Econometrica Supplementary Material

SUPPLEMENT TO "IDENTIFICATION AND INFERENCE IN ASCENDING AUCTIONS WITH CORRELATED PRIVATE VALUES"

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APPENDIX B

B.1. *Overview*

THE COVARIATES THAT MAKE UP X (SALEVAL, MFGCOST, HARVCOST, CONCENTR, INVENTORY, and APPRICE) are treated here as continuously distributed. v_0 is assumed throughout to be a deterministic function of X, so when we condition on X, we condition on the corresponding v_0; and when we average over X, we implicitly also average over v_0.

We use kernel-weighted nonparametric estimators, employing a kernel function K and bandwidth sequence h_L with features to be described below. Let $f_X(x)$ denote the density of X, let $p_N(n|x) = \Pr(N = n|X = x)$, and let $q_{X,N}(x, n) = p_N(n|x) \cdot f_X(x)$. B will denote transaction price, which is equal to $V_{n-1:n}$ by Assumption 2.[42] Let $T_n(r|X) = E_{B|X,N}[\max\{B, r\}|X, N = n]$.

The assumptions in the text are maintained throughout. We will maintain the following additional assumptions:

ASSUMPTION 4:
(i) *The observed data* $U_i \equiv (B_i, N_i, X_i)_{i=1}^L$ *is an i.i.d. sample*, $X_i \in \mathbb{R}^z$ (*with* $z = 6$ *in our empirical analysis*) *is continuously distributed*, *and* $\mathrm{Supp}(N)$ *is a compact set of the form* $\{2, \dots, \overline{n}\}$ (*with* $\overline{n} = 11$ *in our empirical analysis*).
(ii) *There exist* $\underline{q} > 0$, $\overline{q} < \infty$, $\underline{F} > 0$, *and* $\overline{F} < 1$ *such that for every auction* (x, n) *and reserve price* r *that we consider*, *the following statements hold*:
(a) $\underline{q} \leq q_{X,N}(x, n) \leq \overline{q}$ *and* $\underline{F} \leq F_{n-1:n}(r|x) \leq \overline{F}$.
(b) *In a neighborhood of* x, $f_X(X)$, $p_N(n|X)$, $F_{n-1:n}(r|X)$, *and* $T_n(r|X)$ *are twice differentiable with respect to* X *with bounded derivatives*.
(iii) *The kernel* $K : \mathbb{R}^z \longrightarrow \mathbb{R}$ *is a nonnegative function of bounded variation*, *satisfies* $\int K(\psi)\, d\psi = 1$, *has compact support*, *and is symmetric around zero*.
(iv) *The bandwidth sequence* h_L *is nonnegative and satisfies* $h_L \longrightarrow 0$. *In addition*, $\exists \overline{\delta} > 0$ *for which* $L^{1-\overline{\delta}} \cdot h_L^z \longrightarrow \infty$ *and* $L^{1+\overline{\delta}} \cdot h_L^{z+4} \longrightarrow 0$.

The smoothness and regularity restrictions described in Assumption 4 are fairly standard in nonparametric models. The same is true for the restrictions imposed on the kernel and bandwidth. Assuming that $F_{n-1:n}(r|x)$ is bounded

[42]Everything that follows can be adapted to the incomplete model of Haile and Tamer (2003), using the bounds presented in Appendix A.1.

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away from 0 and 1 ensures that the mapping $\phi_n(\cdot)$ is smooth and differentiable[43] at $F_{n-1:n}(r|x)$. We maintain that Assumption 4 holds for each (n, r, x) in the range of values depicted in the figures in the text. The specific kernel and bandwidth we used are described in Section B.7 below.

B.2. *Expected Profits Conditional on N and X*

Let $K_h(\xi) \equiv K(\xi/h)$. For a given (n, r, x), let

$$\widehat{T}_n(r|x) = \frac{\sum_{i=1}^{L} \max\{r, B_i\} \cdot K_h(X_i - x) \cdot \mathbb{1}\{N_i = n\}}{\sum_{i=1}^{L} K_h(X_i - x) \cdot \mathbb{1}\{N_i = n\}},$$

$$\widehat{F}_{n-1:n}(r|x) = \frac{\sum_{i=1}^{L} \mathbb{1}\{B_i \leq r\} \cdot K_h(X_i - x) \cdot \mathbb{1}\{N_i = n\}}{\sum_{i=1}^{L} K_h(X_i - x) \cdot \mathbb{1}\{N_i = n\}}$$

be kernel-based sample analog estimators of $T_n(r|x)$ and $F_{n-1:n}(r|x)$, and let

$$\widehat{\underline{F}}_{n:n}(r|x) = \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} \widehat{F}_{m-1:m}(r|x) + \frac{n}{\bar{n}} \left(\phi_{\bar{n}}\left(\widehat{F}_{\bar{n}-1:\bar{n}}(r|x)\right)\right)^{\bar{n}},$$

$$\widehat{\overline{F}}_{n:n}(r|x) = \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} \widehat{F}_{m-1:m}(r|x) + \frac{n}{\bar{n}} \widehat{F}_{\bar{n}-1:\bar{n}}(r|x),$$

$$\widehat{F}^{\mathrm{IPV}}_{n:n}(r|x) = \left(\phi_n\left(\widehat{F}_{n-1:n}(r|x)\right)\right)^n$$

be the corresponding estimators for the lower bound, upper bound, and IPV point estimate of $F_{n:n}(r|x)$. Our estimators for expected profit given (n, r, x) are then

$$\widehat{\underline{\pi}}_n(r|x) = \widehat{T}_n(r|x) - v_0 - (r - v_0) \cdot \widehat{\overline{F}}_{n:n}(r|x),$$

$$\widehat{\overline{\pi}}_n(r|x) = \widehat{T}_n(r|x) - v_0 - (r - v_0) \cdot \widehat{\underline{F}}_{n:n}(r|x),$$

$$\widehat{\pi}^{\mathrm{IPV}}_n(r|x) = \widehat{T}_n(r|x) - v_0 - (r - v_0) \cdot \widehat{F}^{\mathrm{IPV}}_{n:n}(r|x).$$

[43]The mapping ϕ_n fails to be Lipschitz continuous at 0 and 1, which introduces an irregularity into the estimation problem at the boundary of the support of valuations. Therefore, we restrict attention to the interior of the support. This type of boundary issue in the estimation of ascending auctions is studied in detail in Menzel and Morganti (2012).

To estimate the standard errors, we first define (for (n, r, x) such that $0 < F_{n-1:n}(r|x) < 1$)

$$\nabla\phi_n(r|x) = \frac{\phi_n(F_{n-1:n}(r|x))}{n(n-1)(1-\phi_n(F_{n-1:n}(r|x)))}$$

and let

$$\begin{aligned}
\psi_T(r, U_i|x, n) &= \frac{(\max\{r, B_i\} - T_n(r|x))}{q_{X,N}(x, n)} \mathbb{1}\{N_i = n\}, \\
\psi_F(r, U_i|x, n) &= \frac{(\mathbb{1}\{B_i \leq r\} - F_{n-1:n}(r|x))}{q_{X,N}(x, n)} \mathbb{1}\{N_i = n\}, \\
\overline{\psi}_F(r, U_i|x, n) &= \sum_{m=n+1}^{\overline{n}} \frac{n}{(m-1)m} \psi_F(r, U_i|x, m) + \frac{n}{\overline{n}} \psi_F(r, U_i|x, \overline{n}), \\
\underline{\psi}_F(r, U_i|x, n) &= \sum_{m=n+1}^{\overline{n}} \frac{n}{(m-1)m} \psi_F(r, U_i|x, m) \\
&\quad + n \cdot \nabla\phi_{\overline{n}}(r|x) \cdot \psi_F(r, U_i|x, \overline{n}), \\
\psi_F^{\mathrm{IPV}}(r, U_i|x, n) &= n \cdot \nabla\phi_n(r|x) \cdot \psi_F(r, U_i|x, n)
\end{aligned}$$

and

$$\begin{aligned}
\underline{\psi}_\pi(r, U_i|x, n) &= \psi_T(r, U_i|x, n) - (r - v_0) \cdot \overline{\psi}_F(r, U_i|x, n), \\
\overline{\psi}_\pi(r, U_i|x, n) &= \psi_T(r, U_i|x, n) - (r - v_0) \cdot \underline{\psi}_F(r, U_i|x, n), \\
\psi_\pi^{\mathrm{IPV}}(r, U_i|x, n) &= \psi_T(r, U_i|x, n) - (r - v_0) \cdot \psi_F^{\mathrm{IPV}}(r, U_i|x, n).
\end{aligned} \tag{6}$$

Let $F_{n:n}^{\mathrm{IPV}}(r|x) = (\phi_n(F_{n-1:n}(r|x)))^n$ and $\pi_n^{\mathrm{IPV}}(r|x) = T_n(r|x) - v_0 - (r - v_0) \cdot F_{n:n}^{\mathrm{IPV}}(r|x)$. A second-order Taylor expansion of $\widehat{\underline{\pi}}_n(r|x)$, $\widehat{\overline{\pi}}_n(r|x)$, and $\widehat{\pi}_n^{\mathrm{IPV}}(r|x)$ around the true values $\underline{\pi}_n(r|x)$, $\overline{\pi}_n(r|x)$ and $\pi_n^{\mathrm{IPV}}(r|x)$ gives the following result.[44]

[44]A second-order Taylor expansion, along with Assumption 4, yields

$$\begin{aligned}
\widehat{\underline{\pi}}_n(r|x) &= \underline{\pi}_n(r|x) + \frac{1}{Lh_L^z} \sum_{i=1}^{L} \underline{\psi}_\pi(r, U_i|x, n) \cdot K\left(\frac{X_i - x}{h_L}\right) + o_p\left(\left(Lh_L^z\right)^{-1/2}\right), \\
\widehat{\overline{\pi}}_n(r|x) &= \overline{\pi}_n(r|x) + \frac{1}{Lh_L^z} \sum_{i=1}^{L} \overline{\psi}_\pi(r, U_i|x, n) \cdot K\left(\frac{X_i - x}{h_L}\right) + o_p\left(\left(Lh_L^z\right)^{-1/2}\right),
\end{aligned}$$

RESULT B.1: Under Assumption 4,

$$\sqrt{Lh_L^z} \cdot \big(\widehat{\underline{\pi}}_n(r|x) - \underline{\pi}_n(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \underline{\sigma}_n^2(r|x)\big),$$
$$\sqrt{Lh_L^z} \cdot \big(\widehat{\overline{\pi}}_n(r|x) - \overline{\pi}_n(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \overline{\sigma}_n^2(r|x)\big),$$
$$\sqrt{Lh_L^z} \cdot \big(\widehat{\pi}_n^{\mathrm{IPV}}(r|x) - \pi_n^{\mathrm{IPV}}(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \sigma_n^{\mathrm{IPV}^2}(r|x)\big),$$

where, letting $\mu_K^2 \equiv \int K^2(\xi)\,d\xi$,

$$\underline{\sigma}_n^2(r|x) = E_{U|X}\big[\underline{\psi}_\pi(r, U_i|x, n)^2 \big| X_i = x\big] f_X(x)\mu_K^2,$$
$$\overline{\sigma}_n^2(r|x) = E_{U|X}\big[\overline{\psi}_\pi(r, U_i|x, n)^2 \big| X_i = x\big] f_X(x)\mu_K^2,$$
$$\sigma_n^{\mathrm{IPV}^2}(r|x) = E_{U|X}\big[\psi_\pi^{\mathrm{IPV}}(r, U_i|x, n)^2 \big| X_i = x\big] f_X(x)\mu_K^2.$$

This gives us asymptotic properties of the estimators for the bounds, but we want to do inference on actual profit $\pi_n(r|x)$, which is not point-identified. Imbens and Manski (2004) and Stoye (2009) developed methods for inference on partially identified parameters with point-identified bounds; given the asymptotic normality of our bounds estimators, their approach adapts readily to our non-parametric setting. Let $\widehat{\Lambda}_n^\pi(r|x) = \widehat{\overline{\pi}}_n(r|x) - \widehat{\underline{\pi}}_n(r|x)$. Let $\widehat{\underline{\sigma}}_n(r|x)$ and $\widehat{\overline{\sigma}}_n(r|x)$ be sample analog nonparametric estimators of $\underline{\sigma}_n(r|x)$ and $\overline{\sigma}_n(r|x)$, respectively. To get a confidence interval (CI) for $\pi_n(r|x)$ with asymptotic coverage probability of *at least* $(1-\alpha)$, we use

$$\mathrm{CI}_{1-\alpha}\big(\pi_n(r|x)\big) = \left[\widehat{\underline{\pi}}_n(r|x) - c_\alpha \cdot \frac{\widehat{\underline{\sigma}}_n(r|x)}{\sqrt{Lh_L^z}}, \widehat{\overline{\pi}}_n(r|x) + c_\alpha \cdot \frac{\widehat{\overline{\sigma}}_n(r|x)}{\sqrt{Lh_L^z}}\right], \tag{7}$$

where c_α solves

$$\Phi\left(c_\alpha + \frac{\sqrt{Lh_L^z} \cdot \widehat{\Lambda}_n^\pi(r|x)}{\max\{\widehat{\underline{\sigma}}_n(r|x), \widehat{\overline{\sigma}}_n(r|x)\}}\right) - \Phi(-c_\alpha) = 1 - \alpha \tag{8}$$

(where Φ is the standard normal cumulative distribution function). If $\Lambda_n^\pi(r|x) > 0$, the first term in the left-hand side of (8) converges to 1 and the above critical value is asymptotically equivalent to the one given by

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$$\widehat{\pi}_n^{\mathrm{IPV}}(r|x) = \pi_n^{\mathrm{IPV}}(r|x) + \frac{1}{Lh_L^z}\sum_{i=1}^{L} \psi_\pi^{\mathrm{IPV}}(r, U_i|x, n) \cdot K\left(\frac{X_i - x}{h_L}\right) + o_p\big((Lh_L^z)^{-1/2}\big),$$

where $\underline{\psi}_\pi$, $\overline{\psi}_\pi$, and ψ_π^{IPV} are described in (6). Result B.1 follows from here through Lyapunov's central limit theorem.

$\Phi(-c_\alpha) = \alpha$. However, if $\Lambda_n^\pi(r|x)$ is very small, the latter can provide a poor approximation and lead to under-coverage even in relatively large sample sizes (see Imbens and Manski (2004) and Stoye (2009)). In contrast, the critical value described in (8) is designed to retain good coverage probability even if $\Lambda_n^\pi(r|x)$ is very close to zero. In such cases, the behavior of $\sqrt{Lh_L^z} \cdot \widehat{\Lambda}_n^\pi(r|x)$ merits further discussion. First, the nonnegativity of the kernel K implies $\widehat{\overline{\pi}}_n(r|x) \geq \widehat{\underline{\pi}}_n(r|x)$ with probability 1 (w.p.1) and, therefore, $\widehat{\Lambda}_n^\pi(r|x) \geq 0$ w.p.1. Combining this with the previous asymptotic normality results, the same arguments in the proof of Lemma 3 of Stoye (2009) can be used to show that $\sqrt{Lh_L^z} \cdot (\widehat{\Lambda}_n^\pi(r|x) - \Lambda_n^\pi(r|x)) = o_p(1)$ when $\Lambda_n^\pi(r|x) = 0$.[45] From here, Proposition 1 in Stoye (2009) can be used to show that the CI in (7) has good coverage properties even if $\Lambda_n^\pi(r|x) \approx 0$.[46]

Under IPV, point-identification of $\pi_n(r|x)$ means that we can construct a CI in a straightforward way. Let $(1-\alpha)$ denote our target coverage probability and let κ_α be the value such that $\Phi(\kappa_\alpha) - \Phi(-\kappa_\alpha) = 1-\alpha$. Under IPV, the CI can be estimated as

$$\text{(9)} \qquad \mathrm{CI}_{1-\alpha}\big(\pi_n^{\mathrm{IPV}}(r|x)\big) = \left[\widehat{\pi}_n^{\mathrm{IPV}}(r|x) - \kappa_\alpha \cdot \frac{\widehat{\sigma}_n^{\mathrm{IPV}}(r|x)}{\sqrt{Lh_L^z}}, \widehat{\pi}_n^{\mathrm{IPV}}(r|x) + \kappa_\alpha \cdot \frac{\widehat{\sigma}_n^{\mathrm{IPV}}(r|x)}{\sqrt{Lh_L^z}}\right],$$

where $\widehat{\sigma}_n^{\mathrm{IPV}}(r|x)$ is a sample analog nonparametric estimator of $\sigma_n^{\mathrm{IPV}}(r|x)$.

B.3. *Expected Profits Conditional on X*

Next, we consider the confidence interval for expected profit conditional only on X, that is, in expectation over N. For given (x, r), let

$$\pi_{\overline{N}}(r|x) = E_{N|X}\big[\pi_N(r|x)\big|X = x\big] = \sum_{n=2}^{\overline{n}} p_N(n|x) \cdot \pi_n(r|x).$$

(This is the same as $\pi(r|x)$ in the text.) Using iterated expectations, $\pi_{\overline{N}}(r|x)$ simplifies to

$$\pi_{\overline{N}}(r|x) = T(r|x) - v_0 - F_{\overline{N}:\overline{N}}(r|x) \cdot (r - v_0),$$

[45]This is not hard to show using the influence functions $\underline{\psi}_\pi$ and $\overline{\psi}_\pi$ described in (6).

[46]Note that $\Lambda_n^\pi(r|x) = 0$ can occur only if $F_{\overline{n}-1:\overline{n}}(r|x)$ equals either 0 or 1, and both cases are outside our inferential range of interest. However, using the critical value defined as $\Phi(-c_\alpha) = \alpha$ can lead to undercoverage even in relatively large sample sizes if $\Lambda_n^\pi(r|x)$ is *close* to 0 (see Imbens and Manski (2004) and Stoye (2009)). For this reason, we use the correction given in (8).

where $T(r|x) = E_{B|X}[\max\{r, B\}|X = x]$ and $F_{\overline{N}:\overline{N}}(r|x) = \sum_{n=2}^{\overline{n}} p_N(n|x) \cdot F_{n:n}(r|x)$. Let $\overline{F}_{\overline{N}:\overline{N}}$, $\underline{F}_{\overline{N}:\overline{N}}$, and $F^{\mathrm{IPV}}_{\overline{N}:\overline{N}}$ be the corresponding upper bound, lower bound, and IPV expressions of $F_{\overline{N}:\overline{N}}$, and let $\underline{\pi}_{\overline{N}}$, $\overline{\pi}_{\overline{N}}$, and $\pi^{\mathrm{IPV}}_{\overline{N}}$ the corresponding expressions for $\pi_{\overline{N}}$. The bounds on $F_{\overline{N}:\overline{N}}(r|x)$ simplify to

$$\begin{aligned}
\underline{F}_{\overline{N}:\overline{N}}(r|x) &= \sum_{m=3}^{\overline{n}} \frac{E_{N|X}[N \cdot \mathbb{1}\{N < m\}|X = x]}{(m-1)m} \cdot F_{m-1:m}(r|x) \\
&\quad + \frac{E_{N|X}[N|X = x]}{\overline{n}} \cdot \big(\phi_{\overline{n}}\big(F_{\overline{n}-1:\overline{n}}(r|x)\big)\big)^{\overline{n}}, \\
\overline{F}_{\overline{N}:\overline{N}}(r|x) &= \sum_{m=3}^{\overline{n}} \frac{E_{N|X}[N \cdot \mathbb{1}\{N < m\}|X = x]}{(m-1)m} \cdot F_{m-1:m}(r|x) \\
&\quad + \frac{E_{N|X}[N|X = x]}{\overline{n}} \cdot F_{\overline{n}-1:\overline{n}}(r|x)
\end{aligned}$$

and our estimators are, therefore,

$$\begin{aligned}
\widehat{\underline{\pi}}_{\overline{N}}(r|x) &= \widehat{T}(r|x) - v_0 - \widehat{\overline{F}}_{\overline{N}:\overline{N}}(r|x) \cdot (r - v_0), \\
\widehat{\overline{\pi}}_{\overline{N}}(r|x) &= \widehat{T}(r|x) - v_0 - \widehat{\underline{F}}_{\overline{N}:\overline{N}}(r|x) \cdot (r - v_0), \\
\widehat{\pi}^{\mathrm{IPV}}_{\overline{N}}(r|x) &= \widehat{T}(r|x) - v_0 - \widehat{F}^{\mathrm{IPV}}_{\overline{N}:\overline{N}}(r|x) \cdot (r - v_0),
\end{aligned}$$

where $\widehat{T}(r|x)$ is a kernel-weighted nonparametric estimator for $T(r|x)$ and

$$\begin{aligned}
\widehat{\underline{F}}_{\overline{N}:\overline{N}}(r|x) &= \sum_{m=3}^{\overline{n}} \frac{\widehat{E}_{N|X}[N \cdot \mathbb{1}\{N < m\}|X = x]}{(m-1)m} \cdot \widehat{F}_{m-1:m}(r|x) \\
&\quad + \frac{\widehat{E}_{N|X}[N|X = x]}{\overline{n}} \cdot \big(\phi_{\overline{n}}\big(\widehat{F}_{\overline{n}-1:\overline{n}}(r|x)\big)\big)^{\overline{n}}, \\
\widehat{\overline{F}}_{\overline{N}:\overline{N}}(r|x) &= \sum_{m=3}^{\overline{n}} \frac{\widehat{E}_{N|X}[N \cdot \mathbb{1}\{N < m\}|X = x]}{(m-1)m} \cdot \widehat{F}_{m-1:m}(r|x) \\
&\quad + \frac{\widehat{E}_{N|X}[N|X = x]}{\overline{n}} \cdot \widehat{F}_{\overline{n}-1:\overline{n}}(r|x), \\
\widehat{F}^{\mathrm{IPV}}_{\overline{N}:\overline{N}}(r|x) &= \sum_{n=2}^{\overline{n}} \widehat{p}_N(n|x) \cdot \widehat{F}^{\mathrm{IPV}}_{n:n}(r|x),
\end{aligned}$$

where $\widehat{E}_{N|X}[N|X = x]$, $\widehat{E}_{N|X}[N \cdot \mathbb{1}\{N < m\}|X = x]$, and $\widehat{p}_N(n|x)$ are kernel-weighted nonparametric estimators and $\widehat{F}_{m-1:m}(r|x)$ is defined above in Section B.2. With ψ_F defined above, let

$$
\begin{aligned}
(10)\qquad & \varphi_T(r, U_i|x) = \frac{(\max\{r, B_i\} - T(r|x))}{f_X(x)}, \\
& \varphi_Q(r, U_i|x, n) = \frac{(N_i \cdot \mathbb{1}\{N_i < n\} - E_{N|X}[N \cdot \mathbb{1}\{N < n\}|X = x])}{f_X(x)}, \\
& \overline{\varphi}_F(r, U_i|x) = \sum_{m=3}^{\overline{n}} \Bigg[\frac{E_{N|X}[N \cdot \mathbb{1}\{N < m\}|X = x]}{(m-1)m} \cdot \psi_F(r, U_i|x, m) \\
& \qquad + \frac{F_{m-1:m}(r|x)}{(m-1)m} \cdot \varphi_Q(r, U_i|x, n) \Bigg] \\
& \qquad + \frac{E_{N|X}[N|X = x]}{\overline{n}} \cdot \psi_F(r, U_i|x, \overline{n}) \\
& \qquad + \frac{F_{\overline{n}-1:\overline{n}}(r|x)}{\overline{n}} \cdot \frac{(N_i - E_{N|X}[N|X = x])}{f_X(x)}, \\
& \underline{\varphi}_F(r, U_i|x) = \sum_{m=3}^{\overline{n}} \Bigg[\frac{E_{N|X}[N \cdot \mathbb{1}\{N < m\}|X = x]}{(m-1)m} \cdot \psi_F(r, U_i|x, m) \\
& \qquad + \frac{F_{m-1:m}(r|x)}{(m-1)m} \cdot \varphi_Q(r, U_i|x, n) \Bigg] \\
& \qquad + \nabla \phi_{\overline{n}}(r|x) \cdot E_{N|X}[N|X = x] \psi_F(r, U_i|x, \overline{n}) \\
& \qquad + \frac{\phi_{\overline{n}}(F_{\overline{n}-1:\overline{n}}(r|x))^{\overline{n}}}{\overline{n}} \cdot \frac{(N_i - E_{N|X}[N|X = x])}{f_X(x)}, \\
& \varphi_F^{\mathrm{IPV}}(r, U_i|x) = \sum_{n=2}^{\overline{n}} \Bigg[p_N(n|x) \cdot \psi_F^{\mathrm{IPV}}(r, U_i|x, n) \\
& \qquad + F_{n:n}(r|x) \cdot \frac{(\mathbb{1}\{N_i = n\} - p_N(n|x))}{f_X(x)} \Bigg], \\
& \underline{\varphi}_\pi(r, U_i|x) = \varphi_T(r, U_i|x) - (r - v_0) \cdot \overline{\varphi}_F(r, U_i|x), \\
& \overline{\varphi}_\pi(r, U_i|x) = \varphi_T(r, U_i|x) - (r - v_0) \cdot \underline{\varphi}_F(r, U_i|x), \\
& \varphi_\pi^{\mathrm{IPV}}(r, U_i|x) = \varphi_T(r, U_i|x) - (r - v_0) \cdot \varphi_F^{\mathrm{IPV}}(r, U_i|x).
\end{aligned}
$$

Again, Taylor expansion gives the following result.

RESULT B.2: Under Assumption 4,

$$\sqrt{Lh_L^z}\cdot\left(\widehat{\overline{\pi}}_{\overline{N}}(r|x)-\overline{\pi}_{\overline{N}}(r|x)\right)\xrightarrow{d}\mathcal{N}\left(0,\overline{\sigma}^2(r|x)\right),$$
$$\sqrt{Lh_L^z}\cdot\left(\widehat{\underline{\pi}}_{\overline{N}}(r|x)-\underline{\pi}_{\overline{N}}(r|x)\right)\xrightarrow{d}\mathcal{N}\left(0,\underline{\sigma}^2(r|x)\right),$$
$$\sqrt{Lh_L^z}\cdot\left(\widehat{\pi}_{\overline{N}}^{\mathrm{IPV}}(r|x)-\pi_{\overline{N}}^{\mathrm{IPV}}(r|x)\right)\xrightarrow{d}\mathcal{N}\left(0,\sigma_{\mathrm{IPV}}^2(r|x)\right),$$

where

$$\overline{\sigma}^2(r|x)=E_{U|X}\left[\overline{\varphi}_\pi(r,U_i|x)^2\middle|X_i=x\right]f_X(x)\mu_K^2,$$
$$\underline{\sigma}^2(r|x)=E_{U|X}\left[\underline{\varphi}_\pi(r,U_i|x)^2\middle|X_i=x\right]f_X(x)\mu_K^2,$$
$$\sigma_{\mathrm{IPV}}^2(r|x)=E_{U|X}\left[\varphi_\pi^{\mathrm{IPV}}(r,U_i|x)^2\middle|X_i=x\right]f_X(x)\mu_K^2.$$

From here, confidence intervals for $\pi_{\overline{N}}(r|x)$ are constructed similarly to (7) and (9) above. Let $\widehat{\underline{\sigma}}(r|x)$, $\widehat{\overline{\sigma}}(r|x)$, and $\widehat{\sigma}_{\mathrm{IPV}}(r|x)$ denote sample analog nonparametric estimators of $\underline{\sigma}(r|x)$, $\overline{\sigma}(r|x)$, and $\sigma_{\mathrm{IPV}}(r|x)$, respectively, and let $\widehat{\Lambda}^\pi(r|x)=\widehat{\overline{\pi}}_{\overline{N}}(r|x)-\widehat{\underline{\pi}}_{\overline{N}}(r|x)$. With correlated values, the confidence interval for $\pi_{\overline{N}}(r|x)$ is

$$\mathrm{CI}_{1-\alpha}\left(\pi_{\overline{N}}(r|x)\right)=\left[\widehat{\underline{\pi}}_{\overline{N}}(r|x)-c_\alpha\cdot\frac{\widehat{\underline{\sigma}}(r|x)}{\sqrt{Lh_L^z}},\widehat{\overline{\pi}}_{\overline{N}}(r|x)+c_\alpha\cdot\frac{\widehat{\overline{\sigma}}(r|x)}{\sqrt{Lh_L^z}}\right],$$

where c_α solves $\Phi(c_\alpha+\frac{\sqrt{Lh_L^z}\cdot\widehat{\Lambda}^\pi(r|x)}{\max\{\widehat{\underline{\sigma}}(r|x),\widehat{\overline{\sigma}}(r|x)\}})-\Phi(-c_\alpha)=1-\alpha$; for the IPV case,

$$\begin{aligned}&\mathrm{CI}_{1-\alpha}\left(\pi_{\overline{N}}^{\mathrm{IPV}}(r|x)\right)\\&\quad=\left[\widehat{\pi}_{\overline{N}}^{\mathrm{IPV}}(r|x)-\kappa_\alpha\cdot\frac{\widehat{\sigma}_{\mathrm{IPV}}(r|x)}{\sqrt{Lh_L^z}},\widehat{\pi}_{\overline{N}}^{\mathrm{IPV}}(r|x)+\kappa_\alpha\cdot\frac{\widehat{\sigma}_{\mathrm{IPV}}(r|x)}{\sqrt{Lh_L^z}}\right],\end{aligned}$$

where $\Phi(\kappa_\alpha)-\Phi(-\kappa_\alpha)=1-\alpha$.

B.4. *Effects of Reserve Price Policies*

In Section 4.4 we study the "portfolio-level" impact of various reserve price *policies*. Each policy assigns a reserve price $r(X)$ to a given auction X. (We treat $r(\cdot)$ as *given*, i.e., not as an estimated version of a target policy.) We refer to $r(X)=v_0$ as the *baseline policy*. The effect of each alternative policy is analyzed via the following measures:

(i) *Average profits:* $\mathcal{A}_\pi=E_X[\pi_{\overline{N}}(r(X)|X)|X\in\mathcal{X}]$ (with $\mathcal{A}_{\pi_0}=E_X[\pi_{\overline{N}}(v_0|X)|X\in\mathcal{X}]$).

(ii) *Average change in profits:* $\mathcal{A}_{\Delta\pi}=\mathcal{A}_\pi-\mathcal{A}_{\pi_0}$.

(iii) *Average no-sale probability:* $\mathcal{A}_F = E_X[F_{\overline{N}:\overline{N}}(r(X)|X)|X \in \mathcal{X}]$.
We similarly let $\mathcal{A}_\pi^{\text{IPV}}$, $\mathcal{A}_{\Delta\pi}^{\text{IPV}}$, and $\mathcal{A}_F^{\text{IPV}}$ denote the corresponding measures calculated under the assumption of IPV, that is, based on $\pi_{\overline{N}}^{\text{IPV}}$ and $F_{\overline{N}:\overline{N}}^{\text{IPV}}$. $\mathcal{X} \subset \text{int}(\text{Supp}(X))$ is a compact set chosen such that $q_{X,N}(x,m) > 0$ and $0 < F_{m-1:m}(r(x)|x) < 1$ for all $x \in \mathcal{X}$ and all $m \in \text{Supp}(N)$; we refer to $\mathcal{X}$ as our *inference range*. We estimate the aggregate measures described above using nonparametric sample analogs, which we construct in the manner described above. However, in their construction we now use a *bias-reducing kernel*, which will allow for our estimated measures to be $\sqrt{L}$-consistent. We replace Assumption 4 with the following stronger version.

ASSUMPTION 5: *The first part of Assumption 4 is maintained and, in addition, the following statements hold*:

(i) $\mathcal{X} \subset \text{int}(\text{Supp}(X))$ *is such that* $\forall x \in \mathcal{X}$ *and* $\forall n \in \text{Supp}(N)$, $\exists \underline{f}, \overline{f}: 0 < \underline{f} \le f_X(x) \le \overline{f} < \infty$, $\exists \underline{q}, \overline{q}: 0 < \underline{q} \le q_{X,N}(x,n) \le \overline{q} < \infty$, *and* $\exists \underline{F}, \overline{F}: 0 < \underline{F} \le F_{n-1:n}(r|x) \le \overline{F} < 1$.

(ii) *The reserve price policy* $r(\cdot)$ *is such that* $\Pr[r(X) = V_{n-1:n}|X \in \mathcal{X}] = 0$ *for each* $n \in \text{Supp}(N)$. *In addition*, $r(x)$ *is continuous and has bounded derivatives up to order* $M \ge z+1$ *for almost every* $x \in \mathcal{X}$. *This is also true for* $f_X(x)$, $T(r(x)|x)$, $E[N|X=x]$, *and for* $p_N(n|x)$, $F_{n-1:n}(r(x)|x)$, *and* $E_{N|X}[N \cdot \mathbb{1}\{N < n\}|X = x]$ *for each* $n = 2, \dots, \overline{n}$.

(iii) *Let* M *be the constant mentioned above. The kernel* $K: \mathbb{R}^z \longrightarrow \mathbb{R}$ *is a function of bounded variation that satisfies* $\int K(\xi)\,d\xi = 1$, *has compact support, and is symmetric around zero. It is also a bias-reducing kernel of order* M. *That is, denoting* $\xi \equiv (\xi_1, \dots, \xi_z)$, *then* $\int (\xi_1^{q_1} \cdots \xi_z^{q_z}) K(\xi)\,d\xi_1 \cdots d\xi_z = 0 \; \forall 0 < q_1 + \cdots + q_z < M$ *and* $\int \|\xi\|^M |K(\xi)|\,d\xi < \infty$.

(iv) *The bandwidth sequence* h_L *is nonnegative and satisfies* $h_L \longrightarrow 0$. *In addition*, $\exists \overline{\delta} > 0$ *for which* $L^{1-\overline{\delta}} h_L^{2z} \longrightarrow \infty$ *and* $L^{1+\overline{\delta}} h_L^{M+z} \longrightarrow 0$.

Our choices of $\mathcal{X}$, kernel, and bandwidth are described in Section B.7.

Baseline Policy

Recall that v_0 is assumed to be a deterministic function of X, that is, think of v_0 below as implicitly meaning $v_0(X)$ or $v_0(x)$. $\mathcal{A}_{\pi_0} = E_X[\pi_{\overline{N}}(v_0|X)|X \in \mathcal{X}]$ is point-identified under Assumption 2 and is given by

$$\mathcal{A}_{\pi_0} = E_X\big[T(v_0|X) - v_0 \big| X \in \mathcal{X}\big],$$

where, as before, $T(v_0|X) = E_{B|X}[\max\{v_0, B\}|X]$. Let $\widehat{T}(r|x)$ be as defined above (a kernel-based estimator of $T(r|x)$), but constructed with a kernel and bandwidth sequence satisfying Assumption 5. Let

$$\widehat{P}(X \in \mathcal{X}) = \frac{1}{L} \sum_{i=1}^{L} \mathbb{1}\{X_i \in \mathcal{X}\}$$

and

$$\widehat{\mathcal{A}}_{\pi_0} = \frac{1}{L}\sum_{i=1}^{L} \frac{(\widehat{T}(v_{0,i}|X_i) - v_{0,i}) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}}{\widehat{P}(X \in \mathcal{X})}.$$

Using results from empirical process theory (Nolan and Pollard (1987), Pakes and Pollard (1989), and Sherman (1994a)), under the conditions of Assumption 5 we can show that

$$\sup_{x\in\mathcal{X}} \big|\widehat{T}(v_0|x) - T(v_0|x)\big| = o_p\big(L^{-1/4}\big). \tag{11}$$

Let $P_{\mathcal{X}}$ denote $\Pr(X \in \mathcal{X})$. With φ_T as in (10), define, for any pair of observations i, ℓ in $1, \dots, L$,

$$\begin{aligned}
&\xi_{\pi_0}(U_i, U_\ell) \\
&\quad = \frac{1}{2} \times \left\{ \frac{1}{h_L^z} \cdot \frac{\varphi_T(v_{0,i}, U_\ell|X_i)}{P_{\mathcal{X}}} \cdot K\left(\frac{X_\ell - X_i}{h_L}\right) \right. \\
&\qquad \left. - \frac{(T(v_{0,i}|X_i) - v_{0,i})}{P_{\mathcal{X}}^2} \cdot \big(\mathbb{1}\{X_\ell \in \mathcal{X}\} - P_{\mathcal{X}}\big) \right\} \\
&\qquad \cdot \mathbb{1}\{X_i \in \mathcal{X}\}.
\end{aligned}$$

From (11) and the conditions in Assumption 5, a second order approximation can be used to show that

$$\begin{aligned}
\widehat{\underline{\mathcal{A}}}_{\pi_0} &= \frac{1}{L}\sum_{i=1}^{L} \frac{(T(v_{0,i}|X_i) - v_{0,i})}{P_{\mathcal{X}}} \cdot \mathbb{1}\{X_i \in \mathcal{X}\} \\
&\quad + \binom{L}{2}^{-1} \sum_{i<\ell} \big(\xi_{\pi_0}(U_i, U_\ell) + \xi_{\pi_0}(U_\ell, U_i)\big) + o_p\big(L^{-1/2}\big).
\end{aligned} \tag{12}$$

That is, we can express $\widehat{\underline{\mathcal{A}}}_{\pi_0}$ as the sum of a sample mean, plus a U-statistic of order 2 and a negligible $o_p(L^{-1/2})$ term. Let

$$\begin{aligned}
\zeta_{\pi_0}(U_i) = P_{\mathcal{X}}^{-1} \cdot \big\{ &\big[\big(T(v_{0,i}|X_i) - v_{0,i}\big) - \mathcal{A}_{\pi_0}\big] \cdot \mathbb{1}\{X_i \in \mathcal{X}\} \\
&+ \varphi_T(v_{0,i}, U_i|X_i)\big\}.
\end{aligned}$$

The Hoeffding decomposition or "projection" (see Serfling (1980) and Sherman (1994b)) of the U-statistic described above and the conditions of Assumption 5 yield

$$\widehat{\underline{\mathcal{A}}}_{\pi_0} = \underline{\mathcal{A}}_{\pi_0} + \frac{1}{L}\sum_{i=1}^{L} \zeta_{\pi_0}(U_i) + o_p\big(L^{-1/2}\big). \tag{13}$$

A quick inspection reveals that $E[\zeta_{\pi_0}(U_i)] = 0$. Therefore,

$$\sqrt{L}(\widehat{\mathcal{A}}_{\pi_0} - \mathcal{A}_{\pi_0}) \xrightarrow{d} \mathcal{N}\big(0, \Omega^2_{\pi_0}\big), \tag{14}$$

where $\Omega^2_{\pi_0} = E[\zeta^2_{\pi_0}(U_i)]$. From here, a $(1-\alpha)$ confidence interval for $\mathcal{A}_{\pi_0}$ can be constructed as

$$\mathrm{CI}_{1-\alpha}(\mathcal{A}_{\pi_0}) = \left[\widehat{\mathcal{A}}_{\pi_0} - \kappa_\alpha \cdot \frac{\widehat{\Omega}_{\pi_0}}{\sqrt{L}}, \widehat{\mathcal{A}}_{\pi_0} + \kappa_\alpha \cdot \frac{\widehat{\Omega}_{\pi_0}}{\sqrt{L}}\right],$$

where $\widehat{\Omega}_{\pi_0}$ is a sample analog nonparametric estimator of Ω_{π_0} and $\Phi(\kappa_\alpha) - \Phi(-\kappa_\alpha) = 1 - \alpha$.

Alternative Policies

By definition,

$$\begin{aligned}
E_X\big[\underline{\pi}_{\overline{N}}\big(r(X)|X\big)\big|X \in \mathcal{X}\big] &= \underline{\mathcal{A}}_\pi \le \mathcal{A}_\pi \le \overline{\mathcal{A}}_\pi \\
&= E_X\big[\overline{\pi}_{\overline{N}}\big(r(X)|X\big)\big|X \in \mathcal{X}\big], \\
\underline{\mathcal{A}}_\pi - \mathcal{A}_{\pi_0} = \underline{\mathcal{A}}_{\Delta\pi} &\le \mathcal{A}_{\Delta\pi} \le \overline{\mathcal{A}}_{\Delta\pi} = \overline{\mathcal{A}}_\pi - \mathcal{A}_{\pi_0}, \\
E_X\big[\underline{F}_{\overline{N}:\overline{N}}\big(r(X)|X\big)\big|X \in \mathcal{X}\big] &= \underline{\mathcal{A}}_F \le \mathcal{A}_F \le \overline{\mathcal{A}}_F \\
&= E_X\big[\overline{F}_{\overline{N}:\overline{N}}\big(r(X)|X\big)\big|X \in \mathcal{X}\big].
\end{aligned}$$

Let

$$\begin{aligned}
\widehat{\underline{\mathcal{A}}}_\pi &= \frac{1}{L}\sum_{i=1}^{L} \frac{\widehat{\underline{\pi}}_{\overline{N}}(r(X_i)|X_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}}{\widehat{P}(X \in \mathcal{X})}, \\
\widehat{\overline{\mathcal{A}}}_\pi &= \frac{1}{L}\sum_{i=1}^{L} \frac{\widehat{\overline{\pi}}_{\overline{N}}(r(X_i)|X_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}}{\widehat{P}(X \in \mathcal{X})}, \\
\widehat{\underline{\mathcal{A}}}_F &= \frac{1}{L}\sum_{i=1}^{L} \frac{\widehat{\underline{F}}_{\overline{N}:\overline{N}}(r(X_i)|X_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}}{\widehat{P}(X \in \mathcal{X})}, \\
\widehat{\overline{\mathcal{A}}}_F &= \frac{1}{L}\sum_{i=1}^{L} \frac{\widehat{\overline{F}}_{\overline{N}:\overline{N}}(r(X_i)|X_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}}{\widehat{P}(X \in \mathcal{X})}, \\
\widehat{\underline{\mathcal{A}}}_{\Delta\pi} &= \widehat{\underline{\mathcal{A}}}_\pi - \widehat{\mathcal{A}}_{\pi_0}, \quad \widehat{\overline{\mathcal{A}}}_{\Delta\pi} = \widehat{\overline{\mathcal{A}}}_\pi - \widehat{\mathcal{A}}_{\pi_0}.
\end{aligned} \tag{15}$$

Analogous to (11), empirical process theory can be used to show that, under Assumption 5,

$$
\begin{aligned}
(16)\qquad &\sup_{x\in\mathcal{X}}\big|\widehat{\underline{\pi}}_{\overline{N}}\big(r(x)|x\big)-\underline{\pi}_{\overline{N}}\big(r(x)|x\big)\big|=o_p\big(L^{-1/4}\big),\\
&\sup_{x\in\mathcal{X}}\big|\widehat{\overline{\pi}}_{\overline{N}}\big(r(x)|x\big)-\overline{\pi}_{\overline{N}}\big(r(x)|x\big)\big|=o_p\big(L^{-1/4}\big),\\
&\sup_{x\in\mathcal{X}}\big|\widehat{\underline{F}}_{\overline{N}:\overline{N}}\big(r(x)|x\big)-\underline{F}_{\overline{N}:\overline{N}}\big(r(x)|x\big)\big|=o_p\big(L^{-1/4}\big),\\
&\sup_{x\in\mathcal{X}}\big|\widehat{\overline{F}}_{\overline{N}:\overline{N}}\big(r(x)|x\big)-\overline{F}_{\overline{N}:\overline{N}}\big(r(x)|x\big)\big|=o_p\big(L^{-1/4}\big).
\end{aligned}
$$

With $\underline{\varphi}_\pi$, $\overline{\varphi}_\pi$, $\underline{\varphi}_F$, $\overline{\varphi}_F$, and ζ_{π_0} defined above, let

$$
\begin{aligned}
\underline{\zeta}_\pi(U_i)&=P_{\mathcal{X}}^{-1}\cdot\big\{\big[\underline{\pi}_{\overline{N}}\big(r(X_i)|X_i\big)-\underline{\mathcal{A}}_\pi\big]\cdot\mathbb{1}\{X_i\in\mathcal{X}\}\\
&\qquad+\underline{\varphi}_\pi\big(r(X_i),U_i|X_i\big)\big\},\\
\overline{\zeta}_\pi(U_i)&=P_{\mathcal{X}}^{-1}\cdot\big\{\big[\overline{\pi}_{\overline{N}}\big(r(X_i)|X_i\big)-\overline{\mathcal{A}}_\pi\big]\cdot\mathbb{1}\{X_i\in\mathcal{X}\}\\
&\qquad+\overline{\varphi}_\pi\big(r(X_i),U_i|X_i\big)\big\},\\
\underline{\zeta}_F(U_i)&=P_{\mathcal{X}}^{-1}\cdot\big\{\big[\underline{F}_{\overline{N}:\overline{N}}\big(r(X_i)|X_i\big)-\underline{\mathcal{A}}_F\big]\cdot\mathbb{1}\{X_i\in\mathcal{X}\}\\
&\qquad+\underline{\varphi}_F\big(r(X_i),U_i|X_i\big)\big\},\\
\overline{\zeta}_F(U_i)&=P_{\mathcal{X}}^{-1}\cdot\big\{\big[\overline{F}_{\overline{N}:\overline{N}}\big(r(X_i)|X_i\big)-\overline{\mathcal{A}}_F\big]\cdot\mathbb{1}\{X_i\in\mathcal{X}\}\\
&\qquad+\overline{\varphi}_F\big(r(X_i),U_i|X_i\big)\big\},\\
\underline{\zeta}_{\Delta\pi}(U_i)&=\underline{\zeta}_\pi(U_i)-\zeta_{\pi_0}(U_i),\quad \overline{\zeta}_{\Delta\pi}(U_i)=\overline{\zeta}_\pi(U_i)-\zeta_{\pi_0}(U_i).
\end{aligned}
$$

For each $j\in\{\pi,F,\Delta\pi\}$, the same arguments leading to (12) and (13) apply for $\underline{\mathcal{A}}_j$ and $\overline{\mathcal{A}}_j$, with φ_T replaced with $\underline{\varphi}_j$ and $\overline{\varphi}_j$, respectively (see (10)). The equivalent result to (14) now follows.

RESULT B.3: Under Assumption 5, for each $j\in\{\pi,F,\Delta\pi\}$,

$$
\sqrt{L}(\widehat{\underline{\mathcal{A}}}_j-\underline{\mathcal{A}}_j)\xrightarrow{d}\mathcal{N}\big(0,\underline{\Omega}_j^2\big)\quad\text{and}\quad\sqrt{L}(\widehat{\overline{\mathcal{A}}}_j-\overline{\mathcal{A}}_j)\xrightarrow{d}\mathcal{N}\big(0,\overline{\Omega}_j^2\big),
$$

where $\underline{\Omega}_j^2=E[\underline{\zeta}_j^2(U_i)]$ and $\overline{\Omega}_j^2=E[\overline{\zeta}_j^2(U_i)]$.

Our CIs are obtained analogously to those above, with one difference. Because we now employ bias-reducing kernels, our estimated lower and upper bounds for $\mathcal{A}_\pi$, $\mathcal{A}_F$, and $\mathcal{A}_{\Delta\pi}$ can *cross* with positive probability. For this reason, we follow the prescription in Stoye (2009) and use *shrinkage* estimators for the width of the identified sets in each case. Letting b_L denote a nonnegative sequence $b_L \to 0$ such that $b_L\sqrt{L} \to \infty$, we employ the following estimators for the width of the identified intervals:

$$\widehat{\Upsilon}_j = (\widehat{\overline{\mathcal{A}}}_j - \widehat{\underline{\mathcal{A}}}_j) \cdot \mathbb{1}\{\widehat{\overline{\mathcal{A}}}_j - \widehat{\underline{\mathcal{A}}}_j > b_L\}. \tag{17}$$

Our $(1-\alpha)$ CIs for $\mathcal{A}_\pi$, $\mathcal{A}_F$, and $\mathcal{A}_{\Delta\pi}$ are given by

$$\mathrm{CI}_{1-\alpha}(\mathcal{A}_j) = \left[\widehat{\underline{\mathcal{A}}}_j - c_\alpha^j \cdot \frac{\widehat{\underline{\Omega}}_j}{\sqrt{L}}, \widehat{\overline{\mathcal{A}}}_j + c_\alpha^j \cdot \frac{\widehat{\overline{\Omega}}_j}{\sqrt{L}}\right] \tag{18}$$

for each $j \in \{\pi, F, \Delta\pi\}$, where $\widehat{\underline{\Omega}}_j$ and $\widehat{\overline{\Omega}}_j$ are sample analog nonparametric estimators for $\underline{\Omega}_j$ and $\overline{\Omega}_j$, and $\Phi(c_\alpha^j + \frac{\sqrt{L}\cdot\widehat{\Upsilon}_j}{\max\{\widehat{\underline{\Omega}}_j, \widehat{\overline{\Omega}}_j\}}) - \Phi(-c_\alpha^j) = 1-\alpha$.

Alternative Policies Under IPV

As before, $\mathcal{A}_\pi^{\mathrm{IPV}}$, $\mathcal{A}_F^{\mathrm{IPV}}$, and $\mathcal{A}_{\Delta\pi}^{\mathrm{IPV}}$ are point-identified, and can be estimated as

$$\begin{aligned}
\widehat{\mathcal{A}}_\pi^{\mathrm{IPV}} &= \frac{1}{L}\sum_{i=1}^{L} \frac{\widehat{\pi}_{\overline{N}}^{\mathrm{IPV}}(r(X_i)|X_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}}{\widehat{P}(X \in \mathcal{X})}, \\
\widehat{\mathcal{A}}_F^{\mathrm{IPV}} &= \frac{1}{L}\sum_{i=1}^{L} \frac{\widehat{F}_{\overline{N}:\overline{N}}^{\mathrm{IPV}}(r(X_i)|X_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}}{\widehat{P}(X \in \mathcal{X})}, \\
\widehat{\mathcal{A}}_{\Delta\pi}^{\mathrm{IPV}} &= \widehat{\mathcal{A}}_\pi^{\mathrm{IPV}} - \widehat{\mathcal{A}}_{\pi_0}.
\end{aligned}$$

Under Assumption 5, we can show that

$$\sup_{x\in\mathcal{X}} \left|\widehat{\pi}_{\overline{N}}^{\mathrm{IPV}}\big(r(x)|x\big) - \pi_{\overline{N}}^{\mathrm{IPV}}\big(r(x)|x\big)\right| = o_p\big(L^{-1/4}\big)$$

and

$$\sup_{x\in\mathcal{X}} \left|\widehat{F}_{\overline{N}:\overline{N}}^{\mathrm{IPV}}\big(r(x)|x\big) - F_{\overline{N}:\overline{N}}^{\mathrm{IPV}}\big(r(x)|x\big)\right| = o_p\big(L^{-1/4}\big).$$

With $\varphi_\pi^{\mathrm{IPV}}$ and φ_F^{IPV} defined above, let

$$\begin{aligned}
\zeta_\pi^{\mathrm{IPV}}(U_i) &= P_{\mathcal{X}}^{-1} \cdot \big\{\big[\pi_{\overline{N}}\big(r(X_i)|X_i\big) - \mathcal{A}_\pi\big] \cdot \mathbb{1}\{X_i \in \mathcal{X}\} \\
&\quad + \varphi_\pi^{\mathrm{IPV}}\big(r(X_i), U_i|X_i\big)\big\}, \\
\zeta_F^{\mathrm{IPV}}(U_i) &= P_{\mathcal{X}}^{-1} \cdot \big\{\big[F_{\overline{N}:\overline{N}}\big(r(X_i)|X_i\big) - \mathcal{A}_F\big] \cdot \mathbb{1}\{X_i \in \mathcal{X}\} \\
&\quad + \varphi_F^{\mathrm{IPV}}\big(r(X_i), U_i|X_i\big)\big\}, \\
\zeta_{\Delta\pi}^{\mathrm{IPV}}(U_i) &= \zeta_\pi^{\mathrm{IPV}}(U_i) - \zeta_{\pi_0}(U_i).
\end{aligned}$$

RESULT B.4: Under Assumption 5, for each $j \in \{\pi, F, \Delta\pi\}$, $\sqrt{L}(\widehat{\mathcal{A}}_j^{\mathrm{IPV}} - \mathcal{A}_j^{\mathrm{IPV}}) \xrightarrow{d} \mathcal{N}(0, \Omega_j^{\mathrm{IPV}2})$, where $\Omega_j^{\mathrm{IPV}2} = E[\zeta_j^{\mathrm{IPV}}(U_i)^2]$.

From here, our $(1-\alpha)$ CIs for $\mathcal{A}_\pi$, $\mathcal{A}_F$, and $\mathcal{A}_{\Delta\pi}$ are given by

$$\mathrm{CI}_{1-\alpha}\big(\mathcal{A}_j^{\mathrm{IPV}}\big) = \left[\widehat{\mathcal{A}}_j^{\mathrm{IPV}} - \kappa_\alpha \cdot \frac{\widehat{\Omega}_j^{\mathrm{IPV}}}{\sqrt{L}}, \widehat{\mathcal{A}}_j^{\mathrm{IPV}} + \kappa_\alpha \cdot \frac{\widehat{\Omega}_j^{\mathrm{IPV}}}{\sqrt{L}}\right]$$

for each $j \in \{\pi, F, \Delta\pi\}$, where $\Phi(\kappa_\alpha) - \Phi(-\kappa_\alpha) = 1 - \alpha$ and $\widehat{\Omega}_j^{\mathrm{IPV}}$ is a sample analog nonparametric estimator for Ω_j^{IPV}.

B.5. *Expected Bidders' Surplus Conditional on N and X*

For given (n, r, x), integration by parts allows us to write expected bidders' surplus as $\mathrm{BS}_n(r|x) = \int_r^\infty (F_{n-1:n}(s|x) - F_{n:n}(s|x))\,ds$, giving the bounds/estimate

$$\underline{\mathrm{BS}}_n(r|x) = \int_r^\infty \big(F_{n-1:n}(s|x) - \overline{F}_{n:n}(s|x)\big)\,ds,$$

$$\overline{\mathrm{BS}}_n(r|x) = \int_r^\infty \big(F_{n-1:n}(s|x) - \underline{F}_{n:n}(s|x)\big)\,ds,$$

$$\mathrm{BS}_n^{\mathrm{IPV}}(r|x) = \int_r^\infty \big(F_{n-1:n}(s|x) - F_{n:n}^{\mathrm{IPV}}(s|x)\big)\,ds.$$

Estimation will be simplified by the following assumption.

ASSUMPTION 6: *For each $n \in \mathrm{Supp}(N)$, $V_{n-1:n}$ and $V_{n:n}$ have the same support conditional on $X = x$, and this support is bounded above by $\overline{V} < \infty$.*

If we take a new random variable $S \sim \mathrm{Uniform}[r, \overline{V}]$, then

$$\mathrm{BS}_n(r|x) = (\overline{V} - r) \cdot E_S\big[F_{n-1:n}(S|x) - F_{n:n}(S|x)\big].$$

In fact, we estimate a trimmed version of this using sample analogs

$$\widehat{\underline{\mathrm{BS}}}_n^t(r|x) = (\bar{t} - r) \cdot \frac{1}{L}\sum_{i=1}^{L}\big[\widehat{F}_{n-1:n}(S_i|x) - \widehat{\overline{F}}_{n:n}(S_i|x)\big],$$

$$\widehat{\overline{\mathrm{BS}}}_n^t(r|x) = (\bar{t} - r) \cdot \frac{1}{L}\sum_{i=1}^{L}\big[\widehat{F}_{n-1:n}(S_i|x) - \widehat{\underline{F}}_{n:n}(S_i|x)\big],$$

$$\widehat{\mathrm{BS}}_n^{t,\mathrm{IPV}}(r|x) = (\bar{t} - r) \cdot \frac{1}{L}\sum_{i=1}^{L}\big[\widehat{F}_{n-1:n}(S_i|x) - \widehat{F}_{n:n}^{\mathrm{IPV}}(S_i|x)\big],$$

where $(S_i)_{i=1}^L$ are i.i.d. $\sim$ Uniform$[r, \bar{t}]$, independent of all covariates in the data, with $\bar{t}$ chosen such that $F_{m-1:m}(\bar{t}|x) < 1$ for all $m \in \{2, \ldots, \bar{n}\}$. (These are estimates of the "trimmed integrals" $\underline{\mathrm{BS}}_n^t(r|x) = \int_r^{\bar{t}} (F_{n-1:n}(s|x) - \overline{F}_{n:n}(s|x))\, ds$ and the analogously defined $\overline{\mathrm{BS}}_n^t(r|x)$ and $\mathrm{BS}_n^{t,\mathrm{IPV}}(r|x)$.) The trimming prevents us from reaching the boundary of the support of $V|N = m$, where $\nabla\phi_m(\cdot|x)$ becomes unbounded. (While this would not be a problem for estimation, it complicates inference significantly; see footnote 43.) Note that we can make the trimmed integrals as close as we want to the actual integrals by setting $\bar{t}$ large enough. In our empirical application, we set $\bar{t} = 645$, which covers the entire range of observed values for B_i (transaction price) in our data. The validity of our confidence intervals (described next) depends on the assumption that $F_{m-1:m}(645|x) < 1$ for all $m \in \{2, \ldots, \bar{n}\}$. With ψ_F, $\underline{\psi}_F$, $\overline{\psi}_F$, and ψ_F^{IPV} defined above, let

$$\underline{\psi}_{\mathrm{BS}}(U_i|r, x, n) = \int_r^{\bar{t}} \{\psi_F(s, U_i|X, n) - \overline{\psi}_F(s, U_i|X, n)\}\, ds,$$

$$\overline{\psi}_{\mathrm{BS}}(U_i|r, x, n) = \int_r^{\bar{t}} \{\psi_F(s, U_i|X, n) - \underline{\psi}_F(s, U_i|X, n)\}\, ds,$$

$$\psi_{\mathrm{BS}}^{\mathrm{IPV}}(U_i|r, x, n) = \int_r^{\bar{t}} \{\psi_F(s, U_i|X, n) - \psi_F^{\mathrm{IPV}}(s, U_i|X, n)\}\, ds.$$

RESULT B.5: If $F_{m-1:m}(\bar{t}|x) < 1$ for all $m \in \{2, \ldots, \bar{n}\}$, and Assumptions 4 and 6 hold, then

$$\sqrt{Lh_L^z} \cdot \big(\widehat{\overline{\mathrm{BS}}}_n^t(r|x) - \overline{\mathrm{BS}}_n^t(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \overline{\sigma}_{\mathrm{BS},n}^2(r, x)\big),$$

$$\sqrt{Lh_L^z} \cdot \big(\widehat{\underline{\mathrm{BS}}}_n^t(r|x) - \underline{\mathrm{BS}}_n^t(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \underline{\sigma}_{\mathrm{BS},n}^2(r, x)\big),$$

$$\sqrt{Lh_L^z} \cdot \big(\widehat{\mathrm{BS}}_n^{t,\mathrm{IPV}}(r|x) - \mathrm{BS}_n^{t,\mathrm{IPV}}(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \sigma_{\mathrm{BS},n}^{\mathrm{IPV}^2}(r|x)\big),$$

where

$$\underline{\sigma}_{\mathrm{BS},n}^2(r, x) = E_{U|X}\big[\underline{\psi}_{\mathrm{BS}}^2(U_i|r, x, n)|X_i = x\big] f_X(x)\mu_K^2,$$

$$\overline{\sigma}_{\mathrm{BS},n}^2(r, x) = E_{U|X}\big[\overline{\psi}_{\mathrm{BS}}^2(U_i|r, x, n)|X_i = x\big] f_X(x)\mu_K^2,$$

$$\sigma_{\mathrm{BS},n}^{\mathrm{IPV}^2}(r|x) = E_{U|X}\big[\psi_{\mathrm{BS}}^{\mathrm{IPV}}(U_i|r, x, n)^2|X_i = x\big] f_X(x)\mu_K^2.$$

From here, $(1-\alpha)$ confidence intervals are estimated in the same way as before: letting $\widehat{\underline{\sigma}}_{\mathrm{BS},n}(r|x)$, $\widehat{\overline{\sigma}}_{\mathrm{BS},n}(r|x)$, and $\widehat{\sigma}_{\mathrm{BS},n}^{\mathrm{IPV}}(r|x)$ denote sample analog nonparametric estimators and letting $\widehat{\Lambda}_n^{\mathrm{BS}}(r|x) = \widehat{\overline{\mathrm{BS}}}_n^t(r|x) - \widehat{\underline{\mathrm{BS}}}_n^t(r|x)$, yields

$$\begin{aligned}
&\mathrm{CI}_{1-\alpha}\big(\mathrm{BS}_n^t(r|x)\big) \\
&\quad = \left[\widehat{\underline{\mathrm{BS}}}_n^t(r|x) - c_\alpha \cdot \frac{\widehat{\underline{\sigma}}_{\mathrm{BS},n}(r|x)}{\sqrt{Lh_L^z}}, \widehat{\overline{\mathrm{BS}}}_n^t(r|x) + c_\alpha \cdot \frac{\widehat{\overline{\sigma}}_{\mathrm{BS},n}(r|x)}{\sqrt{Lh_L^z}}\right], \\
&\mathrm{CI}_{1-\alpha}\big(\mathrm{BS}_n^{t,\mathrm{IPV}}(r|x)\big) \\
&\quad = \left[\widehat{\mathrm{BS}}_n^{t,\mathrm{IPV}}(r|x) - \kappa_\alpha \cdot \frac{\widehat{\sigma}_{\mathrm{BS},n}^{\mathrm{IPV}}(r|x)}{\sqrt{Lh_L^z}}, \widehat{\mathrm{BS}}_n^{t,\mathrm{IPV}}(r|x) + \kappa_\alpha \cdot \frac{\widehat{\sigma}_{\mathrm{BS},n}^{\mathrm{IPV}}(r|x)}{\sqrt{Lh_L^z}}\right],
\end{aligned}$$

where c_α solves $\Phi(c_\alpha + \frac{\sqrt{Lh_L^z} \cdot \widehat{\Lambda}_n^{\mathrm{BS}}(r|x)}{\max\{\widehat{\underline{\sigma}}_{\mathrm{BS},n}(r|x), \widehat{\overline{\sigma}}_{\mathrm{BS},n}(r|x)\}}) - \Phi(-c_\alpha) = 1 - \alpha$ and κ_α solves $\Phi(\kappa_\alpha) - \Phi(-\kappa_\alpha) = 1 - \alpha$.

B.6. *Expected Bidders' Surplus Conditional on X*

Let $\mathrm{BS}_{\overline{N}}(r|x) = E_{N|X}[\mathrm{BS}_N(r|x)|X = x] = \sum_{n=2}^{\overline{n}} p_N(n|x) \cdot \mathrm{BS}_n(r|x)$. Maintaining Assumption 6, our estimators are

$$\widehat{\underline{\mathrm{BS}}}_{\overline{N}}^t(r|x) = \sum_{n=2}^{\overline{n}} \widehat{p}_N(n|x) \cdot \widehat{\underline{\mathrm{BS}}}_n^t(r|x),$$

$$\widehat{\overline{\mathrm{BS}}}_{\overline{N}}^t(r|x) = \sum_{n=2}^{\overline{n}} \widehat{p}_N(n|x) \cdot \widehat{\overline{\mathrm{BS}}}_n^t(r|x),$$

$$\widehat{\mathrm{BS}}_{\overline{N}}^{t,\mathrm{IPV}}(r|x) = \sum_{n=2}^{\overline{n}} \widehat{p}_N(n|x) \cdot \widehat{\mathrm{BS}}_n^{t,\mathrm{IPV}}(r|x).$$

With $\underline{\psi}_{\mathrm{BS}}$, $\overline{\psi}_{\mathrm{BS}}$, and $\psi_{\mathrm{BS}}^{\mathrm{IPV}}$ defined above, let

$$\begin{aligned}
\underline{\varphi}_{\mathrm{BS}}(U_i|r,x) = \sum_{n=2}^{\overline{n}} \bigg[& p_N(n|x) \cdot \underline{\psi}_{\mathrm{BS}}(U_i|r,x,n) \\
& + \underline{\mathrm{BS}}_n^t(r|x) \cdot \frac{(\mathbb{1}\{N_i = n\} - p_N(n|x))}{f_X(x)} \bigg], \\
\overline{\varphi}_{\mathrm{BS}}(U_i|r,x) = \sum_{n=2}^{\overline{n}} \bigg[& p_N(n|x) \cdot \overline{\psi}_{\mathrm{BS}}(U_i|r,x,n) \\
& + \overline{\mathrm{BS}}_n^t(r|x) \cdot \frac{(\mathbb{1}\{N_i = n\} - p_N(n|x))}{f_X(x)} \bigg],
\end{aligned}$$

$$\varphi_{\mathrm{BS}}^{\mathrm{IPV}}(U_i|r,x) = \sum_{n=2}^{\overline{n}} \Bigg[p_N(n|x) \cdot \psi_{\mathrm{BS}}^{\mathrm{IPV}}(U_i|r,x,n) + \mathrm{BS}_n^t(r|x) \cdot \frac{(\mathbb{1}\{N_i = n\} - p_N(n|x))}{f_X(x)} \Bigg].$$

RESULT B.6: If $F_{m-1:m}(\overline{t}|x) < 1$ for all $m \in \{2, \dots, \overline{n}\}$ and Assumptions 4 and 6 hold, then

$$\sqrt{Lh_L^z} \cdot \big(\widehat{\overline{\mathrm{BS}}}{}_{\overline{N}}^t(r|x) - \overline{\mathrm{BS}}_{\overline{N}}^t(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \overline{\sigma}_{\mathrm{BS}}^2(r,x)\big),$$
$$\sqrt{Lh_L^z} \cdot \big(\widehat{\underline{\mathrm{BS}}}{}_{\overline{N}}^t(r|x) - \underline{\mathrm{BS}}_{\overline{N}}^t(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \underline{\sigma}_{\mathrm{BS}}^2(r,x)\big),$$
$$\sqrt{Lh_L^z} \cdot \big(\widehat{\mathrm{BS}}{}_{\overline{N}}^{t,\mathrm{IPV}}(r|x) - \mathrm{BS}_{\overline{N}}^{t,\mathrm{IPV}}(r|x)\big) \xrightarrow{d} \mathcal{N}\big(0, \sigma_{\mathrm{BS}}^{\mathrm{IPV}^2}(r|x)\big),$$

where

$$\underline{\sigma}_{\mathrm{BS}}^2(r,x) = E_{U|X}\big[\underline{\varphi}_{\mathrm{BS}}^2(U_i|r,x)\big|X_i = x\big] f_X(x)\mu_K^2,$$
$$\overline{\sigma}_{\mathrm{BS}}^2(r,x) = E_{U|X}\big[\overline{\varphi}_{\mathrm{BS}}^2(U_i|r,x)\big|X_i = x\big] f_X(x)\mu_K^2,$$
$$\sigma_{\mathrm{BS}}^{\mathrm{IPV}^2}(r|x) = E_{U|X}\big[\varphi_{\mathrm{BS}}^{\mathrm{IPV}}(U_i|r,x)^2\big|X_i = x\big] f_X(x)\mu_K^2.$$

From there, confidence intervals are estimated as before,

$$\mathrm{CI}_{1-\alpha}\big(\mathrm{BS}_{\overline{N}}^t(r|x)\big) = \Bigg[\widehat{\underline{\mathrm{BS}}}{}_{\overline{N}}^t(r|x) - c_\alpha \cdot \frac{\widehat{\underline{\sigma}}_{\mathrm{BS}}(r|x)}{\sqrt{Lh_L^z}}, \widehat{\overline{\mathrm{BS}}}{}_{\overline{N}}^t(r|x) + c_\alpha \cdot \frac{\widehat{\overline{\sigma}}_{\mathrm{BS}}(r|x)}{\sqrt{Lh_L^z}}\Bigg],$$

$$\mathrm{CI}_{1-\alpha}^{\mathrm{IPV}}\big(\mathrm{BS}_{\overline{N}}^t(r|x)\big) = \Bigg[\widehat{\mathrm{BS}}{}_{\overline{N}}^{t,\mathrm{IPV}}(r|x) - \kappa_\alpha \cdot \frac{\widehat{\sigma}_{\mathrm{BS}}^{\mathrm{IPV}}(r|x)}{\sqrt{Lh_L^z}}, \widehat{\mathrm{BS}}{}_{\overline{N}}^{t,\mathrm{IPV}}(r|x) + \kappa_\alpha \cdot \frac{\widehat{\sigma}_{\mathrm{BS}}^{\mathrm{IPV}}(r|x)}{\sqrt{Lh_L^z}}\Bigg],$$

where $\widehat{\underline{\sigma}}_{\mathrm{BS}}(r|x)$, $\widehat{\overline{\sigma}}_{\mathrm{BS}}(r|x)$, and $\widehat{\sigma}_{\mathrm{BS}}^{\mathrm{IPV}}(r|x)$ are sample analog nonparametric estimators, $\widehat{\Lambda}^{\mathrm{BS}}(r|x) = \widehat{\overline{\mathrm{BS}}}{}_{\overline{N}}^t(r|x) - \widehat{\underline{\mathrm{BS}}}{}_{\overline{N}}^t(r|x)$, c_α solves $\Phi(c_\alpha + \frac{\sqrt{Lh_L^z} \cdot \widehat{\Lambda}^{\mathrm{BS}}(r|x)}{\max\{\widehat{\underline{\sigma}}_{\mathrm{BS}}(r|x), \widehat{\overline{\sigma}}_{\mathrm{BS}}(r|x)\}}) - \Phi(-c_\alpha) = 1 - \alpha$, and $\Phi(\kappa_\alpha) - \Phi(-\kappa_\alpha) = 1 - \alpha$.

B.7. *Kernels, Bandwidths, and Inference Range Used*

Kernels Employed

For a given (r,x), our approach requires that we estimate $F_{n-1:n}(r|x)$ for *each* $n = \{2, \dots, \overline{n}\}$, where $\overline{n} = 11$ in our empirical application. While our full sample size was $L = 1{,}109$, the number of observations corresponding to each auction size $n \in \{2, \dots, 11\}$ was, naturally, much smaller. Since $X \in \mathbb{R}^6$, this could produce nonparametric estimators that are disproportionately influenced by a handful of observations. In an effort to avoid this, we chose a kernel with bounded, but relatively wide support. We used a multiplicative kernel $K(\psi_1, \dots, \psi_6) = \prod_{\ell=1}^{6} k(\psi_\ell)$, where each $k(\cdot)$ was a quartic kernel of the form

$$k(\psi) = b \cdot \left(s^2 - \psi^2\right)^2 \cdot \mathbb{1}\left\{|\psi| \leq s\right\}.$$

The support of $k(\cdot)$ is the compact set $[-s, s]$, and the constant b was chosen so that $\int_{-s}^{s} k(\psi)\, d\psi = 1$. All individual-auction results are based on $s = 20$. For the reserve price policy counterfactuals, we need to estimate $F_{n-1:n}(r(X_i)|X_i)$ separately for each X_i in our inference range and for each $n = \{2, \dots, 11\}$. In accordance with Assumption 5, we employed a bias-reducing version of the kernel described in the previous paragraph: specifically, we used a multiplicative kernel of the type $K(\psi_1, \dots, \psi_6) = \prod_{\ell=1}^{6} k(\psi_\ell)$, where

$$k(\psi) = \sum_{\ell=1}^{4} b_\ell \cdot \left(s^2 - \psi^2\right)^{2\ell} \cdot \mathbb{1}\left\{|\psi| \leq s\right\}.$$

As in our graphical analysis, we used $s = 20$. The coefficients $b_1, \dots, b_4$ were chosen to ensure that $k(\cdot)$ was bias-reducing of order $M = 8$, which is compatible with Assumption 5. We discuss bandwidth selection below.

Bandwidth Selection

We approach the issue of bandwidth selection along the lines described in Section 4.2. As our *reference* model, we focus on a parametric specification where we assume

$$\log(V_{n-1:n})|X \sim \mathcal{N}\left(\beta_n' X, \exp\left\{\gamma_n' X\right\}\right).$$

By Assumption 2, this implies that

$$\log(B)|X, N = n \sim \mathcal{N}\left(\beta_n' X, \exp\left\{\gamma_n' X\right\}\right),$$

where, as we defined above, B denotes transaction price and X was expanded to include a constant term, so each γ_n includes an "intercept" term. Alternative specifications were considered and fitted, but the maximum likelihood estimator (MLE) estimates produced by the above parametrization proved to be the most robust to alternative starting values and alternative optimization algorithms. In addition, a likelihood ratio statistic comparing our specification against a model including only a constant rejected the latter, indicating that our specification has good explanatory power for the data. Let $(\widetilde{\beta}_n, \widetilde{\gamma}_n)$ denote the MLE estimator of (β_n, γ_n). Figure 11 shows $\underline{\widetilde{\pi}}_n(\cdot|x; \widetilde{\beta}_n, \widetilde{\gamma}_n)$, $\overline{\widetilde{\pi}}_n(\cdot|x; \widetilde{\beta}_n, \widetilde{\gamma}_n)$, and $\widetilde{\pi}_n^{\mathrm{IPV}}(\cdot|x; \widetilde{\beta}_n, \widetilde{\gamma}_n)$ (the resulting estimates for the lower bound, upper bound, and IPV expected profits) conditional on $x = X^{(0.50)}$ and $v_0 = 60$, and

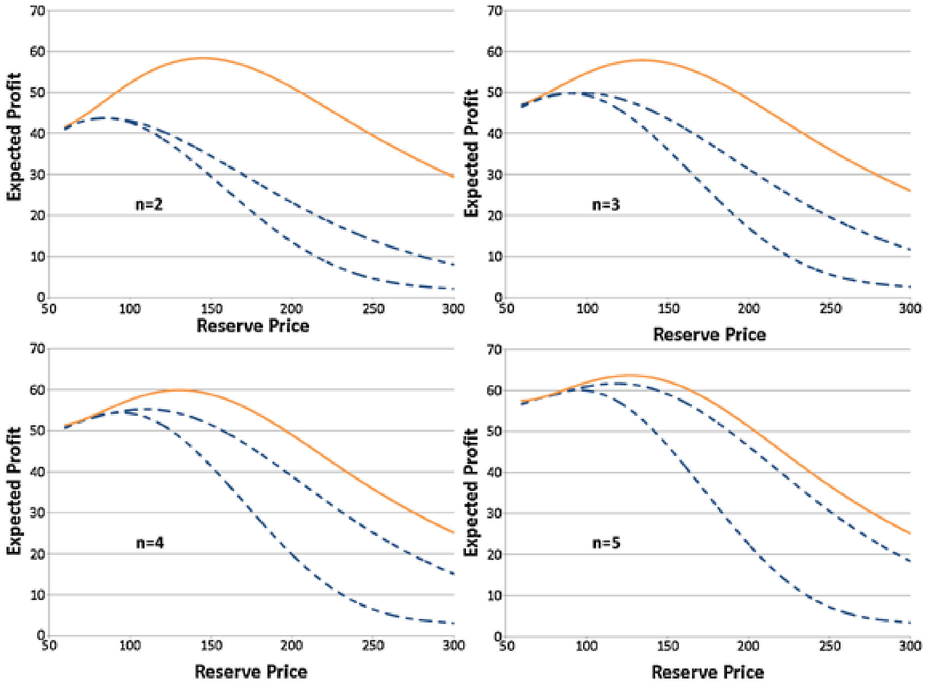

FIGURE 11.—Estimated curves for the lower bound, upper bound, and IPV expected profits produced by our parametric reference model, conditional on $x = X^{(0.50)}$, $v_0 = 60$, and $n = 2, 3, 4, 5$. The solid line depicts IPV profits; the dotted lines depict our bounds.

for $n = 2, 3, 4, 5$. These were obtained as described in Section B.2, using our MLE results in place of $\widehat{T}_n(r|x)$ and $\{\widehat{F}_{m-1:m}(r|x)\}_{m=2}^{\overline{n}}$.

The reference model for profits unconditional on N requires additional parametric assumptions. We parametrized the distribution of N given X, which is used in the estimation of $F^{\mathrm{IPV}}_{\overline{N}:N}$ and $\pi^{\mathrm{IPV}}_{\overline{N}}$, as

$$p_N(n|x) = \frac{\exp\{\delta_n' X\}}{\sum_{m=2}^{\overline{n}} \exp\{\delta_m' X\}} \quad \text{for} \quad n = 2, \ldots, \overline{n},$$

and parametrized the conditional expectations used in the estimation of $\underline{F}_{\overline{N}:\overline{N}}$, $\overline{F}_{\overline{N}:\overline{N}}$, $\underline{\pi}_{\overline{N}}$, and $\overline{\pi}_{\overline{N}}$ as[47]

$$E_{N|X}[N|X] = X'\tau \quad \text{and} \quad E_{N|X}\big[N \cdot \mathbb{1}\{N < m\}|X\big] = X'\zeta_m.$$

[47]Since the reference model is only intended to fit the data, not to structurally estimate model primitives, there is no inconsistency in parametrizing N separately for the two cases, and this allows for quicker computation and a better fit to the data.

In all cases, X was expanded with the inclusion of a constant. Our parametric reference model is, therefore, fully indexed by $\theta \equiv (\{\beta_n, \gamma_n, \delta_n\}_{n=2}^{\overline{n}}, \{\zeta_n\}_{n=3}^{\overline{n}}, \tau)$. The parametric versions of the estimators described in Section B.3 were constructed as described there, replacing the nonparametric estimators with their parametric counterparts.

As discussed in the text, the parametric model is used as a reference in our choice of bandwidth by focusing on "error" measures in estimation with respect to it. Let $B^{(0.99)}$ denote the 99th percentile of B, which is equal to 385 in our data. For a given x and n, consider the following integrated mean squared error measures, all of which are integrated with respect to the empirical distribution in the data (as opposed to analytically):

$$\begin{aligned}
\underline{Q}_{\pi}(x,n) = \widehat{E}_B\big[&\big(\widetilde{\underline{\pi}}_n(B|x;\widetilde{\theta}) - \widehat{\underline{\pi}}_n(B|x)\big)^2 \\
&\cdot \mathbb{1}\{v_0 \le B \le B^{(0.99)}\} \cdot \mathbb{1}\{N=n\}\big], \\
\overline{Q}_{\pi}(x,n) = \widehat{E}_B\big[&\big(\widetilde{\overline{\pi}}_n(B|x;\widetilde{\theta}) - \widehat{\overline{\pi}}_n(B|x)\big)^2 \\
&\cdot \mathbb{1}\{v_0 \le B \le B^{(0.99)}\} \cdot \mathbb{1}\{N=n\}\big], \\
Q_{\pi}^{\mathrm{IPV}}(x,n) = \widehat{E}_B\big[&\big(\widetilde{\pi}_n^{\mathrm{IPV}}(B|x;\widetilde{\theta}) - \widehat{\overline{\pi}}_n^{\mathrm{IPV}}(B|x)\big)^2 \\
&\cdot \mathbb{1}\{v_0 \le B \le B^{(0.99)}\} \cdot \mathbb{1}\{N=n\}\big].
\end{aligned}$$

In each case, v_0 is set equal to appraisal value and is fixed at the corresponding value indicated in x. Next, consider the analogous measures taken unconditional on N:

$$\begin{aligned}
\underline{Q}_{\pi}(x) &= \widehat{E}_B\big[\big(\widetilde{\underline{\pi}}_{\overline{N}}(B|x;\widetilde{\theta}) - \widehat{\underline{\pi}}_{\overline{N}}(B|x)\big)^2 \cdot \mathbb{1}\{v_0 \le B \le B^{(0.99)}\}\big], \\
\overline{Q}_{\pi}(x) &= \widehat{E}_B\big[\big(\widetilde{\overline{\pi}}_{\overline{N}}(B|x;\widetilde{\theta}) - \widehat{\overline{\pi}}_{\overline{N}}(B|x)\big)^2 \cdot \mathbb{1}\{v_0 \le B \le B^{(0.99)}\}\big], \\
Q_{\pi}^{\mathrm{IPV}}(x) &= \widehat{E}_B\big[\big(\widetilde{\pi}_{\overline{N}}^{\mathrm{IPV}}(B|x;\widetilde{\theta}) - \widehat{\pi}_{\overline{N}}^{\mathrm{IPV}}(B|x)\big)^2 \cdot \mathbb{1}\{v_0 \le B \le B^{(0.99)}\}\big].
\end{aligned}$$

Our bandwidths are of the form $h_L = c \cdot \widehat{\sigma}(X) \cdot L^{-\alpha}$, where $\widehat{\sigma}(X)$ is the estimated standard deviation of X and α satisfies the bandwidth convergence restrictions[48] in Assumption 4. This requires $\frac{1}{z+4} < \alpha < \frac{1}{z}$ or $\frac{1}{10} < \alpha < \frac{1}{6}$ for our data. We set $\alpha = \frac{2}{15}$, the midpoint of that range. The choice of value for the constant c is based on the minimization of the various error measures defined above, as we now describe.

Consider the first two empirical analyses in the paper, represented in Figures 3 and 4: expected profits for the "benchmark auction" ($X = X^{(0.50)}$ and

[48] Bandwidths for the counterfactual reserve policy analysis in Section 4.4 must follow the convergence rate conditions in Assumption 5. We will describe the choice of bandwidth for that case below.

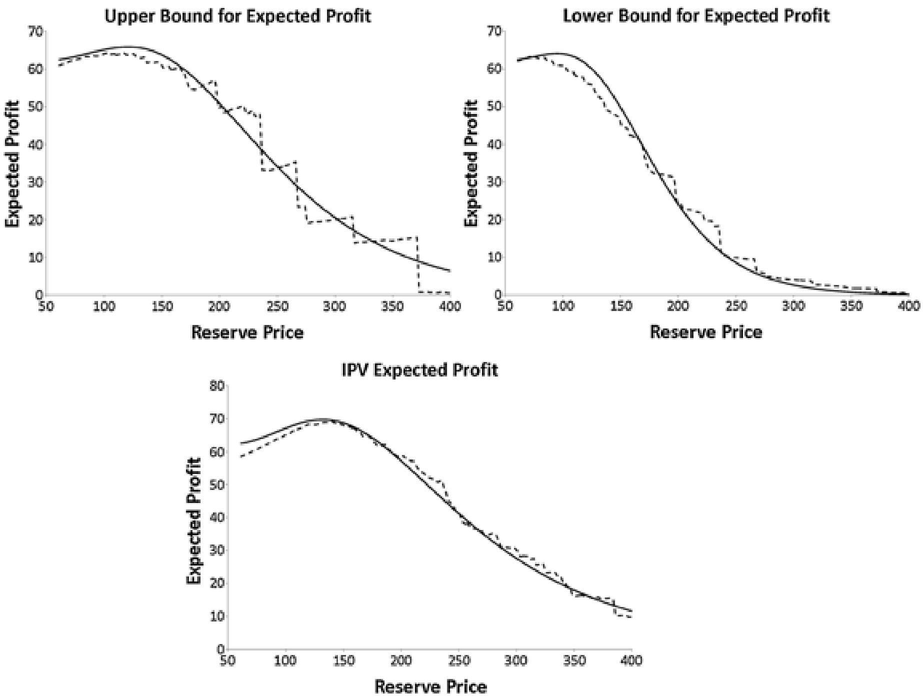

FIGURE 12.—Comparison between the estimated profit curves from the parametric reference model (solid lines) and the nonparametric estimates obtained through our bandwidth selection procedure (dotted lines). Results shown for the benchmark auction, where $x = X^{(0.50)}$ and $v_0 = 60$.

$v_0 = 60$), both conditional and unconditional on N, at various reserve prices. First, consider the latter, expected profit in expectation over N. The error between the nonparametric estimate of $\underline{\pi}_{\overline{N}}(x)$ and the estimate under the reference model, defined above as $\underline{Q}_\pi(x)$, is minimized at a bandwidth of $h_L \approx 0.22 \cdot \widehat{\sigma}(X)$. The error in $\widehat{\overline{\pi}}_{\overline{N}}(x)$ relative to the reference model, $\overline{Q}_\pi(x)$, is minimized at $h_L \approx 0.20 \cdot \widehat{\sigma}(X)$. The error in $\widehat{\pi}^{\mathrm{IPV}}_{\overline{N}}$, Q^{IPV}_π, is minimized at $h_L \approx 0.18 \cdot \widehat{\sigma}(X)$. Figure 12 shows the estimated profit curves that result from these bandwidths (the dotted lines), alongside the parametric estimates from the reference model (the solid lines). The results were very similar at other values of v_0 (the analysis considered in Figure 5). As for expected profit conditional on N (the profit functions illustrated in Figure 3), for $n = 2, \dots, 9$, the measures $\underline{Q}_\pi(x, n)$, $\overline{Q}_\pi(x, n)$, and $Q^{\mathrm{IPV}}_\pi(x, n)$ are all minimized at bandwidths between $0.18 \cdot \widehat{\sigma}(X)$ and $0.26 \cdot \widehat{\sigma}(X)$. Based on all of this, we chose to use bandwidths of $h_L = 0.22 \cdot \widehat{\sigma}(X)$ throughout the individual auction-level analysis (Section 4.3).

Next, we discuss the bandwidths used in the reserve policy counterfactual analysis in Section 4.4. As before, our bandwidth is of the form $h_L = c \cdot \widehat{\sigma}(X) \cdot L^{-\alpha}$, where the rate α is now chosen to satisfy Assumption 5, which requires $\frac{1}{M+z} < \alpha < \frac{1}{2z}$, where M is the order of the kernel used. As we described above, we use $M = 8$, so we need $\frac{1}{14} < \alpha < \frac{1}{12}$. We chose $\alpha = \frac{13}{168}$, the midpoint of this range. Once again, our choice of the constant c was guided by the minimization of the criteria described above. Since our counterfactual analysis requires estimation of the economic measures of interest over a range of x and a range of reserve prices, we focused on the bandwidth that minimized

$$\sum_{x \in \mathcal{I}} \big[\underline{Q}_\pi(x) + \overline{Q}_\pi(x) + Q_\pi^{\mathrm{IPV}}(x)\big],$$

where

$$\mathcal{I} = \big\{X^{(0.25)}, X^{(0.30)}, X^{(0.35)}, \dots, X^{(0.65)}, X^{(0.70)}, X^{(0.75)}\big\}.$$

(Recall that $X^{(\tau)}$ represents the τth percentile of those covariates positively correlated with transaction price and the $(1-\tau)$th percentile of those negatively correlated with transaction price.) In taking this sum, we divided $\underline{Q}_\pi(x)$, $\overline{Q}_\pi(x)$, and $Q_\pi^{\mathrm{IPV}}(x)$ by $\widehat{E}_B[\widetilde{\underline{\pi}}_{\overline{N}}(B|x;\widetilde{\theta})]$, $\widehat{E}_B[\widetilde{\overline{\pi}}_{\overline{N}}(B|x;\widetilde{\theta})]$, and $\widehat{E}_B[\widetilde{\pi}^{\mathrm{IPV}}_{\overline{N}}(B|x;\widetilde{\theta})]$, respectively, as a scale normalization. The criterion function was minimized approximately at $h_L = 0.44 \cdot \widehat{\sigma}(X)$ (see Figure 13), which is, therefore, the bandwidth we used throughout our reserve price policy analysis. Note that this

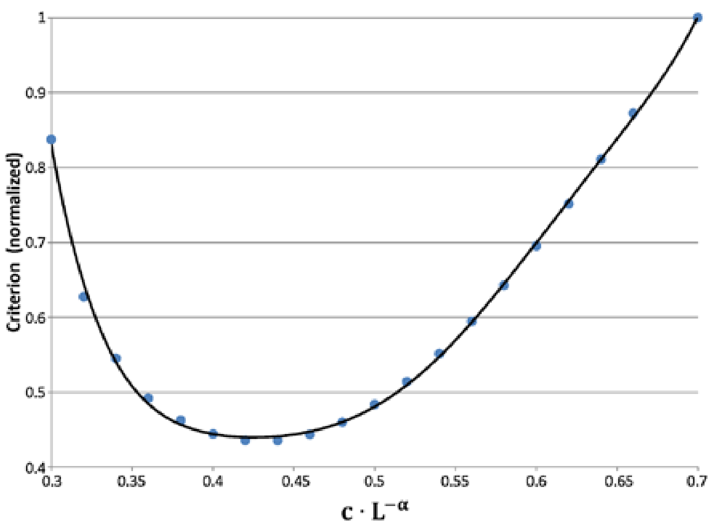

FIGURE 13.—Criterion function for bandwidth selection in reserve policy analysis. Bandwidth is expressed as $h_L = c \cdot \widehat{\sigma}(X) \cdot L^{-\alpha}$.

bound is twice as large as the one used in our auction-level analysis. This result is not surprising since, for a given bandwidth selection criterion, bias-reducing kernels typically admit larger bandwidths compared to non-bias-reducing kernels. Finally, the bandwidth b_L utilized in (17) was set to equal 10^{-8} at our sample size, which made it negligible.

ADDITIONAL REFERENCES

IMBENS, G., AND C. MANSKI (2004): "Confidence Intervals for Partially Identified Parameters," *Econometrica*, 72, 1845–1857. [4,5]

MENZEL, K., AND P. MORGANTI (2012): "Large Sample Properties for Estimators Based on the Order Statistics Approach in Auctions," Working Paper, New York University. [2]

NOLAN, D., AND D. POLLARD (1987): "U-Processes: Rates of Convergence," *Annals of Statistics*, 15, 780–799. [10]

PAKES, A., AND D. POLLARD (1989): "Simulation and the Asymptotics of Optimization Estimators," *Econometrica*, 57 (5), 1027–1057. [10]

SERFLING, R. (1980): *Approximation Theorems of Mathematical Statistics*. New York: Wiley. [10]

SHERMAN, R. (1994a): "Maximal Inequalities for Degenerate U-Processes With Applications to Optimization Estimators," *Annals of Statistics*, 22, 439–459. [10]

——— (1994b): "U-Processes in the Analysis of a Generalized Semiparametric Regression Estimator," *Econometric Theory*, 30, 372–395. [10]

STOYE, J. (2009): "More on Confidence Intervals for Partially Identified Parameters," *Econometrica*, 77, 1299–1315. [4,5,12]

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