Supplement to

TELL ME SOMETHING I DON'T ALREADY KNOW: LEARNING IN LOW AND HIGH-INFLATION SETTINGS

Michael Weber
U. Chicago and NBER

Bernardo Candia
UC Berkeley

Hassan Afrouzi
Columbia University and NBER

Tiziano Ropele
Bank of Italy

Rodrigo Lluberas
Universidad ORT Uruguay

Serafin Frache
Universidad de Montevideo

Brent Meyer
Federal Reserve Bank of Atlanta

Saten Kumar
Auckland University of Technology

Yuriy Gorodnichenko
UC Berkeley and NBER

Dimitris Georgarakos
European Central Bank and CEPR

Olivier Coibion
UT Austin and NBER

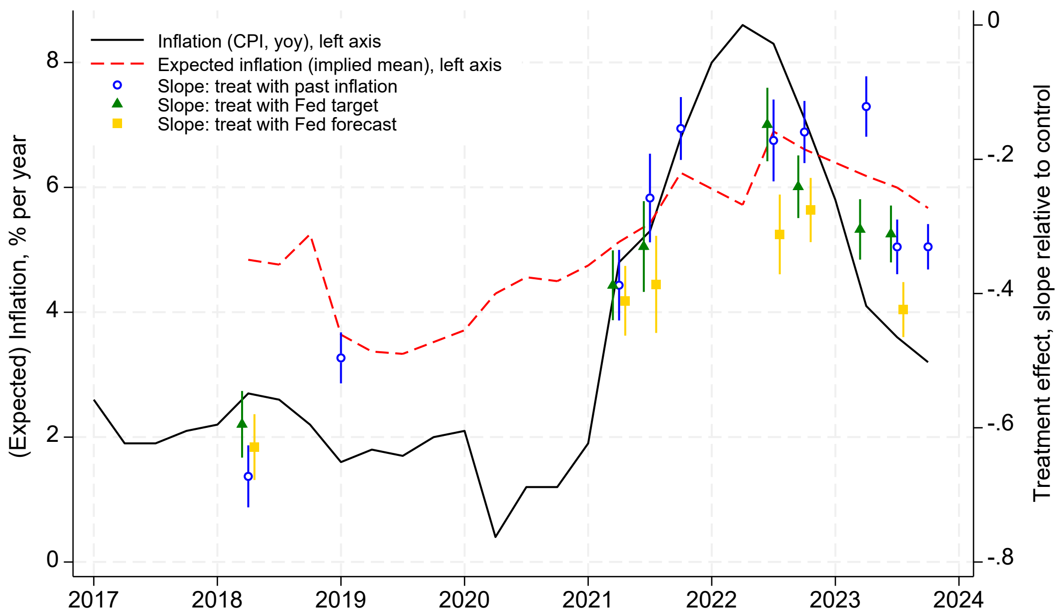
Geoff Kenny
European Central Bank

Jorge Ponce
Central Bank of Uruguay

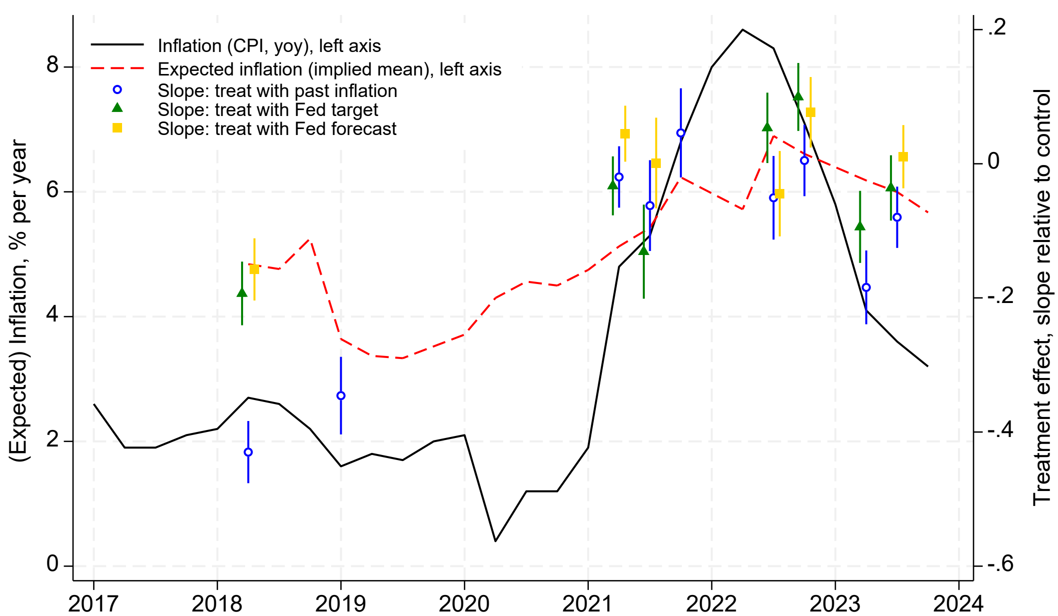
Appendix A. Additional Tables and Figures

Appendix Figure A.1: Not controlling for slope of control group for U.S. households

Panel A: Instantaneous Treatment Effects



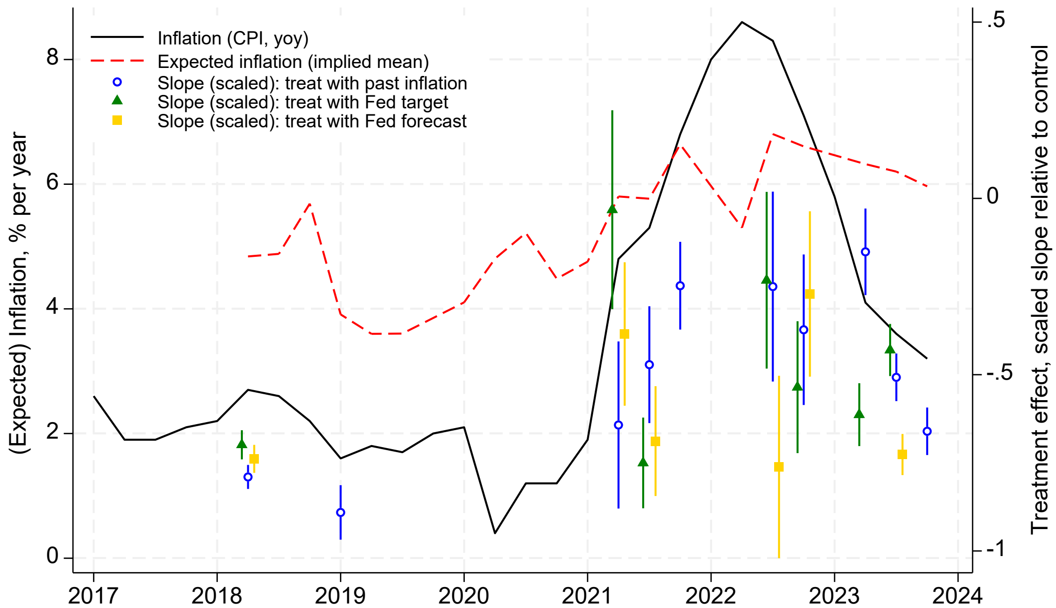
Panel B: Treatment Effects after 3 Months



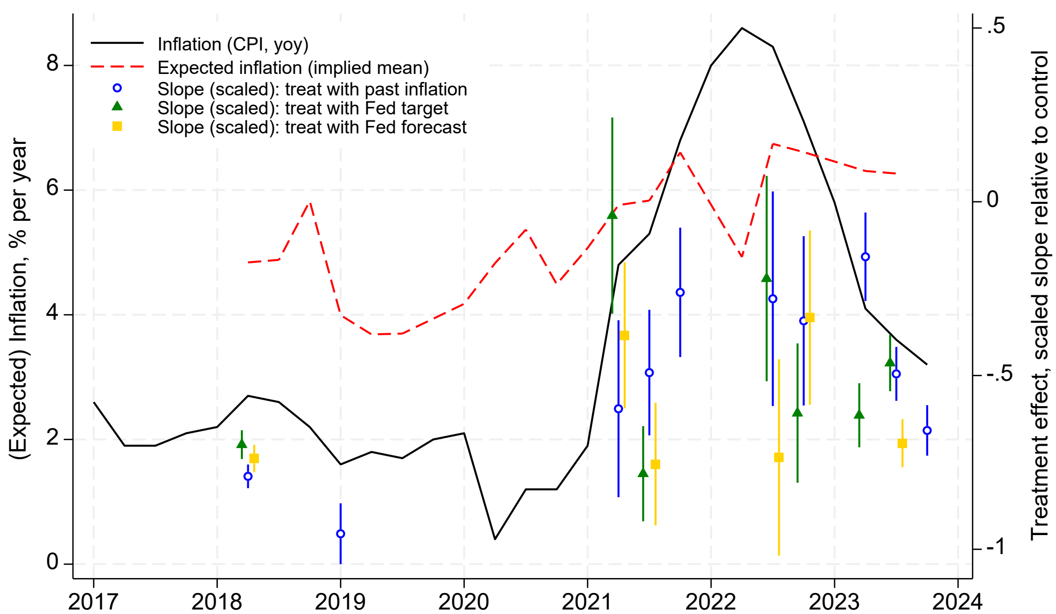
Notes: Each panel shows the time series of actual inflation and average expected inflation as well as the slopes (γ in specification (1) for Panel A and γ in specification (1) with posteriors measured three months later for Panel B) for various treatments across RCTs. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors.

Appendix Figure A.2: Panel Conditioning

Panel A: Subsample of households not participating in previous wave



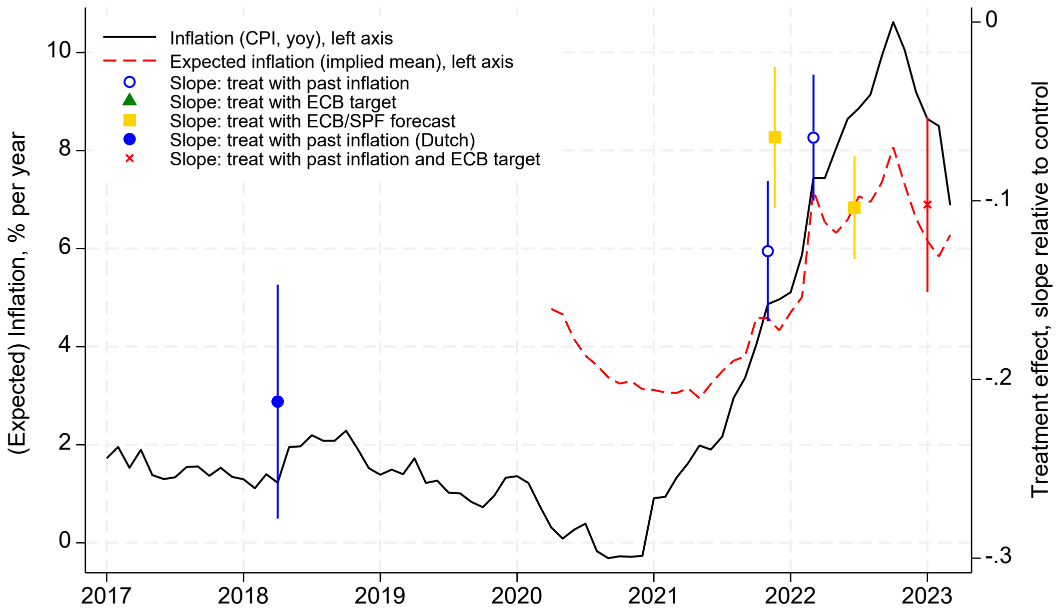
Panel B: Subsample of households not participating in previous 2 waves



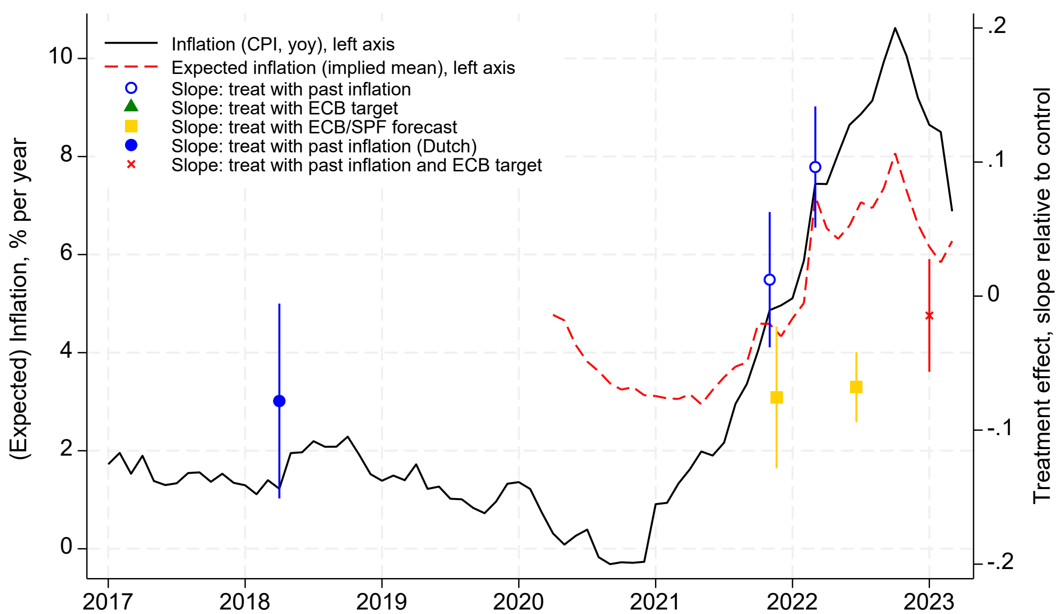
Notes: Each panel shows the time series of actual inflation and average expected inflation as well as the scaled slopes (γ/β in specification (1) for various treatments across RCTs. Panel A shows results when we restrict the sample to households who have not participated in the last wave while Panel B shows results when we restrict the sample to households who have not participated in the last two waves. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors.

Appendix Figure A.3: Not controlling for slope of control group for euro area households

Panel A: Instantaneous Treatment Effect

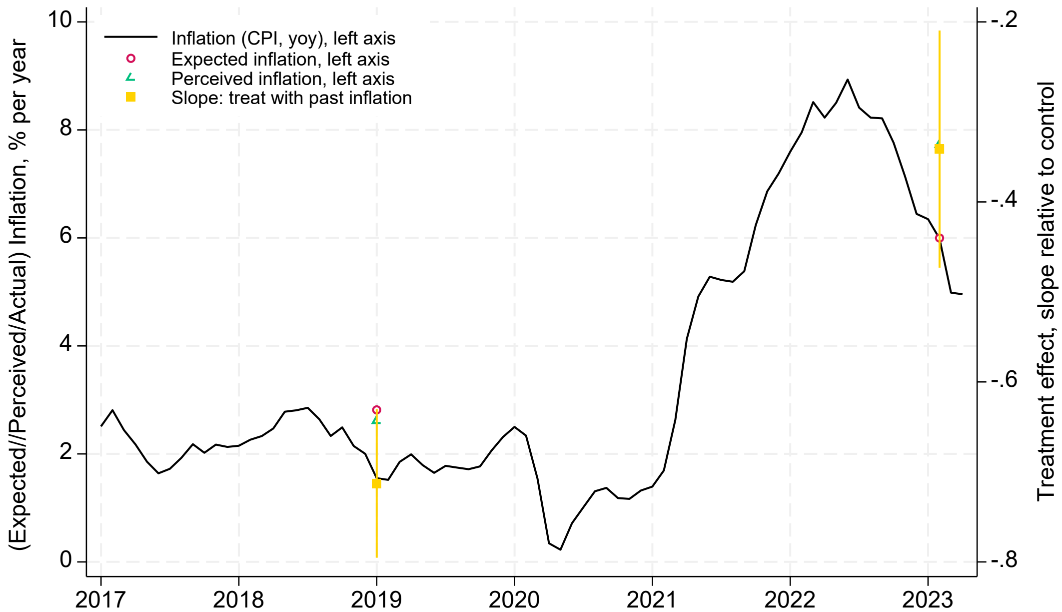


Panel B: Treatment Effect after 3 Months



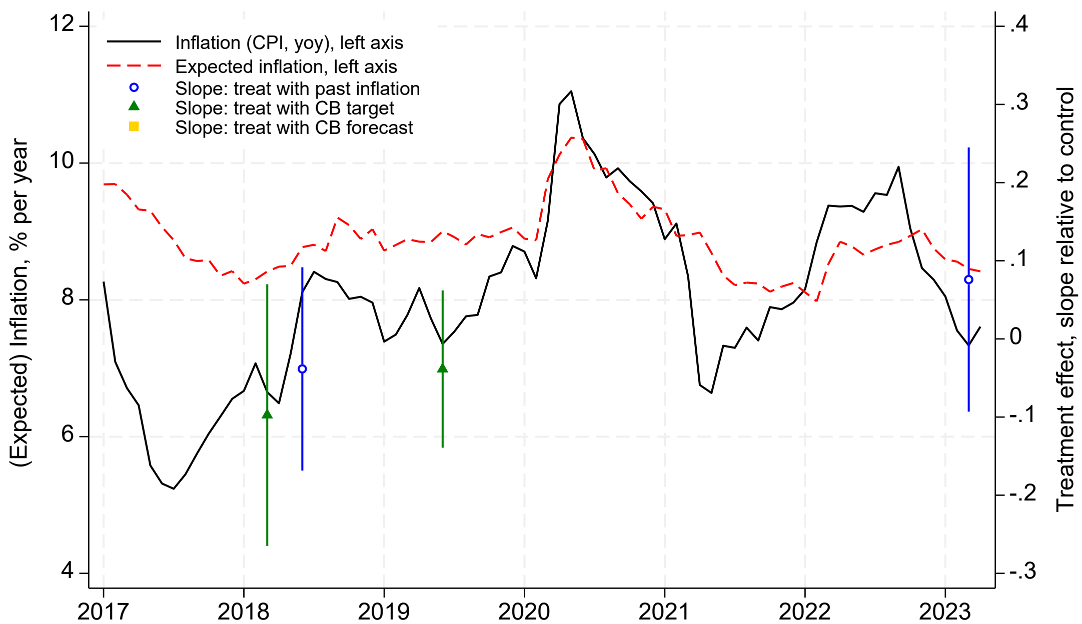
Notes: Each panel shows the time series of actual inflation and average expected inflation as well as the slopes (γ in specification (1) for Panel A and γ in specification (1) with posteriors measured 3 months later for Panel B) for various treatments across RCTs. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors. The last observation in Panel B is one month after the corresponding treatment.

Appendix Figure A.4: Not controlling for slope of control group for U.S. firms



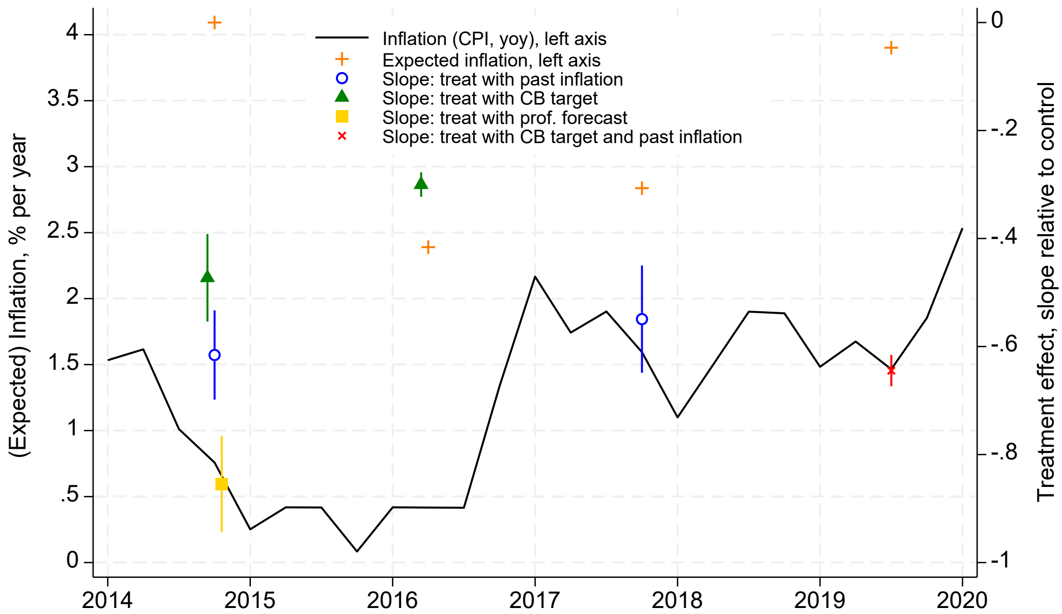
Notes: The figure shows the time series of actual inflation as well as the slopes (γ in specification (1)) for various treatments across RCTs. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors. The figure also reports average expectation and perceived inflation at the time when RCTs were conducted.

Appendix Figure A.5: Not controlling for slope of control group for Uruguayan firms



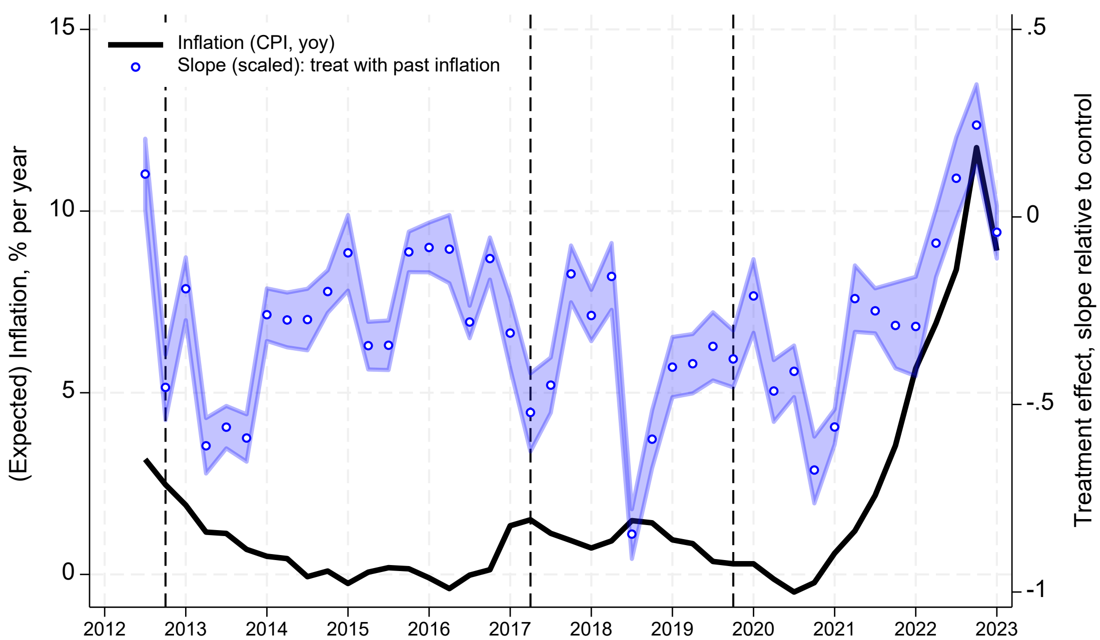
Notes: The figure shows the time series of actual inflation and average expected inflation as well as the slopes (γ in specification (1)) for various treatments across RCTs. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors.

Appendix Figure A.6: Not controlling for slope of control group for New Zealand firms



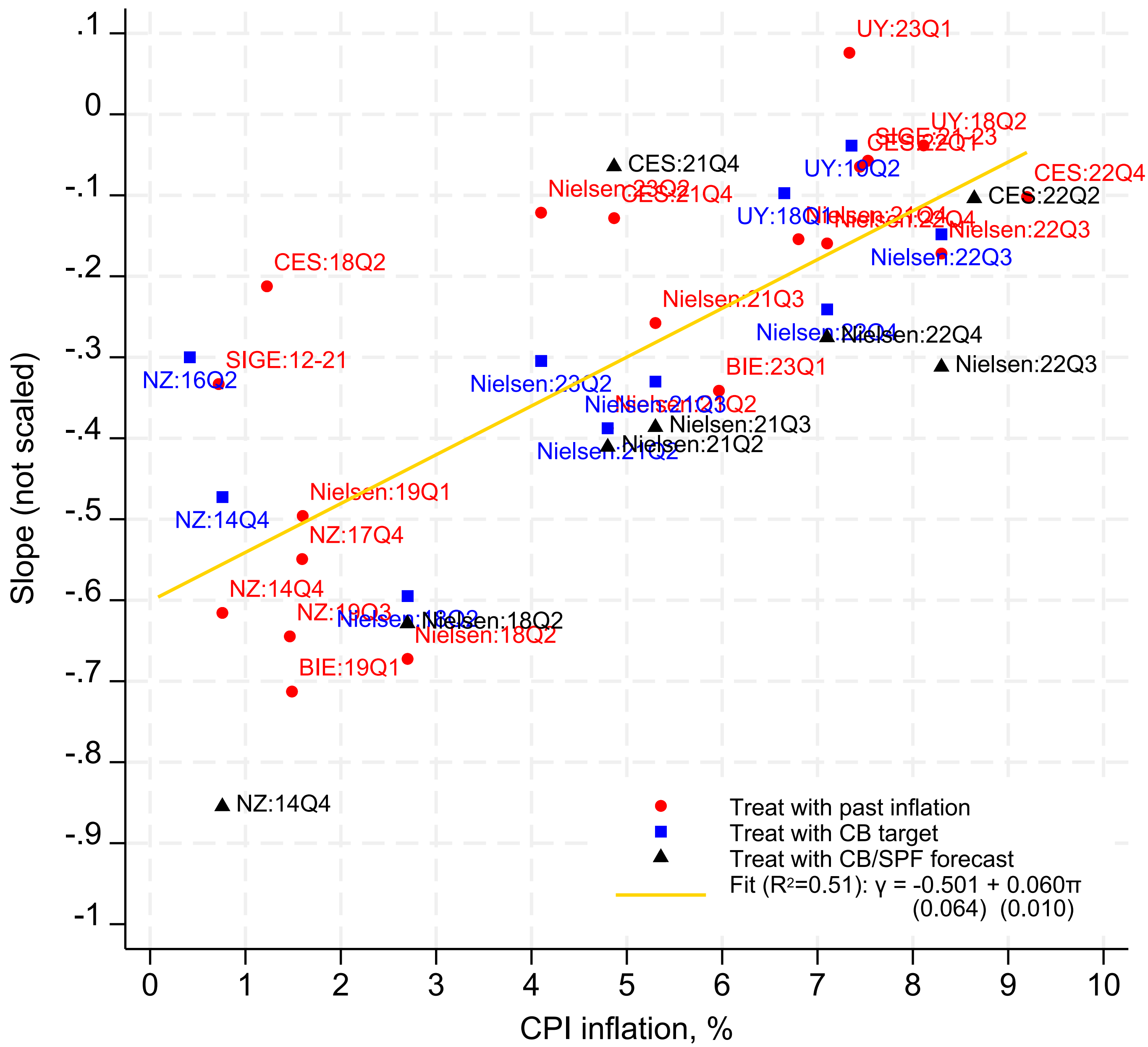
Notes: The figure shows the time series of actual inflation as well as the slopes (γ in specification (1)) for various treatments across RCTs. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors.

Appendix Figure A.7: Not controlling for slope of control group for Italian firms



Notes: The figure shows the time series of actual inflation as well as the slopes (γ in specification (1)) for various treatments across RCTs. The shaded area shows the 90% confidence intervals based on heteroskedasticity robust standard errors. The dashed vertical lines show times when firms were randomly reshuffled into treatment and control groups.

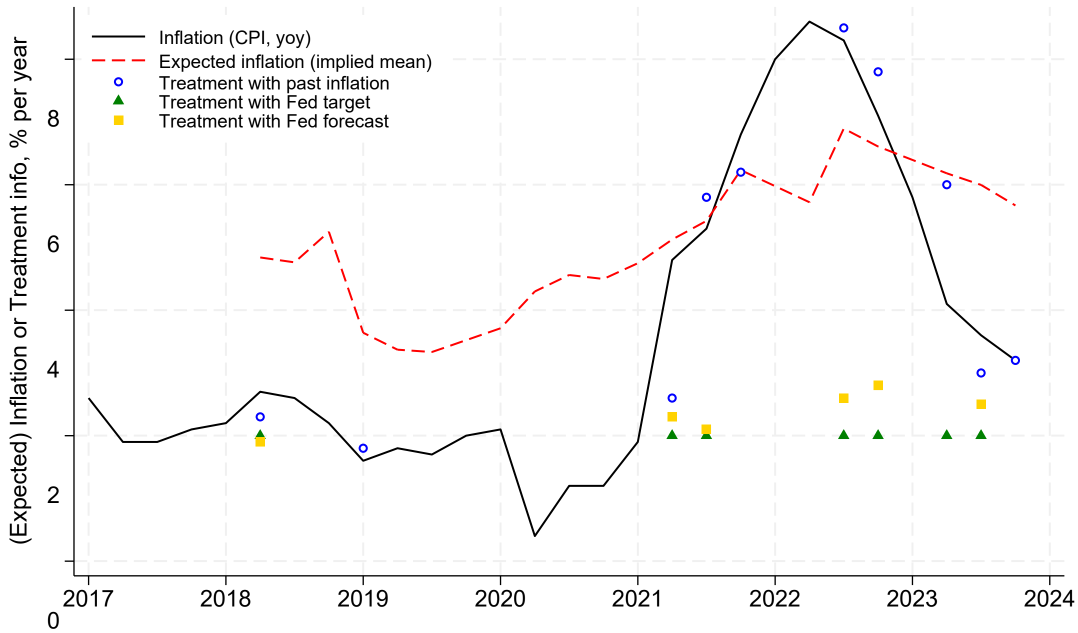
Appendix Figure A.8: Pooling across countries, not controlling for slope of control group



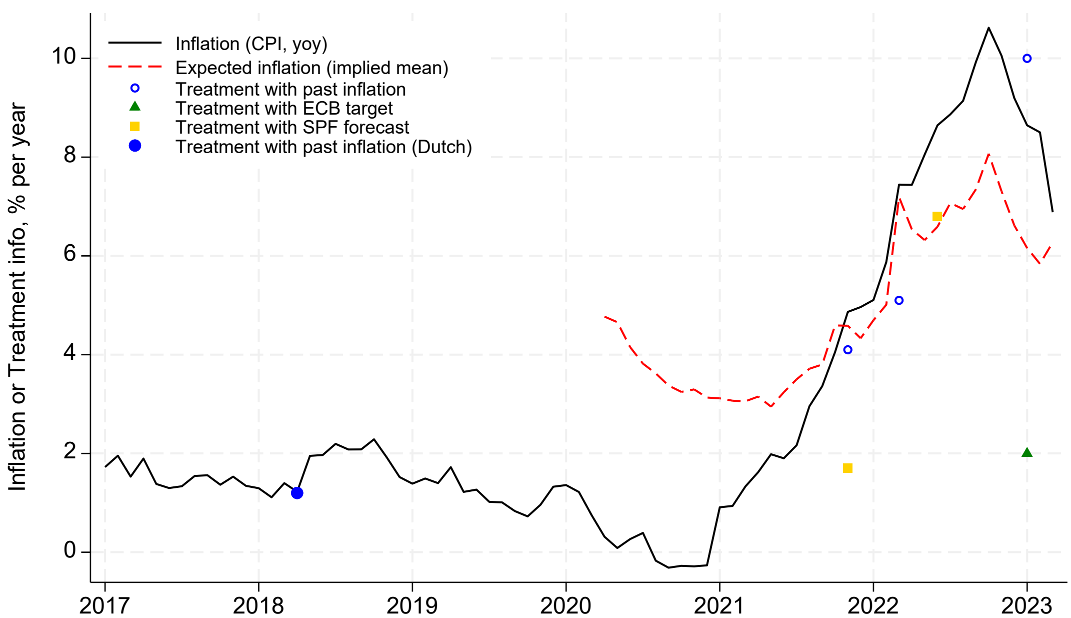
Notes: The figure plots the estimated slopes (γ in specifications (1)) vs. the annual rate of inflation at the time of the corresponding survey. The format of labels is “survey/country: year-quarter”. Surveys/countries are coded as follows: NZ is for New Zealand, CES is for the European Central Bank’s Consumer Expectations Survey (except from CES:18Q2 that is from Dutch National Bank’s household survey), SIGE is for the Bank of Italy’s Survey on Inflation and Growth Expectations, UY is for Uruguay, Nielsen is for the Nielsen Homescan Panel, BIE is the Atlanta Fed’s Business Inflation Expectations survey. Inflation is for the year-quarter when the corresponding survey/RCT was conducted. Data for SIGE are pooled into two “periods”: 2012Q3-2021Q3 and 2021Q4-2023Q1.

Appendix Figure A.9: Information treatments

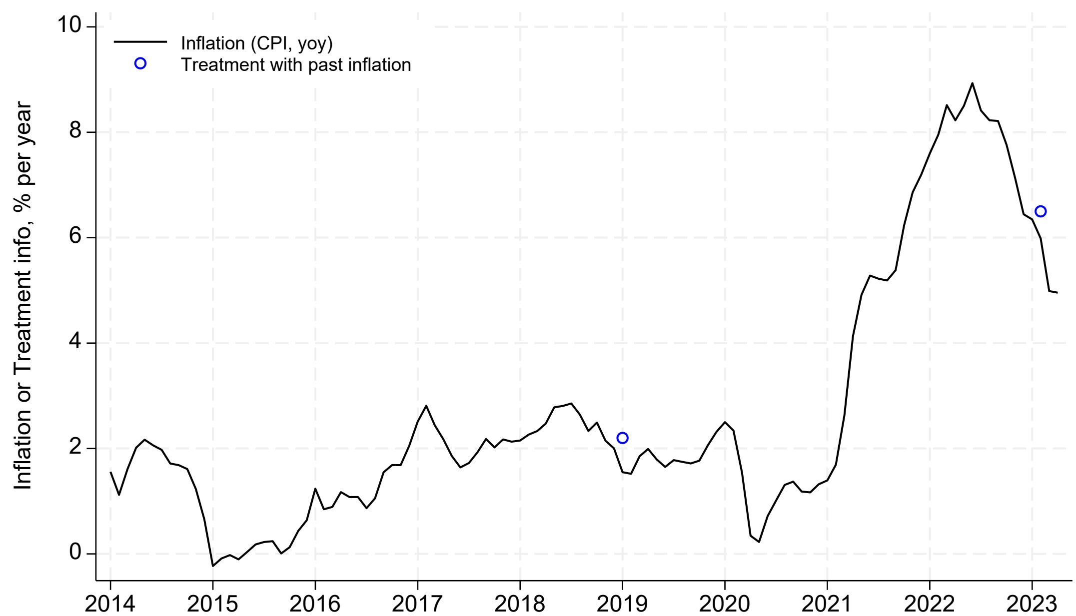
Panel A. Nielsen Homescan Panel



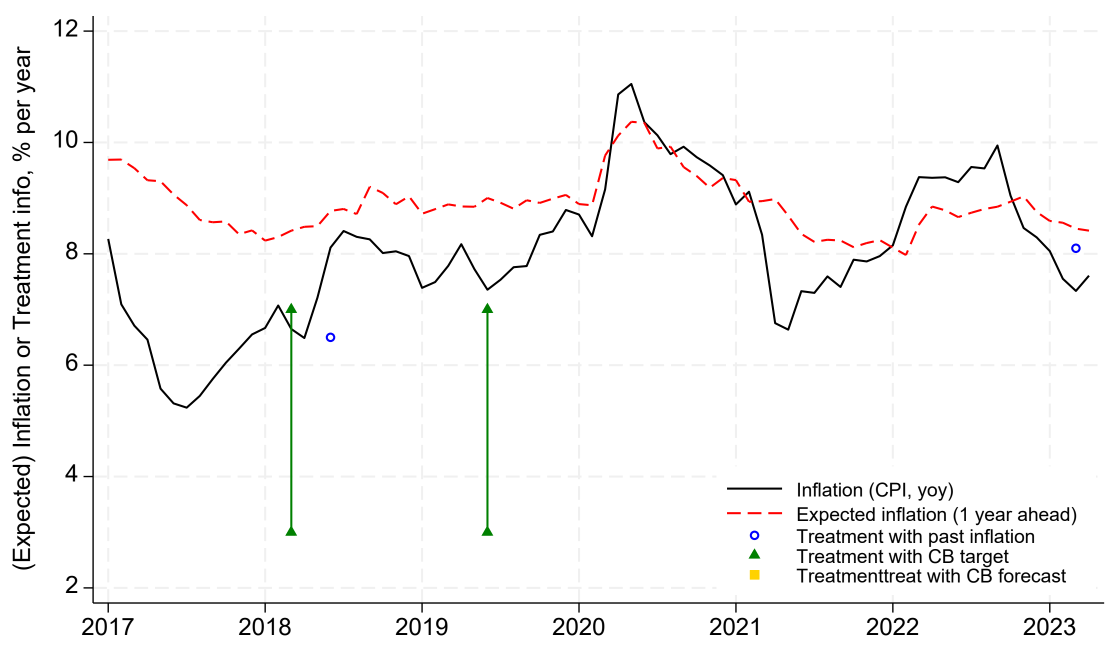
Panel B. ECB's Consumer Expectations Survey (CES)



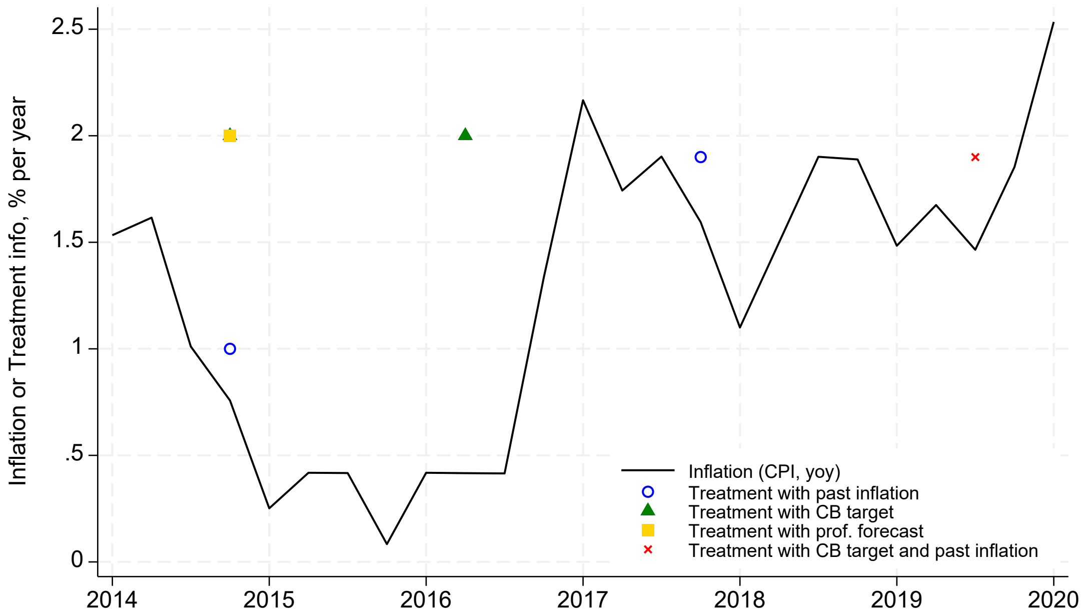
Panel C. Atlanta Fed's Business Inflation Expectations (BIE) Survey



Panel D. Uruguay's Survey of Firms' Expectations



Panel E. New Zealand's Surveys of Firms

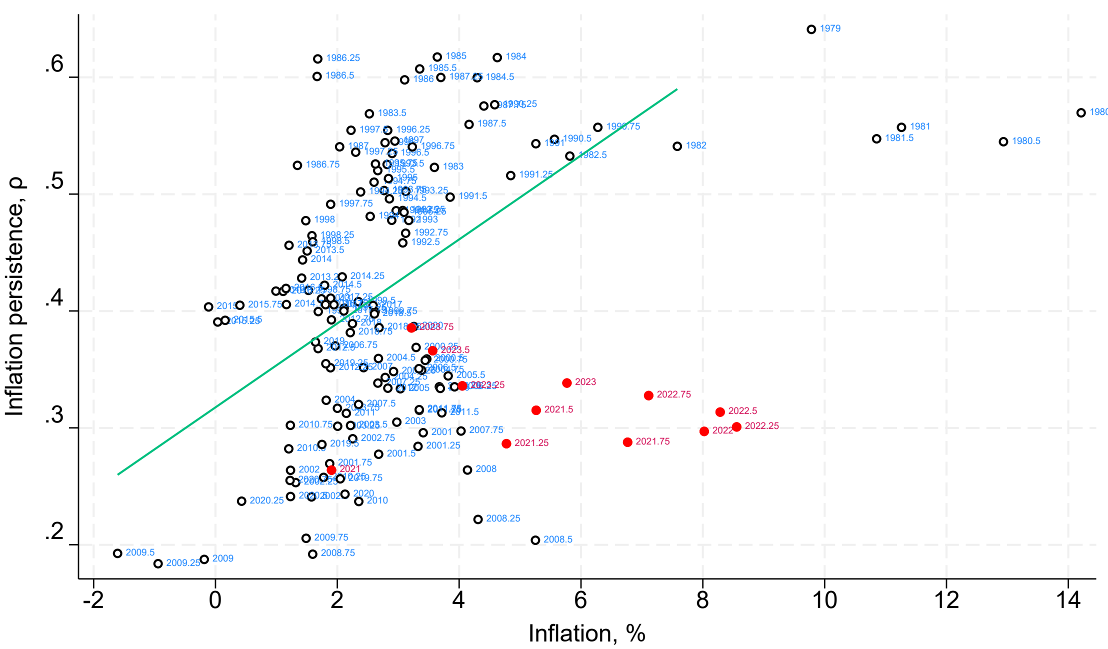


Notes: The figures report statistics that were reported in information treatments.

Appendix Figure A.10: Perceived persistence of inflation

Panel A. Survey of Professional Forecasters

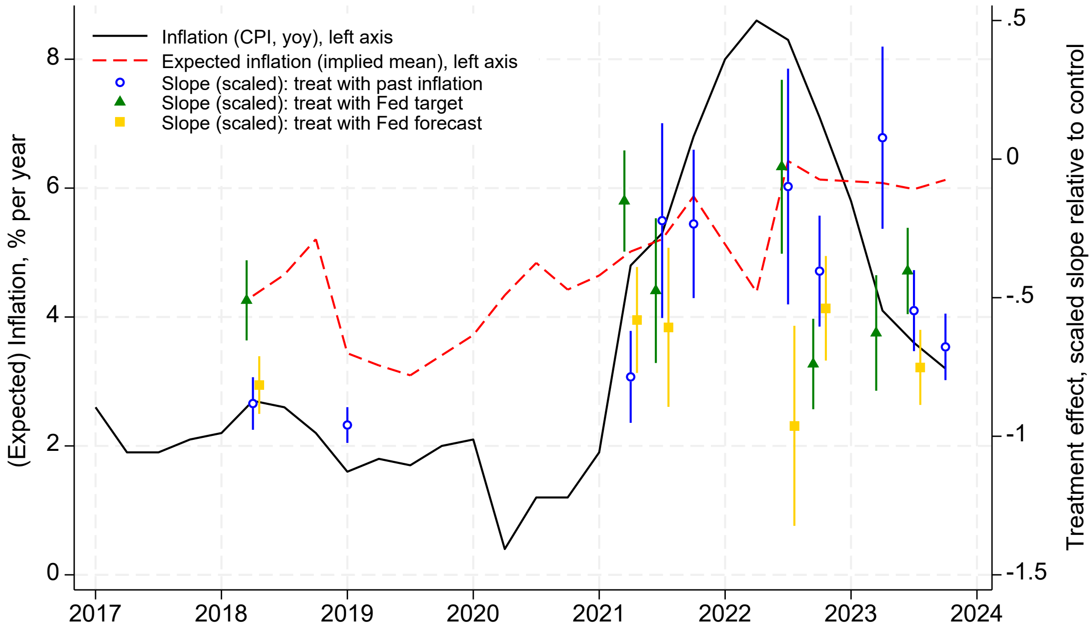
Panel B. Michigan Survey of Consumers



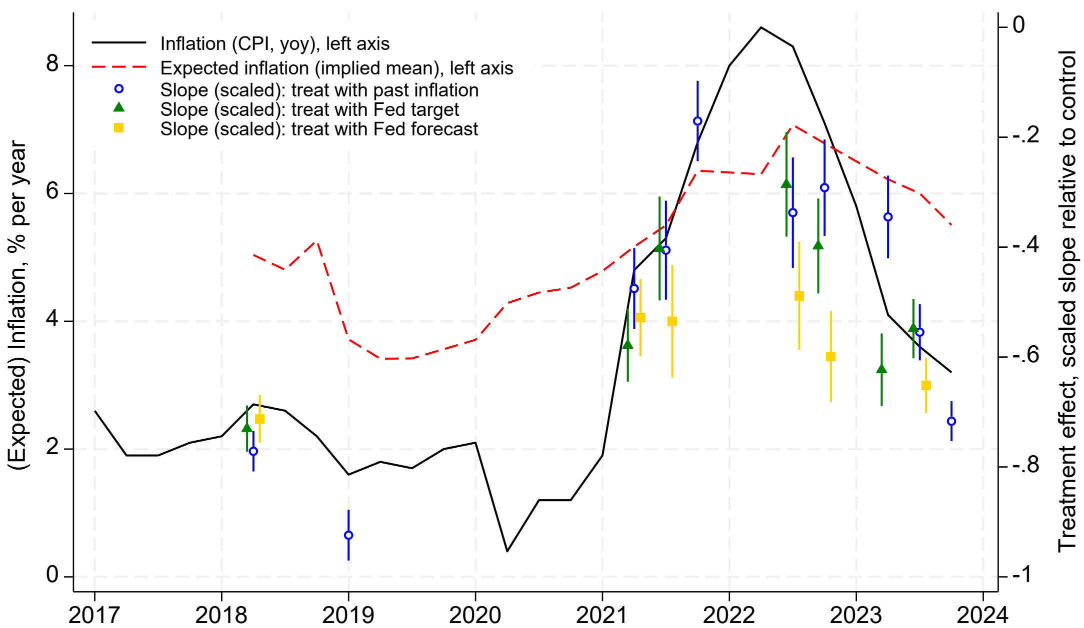
Notes: Following Goldstein and Gorodnichenko (2022), we run the following regression survey wave by survey wave: $F_{i,t}\pi_{t+h} = b_{0,h} + \rho_h \times F_{i,t}\pi_{t+h-1} + error$ where i, t, h index forecasters, time (quarters), and forecast horizons, $F_{i,t}\pi_{t+h}$ is the forecast prepared by forecaster i at time t for period $t+h$. Coefficient $b_{1,h}$ measures the perceived persistence. For professional forecasters we use $h = 4$ (i.e., 4-quarter ahead forecast). For households in the Michigan Survey of Consumers, $F_{i,t}\pi_{t+h}$ is their 5-year-ahead inflation forecast while $F_{i,t}\pi_{t+h-1}$ is their 1-ayear-ahead inflation forecast.

Appendix Figure A.11: Treatment Effects by Age

Panel A. Age<=40



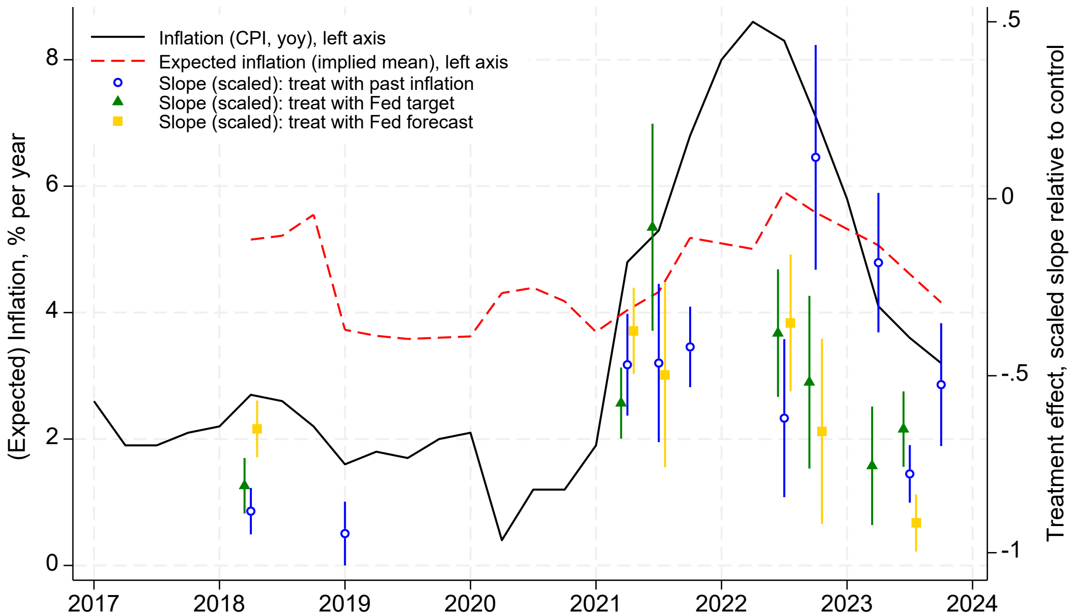
Panel B. Age>40



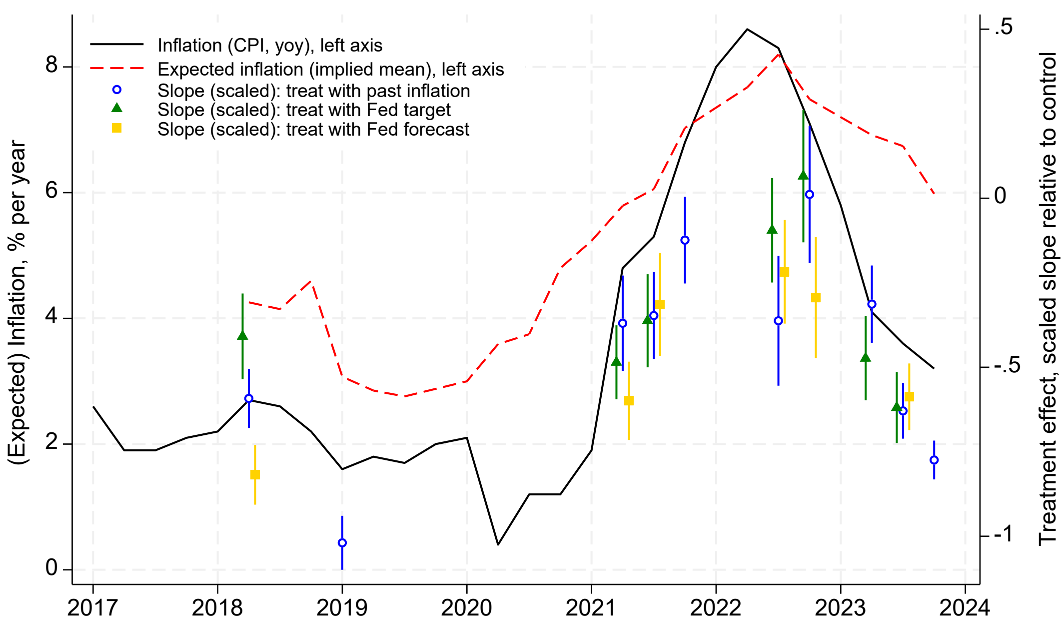
Notes: The figure shows the time series of actual inflation and average expected inflation as well as the scaled slopes (γ/β in specification (1) for various treatments across RCTs by age. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors.

Appendix Figure A.12: Treatment Effects by Political Affiliation

Panel A. Democrats



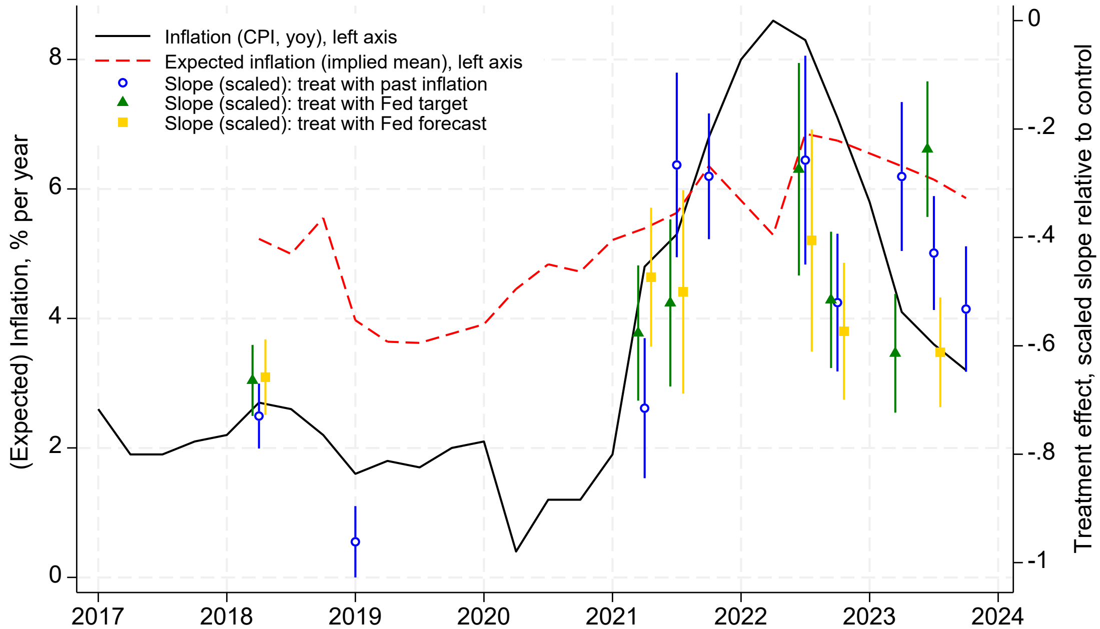
Panel B. Republicans



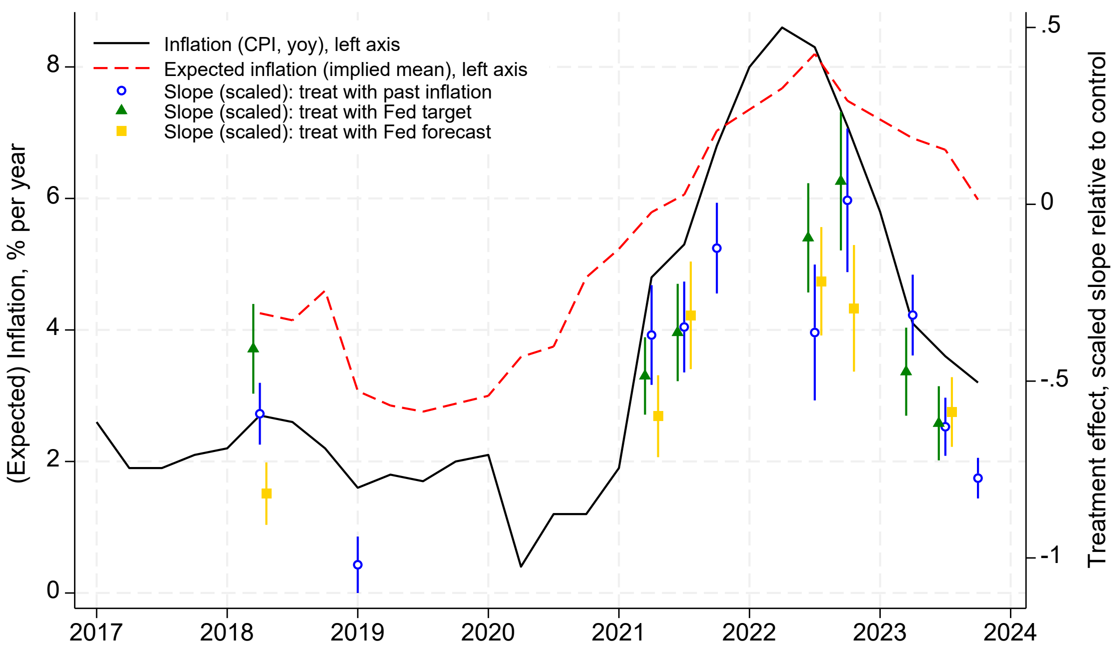
Notes: The figure shows the time series of actual inflation and average expected inflation as well as the scaled slopes (γ/β in specification (1) for various treatments across RCTs by political affiliation. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors.

Appendix Figure A.13: Treatment Effects by Education

Panel A. Assoc. Degree, High school or less



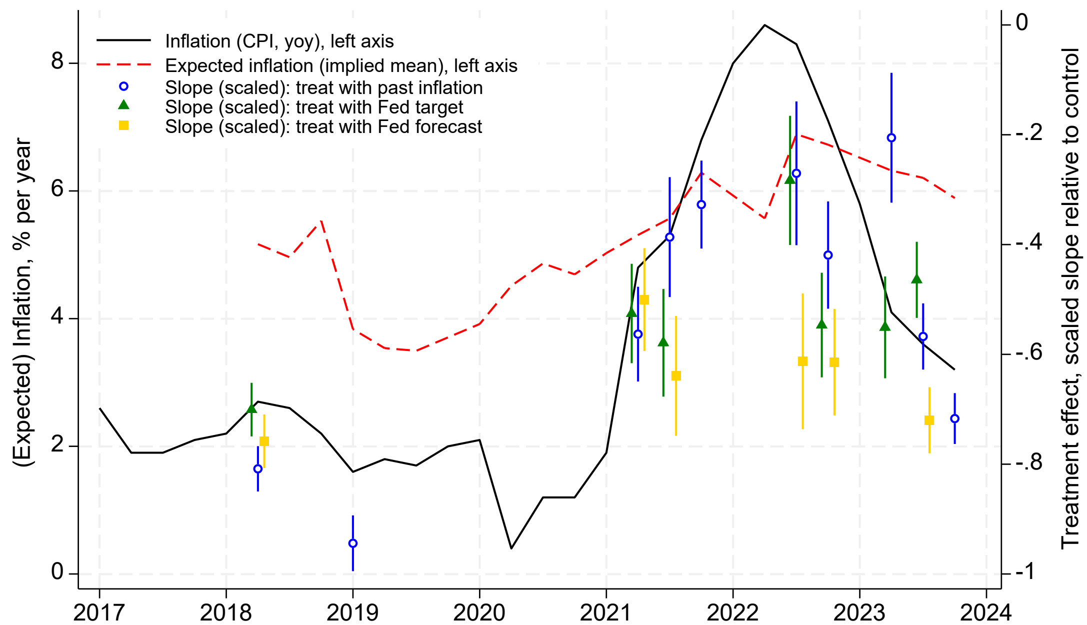
Panel B. College or more



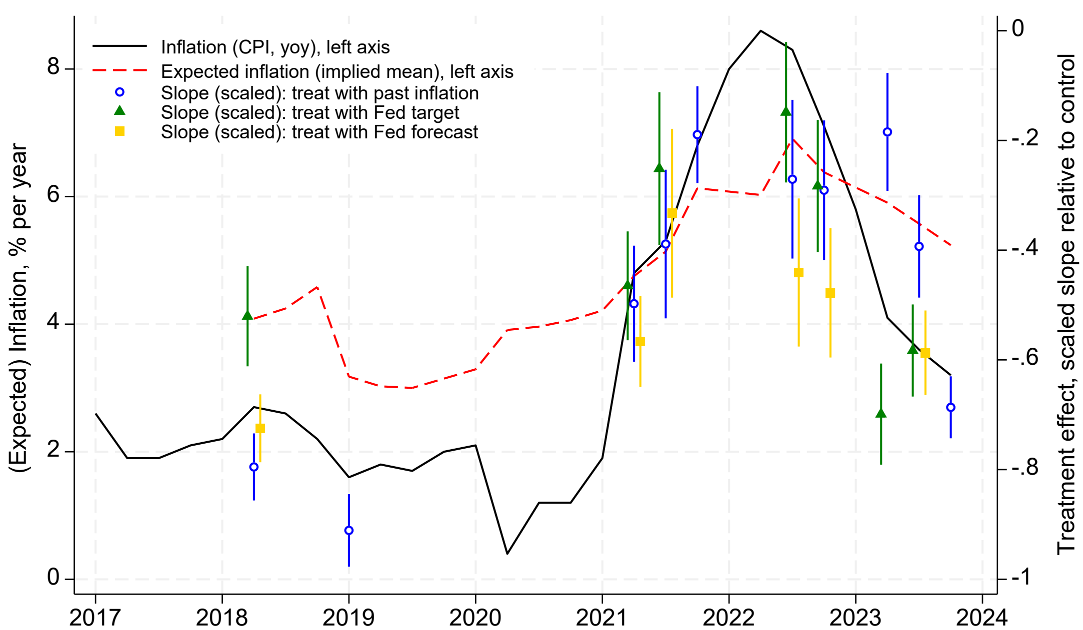
Notes: The figure shows the time series of actual inflation and average expected inflation as well as the scaled slopes (γ/β in specification (1) for various treatments across RCTs by education. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors.

Appendix Figure A.14: Treatment Effects by Gender

Panel A. Female



Panel B. Male



Notes: The figure shows the time series of actual inflation and average expected inflation as well as the scaled slopes (γ/β in specification (1) for various treatments across RCTs by gender. The whiskers show the 90% confidence intervals based on heteroskedasticity robust standard errors.

Appendix Table A.1: Question Formulations in Each Survey

Country	RCT dates	Prior question	Posterior question
United States (Nielsen panel)	2018Q2, 2019Q1, 2021Q2-Q4, 2022Q3-Q4, 2023Q2-Q4	We would like to ask you about the rate of inflation/deflation (Note: inflation is the percentage rise in overall prices in the economy, most commonly measured by the CPI and deflation corresponds to when prices are falling). In this question, you will be asked about the prob. (percent chance) of something happening. The percent chance must be a number between 0 and 100 and the sum of your answers must add up to 100. What do you think is the percent chance that, over the next 12 months the rate of inflation will be (-∞,-12][-12,-8][-8,-4][-4,-2][-2,0][0,2][2,4][4,8][8,12] [12, ∞)	What do you think the inflation rate (as measured by the Consumer Price Index) is going to change over the next 12 months? Please provide an answer as a percentage change from current prices ____% If you think there was inflation, please enter a positive number. If you think there was deflation, please enter a negative number. If you think there was neither inflation nor deflation, please enter zero.
Euro area	2021Q4, 2022Q1-Q2, 2022Q4	How much higher/ lower do you think prices in general will be **12 months from now** in the country you currently live in? Please give your best guess of the change in percentage terms. You can provide a number up to one decimal place. *Show 2 boxes with a decimal point in between.* For prob-bins version question see below [*]	**[2021Q4, 2022Q1-Q2]** How much higher or lower do you think prices in general will be **12 months from now** in the country you currently live in? *Please give your best guess of the change in percentage terms. Use the slider below to indicate the increase or decrease in prices in percentage terms. If you think prices will decrease rather than increase you can provide a negative percentage* **[2022Q4]** Now we would like you to think about what inflation or deflation (the opposite of inflation) in the country you currently live in is likely to be in 12 months from now. We realise that this question may take a little more effort. Below you see 10 possible ways in which inflation or deflation could happen. Please distribute 100 points among them, to indicate how likely you think it is that inflation or deflation will be in that range. The sum of the points you allocate should total 100. The rate of inflation/ deflation will be: (-∞,-12][-12,-8][-8,-4][-4,-2][-2,0][0,2][2,4][4,8][8,12] [12, ∞)

(continued on the next page)

Country	RCT dates	Prior question	Posterior question
Netherlands	2018Q2	How much do you think consumer prices in general will change in the next twelve months in the Netherlands? Please allocate 100 points indicating how likely the listed changes are. (Note that the probabilities in the column should sum to 100) (-∞,-8][-8,-4][-4,-2][-2,-1][-1,1][1,2][2,4][4,8][8, ∞)	How much do you think consumer prices in general will change in the next twelve months in the Netherlands? Please provide an answer in percentage terms. If you think consumer prices on average will decrease, please fill a negative percentage (inset aa minus sign for the number). If you think consumer prices on average will increase, please fill in a positive percentage. If you think consumer prices on average will not change, please fill in 0 (zero).
United States (Atlanta Fed)	2019Q1, 2023Q1	What do you think has been the aggregate rate of inflation in the US over the last 12 months, as measured by the consumer price index? Please prove an answer in percentage terms.	What do you think will be the aggregate inflation rate as measured by the consumer price index, over the next 12 months? Please provide an answer in percentage terms.
Uruguay	2018Q1-Q2, 2019Q2 2023Q1	What do you think the variation in CPI will be in 12 months from now?	What do you think the variation in CPI will be in 12 months from now? (subsequent wave)
New Zealand	2014Q4, 2016Q2, 2018Q1, 2019Q3	Please assign probabilities (from 0-100) to the following ranges of overall price changes in the economy over the next 12 months for New Zealand: (Note that the probabilities in the column should sum to 100). Percentage price changes in 12 months. (-∞,0][0,2][2,4][4,6][6,8][8,10][10,15][15,25][25,∞) (2014Q4) (-∞,-25][-25,-15][-15,-10][-10,-8][-8,-6][-6,-4][-4,-2][-2,0][0,2][2,4][4,6][6,8][8,10][10,15][15,25][25, ∞) (2016Q2, 2018Q1, 2019Q3)	By how much do you think overall prices in the economy will change during the next twelve months? Please provide a precise quantitative answer in percentage terms (2014Q4, 2018Q1, 2019Q3) During the next twelve months, by how much do you think prices will change overall in the economy? Please provide an answer in percentage terms.(2016Q2)
Italy	2012Q3-22Q4	What do you think consumer price inflation in Italy measured by the 12-months change in the harmonized index of consumer prices will be?	What do you think consumer price inflation in Italy measured by the 12-months change in the harmonized index of consumer prices will be? (subsequent wave)

Notes: The table reports actual questions used in each survey.

APPENDIX B: PROOFS

Proof of Proposition 1. For any information set S_i, recall that

$$\begin{aligned} V(S_i) \quad &\equiv \max_{C_0, C_1(\pi_1)} \mathbb{E}[u(C_0) + u(C_1(\pi_1))|S_i] \\ s.t. \quad & C_0 + M \leq W \\ & C_1(\pi_1) \leq \frac{M}{1+\pi_1} \end{aligned}$$

Noting that $u(C) = \frac{C^{1-\psi}-1}{1-\psi}$ is strictly increasing in consumption, we know that the constraints will bind under optimal consumption choice. Thus, we can substitute them in the objective to obtain:

$$V(S_i) = \max_{C_0} \mathbb{E}\left[u(C_0) + u\left(\frac{W - C_0}{1+\pi_1}\right)|S_i\right]$$

Let us define $c_0 \equiv \ln(C_0)$ as log-consumption at period 0, and $v(c_0, \pi_1) \equiv u(e^{c_0}) + u(\frac{W-e^{c_0}}{1+\pi_1})$ as the life time utility of the household for given values of $c_0$and π_1. In particular, consider the consumption value that maximizes the non-stochastic version of this problem with $\pi_1 = 0$:

$$c_0^* \equiv \arg\max_{c_0} u(e^{c_0}) + u(W - c_0) \Longrightarrow c_0^* = \ln(W / 2)$$

i.e., the household perfectly smoothes her consumption in the absence of any shocks. We can then, for any pair of (c_0, π_1), do a quadratic approximation of $v(c_0, \pi_1)$ around the non-stochastic point $(c_0^*, 0)$:

$$\begin{aligned} & v(c_0, \pi) - v(c_0^*, 0) \\ \approx & \frac{\partial v(c_0^*, 0)}{\partial c_0}(c_0 - c_0^*) + \frac{1}{2}\frac{\partial^2 v(c_0^*, 0)}{\partial c_0^{*2}}(c_0 - c_0^*)^2 + \frac{\partial^2 v(c_0^*, 0)}{\partial c_0^* \partial \pi_1}(c_0 - c_0^*)\pi_1 \\ & + g(\pi_1). \end{aligned}$$

where we have only kept terms of up to second order and $g(\pi_1) = \frac{\partial v(c_0^*,0)}{\partial \pi_1}\pi_1 + \frac{1}{2}\frac{\partial^2 v(c_0^*,0)}{\partial \pi_1^2}\pi_1^2$ denotes all such terms that are independent of c_0. We now note that $\frac{\partial v(c_0^*,0)}{\partial c_0^*} = 0$ by definition of c_0^* and observe that

$$\begin{aligned} \frac{\partial v(c_0^*, 0)}{\partial c_0} &= e^{c_0^*}\big(u'(e^{c_0^*}) - u'(W - e^{c_0^*})\big) = 0 \\ \frac{\partial^2 v(c_0^*, 0)}{\partial c_0^{*2}} &= e^{c_0^*}\big(u''(e^{c_0^*})e^{c_0^*} + e^{c_0^*}u''(W - e^{c_0^*})\big) \\ &= -2\psi e^{c_0^*}u'(e^{c_0^*}) \\ \frac{\partial^2 v(c_0^*, 0)}{\partial c_0^* \partial \pi_1} &= e^{c_0^*}\big(u'(W - e^{c_0^*}) + u''(W - e^{c_0^*})(W - e^{c_0^*})\big) \\ &= e^{c_0^*}u'(e^{c_0^*})(1 - \psi) \end{aligned}$$

Therefore, the household's utility given (c_0, π_1) deviates from $v(c_0^*, 0)$, up to second order and in consumption equivalent terms, according to:

$$\frac{v(c_0,\pi_1)-v(c_0^*,0)}{e^{c_0^*}u'(e^{c_0^*})}\approx -\psi(c_0-c_0^*)^2+(1-\psi)(c_0-c_0^*)\pi_1+\tilde{g}(\pi_1),$$

where $\tilde{g}(\pi_1)\equiv\frac{g(\pi_1)}{e^{c_0^*}u'\left(e^{c_0^*}\right)}$. It follows that for any information S_i, if the household is maximizing this quadratic approximation,

$$\begin{aligned}
c_0(S_i) &\equiv \arg\max_{c_0}\ \mathbb{E}[-\psi(c_0-c_0^*)^2+(1-\psi)(c_0-c_0^*)\pi_1|S_i]+\mathbb{E}[\tilde{g}(\pi_1)|S_i]\\
\Longrightarrow \quad & c_0(S_i)-c_0^*=\frac{1-\psi}{2\psi}\mathbb{E}[\rho\pi+u|S_i]=\frac{1-\psi}{2\psi}\rho\mathbb{E}[\pi|S_i]\\
\Longrightarrow \frac{V(S_i)}{e^{c_0^*}u'(e^{c_0^*})} &\approx \frac{\psi(1-\psi^{-1})^2\rho^2}{4}\mathbb{E}[\pi|S_i]^2+\mathbb{E}[\tilde{g}(\pi_1)|S_i]
\end{aligned}$$

which also implies that the ex-ante expected consumption equivalent value is given by

$$\mathbb{E}_0\left[\frac{V(S_i)}{e^{c_0^*}u'(e^{c_0^*})}\right]\approx\frac{\psi(1-\psi^{-1})^2\rho^2}{4}\mathbb{E}_0[\mathbb{E}[\pi|S_i]^2]+\underbrace{\mathbb{E}_0[\mathbb{E}[\tilde{g}(\pi_1)|S_i]]}_{=\mathbb{E}_0[\tilde{g}(\pi_1)]}$$

In particular, note that since $\pi\in\mathbb{S}$, we have that

$$\mathbb{E}_0\left[\frac{V(S_i)}{e^{c_0^*}u'(e^{c_0^*})}\right]\approx\frac{\psi(1-\psi^{-1})^2\rho^2}{4}\mathbb{E}_0[\pi^2]+\mathbb{E}_0[g(\pi_1)]$$

where the equality under the bracket uses the law of iterated expectation. Therefore, ex-ante losses from imperfect information is given by:

$$\begin{aligned}
\mathbb{E}_0\left[\frac{V(S_i)-V(\mathbb{S})}{e^{c_0^*}u'(e^{c_0^*})}\right] &\approx \frac{\psi(1-\psi^{-1})^2\rho^2}{4}\mathbb{E}_0[\mathbb{E}[\pi|S_i]^2-\pi^2]\\
&= \frac{\psi(1-\psi^{-1})^2\rho^2}{4}\mathbb{E}_0\left[\underbrace{\mathbb{E}[\mathbb{E}[\pi|S_i]^2-\pi^2|S_i]}_{=-Var(\pi|S_i)}\right]\\
&= -\frac{\psi(1-\psi^{-1})^2\rho^2}{4}Var(\pi|S_i)
\end{aligned}$$

defining $B\equiv\frac{\psi(1-\psi^{-1})^2}{2}$, we can write this as $-\frac{\rho^2 B}{2}Var(\pi|S_i)$, and we note that $B$is increasing in ψ on its domain of $\psi\in(0,\infty)$, if and only if $\psi>1$:

$$\frac{d\ ln\ (B)}{d\psi}=\frac{2}{\psi-1}-\frac{1}{\psi}=\frac{\psi+1}{(\psi-1)\psi}>0\Leftrightarrow\psi-1>0.$$

Proof of Proposition 2. We consider the two cases of $S_p\in S_i$ and $S_p\notin S_i$ separately:

1. If $S_p\in S_i$, then $\tilde{\pi}_i\equiv\mathbb{E}\left[\pi_1|S_i,S_p\right]=\mathbb{E}[\pi_1|S_i]=\pi_i$.
2. Alternatively, if $S_p\notin S_i$, then given that inflation and the public signal are jointly Gaussian, we have

$$\tilde{\pi}_i=\mathbb{E}\left[\pi_1|S_i,S_p\right]=\pi_i+\frac{\mathrm{Cov}\left(S_p,\pi_1|\vec{S}_i\right)}{\mathrm{Var}\left(S_p|\vec{S}_i\right)}\left(S_p-\mathbb{E}\left[S_p|\vec{S}_i\right]\right)$$

with

$$\text{Cov}(S_p, \pi_1 | \vec{S}_i) = \text{Cov}(\pi + \nu_p, \rho\pi + u | \vec{S}_i) = \rho\text{Cov}(\pi, \pi | \vec{S}_i) = \rho\text{Var}(\pi | \vec{S}_i)$$

where the last equality follows from (1) $\text{Cov}(\pi, u | \vec{S}_i) = \text{Cov}(\nu_p, u | \vec{S}_i) = 0$ because u is only drawn in period 1 and is independent of all information available at period 0, including π and S_i, and (2) $\text{Cov}(\nu_p, \pi | \vec{S}_i) = 0$ by the assumption that $\nu_p \perp (\pi, S_i \setminus \{S_p\})$. Moreover, we also have that

$$\text{Var}(S_p | \vec{S}_i) = \text{Var}(\pi | \vec{S}_i) + \text{Var}(\nu_p | \vec{S}_i) = \text{Var}(\pi | \vec{S}_i) + \sigma_{\nu,p}^2$$

Thus, when $S_p \notin S_i$, we have

$$\tilde{\pi}_i = \pi_i + \frac{\rho\text{Var}(\pi | \vec{S}_i)}{\text{Var}(\pi | \vec{S}_i) + \sigma_{\nu,p}^2}(S_p - \mathbb{E}[S_p | \vec{S}_i]) = \pi_i + \frac{\rho\text{Var}(\pi | \vec{S}_i)}{\text{Var}(\pi | \vec{S}_i) + \sigma_{\nu,p}^2} S_p - \frac{\text{Var}(\pi | \vec{S}_i)}{\text{Var}(\pi | \vec{S}_i) + \sigma_{\nu,p}^2}\pi_i$$

where the second equality follows from the fact that when $S_p \notin S_i$,

$$\mathbb{E}[\rho S_p | \vec{S}_i] = \mathbb{E}[\rho\pi + \rho\nu_p | \vec{S}_i] = \mathbb{E}[\rho\pi + u | \vec{S}_i] = \mathbb{E}[\pi_1 | \vec{S}_i] = \pi_i.$$

Now, for any individual i, regardless of whether they are in the treatment group or not, we can consolidate the above equations as presented in the proposition:

$$\tilde{\pi}_i = \pi_i + \frac{\rho\text{Var}(\pi | \vec{S}_i)}{\text{Var}(\pi | \vec{S}_i) + \sigma_{\nu,p}^2} S_p \times 1_{S_p \notin S_i} \times \mathbb{I}_i - \frac{\text{Var}(\pi | \vec{S}_i)}{\text{Var}(\pi | \vec{S}_i) + \sigma_{\nu,p}^2} 1_{S_p \notin S_i} \times \mathbb{I}_i \times \pi_i$$

where $\mathbb{I}_i$ is the indicator function that explicitly expresses that the last two terms in this equation are relevant when i is in the treatment group; i.e., $i \in T$, and implicitly defines $\tilde{\pi}_i = \pi_i$ for the control group. Moreover, $1_{S_p \notin S_i}$ is the indicator function that is 1 when $S_p \notin S_i$ and zero otherwise. This expression also implies that γ/β is given by the coefficient on $\mathbb{I}_i \times \pi_i$ relative to the coefficient on π_i as:

$$\gamma/\beta = -\frac{\text{Var}(\pi | \vec{S}_i)}{\text{Var}(\pi | \vec{S}_i) + \sigma_{\nu,p}^2} 1_{S_p \notin S_i}$$

Proof of Proposition 3. Consider the agent's problem as specified in the main text, and note that with Gaussian signals, we have the following expression for the information costs, depending on whether S_p is a component of S_i or not:

$$S_p \in S_i \Longrightarrow I(\vec{S}_i; \pi | S_p) = \frac{1}{2}\ln(\text{Var}(\pi | S_p)) - \frac{1}{2}\ln(\text{Var}(\pi | \vec{S}_i))$$

$$S_p \notin S_i \Longrightarrow I(\vec{S}_i; \pi) = \frac{1}{2}\ln(\text{Var}(\pi)) - \frac{1}{2}\ln(\text{Var}(\pi | \vec{S}_i))$$

Thus, as is common in rational inattention problems (see, e.g., Maćkowiak, Matějka, and Wiederholt 2023), we can write the agent's problem as directly choosing the conditional variance $\text{Var}(\pi | \vec{S}_i)$, with the constraint that the optimal $\text{Var}(\pi | \vec{S}_i)$ should not exceed the uncertainty of the agent prior to the acquisition of the new information (commonly referred to as no-forgetting constraints):

$$S_p \in S_i \Longrightarrow \mathrm{Var}(\pi|\vec{S}_i) \leq \mathrm{Var}(\pi|S_p)$$
$$S_p \notin S_i \Longrightarrow \mathrm{Var}(\pi|\vec{S}_i) \leq \mathrm{Var}(\pi) = \sigma_\pi^2$$

Thus, the agent's problem is

$$\min \begin{Bmatrix} \phi + \frac{\omega}{2}\ln\left(\mathrm{Var}\left(\pi\middle|S_p\right)\right) + \frac{1}{2}\min_{\mathrm{Var}(\pi|\vec{S}_i)\leq \mathrm{Var}(\pi|S_p)}\{B\rho^2\mathrm{Var}(\pi|\vec{S}_i) - \omega\ln\left(\mathrm{Var}(\pi|\vec{S}_i)\right)\}, \\ \frac{\omega}{2}\ln\left(\mathrm{Var}(\pi)\right) + \frac{1}{2}\min_{\mathrm{Var}(\pi|\vec{S}_i)\leq \mathrm{Var}(\pi)}\{B\rho^2\mathrm{Var}(\pi|\vec{S}_i) - \omega\ln\left(\mathrm{Var}(\pi|\vec{S}_i)\right)\} \end{Bmatrix}$$

We can then easily confirm that (1) if the solution was interior in either of the inner minimization problems, then $\mathrm{Var}(\pi|\vec{S}_i) = \frac{\omega}{B\rho^2}$ and (2) this would indeed be the optimal solution if both constraints were slack when $\mathrm{Var}(\pi|\vec{S}_i) = \frac{\omega}{B\rho^2}$; i.e.,

$$\mathrm{Var}(\pi|\vec{S}_i) = \frac{\omega}{B\rho^2} < \min\left\{\mathrm{Var}(\pi), \mathrm{Var}(\pi|S_p)\right\} = \mathrm{Var}(\pi|S_p)$$

where the second equality follows from $\mathrm{Var}(\pi|S_p) \leq \mathrm{Var}(\pi)$. Now, since $\frac{\omega}{B\rho^2} < \mathrm{Var}(\pi|S_p)$ holds by assumption of the Proposition, the solution to both inner minimization problems is indeed interior and we have

$$\mathrm{Var}\left(\pi\middle|\vec{S}_i\right) = \frac{\omega}{B\rho^2}$$

regardless of whether $S_p \in S_i$ or not, which concludes the proof of Part 1. To see Part 2, note that under the above posterior variance, the agent's problem reduces to

$$\min \begin{Bmatrix} \phi + \frac{\omega}{2}\ln\left(\mathrm{Var}\left(\pi\middle|S_p\right)\right) + \frac{1}{2}\min_{\mathrm{Var}(\pi|\vec{S}_i)\leq \mathrm{Var}(\pi|S_p)}\{B\rho^2\mathrm{Var}(\pi|\vec{S}_i) - \omega\ln(\mathrm{Var}(\pi|\vec{S}_i))\}, \\ \frac{\omega}{2}\ln\left(\mathrm{Var}(\pi)\right) + \frac{1}{2}\min_{\mathrm{Var}(\pi|\vec{S}_i)\leq \mathrm{Var}(\pi)}\{B\rho^2\mathrm{Var}(\pi|\vec{S}_i) - \omega\ln\left(\mathrm{Var}(\pi|\vec{S}_i)\right)\} \end{Bmatrix}$$

$$= \quad \frac{1}{2}\{\omega - \omega\ln\left(\omega / B\rho^2\right)\} + \min\left\{\phi + \frac{\omega}{2}\ln\left(\mathrm{Var}(\pi|S_p)\right), \frac{\omega}{2}\ln\left(\mathrm{Var}(\pi)\right)\right\}$$

so the agent chooses to observe S_p if and only if

$$\phi + \frac{\omega}{2}\ln\left(\mathrm{Var}(\pi|S_p)\right) \leq \frac{\omega}{2}\ln\left(\mathrm{Var}(\pi)\right)$$
$$\Leftrightarrow \quad \phi \leq \omega \times \frac{1}{2}\ln\left(\frac{\mathrm{Var}(\pi)}{\mathrm{Var}(\pi|S_p)}\right) = \omega I(S_p;\pi)$$

Proof of Proposition 4. Recall from Equation (4) that the treatment effect is given by

$$\left.\frac{\gamma}{\beta}\right|_{i\in T} = \begin{cases} -\frac{\omega}{\omega + B\rho^2\sigma_{v,p}^2} & S_p \notin S_i \\ 0 & S_p \in S_i \end{cases}$$

where, by Proposition 3, $S_p \in S_i$ if and only if $\phi \leq I\left(S_p, \pi\right) = \frac{\omega}{2}\ln\left(1 + \frac{\sigma_\pi^2}{\sigma_{v,p}^2}\right)$. Thus, the size of the treatment effect, in absolute values, is given by

$$|\gamma/\beta| = \begin{cases} 0 & \phi \leq \frac{\omega}{2}\ln\left(1+\frac{\sigma_\pi^2}{\sigma_{v,p}^2}\right) \\ \frac{\omega}{\omega+B\rho^2\sigma_{v,p}^2} & \text{else} \end{cases}$$

Part 1. Suppose we are in the region of the parameter space where $\phi > I(S_p,\pi)$ so that $|\gamma/\beta| = \frac{\omega}{\omega+B\rho^2\sigma_{v,p}^2}$. It follows that in this region:

$$\frac{\partial|\gamma/\beta|}{\partial(\omega)} = \frac{B\rho^2\sigma_{v,p}^2}{\left(\omega+B\rho^2\sigma_{v,p}^2\right)^2} > 0$$

which shows that the size of the treatment effect strictly increases with ω. Similarly, we can see that

$$\frac{\partial|\gamma/\beta|}{\partial\left(B\rho^2\sigma_{v,p}^2\right)} = -\frac{\omega}{\left(\omega+B\rho^2\sigma_{v,p}^2\right)^2} < 0$$

Which shows that the size of the treatment effect strictly decreases with either of the parameters ρ, B or $\sigma_{v,p}^2$.

Part 2. Suppose again that we are in the region of the parameter space where $\phi > I(S_p,\pi) = \frac{\omega}{2}\ln\left(1+\frac{\sigma_\pi^2}{\sigma_{v,p}^2}\right)$ so that $|\gamma/\beta| = \frac{\omega}{\omega+B\rho^2\sigma_{v,p}^2} > 0$ is strictly positive. Then, it follows immediately that if ϕ decreases or $\frac{\sigma_\pi^2}{\sigma_{v,p}^2}$ increases, so much so that $\phi \leq I(S_p,\pi)$ begins to hold, the treatment effect strictly declines to 0.