_i - 1) + \xi + \beta(x_i + \xi) + \cdots + \beta^{l-1}(x_i + \xi)}{1-\beta^l} \\ &= \frac{x_i + \xi}{1-\beta} - \frac{1}{1-\beta^l} < \frac{x_i + \xi}{1-\beta} - 1. \end{aligned}$$

At the same time, if player i always vetoes all proposals in all subsequent periods, his continuation utility would be $\tilde{U}_i^t = \frac{x_i}{1-\beta}$. Since $\frac{\xi}{1-\beta} < \frac{(b+1)\varepsilon}{1-\beta} < 1$, we have $U_i^t < \tilde{U}_i^t$, which implies that player i has a profitable deviation. Hence, it must be that $[\phi_\sigma^j(x)]_i = x_i$ for all $j \geq 1$ and for all $i \in V$.

Since each veto player gets x_i in each period, the equilibrium payoff of each player must equal $U_i^t = \frac{x_i - \delta}{1-\beta}$. However, player i can always guarantee himself $\tilde{U}_i^t = \frac{x_i}{1-\beta}$ by vetoing all proposals. Therefore, he has a profitable deviation, which is impossible in equilibrium. This contradiction completes the proof. *Q.E.D.*

LEMMA A2: *Consider a one-step mapping ϕ, which is independent of protocols, and suppose that the current period is t and the current allocation is $x = x^{t-1}$. Suppose that some*

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player i has $[\phi(y)]_i > [\phi(z)]_i$ *for some* $y, z \in \mathbf{A}$. *Then player i prefers transition to y to transition to z*; *in other words* (*expectations are with respect to realization of protocols*),

$$y_i + \mathbb{E}\xi_i^t + \sum_{\tau=1}^{\infty} \beta^\tau\big(\big[\phi(y)\big]_i + \mathbb{E}\xi_i^{t+\tau}\big) > z_i + \mathbb{E}\tilde{\xi}_i^t + \sum_{\tau=1}^{\infty} \beta^\tau\big(\big[\phi(z)\big]_i + \mathbb{E}\tilde{\xi}_i^{t+\tau}\big), \qquad \text{(A1)}$$

where ξ *and* $\tilde{\xi}$ *reflect the transfers on path that follow acceptance of* y *and* z, *respectively. Furthermore*, *the same is true if* $[\phi(y)]_i = [\phi(z)]_i$, *but* $y_i > z_i$.

PROOF: Suppose $[\phi(y)]_i > [\phi(z)]_i$, but the inequality (A1) does not hold. Since $\xi_i^{t+\tau}, \tilde{\xi}_i^{t+\tau} \in [0, (b+1)\varepsilon]$ for any $\tau \geq 0$, this must imply

$$y_i + \sum_{\tau=1}^{\infty} \beta^\tau\big[\phi(y)\big]_i \leq z_i + \sum_{\tau=1}^{\infty} \beta^\tau\big[\phi(z)\big]_i + \frac{(b+1)\varepsilon}{1-\beta}. \qquad \text{(A2)}$$

Since $[\phi(y)]_i > [\phi(z)]_i$ implies $[\phi(y)]_i - [\phi(z)]_i \geq 1$, this implies

$$y_i + \frac{\beta}{1-\beta} \leq z_i + \frac{(b+1)\varepsilon}{1-\beta}.$$

Given that $z_i - y_i \leq b$, this implies $\frac{\beta-(b+1)\varepsilon}{1-\beta} \leq b$, which, since we assumed $(b+1)\varepsilon < 1-\beta$, implies $\frac{\beta}{1-\beta} \leq b+1$, which is equivalent to $\beta \leq 1 - \frac{1}{b+2}$, a contradiction. This proves the first part of the lemma.

Now suppose that $[\phi(y)]_i = [\phi(z)]_i$, but $y_i > z_i$. As before, assume not, in which case (A2) would hold. Now, given that $y_i - z_i \geq 1$, (A2) would imply $1 \leq \frac{(b+1)\varepsilon}{1-\beta}$, which contradicts our assumption that $(b+1)\varepsilon < 1-\beta$. This contradiction completes the proof. *Q.E.D.*

LEMMA A3: *Suppose that in protocol-free MPE* σ, $x \in \mathbf{A}$ *is such that* $x \neq \phi_\sigma(x) = \phi_\sigma^2(x)$. *Then* $\phi_\sigma(x) \rhd x$.

PROOF: Denote $y = \phi_\sigma(x)$. Let us first prove that $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$. Suppose, to obtain a contradiction, that this is not the case. Take some veto player l and consider protocol π where only player l proposes and does so only once (so $\pi = (l)$). Under this protocol, alternative y must be proposed and subsequently supported at the voting stage by a winning coalition of players. Now consider any agent i such that $y_i < x_i$, which implies $x_i - y_i \geq 1$. If y_i is accepted, agent i gets continuation utility (assuming the current period is t) that satisfies

$$U_i^t \leq y_i + (b+1)\varepsilon + \beta\big(y_i + (b+1)\varepsilon\big) + \cdots = \frac{y_i + (b+1)\varepsilon}{1-\beta}.$$

If, however, y_i is rejected, then the continuation utility satisfies

$$\tilde{U}_i^t \geq x_i + \beta y_i + \beta^2 y_i + \cdots = x_i + \frac{\beta}{1-\beta} y_i.$$

Since $b\varepsilon < 1-\beta$, we have

$$U_i^t - \tilde{U}_i^t \leq \frac{y_i + (b+1)\varepsilon}{1-\beta} - \left(x_i + \frac{\beta}{1-\beta} y_i\right)$$
$$= y_i - x_i + \frac{(b+1)\varepsilon}{1-\beta} \leq \frac{(b+1)\varepsilon}{1-\beta} - 1 < 0.$$

Therefore, such player i prefers the alternative y to fail at the voting stage. This implies that $U_i^t - \tilde{U}_i^t \geq 0$ is possible only if $y_i \geq x_i$, and, by assertion, the set of such players does not form a winning coalition, which means that y cannot be accepted at this voting stage. This contradicts that σ is equilibrium, which proves that $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$.

It remains to prove that for some $i \in V$, $y_i > x_i$ and $\|\phi_\sigma(x)\| \leq \|x\|$. Both results immediately follow from the fact that transition to $\phi_\sigma(x)$ is feasible and is not blocked by any veto player because of transition cost. Now, by definition of the binary relation $\rhd$, we have $\phi_\sigma(x) \rhd x$, which completes the proof. *Q.E.D.*

LEMMA A4: *Every protocol-free equilibrium is simple, that is, for every $x \in \mathbf{A}$, $\phi_\sigma^j(x) = \phi_\sigma(x)$ for all $j \geq 1$.*

PROOF: Suppose that there is a protocol-free equilibrium σ that is not simple, which means that there is $x \in \mathbf{A}$ such that $\phi_\sigma^2(x) \neq \phi_\sigma(x)$. By Lemma A1, σ is acyclic and, therefore, the path starting from x, $\phi_\sigma(x)$, $\phi_\sigma^2(x), \ldots$, stabilizes after no more than $|\mathbf{A}|$ iterations, and thus its limit $\phi_\sigma^\infty(x) = \phi_\sigma^{|\mathbf{A}|}(x)$ is well defined. Denote the set of all such $x \in \mathbf{A}$ by Y, so

$$Y = \left\{x \in \mathbf{A} : \phi_\sigma^2(x) \neq \phi_\sigma(x)\right\} \neq \emptyset.$$

Take allocation $y \in Y$ such that $\phi_\sigma^\infty(y) = \phi_\sigma^2(y)$ (notice that such y exists: indeed, if we take any $x \in Y$ and the minimal number such that $\phi_\sigma^\infty(x) = \phi_\sigma^j(x)$ is $j > 2$, then we can take $y = \phi_\sigma^{j-2}(x)$). Notice that we must have $\sum_{i\in V}[\phi_\sigma^2(y)]_i > \sum_{i\in V}[\phi_\sigma(y)]_i$, for otherwise the transition from $\phi_\sigma(y)$ to $\phi_\sigma^2(y)$ would be blocked by some veto player due to the cost of transition.

Consider veto player l for whom $[\phi_\sigma^2(y)]_l > [\phi_\sigma(y)]_l$. Suppose that in period t where the status quo is y, protocol $\pi = (l)$ is realized. Since σ is protocol-free, this must imply that player l proposes alternative allocation $\phi_\sigma(y)$ and some feasible transfers ξ, and this proposal is subsequently accepted. Now suppose that protocol $\pi' = (l, l)$ is realized and suppose that the game reached the second stage of the protocol. This subgame is isomorphic to one where protocol π has just been realized; consequently, in equilibrium, it must be that $\phi_\sigma(y)$ is proposed, accompanied with transfer ξ, and is accepted.

Let us prove that if in the second stage, the society decides to move to $\phi_\sigma(y)$, then in the first stage player l would be better off proposing $\phi_\sigma^2(y)$ and some feasible vector of transfers $\tilde{\xi}$, which would be accepted. Notice that in the following period, a transition from $\phi_\sigma(y)$ to $\phi_\sigma^2(y)$ would take place, which means that each player would receive a certain expected vector of transfers ξ. On the other hand, if transition to $\phi_\sigma^2(y)$ takes place in the current period, then the next period would have no transition, and in expectation, each veto player would get a transfer $\frac{\varepsilon}{v}$ (since each of them is equally likely to be the last player, who would be able to get the entire budget ε with probability 1). Notice that

$$\|\xi\| + \|\tilde{\xi}\| \leq \left(\max\left(\sum_{i\in V}[\phi_\sigma(y)]_i - \sum_{i\in V} y_i, 0\right) + 1\right)\varepsilon$$
$$+ \left(\max\left(\sum_{i\in V}[\phi^2_\sigma(y)]_i - \sum_{i\in V}[\phi_\sigma(y)]_i, 0\right) + 1\right)\varepsilon$$
$$\leq \left(\sum_{i\in V}[\phi^2_\sigma(y)]_i - \sum_{i\in V} y_i + 2\right)\varepsilon.$$

Take some small value $\alpha > 0$ and define vector χ by $\chi_i = \xi_i + \tilde{\xi}_i - \frac{\varepsilon}{v}\mathbf{I}\{i \in V\} + \alpha_i$, where $\alpha_i = -\alpha$ for $i \neq l$ and $\alpha_l = (n-1)l$. Then $\|\chi\| \leq (\sum_{i\in V}[\phi^2_\sigma(y)]_i - \sum_{i\in V} y_i + 1)\varepsilon$ and $\chi_i \geq 0$ for all i (for $i = l$ this is true because $\xi_l > 0$), so χ is a feasible vector if a transition to $\phi^2_\sigma(y)$ is proposed. If player l proposes such a transition to $\phi^2_\sigma(y)$ and offers feasible vector χ, then all players $i \in N$ who have $[\phi^2_\sigma(y)]_i \geq [\phi_\sigma(y)]_i$ must prefer such a transition to $\phi^2_\sigma(y)$ to happen rather than not. But since the transition from $\phi_\sigma(y)$ to $\phi^2_\sigma(y)$ would happen in a period starting with $\phi_\sigma(y)$, Lemma A3 implies $\phi^2_\sigma(y) \rhd \phi_\sigma(y)$, but this implies that the set of players who are better off if $\phi^2_\sigma(y)$ is accepted at the first stage is a winning coalition. This means that $\phi^2_\sigma(y)$ would be accepted if proposed, which implies that player l has a profitable deviation. This is a contradiction that completes the proof. *Q.E.D.*

LEMMA A5: *If σ is a simple protocol-free MPE, then for all $x \in \mathbf{A}$ either $\phi_\sigma(x) = x$ or $\phi_\sigma(x) \rhd x$.*

PROOF: By Lemma A1, σ is acyclic, and by Lemma A4, it is simple. Then for any $x \in \mathbf{A}$, we must have $\phi^2_\sigma(x) = \phi_\sigma(x)$. Now if $\phi_\sigma(x) = x$, the result is automatically true, and if $\phi_\sigma(x) \neq x$, then it follows immediately from Lemma A3. *Q.E.D.*

LEMMA A6: *Suppose that protocol-free MPE σ is played, and suppose that in period t, $x^{t-1} = x$. Then if there exists $y \in \mathbf{A}$ such that $\phi_\sigma(y) = y$ and $y \rhd x$, then x cannot be stable: $\phi_\sigma(x) \neq x$.*

PROOF: Suppose, to obtain a contradiction, that $\phi_\sigma(x) = x$. Let l be a veto player such that $y_l > x_l$ (such l exists as $y \rhd x$). Consider protocol $\pi = (l)$ (or any protocol ending with l). If a proposal made in this period is rejected, then each player i gets $\tilde{U}^t_i = \frac{x_i}{1-\beta} + \frac{\beta}{1-\beta}\frac{\varepsilon}{v}\mathbf{I}\{i \in V\}$.

Suppose player l makes proposal (y, ξ), where $\xi_i = \frac{(\|y\|-\|x\|+1)\varepsilon-\delta}{n}$. Since $\|y\| - \|x\| \geq 1$ and $\delta < \varepsilon$, we have $\xi_i \geq 0$ for all $i \in N$, so ξ is a feasible transfer. This means that each player i for which $y_i \geq x_i$ would get $\frac{y_i}{1-\beta} + \xi_i + \frac{\beta}{1-\beta}\frac{\varepsilon}{v}\mathbf{I}\{i \in V\}$ if the proposal is accepted, which exceeds $\tilde{U}^t_i$ that he would get if the proposal is rejected. Since $y \rhd x$, such players form a winning coalition, which implies that the proposal (y, ξ) would be accepted if made. Then player l has a profitable deviation, which is impossible. This contradiction completes the proof. *Q.E.D.*

PROOF OF PROPOSITION 2: *Part 1.* Take any simple protocol-free MPE σ and let $\mathbf{S}_\sigma = \{x \in \mathbf{A} : \phi_\sigma(x) = x\}$. By Lemma A1, it is nonempty. Let us prove that it satisfies internal stability. Suppose that for some $x, y \in \mathbf{S}_\sigma$, we have $y \rhd x$. Then by Lemma A6, $\phi_\sigma(y) =$

y implies $\phi_\sigma(x) \neq x$, which contradicts that $x \in \mathbf{S}_\sigma$. This contradiction proves that $\mathbf{S}_\sigma$ satisfies internal stability.

Let us now show that $\mathbf{S}_\sigma$ satisfies external stability. Take $x \notin \mathbf{S}_\sigma$. Then by Lemma A5, $\phi_\sigma(x) \rhd x$. Since σ is simple, $\phi_\sigma(x) \in \mathbf{S}_\sigma$, which shows that there exists $y \in \mathbf{S}_\sigma$ such that $y \rhd x$. This proves that $\mathbf{S}_\sigma$ satisfies external stability. This proves that $\mathbf{S}_\sigma$ is von Neumann–Morgenstern-stable set. *Q.E.D.*

LEMMA A7: *If σ is a protocol-free MPE, then $\|\phi_\sigma(x)\| = \|x\|$ for all $x \in \mathbf{A}$.*

PROOF: Suppose not. Then there exists $x \in \mathbf{A}$ for which $\|\phi_\sigma(x)\| < \|x\|$. Since σ is simple by Lemma A4, we have $\phi_\sigma(x) \in \mathbf{S}$. Take some veto player l and consider the protocol $\pi = (l)$; at this stage, player l must propose $\phi_\sigma(x)$ and it must be accepted. Notice, however, that player l may propose allocation y that has $y_l = [\phi_\sigma(x)]_l + 1$ and $y_i = [\phi_\sigma(x)]_i$ for all $i \neq l$, and split the extra ε of available transfers equally among players. By Proposition 3, such allocation y is stable as well. Consequently, all players would be strictly better off from proposal y (with the corresponding transfers) than the equilibrium proposal $\phi_\sigma(x)$. Thus, if a winning coalition was weakly better off from supporting $\phi_\sigma(x)$, it is strictly better off supporting y. Thus, player l has a profitable deviation at the proposing stage, which is a contradiction that completes the proof. *Q.E.D.*

PROOF OF PROPOSITION 1: *Part* (i). Consider the unique von Neumann–Morgenstern-stable set for dominance relation $\rhd$, $\mathbf{S}$ (its existence and uniqueness follow from Proposition 3 proven in the main text). Take any mapping ϕ such that $\phi(x) = x$ for any $x \in \mathbf{S}$ and for any $x \notin \mathbf{S}$, $\phi(x) \in \mathbf{S}$ and $\phi(x) \rhd x$ (the existence of such a mapping follows from external stability of mapping $\mathbf{S}$ implying that for any $\mathbf{S}$, we can pick such $\phi(x) \in \mathbf{S}$) and, moreover, $\|\phi(x)\| = \|x\|$ (the existence of such ϕ follows from Proposition 3 as well, as otherwise one can add $\|x\| - \|\phi(x)\|$ units to some veto player and get an allocation in $\mathbf{S}$ with the required property). Let us prove the following (stronger) result: there is a protocol-free MPE σ such that $\phi_\sigma = \phi$ (notice that σ will in this case be simple, because $\phi^2 = \phi$.)

We construct equilibrium σ using the following steps. For each possible status quo $x \in \mathbf{A}$ and each protocol $\pi \in \Pi$, we define transfers that each player is supposed to get in that period. We use allocations and these transfer utilities to define continuation utilities. After that, we use these continuation utilities to define strategies players would use for each $x \in \mathbf{A}$ and each $\pi \in \Pi$. We then check that under these strategies, players indeed get the transfers that we defined, and no player has a one-shot deviation. This would prove that σ is MPE, which would be protocol-free by construction.

If $x \notin \mathbf{S}$, then let $V_x = \{i \in V : [\phi(x)]_i = x_i\}$ and let $v_x = |V_x|$. Furthermore, let $Z = \sum_{i \in V} [\phi(x)]_i - \sum_{i \in V} x_i > 0$. Let $l = \pi_{|\pi|}$ be the last proposer, and define transfers $\xi_i(x, \pi)$ as

$$\xi_i(x,\pi) = \begin{cases} 0 & \text{if } i \notin V_x \cup \{l\}, \\ \beta \dfrac{Z\varepsilon}{(1-\beta)v + \beta v_x} & \text{if } i \in V_x \setminus \{l\}, \\ (Z+1)\varepsilon - \displaystyle\sum_{j \neq l} \xi_i(x,\pi) & \text{if } i = l. \end{cases} \tag{A3}$$

If, however, $x \in \mathbf{S}$, then the transfer is defined as $\xi_l(x, \pi) = \varepsilon$ for $l = \pi_{|\pi|}$ and $\xi_i(x, \pi) = 0$ otherwise. Given these transfers, the continuation utilities (at the beginning of the period, before protocol is realized) are given by

$$V_i(x) = \left[\phi(x)\right]_i + \sum_{\pi \in \Pi} \xi_i(x, \pi) + \frac{\beta}{1-\beta}\left(\left[\phi(x)\right]_i + \frac{\varepsilon}{v}\mathbf{I}\{i \in V\}\right). \tag{A4}$$

Let us now define strategies as follows. Suppose that in period t, the current status quo is $x = x^{t-1}$ and protocol π was realized. To define strategies, consider the game with timing from Section 2, but define payoffs in case transition to some $y \in \mathbf{A}$ and set of transfers ξ is decided upon given by

$$U_i(y, \xi) = y_i + \xi_i + \beta V_i(y)$$

(in other words, consider the game truncated at the end of the period, that is, a finite game, but with payoffs coinciding with continuation payoffs of the original game).

Define strategies by proceeding by backward induction, with a few exceptions. In the last stage, the proposer $\pi_{|\pi|}$ proposes to transfer to $\phi(x)$ (or to stay, if $\phi(x) = x$), and offers transfers $\xi_i(x, \pi)$. We require that all players who are at least indifferent vote for this proposal to pass. If any other proposal is made, as well as in all previous stages, we require that players play any strategies consistent with backward induction, except that we require that players vote *no* when indifferent.

Let us show that the players have no incentive to deviate for any strategy that we defined. The one-shot deviation principle applies, so we need to verify that no player has a profitable deviation at any stage. Now consider the two cases $\phi(x) = x$ and $\phi(x) \neq x$ separately.

First, consider the case $\phi(x) \neq x$. Let us check that at the last stage, it is a best response for any player i with $[\phi(x)]_i \geq x_i$ to accept, which would imply that this proposal is indeed accepted. Indeed, both accepting and rejecting results in getting the same allocation $[\phi(x)]_i$ in two periods; thus, if for some player i, $[\phi(x)]_i > x_i$, then by Lemma A2 he is strictly better off if $\phi(x)$ is accepted. Consider a player i with $[\phi(x)]_i = x_i$. If $i \notin V$, then he gets transfer $\xi_i(x, \pi) = 0$ if $\phi(x)$ is accepted, but he gets the same in the following period if the proposal is rejected, which implies that he is indifferent, so supporting $\phi(x)$ is a best response. If $i \in V_x \setminus \{l\}$, then he gets the transfer $\xi_i(x, \pi)$ if the alternative is accepted, and it makes him exactly indifferent between accepting and rejecting. Finally, if $i \notin V_x$ or $i = l$, the player is strictly willing to accept. Thus, for all veto players, it is a best response to support the alternative. Since $\phi(x) \rhd x$, the set of players with $[\phi(x)]_i \geq x_i$ is a winning coalition. Finally, $\|\phi(x)\| = \|x\|$, so the transition is feasible. Consequently, there are best responses that result in $\phi(x)$ being accepted.

Taking one step back, let us verify that it is a best response for player $l = \pi_{|\pi|}$ to propose $\phi(x)$. First, since he prefers $\phi(x)$ to be accepted rather than rejected, he would only propose an alternative y if this alternative would be accepted at the voting stage. Suppose there is such an alternative; it suffices to prove that proposing it does not make the player l better off. By Lemma A2, if $[\phi(y)]_i < [\phi(x)]_i$ for some player i, then this player would be better off if y is rejected. Consequently, for y to be accepted in equilibrium, it is necessary that $[\phi(y)]_i \geq [\phi(x)]_i$ for a winning coalition of players, in particular, for all veto players $i \in V$.

Let us prove that $[\phi(y)]_i = [\phi(x)]_i$ for all $i \in V$. To do so, suppose it is not the case, meaning that for some $j \in V$, the strict inequality $[\phi(y)]_j > [\phi(x)]_j$ holds. In addition, notice that $\|y\| \leq \|x\|$ since transition to y is feasible, but $\|\phi(y)\| \leq \|y\|$ (because transition to $\phi(y)$ would be feasible) and $\|x\| = \|\phi(x)\|$ (by assumption that transition to $\phi(x)$ does not result in waste). This implies $\|\phi(y)\| \leq \|\phi(x)\|$, which, together with $\{i \in N : [\phi(y)]_i \geq [\phi(x)]_i\} \in \mathcal{W}$ and $[\phi(y)]_j > [\phi(x)]_j$, implies $\phi(y) \rhd \phi(x)$. Since $\phi(x), \phi(y) \in \mathbf{S}$, this contradicts internal stability of $\mathbf{S}$, which proves that $[\phi(y)]_i = [\phi(x)]_i$ for all $i \in V$.

Notice that for the proposer, player $l = \pi_{|\pi|}$, to prefer transition to y to transition to $\phi(x)$, it must be that $y_l = [\phi(y)]_l = [\phi(x)]_l$, for otherwise we would get a contradiction with Lemma A2. Consider two possibilities. If $\phi(y) = y$, then for player l to be better off, he needs to get a larger transfer $\chi_l > \xi_l(x, \pi)$. However, since all other veto players in V_x were indifferent between accepting their transfer $\xi_i(x, \pi)$ and rejecting, they need to get at least this transfer as well; since other players need to get $\chi_i \geq 0$ as well, such deviation cannot be profitable. If, however, $\phi(y) \neq y$, then $\phi(y)$ will be reached in the following period. Notice that for each $i \in V$ it must be that $y_i \geq x_i$, for otherwise this player would block the transition. This means, in particular, that for players in V_x, $x_i = y_i = \phi(x_i) = \phi(y_i)$ holds, and they therefore need discounted transfer $\chi_i^t + \beta \mathbb{E} \chi_i^{t+1} \geq \xi_i(x, \pi) + \beta \frac{\varepsilon}{v}$ so as to be willing to accept. However, since the transfers available over the two periods are capped at $(Z + 1)\varepsilon - \delta$, player l cannot be better off from such deviation. Therefore, proposing $\phi(x)$ at the last stage is a best response.

We now prove that for any proposal z made at the previous stage by player $\pi_{|\pi|-1}$, the set of players who strictly prefer transition to z do not form a winning coalition. Indeed, suppose that it is; then by Lemma A2 it must be that for all $i \in V$, $[\phi(z)]_i = z_i = y_i$, for otherwise we would have $\phi(z) \rhd y$, which would contradict internal stability of $\mathbf{S}$. This implies that $z = \phi(z)$, for otherwise transition from z to $\phi(z)$ would be impossible; furthermore, the set of transfers χ proposed at this stage must coincide with $\xi_i(x, \pi)$. If so, if some player $i \notin V$ strictly prefers transition to z, this implies that $z_i > y_i$ for such a player. However, this would contradict the characterization results from Proposition 3. This shows that it is a best response for at least $n - k + 1$ players to vote against proposal z, which implies that there is an equilibrium in this subgame where it is not accepted. Proceeding by backward induction, we can conclude that there is an equilibrium in this finite game where no proposal is accepted until the last stage, where y is accepted.

Now consider the game with $x \in \mathbf{S}$. We allow any strategies, but require that players vote against the proposal when indifferent. Now, again by backward induction, we can conclude that if a winning coalition strictly prefers to accept some proposal z, then either $\phi(z) \rhd x$, which contradicts internal stability of $\mathbf{S}$, or $[\phi(z)]_i = z_i = x_i$ for all $i \in V$, in which case the veto player $\pi_{|\pi|}$ that is the last to propose is actually worse off because of transition cost. Thus, there is an equilibrium in the finite game where no proposal is accepted, so x remains stable.

Last, it is straightforward to check that if these strategies are played, then in every period, transfers are given by $\xi(x, \pi)$ as defined above, and thus the continuation utilities at the beginning of period with x as the status quo are given by $V(x)$. This means that if these strategies are played in the original game Γ, no player has a one-shot deviation. Since by construction the strategies are Markovian and transitions do not depend on the realization of the protocol, then σ is a protocol-free MPE. Moreover, it is simple and efficient by construction, which completes the proof of existence of such equilibrium.

Part (ii). Follows from Lemma A1.

Part (iii). Follows from Lemma A4.

Part (iv). Follows from Lemma A7. *Q.E.D.*

PROOF OF PROPOSITION 4: *Part* (i). Lemma A5 implies that $\phi(y) \rhd y$; in particular, for each $j \in V$, $[\phi(y)]_j \geq y_j$ and for at least one of them the inequality is strict. Suppose, to obtain a contradiction, that $|\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\}| < d - 1$. Then

$|\{j \in M : [\phi(y)]_j < x_j\}| < d$. But we also have that for each $j \in V$, $[\phi(y)]_j \geq x_j$, with at least inequality strict. This means $\phi(y) \rhd x$, which is impossible, given that $x, \phi(y) \in \mathbf{S}$. Now suppose, to obtain a contradiction, that $|\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\}| > d-1$. But then for at least d agents $[\phi(y)]_j < y_j$, which contradicts $\phi(y) \rhd y$. This contradiction proves that $|\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\}| = d - 1$. It remains to prove that $y_i \leq [\phi(y)]_i < x_i$. Suppose not, that is, either $[\phi(y)]_i < y_i$ or $[\phi(y)]_i \geq x_i$. In the first case, we would have that at least d agents have $[\phi(y)]_j < y_j$, contradicting $\phi(y) \rhd y$. In the second case, $[\phi(y)]_i \geq x_i$, coupled with the already established $|\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\}| = d - 1$, would mean $|\{j \in M : [\phi(y)]_j < x_j\}| = d - 1$, and therefore $\phi(y) \rhd x$. This is impossible, and this contradiction completes the proof.

Part (ii). This proof is similar to the proof of internal stability in the proof of Proposition 3. Denote $\phi(y) = z$. Then $z \rhd y$ and $x, z \in \mathbf{S}$. We know that x and z have the group structure by Proposition 3. Then let the r groups be $G_1, \ldots, G_r$ for x and $H_1, \ldots, H_r$ for z, respectively. Without loss of generality, we can assume that each set of groups is ordered so that x_{G_j} and z_{H_j} are nonincreasing in j for $1 \leq j \leq r$. Suppose, to obtain a contradiction, that for some agent $i' \in M$ with $x_{i'} \leq y_i < x_i$, $z_{i'} < y_{i'}$. In that case, among the set $\{j \in M : x_j \geq x_i\}$ there are at most $d-1$ agents with $z_j < y_j$; similarly, among the set $\{j \in M : x_j < x_i\}$ there are at most $d-1$ agents with $z_j < y_j$.

We can now proceed by induction, similarly to the proof of Proposition 3, and show that $x_{G_j} \leq z_{H_j}$ for all j. Base: suppose not. Then $x_{G_1} > z_{H_1}$, and then $x_{G_1} > z_s$ for all $s \in M$. But this means that for all agents $l \in G_1$, we have $x_l > z_l$; since their total number is d, we get a contradiction. Step: suppose $x_{G_l} \leq z_{H_l}$ for $1 \leq l < j$, and suppose, to obtain a contradiction, that $x_{G_j} > z_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \leq l \leq j$ and $j \leq s \leq r$, $x_{G_l} > z_{H_s}$. Consequently, for an agent $i'' \in \bigcup_{l=1}^{j} G_l$ to have $z_{i''} \geq x_{i''}$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j-1)d = d$ agents in $\bigcup_{l=1}^{j} G_l \subset M$, $z_{i''} \geq x_{i''}$ does not hold (denote this set by D). If that is true, it must be that $\bigcup_{l=1}^{j} G_l$ includes all the agents in D, including agents i and i' found earlier, and, in particular, $x_{G_j} \leq y_i < x_i$. But on the other hand, these d agents are not in $\bigcup_{s=1}^{j-1} H_s$. In particular, this implies that for any $i'' \in D$, $z_{i''} < x_{G_j}$, but $x_{i'} \geq x_{G_j}$, which means $z_i < x_{i'}$. But $z_i \geq y_i$ by part (i) of this proposition, so $y_i < x_{i'}$. But this contradicts the way we chose i' to satisfy $x_{i'} \leq y_i < x_i$. This proves that such i' cannot exist, and thus the $d-1$ agents other than i who are made worse off satisfy $x_j \geq x_i$. *Q.E.D.*

PROOF OF PROPOSITION 5: This result immediately follows from the formulas $m = n - v$, $d = n - k + 1$, and $r = \lfloor m/d \rfloor$, and from Proposition 3. *Q.E.D.*

PROOF OF PROPOSITION 6: *Part* (i). If $k < n$, then $d > 1$. An allocation x is stable only if $|\{j \in M : x_j > 0\}|$ is divisible by d. If x is stable and some agent i with $x_i > 0$ is made a veto agent, then the set $|\{j \in M' : x_j > 0\}| = |\{j \in M : x_j > 0\}| - 1$ and is not divisible by d; thus x becomes unstable. At the same time, if $x_i = 0$, then the group structure for all groups with a positive amount is preserved; thus x remains a stable allocation.

Part (ii). In this case, the size of each group in x is $d > 2$, and every positive amount is possessed by either no players or d non-veto players. If k increases by 1, d decreases by 2. Then allocation x becomes unstable, except for the case $x|_M = 0$. *Q.E.D.*

A2. EXAMPLES

In the examples below, we do not explicitly consider costs of transition and transfers explicitly, as they would complicate the exposition. Unless specified otherwise, each of the examples below may be modified to accommodate such factors.

EXAMPLE A1—If Costs of Transition Are Assumed To Be 0: Suppose $n = 3$, $v = 1$, and $k = 2$, so there are three players, one of them a veto player, and the rule is simple majority rule. Assume for simplicity that there is only 1 unit that initially belongs to a non-veto player (say, player #1), so the initial allocation is $(1, 0; 0)$. Then there would be an equilibrium where the veto player (proposing last) would propose to move the unit from player #1 to player #2 if it belongs to player #1, and then propose to move it the other way around if it belongs to player #2. Such a proposal would then be supported by the veto player and the other player who receives the unit.

To complete the description of strategies, we can also assume that any proposal made at a protocol stage before the last one, except for the proposal to transfer the good to the veto player, would be vetoed by the veto player (he is indifferent anyway). On the other hand, if a proposal to transfer the unit to the veto player is ever made, the two non-veto players vote against this proposal. They both have incentives to do so, because the equilibrium play gives them the unit in possession every other period, which is better than having the unit taken away.

Thus, without transaction costs, it is possible to have cyclic equilibria, which do not seem particularly natural.

EXAMPLE A2—Example Where Non-Veto Player Proposes Last: Suppose $n = 11$, $v = 1$, and $k = 9$, so there are 11 players, one of them a veto player, and the rule requires agreement of 9 players. The size of a minimal blocking coalition is then three. In this case, in any protocol-free MPE (where the last proposal is done by the veto player), allocation $(3, 3, 3, 2, 2, 2, 1, 1, 1, 10; 0)$ is unstable, and, in any equilibrium, it results in a transfer to an allocation where all players except for player #10 (the one possessing 10 units in the beginning of the game) are better off. To simplify the following argument, let us focus on the equilibrium where an immediate transition to $(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$ takes place.

Consider, however, what would happen if a protocol has a non-veto player propose last. Specifically, suppose the protocol has two players: first the veto player (player #11) proposes and then the non-veto player #6 proposes. Consider the last stage and suppose that player #6, instead of proposing to move to $(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$ or to stay in the current allocation $(3, 3, 3, 2, 2, 2, 1, 1, 1, 10; 0)$, proposes to transfer to allocation $(3, 3, 1, 2, 2, 3, 1, 1, 2, 4; 6)$; in other words, in addition to moving some units to the veto player, he also proposes to take 2 units from player #3, and takes 1 unit himself and gives the other one to player #9 in order to "buy" his vote. This proposal leads to a stable allocation, and it makes only two players (player #3 and player #10) worse off. It therefore would be accepted; the veto player would agree, because it gives him 6 of the units right away, and he would be able to get the other 4 the following period. (Notice that player #4 might prefer not to get more units for himself in the short run, out of fear that having 4 or more units in the next period would make him a candidate for complete expropriation.)

Taking one step back and consider the stage where the veto player makes the proposal. He would use the opportunity to get the 10 units belonging to him immediately (which hurts player #10). However, he would not be able to make the society move to

$(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$, which they are supposed to do in equilibrium, because doing so would make players #6 and #9, in addition to #10, worse off, and thus such a proposal would not gather the 9 votes needed to pass. This means that by allowing non-veto players to propose, in some examples we would lose the existence of protocol-free MPE.

This example relies on the fact that non-veto players are not indifferent between different stable allocations, and would want to make the society reallocate the units in their favor. As the results in this paper show, these moves cannot happen in protocol-free equilibria studied in the paper. Consequently, we do not view such a possibility to be natural or robust, and we impose the assumption that non-veto players cannot be the last ones in a protocol to avoid such issues and obtain the existence of protocol-free equilibria.

EXAMPLE A3—Example With Fixed Protocol: Suppose $n = 3$, $v = 1$, and $k = 2$, so there are three players, one of them a veto player, and the rule is simple majority. Consider the allocation $(1, 1; 0)$, where the veto player possesses nothing initially. In a protocol-free equilibrium, this allocation would be stable.

Consider a game where the protocol is fixed at $\pi = (1, 3)$ in each period (we can allow the second player to propose in between the other two and get the same result). We claim that the following transitions are possible in an equilibrium. Player #1 is recognized first, and he proposes to move to $(1, 0; 1)$, which is supported by him and the veto player, and in the following period the veto player gets all the surplus, as usual. If the proposal by player #1 is rejected, however, then player #3 is recognized and proposes to move to $(0, 1; 1)$, and this proposal is supported by himself and player #2. Thus, in equilibrium, the society moves from $(1, 1; 0)$ to $(1, 0; 1)$, and then to $(0, 0; 2)$.

The reason why this example works is the following. Player #1 knows that if he does not promise the veto player a transfer of 1 unit, then he would lose his possession immediately (later the same period), whereas delivering the unit to the veto player allows him to postpone for another period. The veto player knows that he cannot take both units at once (as players #1 and #2 would like to stick to them for another period); however, if he allows player #2 to keep his unit, the latter would not mind participating in expropriation of player #1, because in either case he keeps his unit for the current period and loses it in the following one, along the equilibrium path. Furthermore, if these strategies are played, preserving the status quo $(1, 1; 0)$ is not an option. Thus, there is an equilibrium where non-veto players participate in expropriation of each other.

Notice that this transition (from $(1, 1; 0)$ to $(1, 0; 1)$) cannot arise in a protocol-free equilibrium for the following reason. Suppose the protocol only involves the veto player. In such an equilibrium, he needs to propose to transit to $(1, 0; 1)$. But player #2 will oppose it for obvious reasons, and player #1 would know that if he agrees, then he keeps his unit for one extra period (the current one), but if he rejects, then in protocol-free MPE he faces the same transition to $(1, 0; 1)$ the following period, and thus he would be able to keep the unit for two extra periods, which he obviously prefers. Consequently, such transition would be impossible in this protocol, which contributes to the idea that such transitions are not particularly robust.

EXAMPLE A4—Example of Equilibrium That Is Not Markov Perfect: Suppose $n = 3$, $v = 1$, and $k = 2$, so there are three players, one of them a veto player, and the rule is simple majority. Consider the allocation $(1, 1; 0)$, where the veto player possesses nothing initially.

Suppose that the veto player is always the proposer, so the protocol is $\pi = (3)$. Then the following transitions may be supported in equilibrium. As long as the allocation is

$(1, 1; 0)$, the veto player proposes to move to $(1, 0; 1)$ if the period is odd and to move to $(0, 1; 1)$ if the period is even, and the proposal is supported by him and by the non-veto player who keeps the unit (player #1 in odd periods and player #2 in even periods). Once this transition has taken place, in the following period, the veto player gets everything, thus moving to $(0, 0; 2)$.

The rationale for non-veto players to support such proposals is that they get to keep their unit for exactly one extra period, regardless of the outcome of the voting. Thus, they are indifferent in such situations, in which case the veto player is able to allocate a small transfer to break this indifference. As a result, there is a SPE where the society moves to $(1, 0; 1)$ and then to $(0, 0; 2)$; it is supported by the threat of a move to $(0, 1; 1)$ (and then again to $(0, 0; 2)$) if this proposal is rejected.

Two comments are warranted. First, this SPE does not require knowledge of all history, in particular, players' proposals and votes. It only requires that the veto player acts differently in odd and even periods. In particular, this is a dynamic equilibrium (DE) in the sense of Anesi and Seidmann (2015), as if the players are allowed to condition their moves on the past history of alternatives, they of course can make use of the length of this history. Second, such transitions are impossible in a protocol-free equilibrium. Indeed, the proposal to move to $(1, 0; 1)$ made by the veto player would not be accepted if player 1 knew that the veto player would make this very proposal again in the following period, rather than proposing $(0, 1; 1)$.

EXAMPLE A5—Example With Random Recognition of Players but Without Protocol-Free Requirement: Suppose $n = 5$, $v = 2$, and $k = 3$, so there are five players, two of them veto players, and the rule is simple majority. Consider the allocation $(1, 1, 1; 0, 0)$, where the veto players possess nothing initially. In a protocol-free equilibrium, this allocation would be stable.

Consider a game, where in each period, one player is recognized as the proposer. Furthermore, assume for simplicity that only veto players may be recognized, and each of them is recognized with probability 0.5. Then the following strategies would form a MPE. Suppose that player #4, if he is the agenda-setter, proposes to move to $(2, 0, 0; 1, 0)$, and this proposal is supported by the two veto players and player #1. Similarly, if player #5 gets a chance to propose, he proposes to move to $(0, 2, 0; 0, 1)$, which is supported by the two veto players and player #2. If either of the proposals is accepted, then in the following period the society moves to $(0, 0, 0; 2, 1)$, where the veto players possess everything.

To understand why player #1 supports the transition to $(2, 0, 0; 1, 0)$, notice that in this case, he gets payoff 2 in the current period and 0 thereafter. If he rejects, then he keeps 1 in the current period, but in the next period he faces a lottery between 2 and 0, and gets 0 thereafter. His expected continuation payoff is therefore $1 + \beta\frac{2+0}{2} = 1 + \beta < 2$. Consequently, he prefers to agree on the transition to $(2, 0, 0; 1, 0)$. For the same reason, player #2 would support the transition to $(0, 2, 0; 0, 1)$. Notice that neither of the veto players can do better by choosing some other proposal, and therefore these transitions are possible in equilibrium.

Notice that if we impose the requirement that equilibria be protocol-free, which in this case would mean that the transition is the same regardless of the player who gets to make the proposal, such an equilibrium will be ruled out. Thus, the requirement that equilibria do not depend on the protocol is important for our results, but also these equilibria may be considered more robust than the one in this example.

A3. RELATION TO LARGEST CONSISTENT SET

We have proven (Proposition 2) that the set of stable allocations coincides with the vNM-stable set, which is in our case unique. However, as emphasized, for example, in Ray and Vohra (2015), the vNM has the drawback in that it focuses on "static" deviations, that is, those in which a deviating coalition does not foresee the future path of the game. On theoretical grounds, this is a serious objection. One notion to deal with this problem was the largest consistent set, as defined in Chwe (1994). In what follows, we prove that in our setting, the largest consistent set would coincide with a vNM-stable set and thus with the set of stable allocations, that is, the objection concerning farsighted deviation does not apply to our game. In our view, the intuitive reason for this is that in our game, *any* coalition that can make *some* deviation (i.e., a winning coalition) can also make *any* deviation. Coupled with farsightedness (discount factor being high enough), this means that a coalition that would be better off initiating a long path of changing allocations would also be better off transiting immediately to the final allocation in this sequence, and it is also capable of doing so. Thus, allowing for farsighted deviations does not add profitable deviations at states that did not have such deviations. Below we state this result formally.

For any coalition $X \in 2^N \backslash \{\emptyset\}$, define binary relation $\rightarrow_X$ on $\mathbf{A}$: for all $x, y \in \mathbf{A}$, $x \rightarrow_X y$ if and only if $\|y\| \leq \|x\|$ and either $x = y$ or $X \in \mathcal{W}$. In other words, a winning coalition can enforce transition from any x to any y, as long as y contains fewer units, whereas a nonwinning coalition can only preserve the same allocation x. Also, for any coalition $X \in 2^N \backslash \{\emptyset\}$, define binary relation $\prec_X$ on $\mathbf{A}$: for all $x, y \in \mathbf{A}$, $x \prec_X y$ if and only if $X \subset \{i \in N : y_i \geq x_i\}$ and there is $j \in X \cap V$ such that $y_j > x_j$.

We say that x is directly dominated by y, and write $x < y$ if there is coalition X such that $x \rightarrow_X y$ and $x \prec_X y$. We say that state x is indirectly dominated by y, and write $x \ll y$ if there exist $x_0, x_1, \ldots, x_m \in \mathbf{A}$ such that $x_0 = x$ and $x_m = y$, and $X_0, X_1, \ldots, X_{m-1} \in 2^N \backslash \{\emptyset\}$ such that $x_j \rightarrow_{S_j} x_{j+1}$ and $x_j \prec_{S_j} y$ for $j = 0, 1, \ldots, m-1$. We call a set $Q \subset \mathbf{A}$ consistent if $x \in Q$ if and only if for any $y \in \mathbf{A}$ and any coalition $X \in 2^N \backslash \{\emptyset\}$ such that $x \rightarrow_X y$ there exists $z \in Q$ such that $x \nprec_X z$ and either $y = z$ or $y \ll z$. From Chwe (1994), it follows that there is a single largest consistent set, that is, a consistent set P such that for any consistent set Q, $Q \subset P$. We now prove that $P = \mathbf{S}$, that is, the set of stable allocations is the largest consistent set.

PROPOSITION A1: *The set of stable allocations described in Proposition* 3 *is a unique largest consistent set*.

PROOF: First, we need two preliminary observations. First, it is obvious that for any $x, y \in \mathbf{A}$, $x < y$ implies $x \ll y$. In our setup, however, the opposite is also true, so $x < y$ if and only if $x \ll y$. To see this, suppose that $x \ll y$. Take a sequence of states and a sequence of coalitions that establish indirect dominance $x \ll y$. We first notice that $\|x\| = \|x_0\| \geq \|x_1\| \geq \cdots \geq \|x_m\| = \|y\|$, so $\|x\| \geq \|y\|$. Furthermore, $x_0 \prec_{X_0} y$ implies $x \neq y$, for otherwise $j \in X_0 \cap V$ such that $y_j > x_j$ would be impossible. Let $l \geq 0$ be the lowest number such that $x_{l+1} \neq x$; it is well defined and satisfies $l < m$. This means that $x \rightarrow_{X_l} x_{l+1}$, and since $x_{l+1} \neq x$, it must be that $X_l \in \mathcal{W}$. This also means $x_l \prec_{X_l} y$, and thus $x \prec_{X_l} y$; however, since $X_l \in \mathcal{W}$ and $\|x\| \geq \|y\|$, we have $x \rightarrow_{X_l} y$. Now $x \rightarrow_{X_l} y$ and $x \prec_{X_l} y$ by definition imply $x < y$, which proves the equivalence.

Second, we prove that $x < y$ if and only if $y \rhd x$. Indeed, suppose $x < y$. Then for some coalition X, $x \rightarrow_X y$ and $x \prec_X y$; the latter implies $x \neq y$, in which case the former implies $X \in \mathcal{W}$. Furthermore, $x \rightarrow_X y$ implies $\|y\| \leq \|x\|$. We also have $X \subset \{i \in N : y_i \geq x_i\}$, and since $X \in \mathcal{W}$, $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$ as well, in which case $V \subset X$, and then $X \cap V$ is nonempty, so there is $j \in V$ such that $y_j > x_j$. This all implies that $y \rhd x$. Conversely, suppose $y \rhd x$. Let $X = \{i \in N : y_i \geq x_i\}$. Then $X \in \mathcal{W}$, and since $y_j > x_j$ for some $j \in V$ and $V \subset X$, then this is true for some $j \in X \cap V$. This implies that $x \prec_X y$. Now $\|y\| \leq \|X\|$ and $X \in \mathcal{W}$ implies $x \rightarrow_X y$; this means that $x < y$.

Let us now show that $\mathbf{S}$ is consistent.

To show that set $\mathbf{S}$ is consistent, take any $x \in \mathbf{S}$, and then take any $y \in \mathbf{A}$ and any $X \in 2^N \backslash \{\emptyset\}$ such that $x \rightarrow_X y$. We need to prove that there exists $z \in \mathbf{S}$ such that $x \not\prec_X z$ and either $y = z$ or $y \ll z$. If $y \in \mathbf{S}$, then we can take $z = y$ to satisfy this property, because $x \not\prec_X z$. Indeed, this holds trivially if $x = z$, and otherwise follows by contradiction: if $x \rightarrow_X y$ and $x \prec_X y$, then $x < y$, which implies $y \rhd x$, but for two allocations $x, y \in \mathbf{S}$ this would contradict internal stability by Proposition 3. Thus, consider the case $y \notin \mathbf{S}$. Since $x \in \mathbf{S}$, we have $x \neq y$ and thus $X \in \mathcal{W}$. Take any equilibrium σ and any transition mapping $\phi = \phi_\sigma$, and let $z = \phi(y) \in \mathbf{S}$. Notice that it is impossible that this z satisfies $z = y$, since $y \notin \mathbf{S}$. Furthermore, we must have $x \not\prec_X z$, for otherwise we would again contradict Lemma A6 (because $\|z\| \leq \|y\| \leq \|x\|$ and then $x \prec_X z$ coupled with $X \in \mathcal{W}$ would imply $z \rhd x$). It remains to prove that $y \ll z$, for which it suffices to prove that $z \rhd y$, but this follows immediately from Lemma A5. Thus, $z \in \mathbf{S}$ with the required properties exists.

Now take some $x \notin \mathbf{S}$. Let $y = \phi(x)$(again, $\phi = \phi_\sigma$ for some equilibrium σ) and let $X = \{i \in N : y_i \geq x_i\}$. Then $y \in \mathbf{S} \subset \mathbf{A}$, which implies $y \neq x$; furthermore, $y \rhd x$ by Lemma A5 and thus $X \in \mathcal{W}$. We need to prove that there does not exist $z \in \mathbf{S}$ such that $x \not\prec_X z$ and either $y = z$ or $y \ll z$. Suppose, to obtain a contradiction, that such z exists. Then $z \neq y$, because $x \prec_X y$, which is true since $y \rhd x$. Then we must have $x \not\prec_X z$ and $y \ll z$, and the latter is equivalent to $z \rhd y$. However, this violates internal stability of set $\mathbf{S}$, which holds by Proposition 3. This proves that set $\mathbf{S}$ is consistent.

Let us now show that $\mathbf{S}$ is the largest consistent set. Suppose, to obtain a contradiction, that set $P \neq \mathbf{S}$ is the largest consistent set; since $\mathbf{S}$ is consistent, we have $\mathbf{S} \subset P$. As before, take some equilibrium transition mapping ϕ. Take $x \in P \setminus \mathbf{S}$ for which $\|x\|_V = \sum_{j \in V} x_j$ is maximal. Let $y = \phi(x)$ and $X = \{i \in N : y_i \geq x_i\}$. Then $y \in \mathbf{S}$ and $x \prec_X y$. Since P is consistent, there exists $z \in P$ such that $x \not\prec_X z$ and either $y = z$ or $y \ll z$. Notice that $x \not\prec_X z$ implies that $y \neq z$, because $x \prec_X y$. Then $y \ll z$, which is equivalent to $z \rhd y$, but given that $y \in \mathbf{S}$, we must have $z \notin \mathbf{S}$, for otherwise we would get a contradiction to Proposition 3. Thus, $z \in P \setminus \mathbf{S}$, which implies, by the choice of x, that $\|z\|_V \leq \|x\|$. However, we have $y \rhd x$ and $z \rhd y$; thus each $j \in V$ has $z_j \geq y_j \geq x_j$, and for at least one j, one of the inequalities is strict. This implies that $\|z\|_V > \|x\|_V$, a contradiction. This completes the proof that $\mathbf{S}$ is the largest consistent set. *Q.E.D.*

A4. CHARACTERIZATION FOR $n = 3, 4, 5$

The following tables contain a summary of stable allocations if the number of players is small ($n = 3, 4, 5$). The nontrivial cases, where non-veto players form groups and protect each other, are highlighted.

Number of veto players (v)

Voting rule (k)

n = 3	$v = 1$	$v = 2$	$v = 3$
$k = 1$	Non-veto players possess nothing		
$k = 2$	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k = 3$	All allocations are stable	All allocations are stable	All allocations are stable

Number of veto players (v)

Voting rule (k)

n = 4	$v = 1$	$v = 2$	$v = 3$	$v = 4$
$k = 1$	Non-veto players possess nothing			
$k = 2$	**Non-veto players have equal amounts**	Non-veto players possess nothing		
$k = 3$	**Two non-veto players have same amounts, third has nothing**	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k = 4$	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable

Number of veto players (v)

Voting rule (k)

n = 5	$v = 1$	$v = 2$	$v = 3$	$v = 4$	$v = 5$
$k = 1$	Non-veto players possess nothing				
$k = 2$	**Non-veto players have equal amounts**	Non-veto players possess nothing			
$k = 3$	**Three non-veto players have same amounts, fourth has nothing**	**Non-veto players have equal amounts**	Non-veto players possess nothing		
$k = 4$	**Non-veto players form either two classes of two or one class of four**	**Two non-veto players have same amounts, third has nothing**	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k = 5$	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable

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