# SUPPLEMENT TO "IDENTIFICATION AND ESTIMATION OF AVERAGE PARTIAL EFFECTS IN ‘IRREGULAR’ CORRELATED RANDOM COEFFICIENT PANEL DATA MODELS": ADDITIONAL <br> PROOFS AND APPLICATION DETAILS 

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THIS SUPPLEMENT contains proofs of some auxiliary lemmas used to show Theorem 2.1. It also contains a proof of Theorem 2.2 and additional details regarding the empirical application. All notation is as established in the main paper unless noted otherwise. Equation numbering continues in sequence with that established in the main paper. References not included in the bibliography to the main paper are listed below.

## APPENDIX B: Additional Proofs

Proof of Lemma A.3: To verify this result, we must first establish that the defined limit exists. For $h_{N}$ sufficiently small, we can decompose the expectation $\Xi_{N}$ as

$$
\begin{aligned}
\Xi_{N} & =\mathbb{E}\left[\mathbf{1}\left(\left|D_{i}\right| \geq u_{0}\right) \mathbf{X}_{i}^{-1} \mathbf{W}_{i}\right]+\mathbb{E}\left[\mathbf{1}\left(h_{N}<\left|D_{i}\right|<u_{0}\right) \mathbf{X}_{i}^{-1} \mathbf{W}_{i}\right] \\
& =O(1)+\mathbb{E}\left[\mathbf{1}\left(h_{N}<\left|D_{i}\right|<u_{0}\right) D_{i}^{-1} \mathbf{W}_{i}^{*}\right] \\
& =O(1)+\int_{h_{N}}^{u_{0}}\left(\frac{\xi(u)-\xi(-u)}{u}\right) d u,
\end{aligned}
$$

where

$$
\xi(u) \equiv \phi(u) \mathbb{E}\left[W_{i}^{*} \mid D_{i}=u\right]
$$

is twice continuously differentiable for $|u|<u_{0}$ by Assumption 1.2. Using the Taylor's series expansion

$$
\begin{aligned}
& \xi(u)-\xi(-u) \\
& \quad=\xi(0)-\xi(0)+2 \frac{d \xi(0)}{d u} \cdot u+\left[\frac{d^{2} \xi\left(u^{*}\right)}{d u^{2}}-\frac{d^{2} \xi\left(-u^{*}\right)}{d u^{2}}\right] \cdot\left(\frac{u^{2}}{2}\right)
\end{aligned}
$$

for $u^{*}$ some intermediate value between $h_{N}$ and $u_{0}$, it follows that

$$
\begin{aligned}
& \left|1\left(h_{N}<u<u_{0}\right) \cdot\left(\frac{\xi(u)-\xi(-u)}{u}\right)\right| \\
& \quad \leq 1\left(u \leq u_{0}\right)\left[2\left\|\frac{d \xi(0)}{d u}\right\|+\max _{\mid u \leq u_{0}}\left\|\frac{d^{2} \xi(u)}{d u^{2}}\right\| \cdot u_{0}\right],
\end{aligned}
$$

and since the right-hand side is integrable, $\Xi_{N} \rightarrow \Xi_{0}$ by dominated convergence. Then taking $\lambda$ to be an arbitrary (fixed) $q$-vector, verification that $\widehat{\boldsymbol{\Xi}}_{N} \xrightarrow{p} \Xi_{0}$ follows from the convergence of the covariance matrix of the numerator of $\widehat{\Xi}_{N} \lambda$ to zero:

$$
\begin{aligned}
\| \mathbb{V} & {\left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) \mathbf{X}_{i}^{-1} \mathbf{W}_{i} \lambda\right] \| } \\
= & \left\|\mathbb{V}\left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) D_{i}^{-1} \mathbf{W}_{i}^{*} \lambda\right]\right\| \\
\leq & \frac{2\|\lambda\|^{2}}{N} \mathbb{E}\left[\mathbf{1}\left(\left|D_{i}\right|>h_{N}\right)\left|D_{i}\right|^{-2}\left\|\mathbf{W}_{i}^{*}\right\|^{2}\right] \\
= & \frac{2\|\lambda\|^{2}}{N}\left[\mathbb{E}\left[\mathbf{1}\left(\left|D_{i}\right| \geq u_{0}\right)\left|D_{i}\right|^{-2}\left\|\mathbf{W}_{i}^{*}\right\|^{2}\right]\right. \\
& \left.+\mathbb{E}\left[\mathbf{1}\left(h_{N}<\left|D_{i}\right|<u_{0}\right)\left|D_{i}\right|^{-2}\left\|\mathbf{W}_{i}^{*}\right\|^{2}\right]\right] \\
= & O\left(\frac{1}{N}\right)+\frac{2\|\lambda\|^{2}}{N} \int_{h_{N}}^{u_{0}}\left(\frac{\zeta(u)+\zeta(-u)}{u^{2}}\right) d u
\end{aligned}
$$

where $\left\|\mathbf{W}_{i}^{*}\right\|^{2} \equiv \operatorname{tr}\left[\mathbf{W}^{*} \mathbf{W}^{*}\right]$ and

$$
\zeta(u) \equiv \phi(u) \mathbb{E}\left[\left\|W_{i}^{*}\right\|^{2} \mid D_{i}=u\right]
$$

is bounded for $|u|<u_{0}$, so

$$
\begin{aligned}
\| \mathbb{V} & {\left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) \mathbf{X}_{i}^{-1} \mathbf{W}_{i} \lambda\right] \| } \\
& \leq O\left(\frac{1}{N}\right)+\frac{4\|\lambda\|^{2}}{N}\left(\max _{|u| \leq u_{0}}\|\zeta(u)\|\right) \int_{h_{N}}^{u_{0}}\left(\frac{1}{u^{2}}\right) d u \\
& =O\left(\frac{1}{N}\right)+\frac{4\|\lambda\|^{2}}{N}\left(\max _{|u| \leq u_{0}}\|\zeta(u)\|\right)\left[\frac{1}{h_{N}}-\frac{1}{u_{0}}\right]=O\left(\frac{1}{N h_{N}}\right)=o(1)
\end{aligned}
$$

under Assumption 2.5.
Q.E.D.

Proof of Theorem 2.2: Rewriting Equation (30), we have

$$
\begin{equation*}
\widehat{V}=\left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{R}_{i}\right]^{-1} \times\left[\frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \widehat{\mathbf{U}}_{i}^{+} \widehat{\mathbf{U}}_{i}^{+\prime} \mathbf{Q}_{i}\right] \times\left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{R}_{i}\right]^{-1} \tag{54}
\end{equation*}
$$

for $\mathbf{U}_{i}^{+}=\mathbf{Y}_{i}^{*}-\mathbf{R}_{i} \boldsymbol{\theta}_{0}$, with $\widehat{\boldsymbol{\theta}}=\left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{R}_{i}\right]^{-1} \times\left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{Y}_{i}^{*}\right]$ for $\boldsymbol{\theta}=\left(\boldsymbol{\delta}^{\prime}, \boldsymbol{\beta}^{\prime}\right)^{\prime}$ and

$$
\begin{aligned}
\underset{T \times q+p}{\mathbf{Q}_{i}} & =\left(h_{N}^{-1} \mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right) \mathbf{W}_{i}^{*}, \frac{\mathbf{1}\left(\left|D_{i}\right|>h_{N}\right)}{D_{i}} I_{p}\right), \\
\underset{T \times q+p}{\mathbf{R}_{i}} & =\left(\mathbf{W}_{i}^{*}, \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) D_{i} I_{p}\right) .
\end{aligned}
$$

The dependence of $\mathbf{Q}_{i}$ and $\mathbf{R}_{i}$ on $h_{N}$ is suppressed to simplify the notation.

Starting with the Jacobian term in $\widehat{V}$, we get, by the definitions of $\mathbf{Q}_{i}, \mathbf{R}_{i}$, and $\widehat{\boldsymbol{E}}_{N}$,

$$
\begin{aligned}
& \frac{1}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{R}_{i} \\
&=\frac{1}{N} \sum_{i=1}^{N}\binom{h_{N}^{-1} \mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right) \mathbf{W}_{i}^{* \prime}}{\frac{\mathbf{1}\left(\left|D_{i}\right|>h_{N}\right)}{D_{i}} I_{p}}\left(\begin{array}{ll}
\mathbf{W}_{i}^{*} & \left.\mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) D_{i} I_{p}\right) \\
& =\left(\begin{array}{cc}
\frac{1}{N h_{N}} \sum_{i=1}^{N} \mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right) \mathbf{W}_{i}^{* *} \mathbf{W}_{i}^{*} & \underline{0}_{q} \underline{0}_{p}^{\prime} \\
\frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) \mathbf{X}_{i}^{-1} \mathbf{W}_{i} & \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) I_{p}
\end{array}\right) \\
& \xrightarrow[\rightarrow]{p}\left(\begin{array}{cc}
2 \mathbb{E}\left[\mathbf{W}_{i}^{* \prime} \mathbf{W}_{i}^{*} \mid D_{i}=0\right] \phi_{0} & \underline{0} \\
\Xi_{0} & I_{p}
\end{array}\right)
\end{array}\right.
\end{aligned}
$$

by (50) and Lemma A.3. Decomposing the middle term $h_{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \widehat{\mathbf{U}}_{i}^{+} \widehat{\mathbf{U}}_{i}^{+\prime} \mathbf{Q}_{i} / N$ yields

$$
\begin{align*}
& \left\|\frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \widehat{\mathbf{U}}_{i}^{+} \widehat{\mathbf{U}}_{i}^{+\prime} \mathbf{Q}_{i}-\frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{U}_{i}^{+} \mathbf{U}_{i}^{+\prime} \mathbf{Q}_{i}\right\|  \tag{55}\\
& \leq \frac{2 h_{N}}{N} \sum_{i=1}^{N}\left\|\mathbf{U}_{i}^{+}\right\|\left\|\mathbf{Q}_{i}\right\|^{2}\left\|\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}_{0}\right\|+\frac{h_{N}}{N} \sum_{i=1}^{N}\left\|\mathbf{Q}_{i}\right\|^{2}\left\|\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}_{0}\right\|^{2} \\
& =\frac{2}{N h_{N}} \sum_{i=1}^{N}\left(\mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right)\left\|\mathbf{W}_{i}^{*}\right\|^{2}\left\|\mathbf{U}_{i}^{+}\right\|+\mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) \frac{h_{N}^{2}\left\|\mathbf{U}_{i}^{+}\right\|}{\left|D_{i}\right|^{2}}\right) \\
& \quad \times\left\|\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}_{0}\right\|
\end{align*}
$$

$$
\begin{aligned}
& +\frac{1}{N h_{N}} \sum_{i=1}^{N}\left(\mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right)\left\|\mathbf{W}_{i}^{*}\right\|^{2}+\mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) \frac{h_{N}^{2}}{\left|D_{i}\right|^{2}}\right) \\
& \times\left\|\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}_{0}\right\|^{2} .
\end{aligned}
$$

By a similar argument as for (49) above,

$$
\begin{aligned}
\mathbb{E} & {\left[h_{N}^{-1} \mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right)\left\|\mathbf{W}_{i}^{*}\right\|^{2}\left(\left\|\mathbf{U}_{i}^{+}\right\|+1\right)\right] } \\
& =\mathbb{E}\left[h_{N}^{-1} \mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right) \mathbb{E}\left[\left\|\mathbf{W}_{i}^{*}\right\|^{2}\left(\left\|\mathbf{U}_{i}^{+}\right\|+1\right) \mid D_{i}\right]\right] \\
& \rightarrow 2 \mathbb{E}\left[\left\|\mathbf{W}_{i}^{*}\right\|^{2}\left(\left\|\mathbf{U}_{i}^{+}\right\|+1\right) \mid D_{i}=0\right] \phi_{0},
\end{aligned}
$$

and the same reasoning as (43) yields

$$
\begin{aligned}
& \mathbb{E}\left[\mathbf{1}\left(\left|D_{i}\right|>h_{N}\right)\left\|\mathbf{W}_{i}^{*}\right\|^{2}\left(\left\|\mathbf{U}_{i}^{+}\right\|+1\right) \frac{h_{N}}{\left|D_{i}\right|^{2}}\right] \\
& \quad=\mathbb{E}\left[h_{N}^{-1} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right) \frac{h_{N}}{\left|D_{i}\right|^{2}} \mathbb{E}\left[\left\|\mathbf{W}_{i}^{*}\right\|^{2}\left(\left\|\mathbf{U}_{i}^{+}\right\|+1\right) \mid D_{i}\right]\right] \\
& \quad \rightarrow 2 \mathbb{E}\left[\left\|\mathbf{W}_{i}^{*}\right\|^{2}\left(\left\|\mathbf{U}_{i}^{+}\right\|+1\right) \mid D_{i}=0\right] \phi_{0} .
\end{aligned}
$$

Thus, by Markov's inequality, (55) yields

$$
\begin{aligned}
\frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \widehat{\mathbf{U}}_{i}^{+} \widehat{\mathbf{U}}_{i}^{+\prime} \mathbf{Q}_{i} & =\frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{U}_{i}^{+} \mathbf{U}_{i}^{+\prime} \mathbf{Q}_{i}+O_{p}\left(\left\|\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}_{0}\right\|\right) \\
& =\frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{U}_{i}^{+} \mathbf{U}_{i}^{+\prime} \mathbf{Q}_{i}+o_{p}(\mathbf{1})
\end{aligned}
$$

Finally,

$$
\begin{aligned}
& \frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{U}_{i}^{+} \mathbf{U}_{i}^{+\prime} \mathbf{Q}_{i} \\
& =\left(\frac{1}{N h_{N}} \sum_{i=1}^{N} \mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right) \mathbf{W}_{i}^{* \prime}\left(\mathbf{Y}_{i}^{*}-\mathbf{W}_{i}^{*} \boldsymbol{\delta}_{0}\right)\left(\mathbf{Y}_{i}^{*}-\mathbf{W}_{i}^{*} \boldsymbol{\delta}_{0}\right)^{\prime} \mathbf{W}_{i}^{*}\right. \\
& \quad \underline{0}_{p} \underline{0}_{q}^{\prime} \\
& \quad \begin{array}{l}
\underline{0}_{q} \underline{0}_{p}^{\prime}
\end{array} \\
& \left.\quad \frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right)\left\{\mathbf{X}_{i}^{-1}\left(\mathbf{Y}_{i}-\mathbf{W}_{i} \boldsymbol{\delta}_{0}\right)-\boldsymbol{\beta}_{0}\right\}\left\{\mathbf{X}_{i}^{-1}\left(\mathbf{Y}_{i}-\mathbf{W}_{i} \boldsymbol{\delta}_{0}\right)-\boldsymbol{\beta}_{0}\right\}^{\prime}\right)
\end{aligned}
$$

By the calculations that yield (53), the expected value of the first diagonal submatrix is

$$
\begin{aligned}
\mathbb{E} & {\left[h_{N}^{-1} \mathbf{1}\left(\left|D_{i}\right| \leq h_{N}\right) \mathbf{W}_{i}^{* \prime}\left(\mathbf{Y}_{i}^{*}-\mathbf{W}_{i}^{*} \boldsymbol{\delta}_{0}\right)\left(\mathbf{Y}_{i}^{*}-\mathbf{W}_{i}^{*} \boldsymbol{\delta}_{0}\right)^{\prime} \mathbf{W}_{i}^{*}\right] } \\
& =2 \mathbb{E}\left[\mathbf{W}_{i}^{* \prime} \mathbf{X}^{*} \Sigma(\mathbf{X}) \mathbf{X}^{*} \mathbf{W}_{i}^{*} \mid D_{i}=0\right] \phi_{0}+o(1),
\end{aligned}
$$

while the variance of any term in the matrix is $O\left(\left(N h_{N}\right)^{-1}\right)=o(1)$ by Assumptions 2.3 and 2.5 (with $r \leq 8$ ). Similarly, the expectation of the second diagonal submatrix is

$$
\begin{aligned}
\mathbb{E} & {\left[h_{N} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right)\left\{\mathbf{X}_{i}^{-1}\left(\mathbf{Y}_{i}-\mathbf{W}_{i} \boldsymbol{\delta}_{0}\right)-\boldsymbol{\beta}_{0}\right\}\left\{\mathbf{X}_{i}^{-1}\left(\mathbf{Y}_{i}-\mathbf{W}_{i} \boldsymbol{\delta}_{0}\right)-\boldsymbol{\beta}_{0}\right\}^{\prime}\right] } \\
& =2 \mathbb{E}\left[\mathbf{X}^{*} \Sigma(\mathbf{X}) \mathbf{X}^{* \prime} \mid D=0\right] \phi_{0}+o(1) \\
& =2 \Upsilon_{0} \phi_{0}+o(1),
\end{aligned}
$$

while calculations similar to those that lead to (45) yield

$$
\begin{aligned}
\| \mathbb{V} & {\left[h_{N} \mathbf{1}\left(\left|D_{i}\right|>h_{N}\right)\left\{\mathbf{X}_{i}^{-1}\left(\mathbf{Y}_{i}-\mathbf{W}_{i} \boldsymbol{\delta}_{0}\right)-\boldsymbol{\beta}_{0}\right\}\left\{\mathbf{X}_{i}^{-1}\left(\mathbf{Y}_{i}-\mathbf{W}_{i} \boldsymbol{\delta}_{0}\right)-\boldsymbol{\beta}_{0}\right\}^{\prime}\right] \| } \\
& \leq \frac{4 m_{8}(0) \phi_{0}}{N h_{n}}+o\left(\frac{1}{N h_{n}}\right) \\
& =o(1),
\end{aligned}
$$

where $m_{8}(u)$ is defined in Assumption 2.3. Thus

$$
\frac{h_{N}}{N} \sum_{i=1}^{N} \mathbf{Q}_{i}^{\prime} \mathbf{U}_{i}^{+} \mathbf{U}_{i}^{+\prime} \mathbf{Q}_{i} \xrightarrow{p}\left(\begin{array}{cc}
2 \mathbb{E}\left[\mathbf{W}_{i}^{* \prime} \mathbf{X}^{*} \Sigma(\mathbf{X}) \mathbf{X}^{*} \mathbf{W}_{i}^{*} \mid D_{i}=0\right] \phi_{0} & \underline{0}_{q}{\underline{\theta^{\prime}}}_{p}^{\prime} \\
\underline{0}_{p} \underline{0}_{q}^{\prime} & 2 \Upsilon_{0} \phi_{0}
\end{array}\right),
$$

ensuring $\widehat{V} \xrightarrow{p} V_{0}$.
Q.E.D.

## APPENDIX C: Additional Details on Empirical Application

## Data Description

The preparation of our estimation sample from the raw public release data files involved some complex and laborious data processing. We outline the procedures used to construct our sample in this appendix. A sequence of heavily commented STATA do files, which read in the IFPRI (2005) release of the data and output a text file of our estimation sample are available online at http://elsa.berkeley.edu/~bgraham/.

As noted in the main text, we use data collected in conjunction with an external evaluation of the Nicaraguan conditional cash transfer program Red de Protección Social (RPS) (see IFPRI (2005)). The RPS evaluation sample is a panel of 1581 households from 42 rural communities in the departments of Madriz and Matagalpa, located in the northern part of the Central


Figure 2.-International Coffee Organization composite price index, 1998 to 2004. The source data were downloaded from http://www.ico.org/ on 21 June 2011.

Region of Nicaragua. Each sampled household was first interviewed in August/September 2000, with followups attempted in October of both 2001 and 2002. A total of 1359 households were successfully interviewed in all three waves. One of these households reported zero food expenditures (and hence calorie availability) in one wave and was dropped from our sample. Our estimation sample therefore consists of a balanced panel of 1358 households from all three waves.

The survey was fielded using an abbreviated version of the 1998 Nicaraguan Living Standards Measurement Survey (LSMS) instrument. As such it includes a detailed consumption module with information on household expenditure, both actual and in kind, on 59 specific foods, and several dozen other common budget categories (e.g., housing and utilities, health, education, and household goods). The responses to these questions were combined to form an annualized consumption aggregate, $C_{i t}$. In forming this variable, we followed the algorithm outlined by Deaton and Zaidi (2002).

In addition to recording food expenditures, actual quantities of foods acquired are available. Using conversion factors listed in the World Bank (2002) and Instituto Nacional de Estadísticas y Censos (INEC) (2005) (henceforth


Figure 3.-Histogram of the distribution of $D(T=p=3)$. The smallest and largest 10 percent of the $D_{i}$ 's are excluded from the histogram. The two vertical lines correspond to the portion of the sample that is trimmed in our preferred estimates (Table IV, column 3).

INEC), we converted all food quantities into grams. We then used the caloric content and edible percent information in the Instituto de Nutrición de Centro América y Panamá (2000) (henceforth INCAP) food composition tables to construct a measure of daily total calories available for each household. In forming our measure of calorie availability, we followed the general recommendations of Smith and Subandoro (2007). The logarithm of this measure divided by household size, $Y_{i t}$, serves as the dependent variable in our analysis.

The combination of both expenditure and quantity information at the household level also allowed us to estimate unit prices for foods. These unit values were used to form a Paasche cost-of-living index for the $i$ th household in year $t$ of

$$
\begin{equation*}
I_{i t}=\left[S_{i t}\left\{\sum_{f=1}^{F} W_{f, i t}\left(P_{f}^{b} / P_{f, i t}\right)\right\}+\left(1-S_{i t}\right) J_{i t}\right]^{-1} \tag{56}
\end{equation*}
$$

where $S_{i t}$ is the fraction of household spending devoted to food, $W_{f, i t}$ is the share of overall food spending devoted to the $f$ th specific food, $P_{f, i t}$ is the
year $t$ unit price paid by the household for food $f$, and $P_{f}^{b}$ is its base price (equal to the relevant 2001 sample median price). We use 2001 as our base year because it facilitates comparison with information from a nationwide LSMS survey fielded that year. Following the suggestion of Deaton and Zaidi (2002), we replace household-level unit prices with village medians so as to reduce noise in the price data. In the absence of price information on nonfood goods, we set $J_{i t}$ equal to 1 in 2001 and equal to the national consumer price index (CPI) in 2000 and 2002. Our independent variable of interest is real per capita consumption in thousands of cordobas: $\operatorname{Exp}_{i t}=\left(\left[C_{i t} / I_{i t}\right]\right) / M_{i t}$, where $M_{i t}$ is total household size.

## A Nonlinear Model

As we have three periods of data, we can modify our model to allow the calorie elasticity to vary nonlinearly with income. Nonlinearity in the calorie demand curve has been emphasized by Strauss and Thomas $(1990,1995)$ and Subramanian and Deaton (1996). We consider the model

$$
\ln \left(\mathrm{Cal}_{t}\right)=b_{0 t}\left(A, U_{t}\right)+b_{1 t}\left(A, U_{t}\right) \ln \left(\operatorname{Exp}_{t}\right)+b_{2 t}\left(A, U_{t}\right) \operatorname{Exp}_{t}^{-1},
$$

so that a household's period-specific demand elasticity is given by $b_{1 t}\left(A, U_{t}\right)-$ $b_{2 t}\left(A, U_{t}\right) \operatorname{Exp}_{t}^{-1}$. We estimate the average of this elasticity in 2000, 2001, and

TABLE IV
Estimates of the Calorie Engel Curve: Nonlinear Case ${ }^{\text {a }}$

|  | Calorie Demand Elasticities |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
|  | OLS | FE-OLS | I-CRC | I-CRC | I-CRC |
| 2000 Elasticity | 0.6226 | 0.6841 | 0.3720 | 0.2463 | 0.2442 |
|  | $(0.0173)$ | $(0.0329)$ | $(0.2918)$ | $(0.3984)$ | $(0.2737)$ |
| 2001 Elasticity | 0.6269 | 0.6872 | 0.6882 | 1.4738 | 0.7057 |
|  | $(0.0181)$ | $(0.0340)$ | $(0.2825)$ | $(0.3777)$ | $(0.2780)$ |
| 2002 Elasticity | 0.6274 | 0.6875 | 0.4641 | 0.2010 | 0.5689 |
|  | $(0.0182)$ | $(0.0341)$ | $(0.3093)$ | $(0.4340)$ | $(0.2938)$ |
| Percent trimmed | - | - | 12.3 | 5 | 15 |
| Intercept shifts | Yes | Yes | Yes | Yes | Yes |
| Slope shifts | No | No | No | No | No |

[^0]2002 using the approach outlined in Section 3. Unlike in our linear analysis, we only allow for common intercept shifts across periods when fitting the nonlinear model. Figure 3 plots the histogram of $D$ with $\mathbf{X}_{t}=\left(1, \ln \left(\operatorname{Exp}_{t}\right), \operatorname{Exp}_{t}^{-1}\right)^{\prime}$. Because $\ln \left(\operatorname{Exp}_{t}\right)$ and $\operatorname{Exp}_{t}^{-1}$ are highly correlated within units, the density of $D$ is substantial in the neighborhood of zero. The extreme "irregularity" of the augmented model suggests that its estimation will require a substantial amount of trimming.

Table IV reports average elasticity estimates based on the extended model. The average elasticities associated with the OLS and FE-OLS parameter estimates of the nonlinear model are virtually identical to their linear model Table III counterparts. Although the coefficient on $\operatorname{Exp}_{t}^{-1}$ is significant in both models (not reported), the effect of its inclusion on the average elasticity estimates is negligible. Column 3 reports I-CRC estimates with $h_{N}=\frac{c_{D}}{2} N^{-1 / 3}$ (which, given the large density in the neighborhood of zero, results in the trimming of 12 percent of the sample). In contrast to the linear case, the I-CRC estimates are imprecisely determined; they are also more sensitive to variations in the bandwidth (columns 3 and 4). We conclude that we are unable to reliably fit the nonlinear CRC model with the data available.

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[^0]:    ${ }^{\text {a }}$ Estimates based on the balanced panel of 1358 households described in the main text. OLS denotes least squares applied to the pooled 2000, 2001, and 2002 samples; FE-OLS denotes least squares with household-specific intercepts; and I-CRC denotes our irregular correlated random coefficients estimator (now using all three waves). All models include common intercept, but not slope, shifts across periods. The standard errors are computed in a way that allows for arbitrary within-village correlation in disturbances across households and time. The average elasticity estimates in the OLS and FE-OLS columns are computed using the delta method; those in the I-CRC columns are computed as described in Section 3.

