# Supplement to Minimum Wages, Efficiency and Welfare

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Section A provides model parameters and moments as well as the wage distribution in the model and data. Section B provides details of the ***Validation*** exercises described in Section 7. Section C provides ***Robustness*** exercises described in Section 7. Section D contains ***Proofs*** for a simplified monopsony and oligopsony economy that are referred to in Section 3, and an even simpler pedagogical example. This is the Homogeneous worker economy. The *Additional Materials to Minimum Wages, Efficiency and Welfare* (Berger, Herkenhoff, and Mongey, 2024) is available on the authors' websites, and follows this *Supplemental (Online) Appendix*, and provides (i) details on the calibration of $\varphi$, (i) additional figures and tables, (ii) derivations of the equilibrium conditions for the Heterogeneous worker economy, and any other equations in the main text, (iii) algorithm for solving the economy.

## A Additional calibration details and fit

Table A1 provides the full set of parameters for all 12 types of households. Table A2 reports the detailed moments. Figure A1 plots the wage PDF for model v. data in the 12 type economy.

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| **Parameters** | | **NHS** | | | | | **HS** | | | | | **C** | **O** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Wage quintile** | | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | All | All |
| Relative population (%) | $\pi_h / \sum \pi_h$ | 2.6 | 2.6 | 2.6 | 2.6 | 2.6 | 10.7 | 10.7 | 10.7 | 10.7 | 10.7 | 26.1 | 7.0 |
| Relative disutility labor supply | $\overline{\varphi}_h^{-\varphi} / \overline{\varphi}_C^{-\varphi}$ | 39.13 | 6.17 | 3.70 | 2.78 | 1.98 | 4.80 | 2.07 | 1.68 | 1.44 | 1.05 | 1.00 | 0.53 |
| Relative productivity | $\xi_h$ | 0.14 | 0.18 | 0.23 | 0.27 | 0.42 | 0.27 | 0.33 | 0.42 | 0.55 | 0.87 | 1.00 | 0.89 |
| Fraction of capital (%) | $\kappa_h$ | 0.00 | 0.00 | 0.01 | 0.01 | 0.07 | 0.04 | 0.13 | 0.16 | 0.27 | 1.04 | 4.30 | 93.96 |

Table A1: Detailed Parameters

| | **Model** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Targets (* Means Model=Data)** | **NHS** | | | | | **HS** | | | | | **C** | **O** |
| **Wage quintile** | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | All | All |
| Population shares* (CPS, %) | 2.64 | 2.64 | 2.64 | 2.64 | 2.64 | 10.74 | 10.74 | 10.74 | 10.74 | 10.74 | 26.08 | 7.00 |
| Share of agg. labor income* (CPS and SCF, %) | 0.04 | 0.26 | 0.50 | 0.74 | 1.46 | 1.64 | 4.26 | 6.28 | 8.92 | 17.36 | 46.16 | 12.39 |
| Ave. earnings per hour*, (CPS, C=1) | 0.29 | 0.32 | 0.37 | 0.41 | 0.60 | 0.39 | 0.44 | 0.54 | 0.66 | 0.95 | 1.00 | 1.00 |
| Capital income to labor income* (SCF) | 0.00 | 0.01 | 0.02 | 0.01 | 0.02 | 0.01 | 0.02 | 0.01 | 0.02 | 0.03 | 0.05 | 4.00 |

Table A2: Detailed Moments

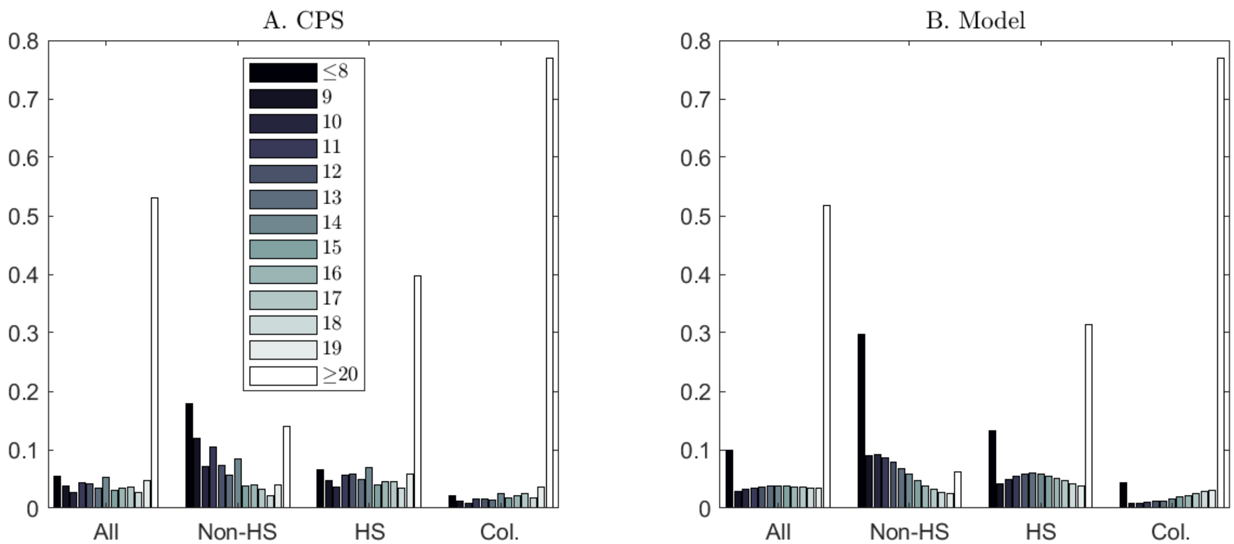


Figure A1: Distribution of wages in model vs. data (CPS)

# B Validation of efficiency channels versus recent empirical evidence

The main text describes three channels through which minimum wages may improve efficiency: (i) direct narrowing of markdowns, (ii) wage spillovers which undo distortions at unconstrained firms, and (iii) reallocation to more productive firms. Recent empirical studies speak directly to these channels: direct effects are measured by Jardim, Long, Plotnick, Van Inwegen, Vigdor, and Wething (2022) and Azar, Huet-Vaughn, Marinescu, Taska, and von Wachter (2023, henceforth AHMTV), spillovers are measured by Engbom and Moser (2022, henceforth EM) and reallocation is measured by Dustmann, Lindner, Schönberg, Umkehrer, and vom Berge (2022, henceforth DLSUB). Two of the studies are from non-U.S. economies due to measurement error concerns for spillovers and lack of comparable reallocation estimates in the U.S. However, with this caveat in mind, we show that the model produces comparable qualitative and quantitative responses.

This implied that the small efficiency gains that we compute in Section 5 are not due to undershooting on any of these mechanisms. Rather, existing reduced form evidence *pointing to* the possibility of efficiency gains can be generated by the theoretical mechanisms suggested in the empirical literature, but nonetheless efficiency gains may be small.

## B.1 Direct effects in Seattle

We replicate the disemployment effects of a high $\underline{w}$ on low wage jobs documented following the Seattle minimum wage increase studied in Jardim et. al. (2022).

**Empirical setting.** Jardim et. al. (2022) study the minimum wage increases in Seattle in 2015 and 2016. These are useful benchmarks as (i) they are minimum wage increases from initially high minimum wages, (ii) the authors have access to hours data which most closely maps into our model concept of $n_{ijh}$ from an efficiency perspective since it is the object that enters production. The authors study two minimum wage increases: *"The minimum wage rose from the state's minimum of $9.47 to as high as $11 on April 1, 2015, and again to as high as $13 on January 1, 2016"* (page 266, and Table 1). The authors compare single-establishment firms in Seattle to those in Washington state, and compute the elasticity of employment in jobs that pay less than $19 per hour, which account for 63 percent of the workforce (page 269, and Table 2). In Tables 6A and 6B the authors present estimates of causal effects in percent changes on wages and hours. Their results vary across specifications. We summarize them as ranges via their text:

1. **Wages** - We associate the first minimum wage increase with wage effects of 1.1 to 2.2 percent, averaging 1.7 percent, the second increase is associated with a larger 3.0 to 3.9 percent, averaging 3.4 percent wage effect. (page 290)

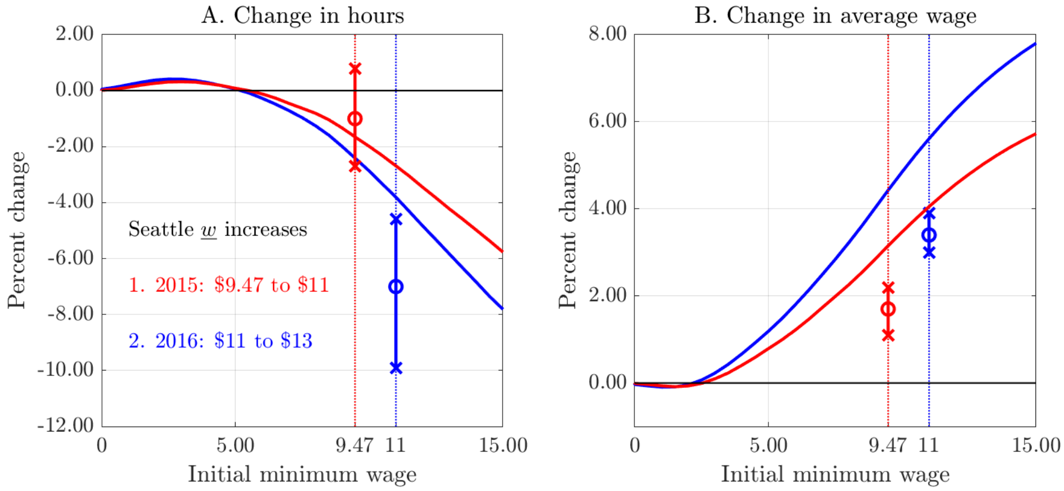


Figure B1: Disemployment effects on low wage employment from high initial minimum wages - Seattle

Notes: **Panel A.** The red line plots percent changes in hours following a $1.53 increase in $\underline{w}$, and the blue line following a $2.00 increase for an initial minimum wage specified on the x-axis. Vertical dashed lined denote the initial minimum wages of $9.47 (red) and $11 (blue). Solid box-whisker lines denote the range of point estimates described by the authors in Jardim et. al. (2022), see text. **Panel B.** Repeats the same exercise as Panel A with average wages.

2. **Hours** - Point estimates for the $11 period range +0.8 and -2.7 percent, averaging -1.0 percent, the subsequent increase to $13 is associated with larger reductions between 4.6 and 9.9 percent, averaging -7.0 percent. (page 292-3)

**Replication.** Our economy is calibrated to 2019, so we first deflate all wages to 2015 levels at 1.55 percent inflation using the 2015-2019 CPI. We take an economy with a $\underline{w}$ of $9.47 to match the pre-2015 baseline. We then consider a $1.53 minimum wage increase, corresponding to the first raise, and $2 minimum wage increase corresponding to the second raise. We keep all jobs of all worker types that had a pre-policy wage less than $20, to match the 63 percent of employment in the study, which applied a very similar cut-off of $19. We then compute the percent change in total employment—which corresponds to hours in their study—and the average wage. A benefit of the model is that we can conduct this for multiple initial minimum wages. We do not recalibrate any other parameters to Seattle data.

**Results.** Figure B1 presents our results. The vertical lines denote the aforementioned ranges of point estimates and average estimate from the authors. The horizontal axis plots the initial minimum wage. The red line plots percent changes in hours and average wage following a $1.53 increase in $\underline{w}$, and the blue line following a $2.00 increase.

First, consistent with the authors we obtain negative effects on hours and positive effects on wages. Second, the model has similar non-linear employment effects as found in the data. Effects on hours are small following the first increase, and large following the second increase. The model understates the large negative effects on the second increase found in Seattle, but would obtain similar estimates from a $2 increase from $13 to $15 per hour. Third, the increase in average wages is larger for the second increase, in roughly the right proportion to the first increase. However, in levels, our response is only about 1 percentage point larger in the model compared to the authors' empirical estimates.

In summary, these results give us confidence that the non-linearities in the model observed in our welfare exercises are consistent with the data, and kick in at the empirically relevant range of minimum wages, around $10 to $13 per hour.

## B.2 Direct employment effects in concentrated markets

We also analyze the direct effects of minimum wages and how they vary by market structure. AHMTV highlight the positive effects of minimum wages on employment in high concentration markets (where concentration is measured by the Herfindahl index of employment in a local labor market), and the negative effects of minimum wages on employment in low concentration markets. We further demonstrate that at low levels of the minimum wage, small changes in the minimum wage generate employment increases nationally. These results suggest that minimum wages may reduce markdowns and induce employment expansions, similar to neoclassical frameworks built on Robinson (1933).

**Empirical setting.** AHMTV compute the response of employment in low wage occupations to changes in state minimum wages, but stratify responses by the concentration of the labor market for each occupation. They estimate statistically significant positive effects in markets in the upper tercile of concentration, and statistically significant negative effects in markets in the lower tercile of concentration. We show that the same results hold in our economy, qualitatively.

Our replication is subject to two caveats. First, AHMTV measure concentration using the Herfindahl of job openings in Burning Glass vacancy data. In a large class of search models with balanced matching, vacancies are proportional to employment. While we do not model job search, we appeal to this intuition and measure concentration using the employment Herfindahl. Second, they restrict their analysis to the retail sector (Stock Clerks, Retail Sales, and Cashiers). The average retail wage is \$16.70[2]. So to align our results with theirs we restrict our analysis to high-school "retail sales" workers in the fourth quintile of earnings whose average wage is \$16.76, and thus maps most closely to AHMTV.

**Statistic.** Holding aggregates fixed, we increase the minimum wage by $\phi$ and compute the increase in employment in each market $j$. Exactly as in AHMTV, we regress the change in market employment $\Delta \log n_j$ on the change in the minimum wage $\Delta \log \underline{w}$, interacted with dummies for DOJ concentration thresholds based on the employment Herfindahl ($HHI^n$):[3]

$$\Delta \log n_j = \psi_L \Delta \log \underline{w} + \psi_H D\Big(HHI_j^n \in [0.25, \infty)\Big) \times \Delta \log \underline{w} + \varepsilon_j.$$

In their sample, the average pre- and post-policy minimum wages are \$7.43 and \$7.83, which we round to $\phi = 50$cents.[4] We use the model to understand heterogeneity by the level of the initial minimum wage, repeating this exercise for initial minimum wages $\underline{w}_0$ between \$2 and \$10 per hour.

**Results.** Figure B2 plots the estimated coefficients for low ($\widehat{\psi}_L$) and high ($\widehat{\psi}_L + \widehat{\psi}_H$) concentration markets, holding the increase in the minimum wage constant (50c), but varying the initial minimum wage $\underline{w}_0$. For $\underline{w}_0$ consistent with the setting the paper studies—i.e. less than \$8.00 per hour—the model is consistent with AHMTV's key empirical findings. High concentration markets experience positive employment effects (solid red), and low concentration markets experience small negative employment effects (dashed green). Our peak employment elasticity in high concentration markets is 0.12. Our point estimate is about 41% of theirs (=0.12/0.29) and very close to the lower bound of the 95% CI of 0.17 (Table 2, Col 2 of AHMTV). Firms in more concentrated markets have more market power, wider markdowns, and hence have larger positive employment gains available in Region II before shrinking in Region III. The expansion of employment

[2]https://www.bls.gov/oes/current/oes412031.htm, accessed September 2023.

[3]Following the methodology of AHMTV, we average the pre- and post- minimum wage Herfindahl.

[4]We thank the authors for sharing these two statistics with us.

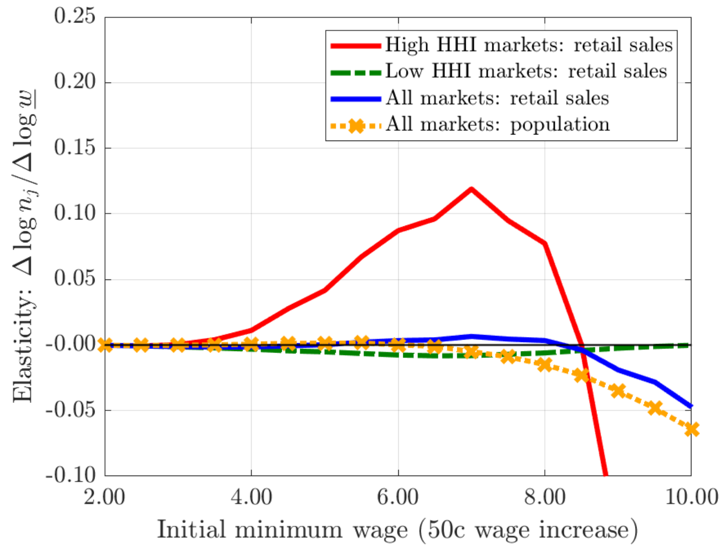


Figure B2: Replication of direct effects by HHI

Notes: Horizontal axis gives the initial minimum wage $\underline{w}_0$. The minimum wage is then increased by 50 cents. Red solid line plots estimated elasticity in high concentration markets ($\widehat{\psi}_L + \widehat{\psi}_H$). Green dashed line plots estimated elasticity in low concentration markets ($\widehat{\psi}_L$). Blue and Orange-Cross lines represent pooled effects for "retail" and the total population, respectively.

in concentrated markets is evidence of direct effects of minimum wages reducing markdowns and inducing employers to expand. In less concentrated markets, firms have initially narrow markdowns and move quickly into Region III, incurring employment losses. Crucially, the positive effects of minimum wages occur in concentrated markets.

Lastly, we run two unconditional regressions to measure the aggregate elasticity of employment:

$$\Delta \log n_j = \psi_{pooled} \Delta \log \underline{w} + \varepsilon_j$$

We estimate $\psi_{pooled}$ for (1) "Clerks" (blue solid), and (2) the overall population (orange crosses). We find broadly similar results: employment expands initially following increases from initially low levels of the minimum wage and then contracts once the initial minimum wage is beyond $8.00 per hour. The employment expansion at low levels of the minimum wage is, through the lens of our model, due to a reduction in markdowns as firms enter Region II. Overall, low concentration markets dominate the response as they employ the most workers.

Among the positive employment elasticities reported among U.S. studies (see Neumark and Shirley (2022) and Clemens and Strain (2021)), our model's small positive employment elasticities fall within the range reported by the literature.

## B.3 Spillovers from competitors' minimum wages

Our next validation exercise is to replicate the spillover effects observed in EM. There are estimates of U.S. spillovers based on survey data, however Autor, Manning, and Smith (2016) argue that measurement error in survey data poses significant issues for inference.[5] EM avoid these issues via administrative data on hours and wages from Brazil. Additionally, we focus on EM since they provide the necessary summary statistics for replication.

**Empirical setting.** EM compute that in 1996 the minimum wage was 30.3 percent of the median wage. It then increased by 128 percent between 1996 and 2012 (EM, page 3813). To replicate this experience we solve our economy under a minimum wage of \$5.50, which is 30.0 percent of the median wage, then increase it to \$12.50 which is a 128 percent increase. We denote these period zero and period one.

**Statistic.** Let $\overline{p}$ be a reference percentile of the wage distribution, and let $w_{p,t}$ be the percentile $p$ wage in period $t$. We compute spillovers at $p$ by

$$Spillover_p = \frac{\log(w_{p,1}/w_{\overline{p},1}) - \log(w_{p,0}/w_{\overline{p},0})}{\log(\underline{w}_1/w_{\overline{p},1}) - \log(\underline{w}_0/w_{\overline{p},0})} \tag{B1}$$

By construction $Spillover_{\overline{p}} = 0$. If wages below $\overline{p}$ compress upward, then $Spillover_p > 0$. If wages above $\overline{p}$ compress upward, then $Spillover_p < 0$. EM use a regression framework to obtain estimates of $Spillover_p$, whereas we simply compute $Spillover_p$ non-parametrically via (B1). As shown by (EM, Figure A2), even within the 70th percentile of the earnings distribution more than 80 percent of workers have not completed high school in Brazil.[6] We therefore compute results for non-High school workers.

**Results.** Figure B3 plots $Spillover_p$ for $p \in [10, 12, \ldots, 90]$ and compares estimates to those from (EM, Figure 4) where the reference percentile is $\overline{p} = 50$.[7] We find very similar qualitative and quantitative patterns of spillovers, with compression far up into the wage distribution. At the 30th percentile, wages compress by

[5] Autor, Manning, and Smith (2016)'s concern is that measurement error in U.S. survey data can explain the majority of measured spillover effects. See Section IV of Autor, Manning, and Smith (2016).

[6] Another statistic that reflect this is as follows: at the 90th percentile of the earnings distribution, only 10 percent of workers have a college degree.

[7] We compare our results to their IV specification that controls for state-level trends and state fixed effects. This delivers similar results to their specification with state-level fixed effects only.

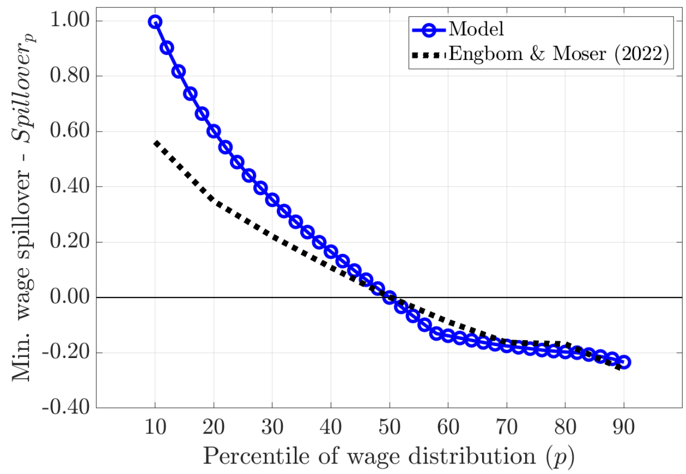


Figure B3: Replication of wage spillovers

Notes: Consistent with the minimum wage in Brazil in 1996, the initial minimum wage is 30 percent of the median wage. Consistent with the minimum wage increase in Brazil from 1996 to 2012, the minimum wage increases by 128 percent. These statistics are reported in Engbom and Moser (2022, page 12). We compare model results to those of Engbom and Moser (2021), Figure 4, under the 'State Fixed Effects plus Trends' OLS specification.

22% in the data versus 35% in the model. By construction the spillover is zero at $\overline{p} = 50$. At the 80th percentile, wages compress by 17% in the data versus 20% in the model. While the US labor market is subject to very different institutions than the Brazilian labor market, we view Figure B3 as a validation of our model's mechanisms on the best available data.

**Additional replication.** In BHM we quantitatively replicated Staiger, Spetz, and Phibbs (2010), which documented how competing hospitals raised nurse's wages following the imposition of a wage floor at Veteran's Affairs hospitals in 1991.

## B.4 Reallocation effects of minimum wages

Our final validation exercise replicates DLSUB, *"Reallocation Effects of the Minimum Wage,"* who study the effect of the introduction of a minimum wage in Germany and its impact on the cross-section of workers and firms. In January 2015, a national minimum wage of 8.50 euros was introduced into an environment with no pre-existing minimum wage. This corresponds to $10.40/hr in 2019 US dollars. The minimum wage introduced in Germany was large: pre-reform, 15 percent of workers earned below 8.50, which was 48 percent of the median wage. The key finding is employment reallocation: small firms exit, and larger more productive firms expand, increasing average firm size.

**Empirical setting.** DLSUB consider a number of empirical approaches. The one we focus on computes the elasticity of firm characteristics with respect to minimum wage exposure. The authors compute a *Gap* measure: the percent increase in total earnings required to satisfy the new minimum wage, holding employment and hours fixed at their pre-reform level. Let workers be indexed by $\ell \in \{1, \ldots, n\}$. DLSUB define *Gap* using workers' pre-reform hours $h_\ell$ and wages $w_\ell$:

$$Gap := \left[\sum_\ell \max\left\{\underline{w} - w_\ell, 0\right\} h_\ell\right] \Big/ \left[\sum_\ell w_\ell h_\ell\right]$$

The authors group firms by geographic regions $r$, and regress changes in region outcomes on $Gap_r$. We focus on two dependent variables studied: (i) total number of operating firms, and (ii) average firm size. Their results are in Table 7, page 54.

**Replication.** To an economy with no minimum wage, we introduce a minimum wage of \$9.85/hr. This is relatively low, but equals 48 percent of the pre-reform median wage. The empirical setting is a national reform, so we solve the pre- and post-reform economy in general equilibrium. The regions considered in DLSUB, comprise all industries in multiple commuting zones and rural areas. These are much larger than markets $j$ in our model. We therefore treat our whole economy as one region, which generates a single *Gap* measure directly comparable to theirs:

$$Gap = \left[\sum_h \int \sum_i \max\{\underline{w} - w_{ijh}, 0\} n_{ijh}\, dj\right] \Big/ \left[\sum_h \int \sum_i w_{ijh} n_{ijh}\, dj\right]. \tag{B2}$$

To compute the elasticity of variable $x$ with respect to *Gap*, we divide economy-wide $\Delta \log x$ by *Gap*.

**Results.** Figure B4 gives the results.[8] We plot results for a range of minimum wages $\underline{w}_1$, indexed by the ratio of $\underline{w}_1$ to the pre-reform median wage $w_0^{p50}$. The vertical line marks the $\underline{w}_1 / w_0^{p50} = 0.48$ corresponding to DLSUB.

Consistent with the new reallocation facts in DLSUB, Panel A shows that reallocation causes average firm size to grow and Panel B shows that small firms exit. In the model all firms still operate due to decreasing returns and since $n_{ijh}$ is continuous it can go below one (recall Figure 1D). To compare our model to DLSUB, we classify a firm as 'operating' when their employment is above one worker.

The model's elasticity of average firm size with respect to minimum wage exposure is positive and in line with the data (Figure B4A). The increase in average firm

[8] There are two sets of the authors' results: 'Data 1' and 'Data 2'. Both feature controls that account for observable regional differences (e.g. average age) and region specific trends in the moments. 'Data 2' additionally interacts these trends with year fixed effects.

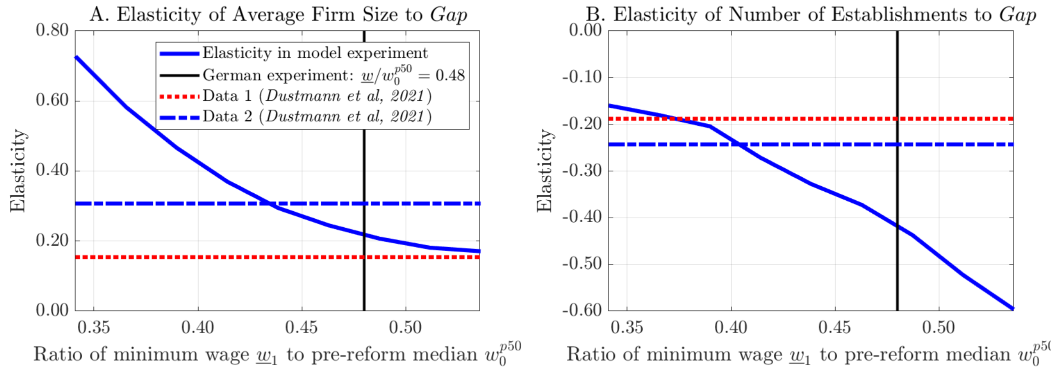


Figure B4: Replication of DLSUB (2021) - Reallocation effects of minimum wages

Notes: Corresponding data estimates for "Data 1" and "Data 2" are respectively taken from p.54 of DLSUB, Table 7, Columns (2) [regional controls and region specific linear trend] and (4) [regional controls interacted with year fixed effects]. The solid blue line plots the elasticity of the relevant moment to the minimum wage *Gap*, computed as in equation (B2). The horizontal axis plots the minimum wage in the policy experiment simulated in the model as a fraction of the pre-reform median wage in the model.

size represents reallocation, and is moderated at larger minimum wage increases due to firms shrinking in Region III, consistent with positive gains from reallocation being limited to small minimum wage increases. The elasticity of the number of operating firms with respect to *Gap* is negative and thus correctly signed, but more responsive compared to the data (Figure B4AB.

**Interpretation.** One of the key take-aways of DLSUB is that minimum wage increases have heterogeneous effects across firms. Low productivity firms exit, but their workers do not move out of the labor market. Jobs which existed due to the small amount of market power at these low productivity firms are destroyed, but workers are reallocated to larger, more productive firms. This can improve allocative efficiency. Our model generates dynamics consistent with these observations.

# C Robustness exercises

## C.1 Varying Frisch elasticity $\varphi$

Our main results are robust to the Frisch elasticity of labor supply. We consider two values of $\varphi$ either side of the baseline value of 0.62. These values are informed by our exercise in Appendix E using data from Golosov et. al. (2021). Their results imply larger $\varphi$ for high income households (lower MPC, higher MPE) than low income households (higher MPC, lower MPE). We consider values that match data for both groups: $\varphi \in \{0.30, 0.86\}$.[9] We recalibrate all other 'shifter' parameters to match data in Table A1. Appendix Tables C1 and C2 show that levels of $\varphi$ have essentially zero effect on our calculations.

| **A. Maximize utilitarian welfare,** $\Lambda_\pi(\underline{w})$ | | **Alt. Economy** | | **Baseline** | |
|---|---|---|---|---|---|
| Optimal minimum wage ($) | $\underline{w}^*$ | 10.93 | | 10.95 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^*)$ | 2.77 | | 2.79 | |
| Aggregate efficiency (%) | $AE(\underline{w}^*)$ | -0.13 | | -0.07 | |
| **B. Maximize aggregate efficiency,** $AE(\underline{w})$ | | **Alt. Economy** | | **Baseline** | |
| Optimal minimum wage ($) | $\underline{w}^{*,AE}$ | 7.01 | | 7.35 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^{*,AE})$ | 1.40 | | 1.55 | |
| Aggregate efficiency (%) | $AE(\underline{w}^{*,AE})$ | 0.08 | | 0.09 | |
| **C. Moments in alternative economy** | | **NHS** | **HS** | **C** | **O** |
| Relative population* (%) | $\pi_h / \sum \pi_h$ | 13.22 | 53.70 | 26.08 | 7.00 |
| Average earnings per hour*, (C=1) | | 0.40 | 0.59 | 1.00 | |
| Share of aggregate labor income* (CPS and SCF, %) | | 3.0 | 38.5 | 46.2 | 12.4 |
| Binding at $15, all* (%) | | —— 30.57 —— | | | |

Table C1: Robustness exercise - $\varphi = 0.30$

| **A. Maximize utilitarian welfare,** $\Lambda_\pi(\underline{w})$ | | **Alt. Economy** | | **Baseline** | |
|---|---|---|---|---|---|
| Optimal minimum wage ($) | $\underline{w}^*$ | 10.97 | | 10.95 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^*)$ | 2.80 | | 2.79 | |
| Aggregate efficiency (%) | $AE(\underline{w}^*)$ | -0.03 | | -0.07 | |
| **B. Maximize aggregate efficiency,** $AE(\underline{w})$ | | **Alt. Economy** | | **Baseline** | |
| Optimal minimum wage ($) | $\underline{w}^{*,AE}$ | 7.54 | | 7.35 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^{*,AE})$ | 1.64 | | 1.55 | |
| Aggregate efficiency (%) | $AE(\underline{w}^{*,AE})$ | 0.10 | | 0.09 | |
| **C. Moments in alternative economy** | | **NHS** | **HS** | **C** | **O** |
| Relative population* (%) | $\pi_h / \sum \pi_h$ | 13.22 | 53.70 | 26.08 | 7.00 |
| Average earnings per hour*, (C=1) | | 0.40 | 0.59 | 1.00 | |
| Share of aggregate labor income* (CPS and SCF, %) | | 3.0 | 38.5 | 46.2 | 12.4 |
| Binding at $15, all* (%) | | —— 30.57 —— | | | |

Table C2: Robustness exercise - $\varphi = 0.86$

## C.2 Heterogeneous region calibration

We split our economy into three separate regions, denoted $r$ and consider a separate household type for each region.[10] We calibrate each region to data from three

[9]This range subsumes the range used by the Congressional Budget Office when modeling policy, which is around 0.30 to 0.53. See the following (link).

[10]We make the simplifying assumption that labor is immobile across regions. Capital and consumption goods are traded at the same rental rate and price across all regions.

sets of US states, grouped by median household income. Each region contains approximately one third of the civilian labor force.[11] Across regions, we keep some preference and technology parameters the same, as well as the distribution of number of firms in a market: $\{\beta, \theta, \eta, \delta, \alpha, \gamma, G(M_j)\}$. Region $r$ shifters $(\overline{\varphi}_r, \overline{Z}_r)$, $\{\overline{\varphi}_{kr}, \xi_{kr}\}_{k=1,r=1}^{K,R}$ and measures $\{\pi_{kr}\}_{k=1,r=1}^{K,R}$, are chosen to match CPS data from each region, following Table A1. Since the SCF does not identify an individual's state, we impose three further restrictions across regions. We keep constant (i) the ratio of household capital to labor income,[12] (ii) the fraction of households that are owners,[13] (iii) average firm size, which determines $\overline{\varphi}_r$.

Tables C3 and C4 show that relative to High income states, Low income states have significantly lower wages (last row). A \$15 minimum wage binds for 34% of low income state workers, and only 27% for high income state workers.

The greater rationing among low income states puts downward pressure on both the Utilitarian and efficiency maximizing minimum wages. The high income states have a marginally higher Utilitarian minimum wage (+\$0.91) and a marginally higher efficiency maximizing minimum wage (+\$1.17). However, aggregate efficiency gains are still less than 0.10% in both.

We repeat this exercise for Mississippi (MS) in Table C5, recalibrating to match 41.3% of workers below \$15 an hour in MS. The optimal minimum wage falls by \$1.71 due to the stronger degree of rationing and greater share of non-highschool workers. The efficiency maximizing minimum wage also falls by \$1.40 for similar reasons. As in our other regional exercises, firm heterogeneity and rationing of the lowest wage workers mutes the efficiency gains from minimum wages.

Overall we note that efficiency maximizing minimum wages are below \$8 across these exercises, and efficiency gains are less than 0.10%.

[11] States are allocated to regions as followed, ordered by 2019 median household income within each region. Low income states: MS, LA, NM, WV, AR, KY, AL, TN, GA, FL, OK, MT, MS, NC, SC, MI, SD. Medium income states: OH, WY, ID, IA, ME, IN, WI, TX, ND, RI, PA, AZ, NV, NY, CO, NE, KS, DE, VT. High income states: IL, OR, CA, AK, VA, MN, WA, UT, NH, CT, MA, NJ, HI, DC, MD.

[12] Since other parameters change, we recalibrate the share parameters $\{\kappa_{kr}\}_{k=1,r=1}^{K,R}$ to match the benchmark targets.

[13] For example, if Region A has 37% of workers with a college degree, and Region B has 29%, then in both Region A and Region B we maintain that 7% of households are *college-owners* (Table A1) and set the share of households that are *college-workers* to 30% in Region A and 22% in Region B.

| A. Maximize utilitarian welfare, $\Lambda_\pi(\underline{w})$ | | Alt. Economy | | Baseline | |
|---|---|---|---|---|---|
| Optimal minimum wage ($) | $\underline{w}^*$ | 10.68 | | 10.95 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^*)$ | 2.91 | | 2.79 | |
| Aggregate efficiency (%) | $AE(\underline{w}^*)$ | -0.13 | | -0.07 | |
| **B. Maximize aggregate efficiency**, $AE(\underline{w})$ | | **Alt. Economy** | | **Baseline** | |
| Optimal minimum wage ($) | $\underline{w}^{*,AE}$ | 6.76 | | 7.35 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^{*,AE})$ | 1.42 | | 1.55 | |
| Aggregate efficiency (%) | $AE(\underline{w}^{*,AE})$ | 0.10 | | 0.09 | |
| **C. Moments in alternative economy** | | **NHS** | **HS** | **C** | **O** |
| Relative population* (%) | $\pi_h / \sum \pi_h$ | 13.05 | 57.19 | 22.75 | 7.00 |
| Average earnings per hour*, (C=1) | | 0.42 | 0.62 | 1.00 | |
| Share of aggregate labor income* (CPS and SCF, %) | | 3.2 | 44.6 | 39.9 | 12.3 |
| Binding at $15, all* (%) | | —— 34.21 —— | | | |

Table C3: Robustness exercise - States in lowest tercile of income

| A. Maximize utilitarian welfare, $\Lambda_\pi(\underline{w})$ | | Alt. Economy | | Baseline | |
|---|---|---|---|---|---|
| Optimal minimum wage ($) | $\underline{w}^*$ | 11.59 | | 10.95 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^*)$ | 2.80 | | 2.79 | |
| Aggregate efficiency (%) | $AE(\underline{w}^*)$ | -0.06 | | -0.07 | |
| **B. Maximize aggregate efficiency**, $AE(\underline{w})$ | | **Alt. Economy** | | **Baseline** | |
| Optimal minimum wage ($) | $\underline{w}^{*,AE}$ | 7.93 | | 7.35 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^{*,AE})$ | 1.62 | | 1.55 | |
| Aggregate efficiency (%) | $AE(\underline{w}^{*,AE})$ | 0.09 | | 0.09 | |
| **C. Moments in alternative economy** | | **NHS** | **HS** | **C** | **O** |
| Relative population* (%) | $\pi_h / \sum \pi_h$ | 13.33 | 50.34 | 29.33 | 7.00 |
| Average earnings per hour*, (C=1) | | 0.40 | 0.57 | 1.00 | |
| Share of aggregate labor income* (CPS and SCF, %) | | 3.1 | 33.6 | 51.0 | 12.2 |
| Binding at $15, all* (%) | | —— 27.26 —— | | | |

Table C4: Robustness exercise - States in highest tercile of income

## C.3 Fixed capital: Short run vs. long run

In comparing steady-states we are implicitly studying the long-run effects of the minimum wage. Our theory suggests a smaller optimal minimum wage in the short-run if the cost of labor increases but the level and distribution of capital across workers in each firm is slow to adjust. If we assume maximal stickiness in reallocation of capital across-workers within-firm (fixed capital) a minimum wage causes exit, but we find these effects are quantitatively small. When capital is fixed, the optimal minimum wage under Utilitarian weights declines by 80 cents (Table C6). With sharper decreasing returns in the short-run, the range of productivity for which firms are in Region II shrinks (Figure C1), reducing poten-

| A. Maximize utilitarian welfare, $\Lambda_\pi(\underline{w})$ | | Alt. Economy | | Baseline | |
|---|---|---|---|---|---|
| Optimal minimum wage ($) | $\underline{w}^*$ | 9.24 | | 10.95 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^*)$ | 2.75 | | 2.79 | |
| Aggregate efficiency (%) | $AE(\underline{w}^*)$ | -0.09 | | -0.07 | |
| **B. Maximize aggregate efficiency,** $AE(\underline{w})$ | | **Alt. Economy** | | **Baseline** | |
| Optimal minimum wage ($) | $\underline{w}^{*,AE}$ | 5.96 | | 7.35 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^{*,AE})$ | 1.42 | | 1.55 | |
| Aggregate efficiency (%) | $AE(\underline{w}^{*,AE})$ | 0.10 | | 0.09 | |
| **C. Moments in alternative economy** | | **NHS** | **HS** | **C** | **O** |
| Relative population* (%) | $\pi_h / \sum \pi_h$ | 15.98 | 63.01 | 14.01 | 7.00 |
| Average earnings per hour*, (C=1) | | 0.45 | 0.68 | 1.00 | |
| Share of aggregate labor income* (CPS and SCF, %) | | 5.4 | 52.9 | 27.8 | 13.9 |
| Binding at $15, all* (%) | | —— 41.43 —— | | | |

Table C5: Robustness exercise - Mississippi

tial efficiency gains. Quantitatively, short- and long-run elasticities in our model are similar which is reassuring for our mapping to empirical studies of short-run changes.

We provide the theory and details for the short vs. long-run exercise above. We increase the minimum wage but keep firm-worker specific installations of capital fixed at the allocation $\overline{k}_{ijh}$ under a zero minimum wage. Firm profits from each type are as follows:

$$\pi_{ijh} = \overline{Z}\xi_h \left( z_{ij}\overline{k}_{ijh}^{(1-\gamma)\alpha} \right) n_{ijh}^{\gamma\alpha} - w_{ijh}n_{ijh} - R\overline{k}_{ijh}.$$

First, with fixed capital, the production function has sharper decreasing returns in labor: $\gamma\alpha < \widetilde{\alpha}$. Second, firms face overheard costs of pre-installed capital, $R\overline{k}_{ijh}$, which will cause termination of non-profitable jobs at high minimum wages. We therefore add an endogenous margin of operation into the solution of the model.[14] Third, equilibrium conditions are as before, minus the capital demand condition. Capital supply remains infinitely elastic at $R = 1/\beta + (1-\delta)$, but demand is

[14]Market-by-market we first assume that all firms enter, and then solve the Nash equilibrium of the market and general equilibrium of the economy. We then compute firm-type profits $\pi_{ijh}$, which account for fixed capital costs. If any firm has profits $\pi_{ijh} < 0$, we drop the lowest productivity firm in the market and then solve the market equilibrium again. With fewer firms, labor market power of the remaining firms increases, which increases profits, hence the need to remove only one firm at a time. We continue in this way until we reach a Cournot Nash equilibrium: no firm with shut-down jobs wishes to re-open them given competitor's operation and intensive margin labor decisions.

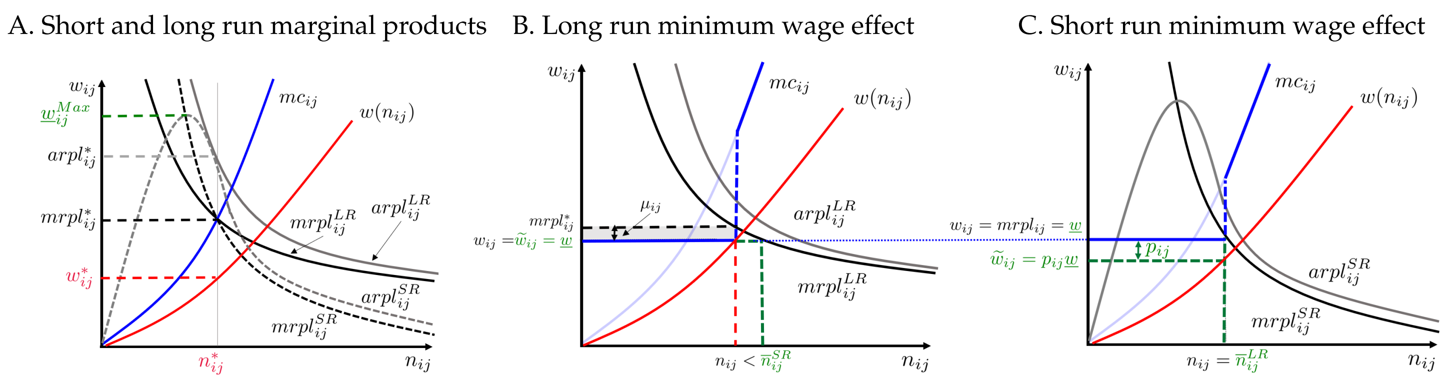


Figure C1: Partial equilibrium theory of minimum wage in the short-run

pinned down at $\overline{K}(\underline{w}) = \sum_h \int \sum_i \chi_{ijh}(\underline{w})\overline{k}_{ijh}\, dj$, where $\chi_{ijh}(\underline{w}) \in \{0,1\}$ indicates whether the firm operates worker-type-$k$ capital in equilibrium under minimum wage $\underline{w}$.

Figure C1 characterizes the mechanism behind a lower optimal minimum wage in this environment. Panel A considers a firm in an economy without a minimum wage, where capital is fixed at the allocation consistent with long-run employment $n_{ij}^{*}$. Short-run marginal and average products coincide with long-run values at this point. Away from $n_{ij}^{*}$, short-run $mrpl_{ij}^{SR}$ is steeper due to sharper decreasing returns with fixed capital: if $n_{ij} > n_{ij}^{*}$, then $mrpl_{ij}^{SR} < mrpl_{ij}^{LR}$. With fixed overhead capital, the $arpl_{ij}^{LR}$ goes to zero as $n_{ij}$ goes to zero and overhead per worker explodes. The peak in $arpl_{ij}^{SR}$ intersects $mrpl_{ij}^{SR}$ and gives the maximum $\underline{w}$ the firm could afford and still operate type-$k$ capital: $\underline{w}_{ij}^{Max}$. At $\underline{w} > \underline{w}_{ij}^{Max}$, equating $\underline{w} = mrpl_{ij}^{SR}$ would imply $arpl_{ij}^{SR} < \underline{w}$ and shutdown is optimal.

Panels B and C show how these differences constrain the positive efficiency gains from narrowing $\widetilde{\mu}_h$. Take the firm in Panel A, in the long run, at the minimum wage pictured in Panel B, the firm is in Region II: employment is non-rationed ($n_{ij} < \overline{n}_{ij}^{SR}$), and wages are a narrower markdown on $mrpl_{ij}^{LR}$. A small increase in the minimum wage *increases employment* and *narrows shadow markdowns*. Panel C considers the short run, at the same minimum wage. The lower $mrpl_{ij}^{SR}$ places the firm in Region III, where employment is rationed. A small increase in the minimum wage *decreases employment* and *widens shadow markdowns*. In the short run, the range of $\underline{w}$ over which firms are in Region II is smaller. This constrains the efficiency gains from improvements in $\widetilde{\mu}_h$.

Table C6 reports the results from fixing capital. The short-run optimal mini-

| **A. Maximize utilitarian welfare**, $\Lambda_\pi(\underline{w})$ | | **Alt. Economy** | **Baseline** | |
|---|---|---|---|---|
| Optimal minimum wage ($) | $\underline{w}^*$ | 10.12 | 10.95 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^*)$ | 2.33 | 2.79 | |
| Aggregate efficiency (%) | $AE(\underline{w}^*)$ | -0.03 | -0.07 | |
| **B. Maximize aggregate efficiency**, $AE(\underline{w})$ | | **Alt. Economy** | **Baseline** | |
| Optimal minimum wage ($) | $\underline{w}^{*,AE}$ | 7.08 | 7.35 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^{*,AE})$ | 1.42 | 1.55 | |
| Aggregate efficiency (%) | $AE(\underline{w}^{*,AE})$ | 0.08 | 0.09 | |
| **C. Moments in alternative economy** | | **NHS** | **HS** | **C** | **O** |
| Relative population* (%) | $\pi_h / \sum \pi_h$ | 13.22 | 53.70 | 26.08 | 7.00 |
| Average earnings per hour*, (C=1) | | 0.40 | 0.59 | 1.00 | |
| Share of aggregate labor income* (CPS and SCF, %) | | 3.0 | 38.5 | 46.2 | 12.4 |
| Binding at $15, all* (%) | | —— 30.57 —— | | | |

Table C6: Robustness - Fixed capital / Short-run

mum wage under Utilitarian weights declines by about 80 cents. Likewise, the aggregate efficiency maximizing minimum wage is roughly 20 cents lower. Thus with sharper decreasing returns in the short-run, there is a smaller range of productivity for which firms are in Region II and efficiency gains decline.

## C.4 Labor-labor substitution

Despite the additive nature of our production function, type-level decreasing returns implies different types of labor are not perfect substitutes. As shown in equation (9), the elasticity of substitution between different education groups is $(1 - \alpha(1 - \gamma))/(1 - \alpha)$. Baseline $\alpha = 0.94$ and $\gamma = 0.81$ implies an elasticity of 13.7, which is high relative to the literature.[15] We consider an alternative calibration with $\alpha = 0.70$ which delivers an elasticity of substitution of 2.9, close to the value estimated by Acemoglu and Autor (2011) which extended Katz and Murphy (1992) through 2008. Qualitatively, as in the above short-run exercise, a lower $\alpha$ steepens the labor demand curve, reducing the range over which a firm will be found in Region II, choking off the *Direct effect*. The efficiency maximizing minimum wage falls to $6.82 and the welfare gains from improvements in efficiency fall slightly from 0.09% to 0.08%.

[15] See for example, Katz and Murphy (1992), Card and Lemieux (2001), and Acemoglu and Autor (2011) which falls in the range of 1.5 to 2.9.

| A. Maximize utilitarian welfare, $\Lambda_\pi(\underline{w})$ | | Alt. Economy | | Baseline | |
|---|---|---|---|---|---|
| Optimal minimum wage ($) | $\underline{w}^*$ | 10.99 | | 10.95 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^*)$ | 2.56 | | 2.79 | |
| Aggregate efficiency (%) | $AE(\underline{w}^*)$ | -0.01 | | -0.07 | |
| **B. Maximize aggregate efficiency**, $AE(\underline{w})$ | | **Alt. Economy** | | **Baseline** | |
| Optimal minimum wage ($) | $\underline{w}^{*,AE}$ | 6.82 | | 7.35 | |
| Welfare (%) | $\Lambda_\pi(\underline{w}^{*,AE})$ | 1.47 | | 1.55 | |
| Aggregate efficiency (%) | $AE(\underline{w}^{*,AE})$ | 0.08 | | 0.09 | |
| **C. Moments in alternative economy** | | **NHS** | **HS** | **C** | **O** |
| Relative population* (%) | $\pi_h / \sum \pi_h$ | 13.22 | 53.70 | 26.08 | 7.00 |
| Average earnings per hour*, (C=1) | | 0.40 | 0.59 | 1.00 | |
| Share of aggregate labor income* (CPS and SCF, %) | | 3.0 | 38.5 | 46.2 | 12.4 |
| Binding at $15, all* (%) | | —— 30.61 —— | | | |

Table C7: Robustness - Labor-labor substitution

## C.5 Capital-labor substitution

Our benchmark model features an elasticity of substitution between capital and labor of 1.0. Prominent existing studies estimate lower elasticities around 0.7 (Oberfield and Raval, 2021) and high elasticities around 1.2 (Karabarbounis and Neiman, 2014), with the majority of studies pointing to estimates less than 1.0 (Gechert, Havranek, Irsova, and Kolcunova, 2022). Our baseline value of a unitary elasticity is within this range. It is important to note that lower elasticities do little to our main result, namely that the efficiency maximizing minimum wage is relatively small. The above short-run exercise incorporates an extreme elasticity of substitution of zero and finds the efficiency maximizing minimum wage is unaltered.

# D Proofs

We characterize the equilibrium of a simple oligopsony economy without capital or worker heterogeneity in order to ease exposition. Firms and market structure are identical to the economy in the text, except they do not rent capital. Workers have linear preferences over consumption to simplify the labor supply system. The proofs generalize to economies that do not make these assumptions.

**Preferences.** Preference over consumption are linear with an additive disutility of supplying labor identical to the main text.

$$\mathcal{U} = C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad , \quad N := \left[ \int_0^1 n_j^{\frac{\theta+1}{\theta}} \, dj \right]^{\frac{\theta}{\theta+1}} \quad , \quad n_j := \left[ \sum_{i=1}^{M_j} n_{ij}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}} \quad , \quad \eta \geq \theta. \tag{D1}$$

**Labor market competition.** As in the text, firms take actions taking their competitors' employment as given. That is they Cournot compete, and understand that they influence market-level outcomes (i.e. firms are oligopsonists). Actions consist of choosing their own quantity of employment, wage and rationing constraint. Labor market $j$ is infinitesimal with respect to other labor markets in the economy, so firms take quantities and wages outside of their labor market as given.

**Household problem.** The household takes rationing constraints $\{\overline{n}_{ij}\}$, wages $\{w_{ij}\}$ and profits $\Pi$ as given (which are accordingly omitted from the optimization problem below). The household chooses employment $\{n_{ij}\}$ at each firm $ij$ to maximize:

$$\max_{\{n_{ij}\}_{i \in \{0,M_j\}, j \in [0,1]}} \int_0^1 \sum_{i=1}^{M_j} w_{ij} n_{ij} dj - \frac{N^{1+1/\varphi}}{1+1/\varphi} \tag{D2}$$

subject to $n_{ij} \leq \overline{n}_{ij}$ for all $i \in \{1, \ldots, M_j\}$ and $j \in [0,1]$. Let $\nu_{ij}$ be the multiplier on the rationing constraint. The following optimality conditions characterizes the labor supply decision of the household:

$$w_{ij} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} + \nu_i \quad , \quad \nu_i(\overline{n}_i - n_i) = 0. \tag{D3}$$

We can combine the conditions in equation (D3) to obtain *the inverse labor supply schedule*, which equates the wage to the marginal disutility of labor:

$$w\Big(n_{ij}, \overline{n}_{ij}, n_j, N\Big) = \begin{cases} \left(\frac{n_{ijt}}{n_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{n_{jt}}{N_t}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} & , n_{ijt} \in [0, \overline{n}_{ijt}) \\ \in \left[ \left(\frac{\overline{n}_{ijt}}{n_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{n_{jt}}{N_t}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}, \infty \right) & , n_{ijt} = \overline{n}_{ijt} \end{cases} \tag{D4}$$

Note that this does not directly depend on the minimum wage. This is a correspondence at $\overline{n}_{ij}$. Given any wage greater than the marginal disutility of labor at $\overline{n}_{ij}$, *in red*, the household will supply $\overline{n}_{ij}$. Note, also, that the marginal disutility of labor at any firm doesn't depend on the *rationing constraints* at other firms, but simply the labor employed at other firms. This is for the standard reason that the first order condition for labor at firm $i$ is a *partial* derivative.

**Firm problem.** Firm $i$ in market $j$ takes as given local competitors' employment levels $\{n_{-ij}\}$ the aggregate employment index $N$ and chooses its (i) wage $w_{ij}$, (ii) employment $n_{ij}$, and (iii) rationing constraint $\overline{n}_{ij}$ in order to maximize profits. The firm is constrained by (a) the minimum wage $w_{ij} \geq \underline{w}$, (b) its rationing constraint $n_{ij} \leq \overline{n}_{ij}$, and (c) the inverse labor supply schedule (D5). Therefore the firm problem is given by,

$$\max_{\overline{n}_{ij}, n_{ij}, w_{ij}} z_{ij} n_{ij}^{\alpha} - w_{ij} n_{ij} \quad \text{subject to} \quad w_{ij} \geq \underline{w} \ , \ n_{ij} \leq \overline{n}_{ij} \ , \ w_{ij} = w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right). \tag{D5}$$

The firm understands, directly, $\partial w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)/\partial n_{ij} \neq 0$ and, indirectly, via $\partial n_j / \partial n_{ij} \neq 0$ (equation D1), yielding *oligopsonistic* behavior.

**Equilibrium.** Given a minimum wage $\underline{w}$, an *oligopsonistic Nash-Cournot* equilibrium is (i) a household inverse labor supply curve $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$, (ii) wages $\{w_{ij}\}$, (iii) quantities of labor $\{n_{ij}\}$, (iv) rationing constraints $\{\overline{n}_{ij}\}$, (v) profits $\Pi$, and (vi) aggregate employment index $N$ and market level employment indices $\{n_j\}$ such that (1) given wages $\{w_{ij}\}$, rationing constraints $\{\overline{n}_{ij}\}$, and profits $\Pi$, household optimization implies the inverse labor supply curve $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$, (2) for every firm $i$ in market $j$: given competitor employment $\{n_{-ij}\}$, the aggregate employment index $N$, and the household inverse labor supply curve, firm $ij$'s optimization yields rationing constraint $\overline{n}_{ij}$, wage $w_{ij}$ and employment $n_{ij}$, (3) firm employment decisions are consistent with the aggregate and market employment indices, $N$, $\{n_j\}$, as well as profits, $\Pi$, and (4) markets clear $w_i = w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) \ \forall i$.

The remainder of the appendix provides detailed derivations of (1) the firm's perceived inverse labor supply curve and (2) optimal rationing constraint.

**(1) Perceived labor supply curve.** We proceed via three Lemmas.

**Lemma 0** - *Given competitor employment $\{n_{-ij}\}$, competitor rationing constraints $\{\overline{n}_{-ij}\}$ are payoff irrelevant for firm ij.*

*Proof:* $\{\overline{n}_{-ij}\}$ do not enter the Cournot oligopsony firm problem. ■

**Lemma 1** - *Consider some level of employment $n_{ij} \leq \overline{n}_{ij}$. Given competitor employment $\{n_{-ij}\}$, a firm would never pay a wage that is greater than the lowest legal wage necessary to deliver $n_{ij}$.*

*Proof:* Conditional on $n_{ij}$ and $\{n_{-ij}\}$, profits are strictly decreasing in $w_{ij}$. ■

**Lemma 2** - *Consider some level of employment $n_{ij} \leq \overline{n}_{ij}$. Given competitor employment $\{n_{-ij}\}$, the lowest legal wage that delivers $n_{ij}$, is given by*

$$\max\left\{\underline{w}, \min\left\{w_L\left(n_{ij}, n_j, N\right), w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)\right\}\right\} \qquad (*)$$

*where $w_L\left(n_{ij}, n_j, N\right) = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$.*

*Proof:* Given competitor employment $\{n_{-ij}\}$, the mapping from wages to employment is one-to-one except when $\overline{n}_i = n_i$. In that case $\min\left\{w_L\left(n_{ij}, n_j, N\right), w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)\right\}$ is the lowest wage that delivers $n_{ij} = \overline{n}_{ij}$ employees. This wage may not be legal. The lowest legal wage that delivers $n_{ij} = \overline{n}_{ij}$ employees is therefore given by (∗). ■

Lemma 2 maps employment to legal wages. We call this mapping the firm's *perceived inverse labor supply curve*, which is the inverse labor supply curve that the firm faces *conditional on* a choice $\overline{n}_{ij}$, and also taking account of the minimum wage. Given competitor employment $\{n_{-ij}\}$ and substituting the firms choice of $w_{ij}$ conditional on choices of $\left(n_{ij}, \overline{n}_{ij}, n_j\right)$, we can write the firm's problem as

$$\max_{\overline{n}_{ij}, n_{ij}} z_{ij} n_{ij}^{\alpha} - w_{ij} n_{ij} \quad , \quad \text{subject to}$$

$$w_{ij} = w^p\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \begin{cases} \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & , \quad n_{ij} \in \left[0, \overline{n}_{ij}\right) \\ \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & , \quad n_{ij} = \overline{n}_{ij} \end{cases}$$

Using the monotonicity of $\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$ in $n_{ij}$,[16] we know that the *highest wage possible* for $n_{ij} \in \left[0, \overline{n}_{ij}\right]$ is at $\overline{n}_{ij}$. If $\left(\frac{\overline{n}_{ij}}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$ is less than $\underline{w}$, then it must be the case that $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} < \underline{w}$ for all $n_{ij} \in \left[0, \overline{n}_{ij}\right]$. Define the function $n_j\left(n_{ij}\right)$ as follows:

$$n_j\left(n_{ij}\right) := \left[n_{ij}^{\frac{\eta}{\eta+1}} + \sum_{k \neq i}^{M_j} n_{kj}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}.$$

Using this, and given $\overline{n}_{ij}$, we can write the perceived labor supply curve on $n_{ij} \in$

[16] Note $w\left(n_{ij}, \{n_{-ij}\}\right) = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} = n_{ij}^{\frac{1}{\eta}} n_j^{\frac{1}{\theta} - \frac{1}{\eta}} N^{\frac{1}{\varphi} - \frac{1}{\theta}}$ and that $\partial n_j / \partial n_{ij} > 0$ and $\eta > \theta$. Therefore, given competitor employment $\{n_{-ij}\}$, $\partial w\left(n_{ij}, \{n_{-ij}\}\right) / \partial n_{ij} > 0$.

$[0,\overline{n}_{ij}]$ as follows

$$w^p\left(n_{ij},\overline{n}_{ij},n_j,N\right)=\begin{cases}\underline{w} & \text{if } \underline{w}>\left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}\\ \max\left\{\underline{w},\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w}\leq\left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}.\end{cases}$$

Note that the perceived labor supply curve is not a function of $w_{ij}$. Given competitor employment $\{n_{-ij}\}$, the Cournot firm problem becomes,

$$\max_{\overline{n}_{ij},n_{ij}} z_{ij}n_{ij}^{\alpha}-w_{ij}n_{ij}$$

subject to $n_{ij}\leq\overline{n}_{ij}$ and the *perceived inverse labor supply curve* defined over $[0,\overline{n}_{ij}]$:

$$w^p\left(n_{ij},\overline{n}_{ij},n_j,N\right)=\begin{cases}\underline{w} & \text{if } \underline{w}>\left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}\\ \max\left\{\underline{w},\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w}\leq\left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}.\end{cases}$$

**(2) Optimal rationing constraint.** Consider the case of $\overline{n}_{ij}=\infty$. Given competitor employment $\{n_{-ij}\}$, the firm solves the following problem:

$$\max_{n_{ij}} z_{ij}n_{ij}^{\alpha}-\max\left\{\underline{w},\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}\right\}n_{ij}.$$

We can partition $n_{ij}$ into two sets. Let $\widetilde{n}_{ij}$ be such that

$$\underline{w}=\left(\frac{\widetilde{n}_{ij}}{n_j\left(\widetilde{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_j\left(\widetilde{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}$$

where

$$\max\left\{\underline{w},\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}\right\}=\begin{cases}\underline{w} & \text{if } n_{ij}<\widetilde{n}_{ij}\\ \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}} & \text{if } n_{ij}\geq\widetilde{n}_{ij}.\end{cases}$$

Note that the marginal revenue product is well defined and differentiable for all $n_{ij}$. Total labor costs are differentiable everywhere except at $n_{ij}=\widetilde{n}_{ij}$. However, the unconstrained labor supply curve is differentiable everywhere (note that $n_j$ depends on $n_{ij}$ and we suppress dependence on aggregates and competitor employment, both of which are taken as given),

$$\widehat{w}\left(n_{ij}\right)=\left(\frac{n_{ij}}{n_j\left(n_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_j\left(n_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}.$$

There are three possible first-order conditions, depending on the firm's optimal choice of $n_{ij}$ relative to $\widetilde{n}_{ij}$,

$$mrpl\left(n_{ij}\right)=\begin{cases}\underline{w} & n_{ij}<\widetilde{n}_{ij}\\ \in\left[\underline{w},\widehat{w}'\left(n_{ij}\right)n_{ij}+\widehat{w}\left(n_{ij}\right)\right] & n_{ij}=\widetilde{n}_{ij}\\ \widehat{w}'\left(n_{ij}\right)n_{ij}+\widehat{w}\left(n_{ij}\right) & n_{ij}>\widetilde{n}_{ij}.\end{cases}$$

Lemma 3 characterizes the firm's optimal choice of $n_{ij}$.

**Lemma 3** - *Given competitor employment $\left\{n_{-ij}\right\}$, the firm's optimal choice of $n_{ij}$ satisfies $mrpl\left(n_{ij}\right)\geq\underline{w}$ .*

*Proof:* If $n_{ij}<\widetilde{n}_{ij}$ then $mrpl\left(n_{ij}\right)=\underline{w}$. If $n_{ij}=\widetilde{n}_{ij}$ then $mrpl\left(n_{ij}\right)\geq\underline{w}$ where we have used $\widehat{w}'\left(\widetilde{n}_{ij}\right)\widetilde{n}_{ij}+\widehat{w}\left(\widetilde{n}_{ij}\right)=\widehat{w}'\left(\widetilde{n}_{ij}\right)\widetilde{n}_{ij}+\underline{w}>\underline{w}$. If $n_{ij}>\widetilde{n}_{ij}$, we need to show that show that $\widehat{w}'\left(n_{ij}\right)n_{ij}+\widehat{w}\left(n_{ij}\right)$ is also increasing in $n_{ij}$, therefore $mrpl\left(n_{ij}\right)=\widehat{w}'\left(n_{ij}\right)n_{ij}+\widehat{w}\left(n_{ij}\right)>\widehat{w}'\left(\widetilde{n}_{ij}\right)\widetilde{n}_{ij}+\widehat{w}\left(\widetilde{n}_{ij}\right)>\underline{w}$. We can rewrite the marginal cost the firm as follows:

$$mc\left(n_{ij}\right)=w'\left(n_{ij}\right)n_{ij}+w\left(n_{ij}\right)=\left[\frac{w'\left(n_{ij}\right)n_{ij}}{w\left(n_{ij}\right)}+1\right]w\left(n_{ij}\right)=\left[\varepsilon^{Inv}\left(n_{ij}\right)+1\right]w\left(n_{ij}\right).$$

We can then show that $mc'(n_{ij})>0$ so long as $\varepsilon^{Inv\prime}(n_{ij})>0$:

$$mc'\left(n_{ij}\right)=\underbrace{\varepsilon^{Inv\prime}\left(n_{ij}\right)}_{\text{RHS positive if this is positive}}w\left(n_{ij}\right)+\left[\varepsilon^{Inv}\left(n_{ij}\right)+1\right]w'\left(n_{ij}\right)$$

Following the derivations in BHM this is true in the Cournot oligopsony problem of the firm since we have

$$\varepsilon^{Inv}\left(n_{ij}\right)=\frac{1}{\theta}s_{ij}+\left(1-s_{ij}\right)\frac{1}{\eta}$$

which is increasing (holding $n_{-ij}$ fixed), as higher $n_{ij}$ increases also $w_{ij}$, which increases $s_{ij}$, which pushes toward the larger $1/\theta$ term which is > $1/\eta$.■

Define $\overline{n}_{ij}$ by $mrpl\left(\overline{n}_{ij}\right)=\underline{w}$. Then by Lemma 3 we know that $mrpl\left(n_{ij}\right)\geq\underline{w}=mrpl\left(\overline{n}_{ij}\right)$, and hence $n_{ij}\leq\overline{n}_{ij}$. Therefore we can always set $\overline{n}_{ij}$ by $mrpl\left(\overline{n}_{ij}\right)=\underline{w}$ and this rationing constraint is non-binding away from the minimum wage, and weakly binding at the optimal value of employment when the firm is constrained by the minimum wage. Lemma 4 formally proves this result.

**Lemma 4** - *It is (weakly) optimal for the firm to choose a rationing constraint $\overline{n}_{ij}=\left(\alpha z_{ij}/\underline{w}\right)^{\frac{1}{1-\alpha}}$.*

*Proof:* As above, define $\overline{n}_{ij}$ by $mrpl\left(\overline{n}_{ij}\right) = \underline{w}$. Note that $mrpl\left(n_{ij}\right) = \alpha z_{ij} n_{ij}^{\alpha-1}$, thus $\underline{w} = \alpha z_{ij} \overline{n}_{ij}^{\alpha-1}$, and $mrpl\left(\overline{n}_{ij}\right) = \underline{w}$. Also note that $mrpl\left(n\right)$ is *decreasing* in $n$. Conditional on competitor employment, we define three regions (I,II, and III) on the perceived inverse labor supply curve:

$$w^p\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \begin{cases} \underbrace{\underline{w}}_{\text{Region III}} & \text{if } \underline{w} > \left(\frac{\overline{n}_{ij}}{n_j(\overline{n}_{ij})}\right)^{\frac{1}{\eta}} \left(\frac{n_j(\overline{n}_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\ \max\left\{ \underbrace{\underline{w}}_{\text{Region II}} , \underbrace{\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}}_{\text{Region I}} \right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j(\overline{n}_{ij})}\right)^{\frac{1}{\eta}} \left(\frac{n_j(\overline{n}_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}. \end{cases}$$

Let **Region I** be the case that $\left(w_{ij}, n_{ij}, \overline{n}_{ij}, n_j\right)$ are such that the firm is on the *second* part of the second branch of $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$. Let **Region II** be the case that $\left(w_{ij}, n_{ij}, \overline{n}_{ij}, n_j\right)$ are such that the firm is on the *first* part of the second branch of $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$. Let **Region III** be the case that $\left(w_{ij}, n_{ij}, \overline{n}_{ij}, n_j\right)$ are such that the firm is on the first branch of $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$ (note that this *does not* require that $n_{ij} = \overline{n}_{ij}$, although this will be the case under firm optimality). We proceed by solving for the optimal $n_{ij}$ in each Region and show that $\overline{n}_{ij}$ is weakly binding, and thus weakly optimal.

**Region I.** Suppose the firm is in Region I, then it is solving the problem (taking competitor employment as given),

$$\max_{n_{ij} \leq \overline{n}_{ij}} z_{ij} n_{ij}^{\alpha} - \widehat{w}\left(n_{ij}\right) n_{ij} \quad , \quad \widehat{w}\left(n_{ij}\right) = \left(\frac{n_{ij}}{n_j\left(n_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(n_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

and hence has first order condition

$$\begin{aligned} mrpl\left(n_{ij}^*\right) &= \widehat{w}'\left(n_{ij}^*\right) n_{ij}^* + \widehat{w}\left(n_{ij}^*\right) && , \quad \text{then since } \widehat{w}'\left(n_i\right) > 0 \\ &> \widehat{w}\left(n_{ij}^*\right) && , \quad \text{then since in Region I, then } \widehat{w}\left(n_{ij}^*\right) > \underline{w} \\ &> \underline{w} = mrpl(\overline{n}_{ij}) && , \quad \text{by the conjectured } \overline{n}_{ij},\ \underline{w} = mrpl(\overline{n}_{ij}). \end{aligned}$$

Since $mrpl(n_{ij}^*) > mrpl(\overline{n}_{ij})$, and $mrpl$ is decreasing, $n_{ij}^* < \overline{n}_{ij}$. Therefore the constraint is slack. Note also that the value is independent of $\overline{n}_{ij}$.

**Region II.** Suppose the firm is in Region II, then $w_{ij} = \underline{w}$ and $\underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j(\overline{n}_{ij})}\right)^{\frac{1}{\eta}} \left(\frac{n_j(\overline{n}_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$. Define $\widetilde{n}_{ij}$ such that

$$\underline{w} = \left(\frac{\widetilde{n}_{ij}}{n_j\left(\widetilde{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\widetilde{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}.$$

Note that for $n_{ij} \leq \widetilde{n}_{ij}$, then $\max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j(n_{ij})}\right)^{\frac{1}{\eta}} \left(\frac{n_j(n_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} = \underline{w}$, and hence the firm is in Region II, while the firm is not in Region II for $n_{ij} > \widetilde{n}_{ij}$. Since the firm is in Region II, then also have $\underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j(\overline{n}_{ij})}\right)^{\frac{1}{\eta}} \left(\frac{n_j(\overline{n}_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$, which by monotonicity of the labor supply curve implies that $\widetilde{n}_{ij} \leq \overline{n}_{ij}$. Therefore the $n_{ij}$ for which the firm is in Region II are all weakly less than $\overline{n}_{ij}$. Note that this *does not* require knowing anything about the $mrpl_{ij}$, its simply by definition of Region II. Note also that the value is independent of $\overline{n}_{ij}$.

**Region III.** Suppose the firm is in Region III, then $w_{ij} = \underline{w}$ for all $n_{ij} \leq \overline{n}_{ij}$. Therefore the firm is solving:

$$\max_{n_{ij}} z_{ij} n_{ij}^{\alpha} - \underline{w} n_{ij}$$

and hence has the first order condition $mrpl\left(n_{ij}^{*}\right) = \underline{w} = mrpl\left(\overline{n}_{ij}\right)$ therefore the constraint is weakly binding.■

Applying Lemma 4, we can write the firm problem with the constraint $\overline{n}_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}$ imposed:

$$\max_{n_{ij}} z_{ij} n_{ij}^{\alpha} - w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) n_{ij}$$

subject to $n_{ij} \leq \overline{n}_{ij}$ and $\overline{n}_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}$ and the *perceived labor supply curve*

$$w^{p}\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \begin{cases} \underline{w} & \text{if } \underline{w} > \left(\frac{\overline{n}_{ij}}{n_j(\overline{n}_{ij})}\right)^{\frac{1}{\eta}} \left(\frac{n_j(\overline{n}_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j(\overline{n}_{ij})}\right)^{\frac{1}{\eta}} \left(\frac{n_j(\overline{n}_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \end{cases}$$

This is the problem described in the main text.