

SUPPLEMENT TO “RETIREMENT FINANCING: AN OPTIMAL REFORM APPROACH”

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S1. CONSTRUCTION OF TAX SCHEDULES

IN THIS SECTION, we describe how to back out the optimal taxes from the optimal allocations and wedges discussed in the paper. Consider the following budget constraint for the individual:

$$\begin{aligned}
 (1 + \tau_c)c_j + a_{j+1} &= (w_{t+j}\varphi_j l_j - T_{y,j,t+j}(w_{t+j}\varphi_j l_j) + \text{Tr}_{j,t+j})\mathbf{1}[j < R] \\
 &\quad + (1 + r_{t+j})a_j - T_{a,j,t+j}((1 + r)a_j) \\
 &\quad + S_{j,t+j}(\mathcal{E}_t)\mathbf{1}[j \geq R] + B_{t+j}, \\
 a_{j+1} &\geq 0.
 \end{aligned} \tag{S1}$$

The following lemma and its proof illustrate the construction of a tax and transfer schedule as in (S1) such that individual optimizations’ first-order conditions are satisfied (in what follows, we adopt the following notation to avoid clutter: $u_{c,j}(\theta) \equiv u'(c_j(\theta))$ and $v_{l,j}(\theta) \equiv v'(l_j(\theta))$):

LEMMA S1: Consider an allocation $\{c_j(\theta), l_j(\theta)\}$ that satisfies the following implementability constraint:

$$\begin{aligned}
 U'(\theta) &= \sum_{j=0}^J \beta(\theta)^j P_j(\theta) \frac{\varphi'_j(\theta) l_j(\theta)}{\varphi_j(\theta)} v'(l_j(\theta)) \\
 &\quad + \sum_{j=0}^J \left(\frac{j\beta'(\theta)}{\beta(\theta)} + \frac{P'_j(\theta)}{P_j(\theta)} \right) \beta^j P_j(\theta) [u(c_j(\theta)) - v(l_j(\theta))].
 \end{aligned} \tag{S2}$$

Moreover, assume that $(\varphi_j(\theta) l_j(\theta))' > 0$ and

$$\sum_{s=j}^J \beta(\theta)^s P_s(\theta) [u_{c,s}(\theta) c'_s(\theta) - v_{l,s}(\theta) (\varphi_s(\theta) l'_s(\theta))'] > 0.$$

Then tax and transfer functions $T_{a,j}(\cdot)$, $T_{y,j}(\cdot)$, S_j together with asset holdings $a_j(\theta)$ exist so that the allocations $\{c_j(\theta), l_j(\theta), a_j(\theta)\}$ satisfy the budget constraints (S1) and the first-order conditions associated with the individual optimization.

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PROOF: We start by writing the first-order conditions for the maximization problem above for an individual of type θ :

$$1 - T'_{y,j}(\varphi_j(\theta)l_j(\theta)) = \frac{v'(l_j(\theta))}{\varphi_j(\theta)u_{c,j}(\theta)}, \quad (\text{S3})$$

$$u_{c_j} = \beta(1+r)p_{j+1}(\theta)(1 - T'_{a,j+1})u_{c_{j+1}}. \quad (\text{S4})$$

Equation (S3) is the individual intratemporal optimality condition and equation (S4) is the individual Euler equation.

We can use equation (S3) to back out the optimal marginal taxes on labor earnings at each age. This is possible because the efficient allocations of consumption and hours come directly from solving the planning problem. Thus, the earnings taxes can simply be defined by integrating over the implied marginal rate in (S3)—this is well-defined since output in each age is increasing in θ .

The calculation of optimal asset taxes, however, is not straightforward. The level of assets a cannot be pinned down independent from the marginal taxes $T'_{a,j+1}$. Therefore, we are going to assume that asset holdings of the lowest type are zero for all ages. This implies that in the equilibrium that decentralizes efficient allocations, the poorest individual is hand-to-mouth in all ages. Given this restriction, we can use the following procedure to find the optimal asset taxes.

We can combine equations (S3) and (S4) together with (S1) and use extensive algebra to show that the derivative of asset holdings with respect to θ , a'_j , satisfies

$$a'_j(\theta) = \frac{1}{u_{c,j}(\theta)} \sum_{s=j}^T \beta^{s-j} \frac{P_s(\theta)}{P_j(\theta)} [u_{c,s}(\theta)c'_s(\theta) - v_{l,s}(\theta)(\varphi_s(\theta)l_s(\theta))'].$$

Since by assumption $a_j(\underline{\theta}) = 0$, the above determines the level of asset holdings at each age and for each type:

$$a_j(\theta) = a_j(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} a'_j(\theta) d\theta.$$

Finally, using the Euler equation (S4), we must have

$$1 - T'_{a,j+1} = \frac{u_{c_j}}{\beta(1+r)p_{j+1}u_{c_{j+1}}}.$$

The above formula determines the marginal tax rate on asset holdings, and since $a'_j > 0$, a well-defined tax function on asset holdings must exist. This completes the construction.

Q.E.D.

The construction of the taxes and asset holdings is somewhat standard. In particular, earnings and asset taxes can be constructed from integrating the labor and saving wedges as defined above. Furthermore, fixing the intercept of taxes at each age determines the asset holdings of individuals. Finally, the assumptions imposed on allocations in the lemma ensure that assets and earnings at each age are increasing in θ and thus the tax functions constructed are well-defined.

We cannot derive a closed form formula for optimal taxes. However, our implementation procedure as discussed in the above paragraph provides a guideline on how to numerically compute the optimal tax functions.

Finally, note that the monotonicity constraints in Lemma S1 are necessary for existence of a tax function. While we have no way of theoretically checking that they are satisfied, our numerical simulations always involve a check that ensures that they are indeed satisfied. Needless to say, in all of our simulations the monotonicity constraints are satisfied.

S2. EXTENSIONS AND ROBUSTNESS

In this section, we provide supplementary material to the extensions discussed in Section 7 of the main paper.

S2.1. *Alternative Policy Reforms*

S2.1.1. *Optimal Privatization Reform*

In this section, we examine the importance of the asset subsidies. More precisely, we find the best reform policies that feature no old age transfers and no asset taxes/subsidies. In this regard, the efficiency gains from these policies can be viewed as an upper bound on what can be gained through privatization policies.

To carry out this exercise, we need to put restrictions on the type of policies available to the government. First, note that in the absence of asset taxes and subsidies, the individuals' consumption allocations must satisfy the following equation:

$$\frac{P_j(\theta)u'(c_j(\theta))}{P_{j+1}(\theta)\beta(\theta)(1+r)u'(c_{j+1}(\theta))} = 1 \quad (\text{S5})$$

for all ages j and for all ability types θ . This equation is simply the restriction that individuals do not face taxes/subsidies on their risk-free asset returns.

In order to find the best policies that respect the no tax/subsidy restrictions, we solve the planning problem subject to constraints (12)–(14), and the no tax/subsidy constraint (S5). Imposing constraint (S5) guarantees that the allocation can be implemented without the need for asset taxes/subsidies. However, these allocations cost more than the allocations that result under fully optimal reform policies discussed in the main text. Note that we only impose restrictions on asset taxes and do not impose any restriction on earnings taxes.

Table SI shows the changes in the aggregate variables. Note that under privatization policies, the present discounted value of consumption, net of labor income, rises relative to the status quo under all scenarios regarding prices and demographics. These policies lead to a slightly lower stock of capital. Consumption and output are slightly higher due to higher labor supply. The labor supply will be higher because of the lower tax on labor income (see Figure S1).

We should note that in a related paper, McGrattan and Prescott (2017) showed that a Pareto improving privatization policy does exist. Their model and calibration are different from this paper in many key dimensions. One of these differences is that they assumed Frisch elasticity of labor supply of 2.5, which is much higher than 0.5 that we use. We repeat our optimal reform exercise using Frisch elasticity of labor supply of 2.5 and report results in Section S2.2 of this supplement. We find that in a model with higher labor supply elasticity, optimal privatization policy is indeed Pareto improving. Therefore, we arrive at the same qualitative conclusion as McGrattan and Prescott (2017) when we adopt their value for labor supply elasticities. We should note that, while positive, the gains from privatization are significantly smaller than those of optimal reforms.

TABLE S1
AGGREGATE EFFECTS OF PRIVATIZATION^a

	Current U.S. (1)	Continue (2)	Optimal privatization		
			(3)	(4)	(5)
Factor prices					
Interest rate (%)	4.05	3.37	4.05	4.06	3.44
Wage	1	1.08	1	0.99	1.07
Values relative to GDP					
Consumption	0.69	0.69	0.69	0.69	0.69
Capital	4.00	4.41	3.99	3.99	4.37
Tax revenue (total)	0.26	0.29	0.23	0.23	0.23
Earnings tax	0.14	0.14	0.11	0.11	0.10
Consumption tax	0.04	0.07	0.04	0.04	0.07
Capital (corporate) tax	0.08	0.07	0.08	0.08	0.08
Transfers	0.16	0.19	0.05	0.05	0.06
To retirees	0.08	0.12	0.00	0.00	0.01
To workers	0.08	0.07	0.05	0.05	0.05
Asset subsidies	0.00	0.00	0.00	0.00	0.00
Change (%) (relative to status quo)					
GDP	–	–2.13	0.38	0.47	–2.14
Consumption	–	–2.38	0.50	0.53	–2.06
Capital	–	7.96	0.07	0.33	6.88
Labor input	–	–9.26	0.62	0.57	–8.56
PDV of net resources	–	–	4.94	5.59	5.06
Consumption equivalence	–	–	–0.37	–0.41	–0.64

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of U.S. status quo policies (with consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal privatization policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal privatization policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal privatization policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

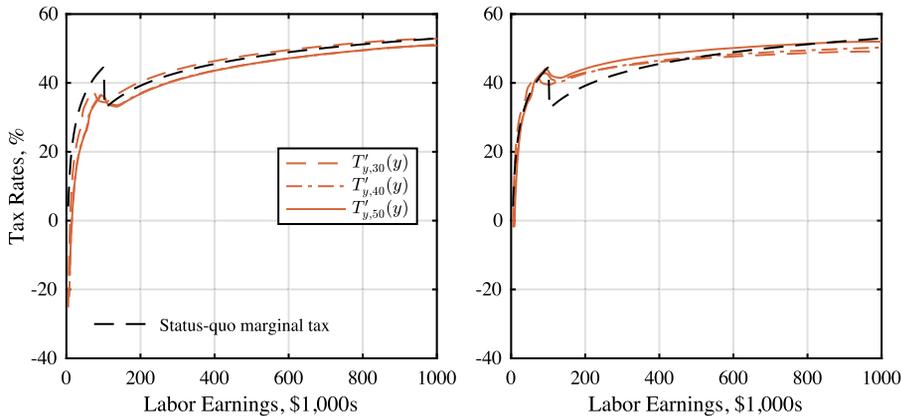


FIGURE S1.—Optimal labor income tax functions with *privatization* (no old age transfers and no asset subsidies). The left panel is optimal marginal taxes under *privatization*, while the right panel shows the same for the benchmark calibration. The black dashed line is the effective status quo tax schedule.

S2.1.2. *Optimal Linear Asset Subsidies*

In this section, we illustrate the importance of the progressivity in subsidies. In particular, in our optimal policy problem, we impose that distortions to the intertemporal margin be equated across all individuals of the same age; variations with age are allowed.

The linearity restriction on asset taxes or subsidies imposes restrictions on the set of implementable allocations. To be more precise, in order for allocations to be implementable by linear asset taxes or subsidies, the following condition must hold:

$$\frac{P_j(\theta)u'(c_j(\theta))}{P_{j+1}(\theta)\beta(\theta)(1+r)u'(c_{j+1}(\theta))} = \frac{P_j(\theta')u'(c_j(\theta'))}{P_{j+1}(\theta')\beta(\theta')(1+r)u'(c_{j+1}(\theta'))} \quad (\text{S6})$$

for all ages j and for all types θ and θ' . This equation simply implies that the intertemporal marginal rate of substitution must be equal across all types (and therefore, all asset levels).

In order to find the best policies that respect the linear asset tax or subsidy restrictions, we solve the planning problem with constraints (12)–(14) in the paper, and the linear tax constraint (S6). Imposing constraint (S6) guarantees that the allocation can be implemented by a linear set of asset taxes or subsidies.

It is important to remember that in our model, even in the absence of differential mortality and differential discount factor, the optimal subsidies on assets are not 0. If there is no annuity market, the asset subsidies are needed to correct the inefficiency due to incomplete markets. In that case, these subsidies will be linear and the rate will be equal to the average population mortality at each age. In Figure S2 (right panel), we plot the optimal linear asset subsidies in our model along with the average marginal taxes in a fully optimal system (with nonlinear subsidies) and average population mortality index. The figure shows a large difference in these three measures. The optimal linear subsidies are much lower than the average mortality in the population. This implies that, even in deriving simple policies with linear subsidies, the differential mortality cannot be ignored. In other words, we still need to include these features in the model to correctly capture the effect of heterogeneity in mortality on optimal policies.

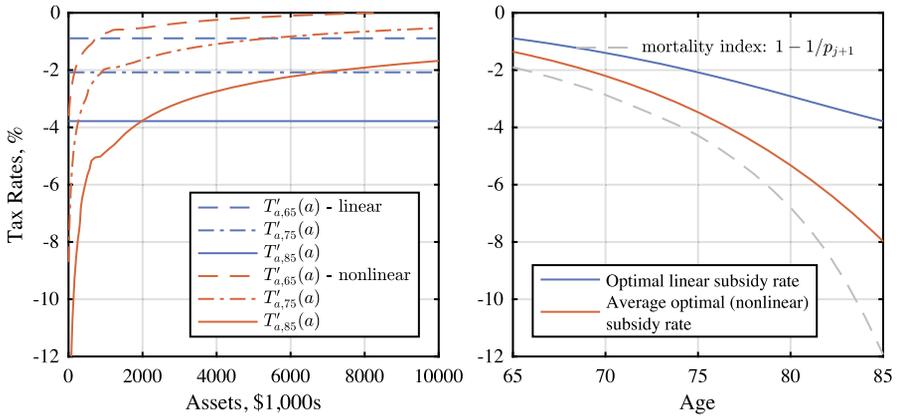


FIGURE S2.—Optimal asset tax functions: linear subsidies versus nonlinear subsidies. The left panel shows marginal taxes over all asset levels at ages 65, 75, and 85, while the right panel shows average marginal rates at each age from 65 to 85. Blue lines are optimal linear subsidies. Red lines are fully optimal nonlinear subsidies. The dashed line is the population average mortality index.

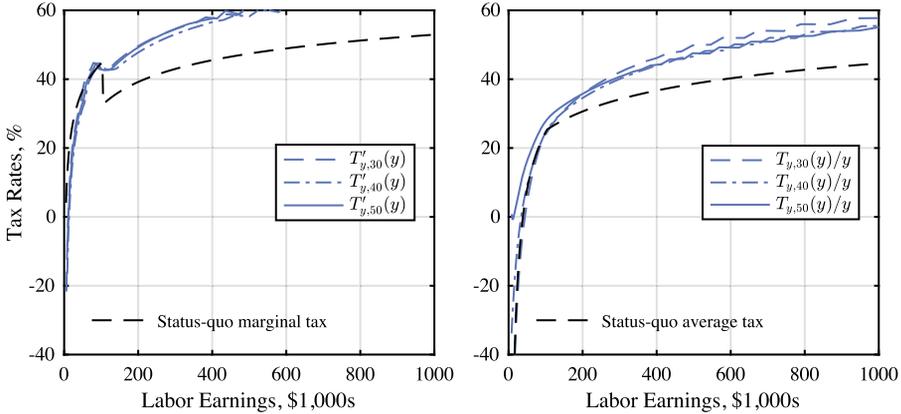


FIGURE S3.—Optimal earnings tax functions with linear asset subsidies. The left panel is marginal taxes, while the right panel is average taxes. The black dashed line is the effective status quo tax schedule.

Figure S3 shows the marginal labor income tax functions. Note that the linearity restriction on asset taxes or subsidies results in a negative marginal tax on labor income for the poorest individuals (left panel). When subsidies are linear, the marginal rates are much lower for the poorest individuals (relative to the ones that result from fully optimal policies). Therefore, imposing restrictions on proper asset subsidies puts the burden of the redistribution on the labor income tax. In the absence of proper asset subsidies, the consumption for the poor is more front-loaded. To accommodate high consumption, the labor income tax must be low (even negative). Also, note that the tax rates at the top are higher under policies with a linear subsidy restriction. Linear subsidies imply that high-income individuals receive too much asset subsidies (relative to the full optimal). Therefore, again, the burden of redistribution is on the labor income tax to correct this excess subsidization of the high-income workers.

Table SII shows the effect on aggregate variables. Aggregate output, consumption, and capital are affected similarly to the fully optimal reform. However, restricted policies only achieve a fraction of the cost savings that are achieved by the fully optimal reform policies.

S2.2. High Labor Supply Elasticity

In our benchmark calibration, we assume the Frisch elasticity of labor supply is $\varepsilon = 0.5$. This value is in the range estimated in micro studies and very common in quantitative life-cycle macroeconomic models.¹ In this section, we report our results for $\varepsilon = 2.5$, which is more in line with values calibrated using macro aggregates in the real business-cycle studies. We recalibrate our model using this value for ε . The calibrated parameters are presented in Table SIII.

We first check the optimality of the status quo policies using the conditions derived in Proposition 4 of the main text. We compute the intertemporal and intratemporal distortions for the status quo allocations and check how much equations (16), (17), (19) and inequality (18) are violated.

The results are demonstrated in Figures S4 and S5. The left panel in Figure S4 shows the right-hand side (black line) and left-hand side (dashed red line) of inequality (18).

¹See Keane (2011), Chetty et al. (2011), and Chetty (2012).

TABLE SII
AGGREGATE EFFECTS OF LINEAR SUBSIDIES^a

	Current U.S. (1)	Continue (2)	Optimal linear subsidies		
			(3)	(4)	(5)
Factor prices					
Interest rate (%)	4.05	3.37	4.05	3.80	3.30
Wage	1	1.08	1	1.03	1.09
Values relative to GDP					
Consumption	0.69	0.69	0.67	0.68	0.68
Capital	4.00	4.41	4.33	4.15	4.46
Tax revenue (total)	0.26	0.29	0.25	0.25	0.26
Earnings tax	0.14	0.14	0.12	0.13	0.11
Consumption tax	0.04	0.07	0.04	0.04	0.07
Capital (corporate) tax	0.08	0.07	0.09	0.08	0.07
Transfers	0.16	0.19	0.09	0.09	0.08
To retirees	0.08	0.12	0.00	0.00	0.00
To workers	0.08	0.07	0.02	0.02	0.03
Asset subsidies	0.00	0.00	0.07	0.07	0.05
Change (%) (relative to status quo)					
GDP	–	–2.13	5.08	2.26	–1.19
Consumption	–	–2.38	2.19	1.02	–1.80
Capital	–	7.96	13.68	5.97	10.14
Labor input	–	–9.26	–1.55	–0.52	–9.12
PDV of net resources	–	–	–6.21	–26.05	–6.18
Consumption equivalence			0.46	1.93	0.79

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of U.S. status quo policies (with consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal linear subsidy policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal linear subsidy policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal linear subsidy policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

Note that the inequality fails to hold not only at the Social Security earnings cap but also at the very top income levels. This is expected. When the elasticity of labor supply is high, it is more likely that for some workers there is a Laffer effect. Therefore, it is possible to

TABLE SIII
CALIBRATED PARAMETERS—HIGH LABOR SUPPLY ELASTICITY

Parameters	Description	Values
β_0	discount factor: level	0.975
β_1	discount factor: elasticity w.r.t θ	0.009
ψ	weight on leisure	0.644
<hr/>		
Targeted Moments	Data	Model
Wealth-income ratio	4.00	4.00
Wealth Gini	0.78	0.78
Average annual hours	2000	2000

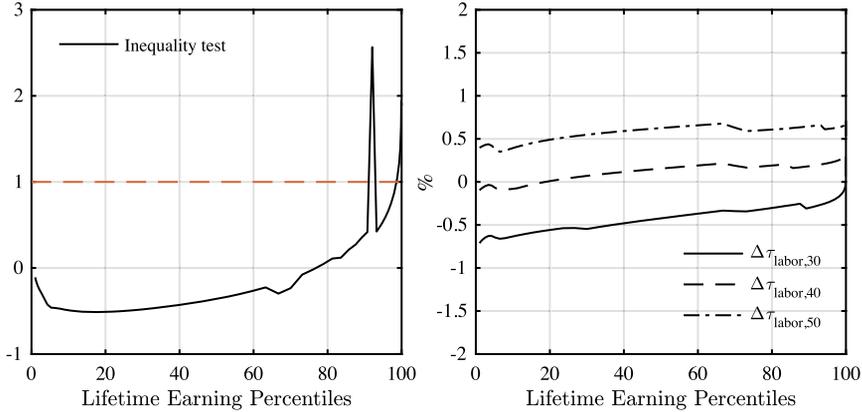


FIGURE S4.—Test of Pareto optimality for status quo policies—high labor supply elasticity . The left panel plots the two sides of inequality (18). The right panel depicts the change in earnings wedges required for equation (19) to hold.

reduce the tax rates for some individuals (and improve their welfare) while increasing tax revenue. However, this applies to a small fraction of the earnings distribution. As we see below, the optimal tax rates at the top are indeed much lower than the status quo.

The right panel of Figure S4 shows the changes in earnings wedges required for equation (19) to hold. As in the benchmark calibration, these deviations from efficiency are small. Figure S5 shows the changes in savings wedges required for equations (16) and (17) to hold. As in the baseline calibration, these figures show that the savings wedges are further away from efficiency, particularly at older ages.

Figures S6 and S7 show the optimal asset taxes and optimal labor income taxes, respectively. The only notation difference here is that optimal labor taxes on labor income deviate significantly from status quo, particularly at the top. This is mainly due to a Laffer effect. Since labor supply is more elastic, it is possible to reduce tax rates and expand the tax base without sacrificing welfare.

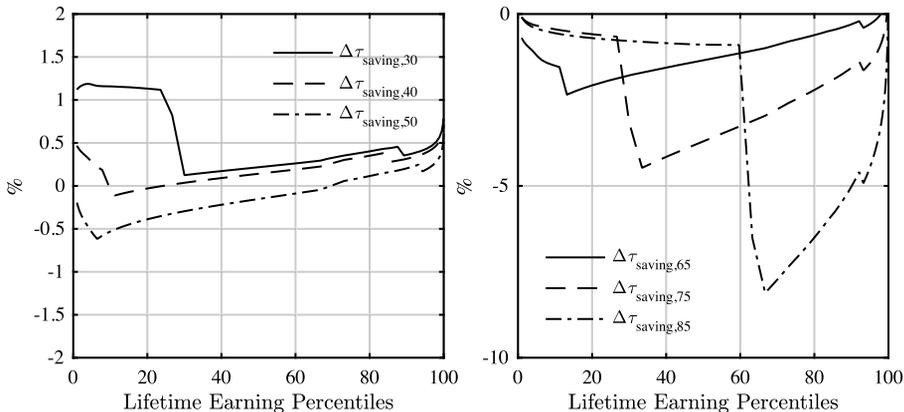


FIGURE S5.—Test of Pareto optimality for status quo policies—high labor supply elasticity. The left panel depicts the required change in savings wedges so that (16) holds at ages 30, 40, and 50. The right panel depicts the required change in savings wedges so that (17) holds at ages 65, 75, and 85.

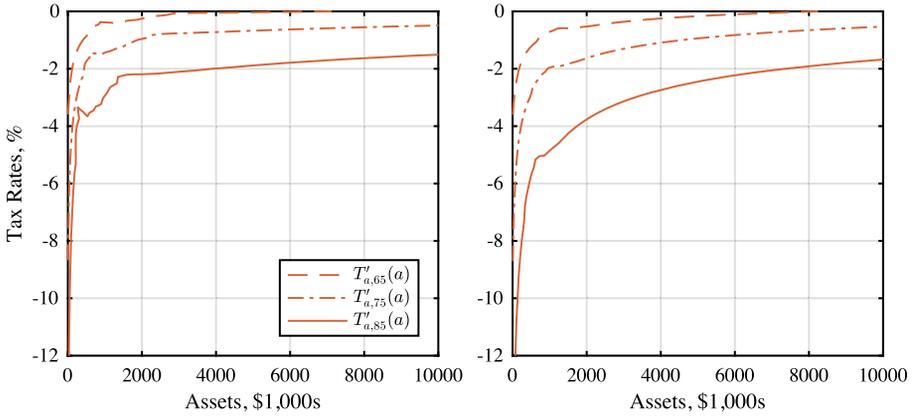


FIGURE S6.—Optimal asset tax functions—high labor supply elasticity. The left panel shows optimal marginal taxes with labor supply elasticity of 2.5 over all asset levels at ages 65, 75, and 85. The right panel shows the same for the benchmark model.

We report the aggregate effects of full optimal reform in Table SIV. To compare and contrast with the benchmark calibration, we also repeat the privatization exercise. These results are reported in Table SV. As we see, when elasticity of labor supply is high enough, even privatization yields efficiency gains, although the gains are smaller than those of the fully optimal reform.

To summarize, asset subsidies remain an integral part of Pareto optimal policies even in a specification with high labor supply elasticity. In this case, under optimal policies, major changes in earnings taxes apply only to top earning levels, while large asset subsidies apply to all individuals.

S2.3. Alternative Status quo Tax Function

In this section, we report our results with a non-smooth approximation of the status quo tax function. We use effective marginal federal (and state and local) tax rates for

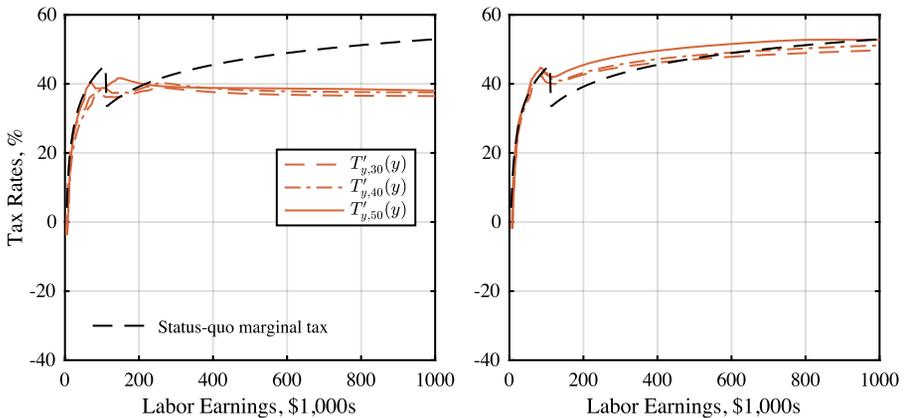


FIGURE S7.—Optimal earnings tax functions—high labor supply elasticity. The left panel shows optimal marginal taxes with labor supply elasticity of 2.5. The right panel shows the same for the benchmark model. The black dashed line is the effective status quo tax schedule.

TABLE SIV
 AGGREGATE EFFECTS OF OPTIMAL POLICIES—HIGH LABOR SUPPLY ELASTICITY^a

	Current U.S.	Continue	Optimal reform		
	(1)	(2)	(3)	(4)	(5)
Factor prices					
Interest rate (%)	4.05	3.48	4.05	3.89	3.48
Wage	1.00	1.06	1.00	1.02	1.06
Values relative to GDP					
Consumption	0.69	0.69	0.67	0.68	0.69
Capital	4.00	4.34	4.27	4.09	4.34
Tax revenue (total)	0.26	0.29	0.26	0.27	0.27
Earnings tax	0.14	0.15	0.14	0.15	0.13
Consumption tax	0.04	0.07	0.04	0.04	0.07
Capital (corporate) tax	0.08	0.08	0.09	0.08	0.08
Transfers	0.17	0.20	0.14	0.14	0.13
To retirees	0.08	0.12	0.02	0.03	0.04
To workers	0.09	0.08	0.07	0.07	0.06
Asset subsidies	0.00	0.00	0.05	0.05	0.03
Change (%) (relative to status quo)					
GDP	–	–0.56	4.44	2.16	0.87
Consumption	–	–0.29	2.03	1.38	1.16
Capital	–	7.94	11.57	4.46	9.41
Labor input	–	–6.66	–1.05	0.42	–5.26
PDV of net resources	–	–	–8.46	–17.76	–5.01
Consumption equivalence			0.83	1.73	0.73

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of the U.S. status quo policies (with the consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal reform policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal reform policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal reform policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

head of household with one child as reported in [Harris \(2005\)](#). We plot these effective marginal rates and implied average tax rates in [Figure S8](#). The left panel shows the tax rates excluding payroll taxes. The right panel includes payroll taxes. In contrast to the benchmark tax function that we use, we see many jumps in tax rates, especially at lower incomes.

[Figures S9](#) and [S10](#) show the result of the efficiency test. The left panel of [Figure S9](#) shows the inequality ([S10](#)). Despite a non-smooth status quo tax function, the earnings tax function fails the inequality ([S10](#)) only at the Social Security maximum taxable income. However, it comes close to being violated at a lower income level as well. This is the large drop in the effective marginal tax rate due to transition from EITC phase-out (which implies the effective marginal rate of 31%) to the 15% bracket (see [Harris \(2005\)](#) for more details). Additionally, the deviations from the tax-smoothing equations (equations (16), (17), and (19)) vary a lot more relative to the benchmark. [Figure S11](#) depicts the optimal savings taxes. There is no significant change in policy with regard to asset subsidies.

Finally, [Table SVI](#) shows the aggregate implications of the reform under various scenarios regarding partial or general equilibrium and current versus future demographics. The aggregate implications are very similar to the ones reported in the paper for the main model.

TABLE SV
 AGGREGATE EFFECTS OF OPTIMAL PRIVATIZATION POLICIES—HIGH LABOR SUPPLY ELASTICITY^a

	Current U.S. (1)	Continue (2)	Optimal privatization		
			(3)	(4)	(5)
Factor prices					
Interest rate (%)	4.05	3.35	4.05	3.81	3.449
Wage	1.00	1.06	1.00	1.03	1.06
Values relative to GDP					
Consumption	0.69	0.69	0.68	0.69	0.69
Capital	4.00	4.34	4.12	4.04	4.34
Tax revenue (total)	0.26	0.29	0.25	0.25	0.25
Earnings tax	0.14	0.15	0.13	0.13	0.13
Consumption tax	0.04	0.07	0.04	0.04	0.07
Capital (corporate) tax	0.08	0.08	0.08	0.08	0.08
Transfers	0.17	0.20	0.12	0.12	0.13
To retirees	0.08	0.12	0.01	0.01	0.03
To workers	0.09	0.08	0.11	0.11	0.10
Asset subsidies	0.00	0.00	0.00	0.00	0.00
Change (%) (relative to status quo)					
GDP	–	–0.56	3.38	2.49	1.63
Consumption	–	–0.29	2.36	2.15	1.96
Capital	–	7.94	6.41	3.49	10.16
Labor input	–	–6.66	1.05	1.73	–4.48
PDV of net resources	–	–	–2.66	–6.42	–1.67
Consumption equivalence	–	–	0.26	0.63	0.25

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of the U.S. status quo policies (with the consumption tax adjusted to balance the government’s budget constraint). Column (3) is the optimal privatization policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal privatization policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal privatization policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

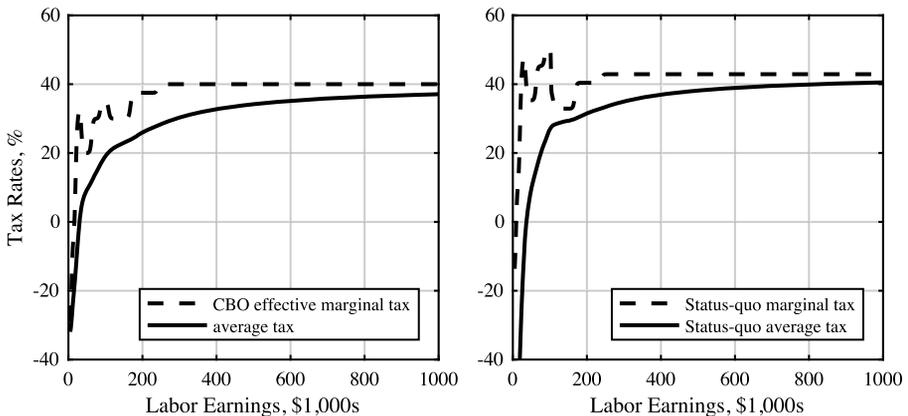


FIGURE S8.—Alternative tax functions. The left panel is the effective marginal (and average) tax rates for head of household with one child as reported in Harris (2005), excluding payroll taxes. The right panel is the total effective tax rates including payroll taxes.

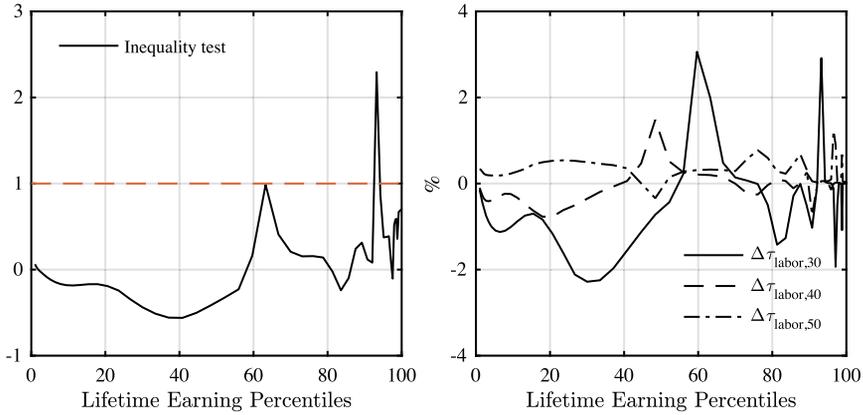


FIGURE S9.—Test of Pareto optimality for status quo policies with CBO effective tax rates. The left panel plots the two sides of the inequality (18). The right panel depicts the change in earnings wedges required for (19) to hold.

S2.4. *Small Open Economy*

In the main text, we described the transition from the current steady state in the U.S. economy to a new steady state with new demographic parameters. In performing that exercise, we maintained the closed economy assumption and determined factor prices in general equilibrium. In this section, we report the steady-state result under a small open economy (fixed factor price assumptions). Figures S12 and S13 show the optimal earnings and asset tax functions at the new steady state, respectively. Table SVII reports the aggregate implications. There is no significant difference between these numbers and the ones reported for the main exercise.

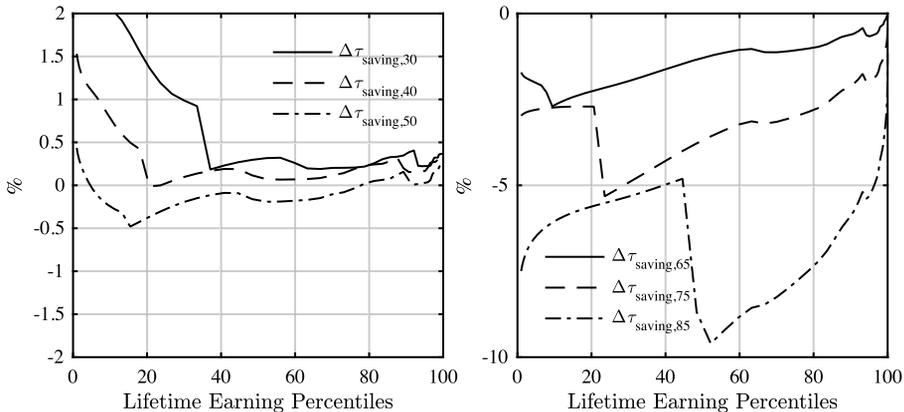


FIGURE S10.—Test of Pareto optimality for status quo policies with CBO effective tax rates. The left panel depicts the required change in savings wedges so that (16) holds at ages 30, 40, and 50. The right panel depicts the required change in savings wedges so that (17) holds at ages 65, 75, and 85.

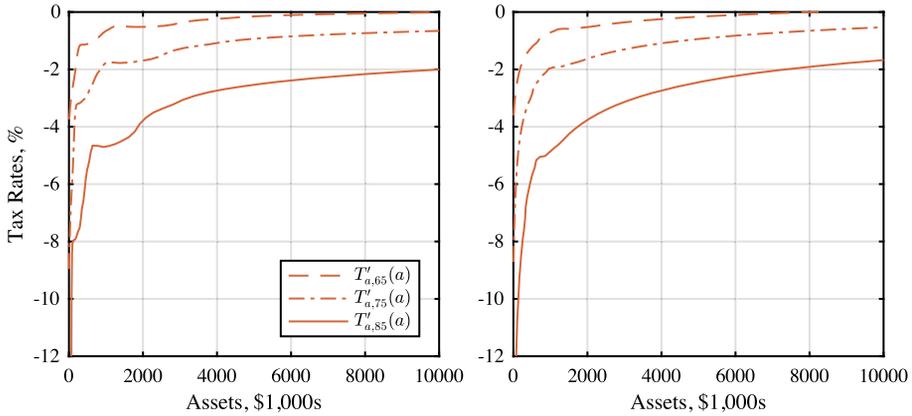


FIGURE S11.—Optimal asset tax functions for the alternative status quo tax policy (CBO effective tax rates). The left panel shows the optimal marginal taxes for calibration with CBO effective tax rates at ages 65, 75, and 85. The right panel shows the same for the benchmark model.

TABLE SVI

AGGREGATE EFFECTS OF OPTIMAL POLICIES—ALTERNATIVE (STATUS QUO) EARNINGS TAX FUNCTION^a

	Current U.S. (1)	Continue (2)	Optimal reform		
			(3)	(4)	(5)
Factor prices					
Interest rate (%)	4.05	3.38	4.05	3.83	3.33
Wage	1	1.08	1	1.02	1.08
Values relative to GDP					
Consumption	0.69	0.69	0.67	0.68	0.68
Capital	4.00	4.40	4.29	4.13	4.44
Tax revenue (total)	0.26	0.29	0.27	0.27	0.28
Earnings tax	0.14	0.14	0.14	0.15	0.14
Consumption tax	0.04	0.07	0.04	0.04	0.07
Capital (corporate) tax	0.08	0.07	0.09	0.08	0.07
Transfers	0.16	0.19	0.15	0.15	0.14
To retirees	0.08	0.12	0.02	0.02	0.03
To workers	0.08	0.07	0.05	0.05	0.05
Asset subsidies	0.00	0.00	0.08	0.08	0.06
Change (%) (relative to status quo)					
GDP	—	-2.13	4.07	1.64	-1.41
Consumption	—	-2.30	1.57	0.59	-1.87
Capital	—	7.67	11.45	4.76	9.39
Labor input	—	-9.08	-1.63	-0.70	-8.98
PDV of net resources	—	—	-10.81	-28.48	-7.17
Consumption equivalence	—	—	0.79	2.09	0.92

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of the U.S. status quo policies (with the consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal reform policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal reform policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal reform policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

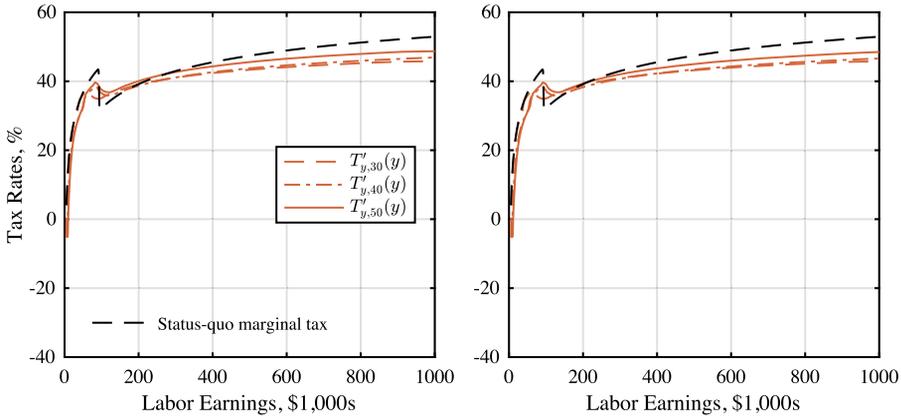


FIGURE S12.—Optimal labor income tax functions—small open economy assumption. The left panel shows the marginal taxes in a small open economy, while the right panel shows the same for the benchmark model. The black dashed line is the effective status quo tax schedule. Both panels show the tax functions in the new steady state after demographics transition.

S2.5. Out-of-Pocket Medical Expenditures

In this section, we describe the details of our estimation of out-of-pocket medical expenditure, as well as the calibration and optimal policy exercise in this extension of our model.

Out-of-Pocket Medical Expenditure

The estimation of out-of-pocket medical expenditure profiles closely follows [De Nardi, French, and Jones \(2010\)](#), who used a sample of 3259 single retired individuals in the

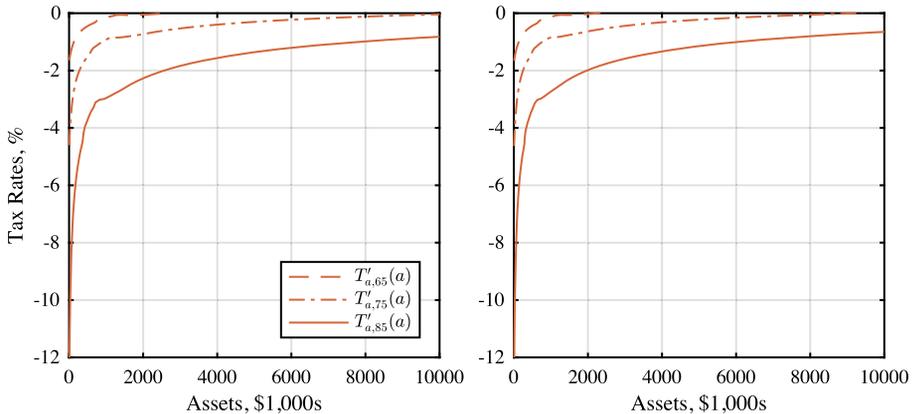


FIGURE S13.—Optimal asset tax functions—small open economy assumption. The left panel shows the marginal taxes in a small open economy over all asset levels at ages 65, 75, and 85, while the right panel shows the same for the benchmark calibration. Both panels show the tax functions in the new steady state after demographics transition.

TABLE SVII
 AGGREGATE EFFECTS OF OPTIMAL POLICIES—SMALL OPEN ECONOMY^a

	Current U.S.	Continue	Optimal reform	
	(1)	(2)	(3)	(4)
Factor prices				
Interest rate (%)	4.05	4.05	4.05	4.05
Wage	1.00	1.00	1.00	1.00
Values relative to GDP				
Consumption	0.69	0.80	0.67	0.82
Capital	4.00	4.00	4.31	4.00
Tax revenue (total)	0.26	0.30	0.27	0.31
Earnings tax	0.14	0.14	0.14	0.14
Consumption tax	0.04	0.09	0.04	0.09
Capital (corporate) tax	0.08	0.08	0.09	0.08
Transfers	0.16	0.21	0.15	0.18
To retirees	0.08	0.13	0.02	0.04
To workers	0.08	0.08	0.05	0.05
Asset subsidies	0.00	0.00	0.08	0.09
Change (%) (relative to status quo)				
GDP	–	–10.78	4.33	–11.39
Consumption	–	3.40	1.66	5.52
Capital	–	–10.78	12.29	–11.39
Labor input	–	–10.78	–1.80	–11.39
PDV of net resources	–	–	–11.08	–5.59
Consumption equivalence			0.82	0.56

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of the U.S. status quo policies (with the consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal reform policies with demographics fixed at the current U.S. values. Column (4) is the optimal reform with future demographics. In column (3), the percentage change in PDV is calculated relative to column (1). In column (4), the percentage change in PDV is calculated relative to column (2).

AHEAD survey between 1996 and 2006.² All individuals in the sample are aged 70 or older in 1994 (the year they enter the AHEAD survey). The measure of medical expenditure includes out-of-pocket expenditures on insurance premia, drug costs, hospital stays, nursing home care, doctor visits, dental visits, and outpatient care. The measure of income includes the value of Social Security benefits, defined pension benefits and annuities, veterans' benefit, welfare, and food stamps. The permanent income is defined as the average of all income that an individual receives over the period he or she is in the sample. The measure of permanent income is then used to compute each person's rank in the income distribution (which we refer to as permanent income ranking). In the estimation, this variable is used to capture the dependence of medical expenditure on income.

The estimation procedure is as follows. The mean of logged medical expenditure is modeled as a polynomial in age, a quadratic in the individual's permanent income ranking, and permanent income ranking interacted with age. We estimate the profiles using a fixed-effect estimator.³ The resulting estimation gives us the following model for the log

²The replication data, codes, and estimation results for De Nardi, French, and Jones (2010) are available at http://users.nber.org/~denardim/research/De_Nardi_French_Jones.zip. We would like to thank John Jones for providing details and patiently walking us through the estimation procedure.

³Here we depart from De Nardi, French, and Jones (2010) and do not include gender and health status in the estimation since we do not have these variables in our model.

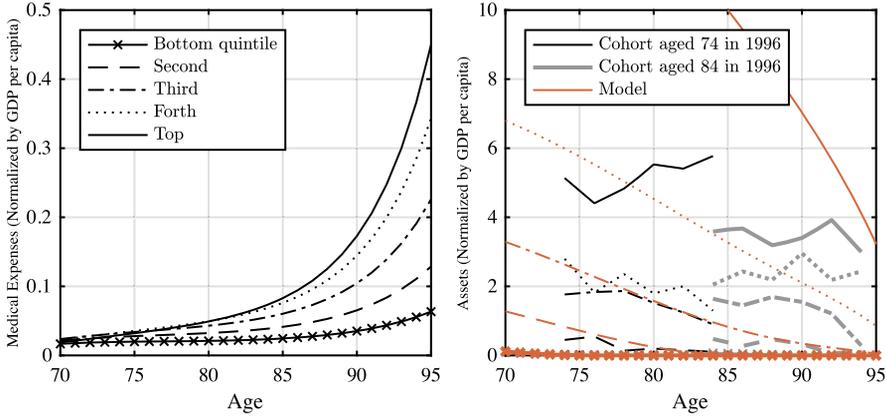


FIGURE S14.—The left panel shows the average out-of-pocket medical expenditures by permanent income. The right panel shows the median assets by permanent income quintile in the model (solid line) and data (dashed line). Data source: [De Nardi, French, and Jones \(2010\)](#) calculations for AHEAD cohorts who were 74 and 84 years old in 1996. Note that the lowest quintile has 0 assets in the model and in the data.

of medical expenditure:

$$\log(\text{medical expenditure at age } j) = \text{polynomial in } j + f(\text{permanent income ranking}, j),$$

where $f(\cdot, \cdot)$ is the sum of a quadratic function of permanent income ranking and permanent income ranking interacted with age.

We can now use our estimation to generate the profiles of age-dependent medical expenditures $m_j(\theta)$. In our framework, there is a one-to-one correspondence between permanent (or lifetime) income and ability type θ . Therefore, for each θ , we use $H(\theta)$ as the permanent income ranking. The left panel in Figure S14 shows the simulated mean medical expenditure profiles for each permanent income quintile normalized by GDP per capita in 1998. This corresponds to Figure 3 in [De Nardi, French, and Jones \(2010\)](#).

Preferences

In order to better capture the pattern of asset decumulation, we assume a curvature of $\sigma = 2$ for utility over consumption. Finally, we fix the value β_1 , the gradient of the discount factor with respect to ability θ , at the benchmark calibrated value and, therefore, we do not target the wealth Gini in cross section.⁴ As before, we choose β_0 (the location parameter for the discount factor) to match a capital to output ratio of 4. We also choose ψ (disutility of work) to match the average annual hours worked of 2000. Table SVIII shows the new calibration results.

To show how well the model captures the pattern of dissaving in retirement, we plot the median assets by permanent income quintile in the model as well as the median assets by permanent income quintile in the AHEAD data. The data are based on [De Nardi, French, and Jones \(2010\)](#) calculations for the AHEAD cohorts who were 74 and 84 years old in 1996. As we see, the model (solid line) captures the pattern of dissaving very well except for the assets of the top income quintile. However, under this calibration, the wealth Gini is 0.67. Therefore, the model fails to capture the cross section of wealth inequality.

⁴Our attempt to match the wealth Gini leads to a very slow decumulation of wealth at the top income quintile.

TABLE SVIII
PARAMETERS CALIBRATED USING THE MODEL—WITH
OUT-OF-POCKET MEDICAL EXPENDITURES

Parameters	Description	Values
β_0	discount factor: level	0.987
ψ	weight on leisure	0.037

Targeted Moments	Data	Model
Capital-output ratio	4.00	4.00
Average annual hours	2000	2000

Using the calibrated model, we compute the optimal earnings tax and asset subsidies. These are presented in Figures S15 and S16. As these figures demonstrate, there are no significant differences between the optimal policies derived in the model with out-of-pocket medical expenditures and those in our main exercise.

Table SIX shows the effect of the optimal policies on aggregate quantities. The last two rows present the efficiency gains. The second to last row is a decline in the present discounted value of lifetime consumption net of labor income for each cohort. The last row is the percentage decline that is required in non-medical consumption under the status quo policies to make the cost of allocations (in net PDV) equal across the two economies. As we see, the magnitude of cost savings is not very different than the ones in the main exercise. For the partial equilibrium calculation (column 3), they are lower, indicating that the value consumption smoothing over life cycle is lower in this model. However, in the general equilibrium setup with new demographics (column 5), the magnitudes are slightly greater. This is because with an aging population, medical expenditures provide even a stronger motive for saving. This leads to high stock of capital and low interest rate. This in effect magnifies the present discounted values.

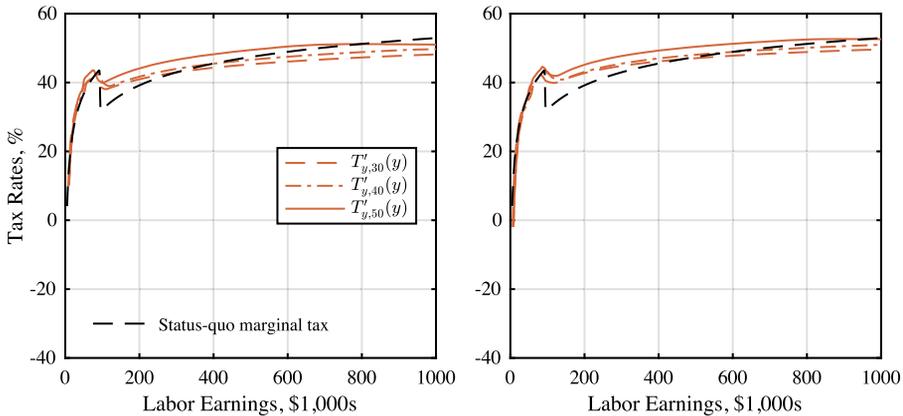


FIGURE S15.—Optimal labor income tax functions with out-of-pocket medical expenditures. The left panel is optimal marginal taxes with out-of-pocket medical expenditures. The right panel shows the same in the benchmark model. The black dashed line is the effective status quo tax schedule.

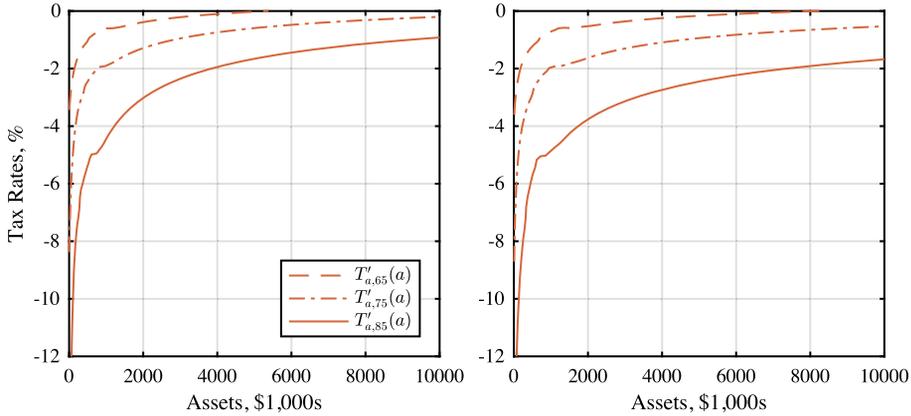


FIGURE S16.—Optimal asset tax functions with out-of-pocket medical expenditures. The left panel shows the optimal marginal taxes with out-of-pocket medical expenditures at ages 65, 75, and 85. The right panel shows the same in the benchmark model.

TABLE SIX
AGGREGATE EFFECTS OF OPTIMAL POLICIES—WITH OUT-OF-POCKET MEDICAL EXPENDITURES^a

	Current U.S. (1)	Continue (2)	Optimal reform		
			(3)	(4)	(5)
Factor prices					
Interest rate (%)	4.05	3.14	4.05	3.84	3.06
Wage	1	1.11	1	1.02	1.12
Values relative to GDP					
Consumption	0.69	0.68	0.68	0.68	0.67
Capital	4.00	4.57	4.21	4.12	4.63
Tax revenue (total)	0.24	0.27	0.26	0.26	0.27
Earnings tax	0.12	0.13	0.14	0.15	0.13
Consumption tax	0.04	0.07	0.04	0.04	0.07
Capital (corporate) tax	0.08	0.07	0.09	0.08	0.07
Transfers	0.15	0.18	0.18	0.17	0.16
To retirees	0.08	0.12	0.05	0.04	0.06
To workers	0.07	0.06	0.06	0.06	0.04
Asset subsidies	0.00	0.00	0.07	0.07	0.06
Change (%) (relative to status quo)					
GDP	—	1.70	2.18	0.9	1.92
Consumption	—	0.19	0.34	-0.31	-0.06
Capital	—	16.21	7.63	3.95	17.97
Labor input	—	-8.24	-2.02	-1.39	-8.93
PDV of net resources	—	—	-9.67	-28.76	-7.94
Consumption equivalence	—	—	0.66	1.97	0.99

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of the U.S. status quo policies (with the consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal reform policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal reform policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal reform policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

In summary, the inclusion of out-of-pocket medical expenditure results in a richer model that is able to capture more details in patterns of asset decumulation at older ages. However, the model's implication for an optimal policy does not change. Moreover, the efficiency gains from implementing optimal policies are still significant, but lower compared to the benchmark model without out-of-pocket medical expenditure.

S2.6. *Asset Tax Paid Directly by Households*

In this section, we consider an alternative calibration in which households directly pay taxes on their assets. We consider two alternatives: a flat tax on asset income and a progressive tax on asset income.

S2.6.1. *Flat Tax on Asset Income*

In this section, we assume that taxes on capital are not entirely paid by firms, and a portion is paid by individuals. In particular, there is a flat asset tax rate of 27 percent on individual asset income, while the corporate income tax rate is 8.2 percent. These numbers are chosen so that the after-tax return on savings is the same as in the benchmark model, and that corporate income tax revenue is 2 percent of GDP. As in the benchmark model, we reform the savings taxes, but not the corporate income tax rate. The rest of the model parameters are not changed.

Within this setup, we solve for optimal policies. In Figure S17, we compare the optimal asset subsidies under this setup (left panel) to those in the main exercise (right panel). Notice that at the bottom of the asset distribution, they are very similar. Inclusion of the asset tax in the calibration does not affect very-low-income individuals under the status quo, since they do not hold any asset (regardless of whether taxes are paid by them or by the firm). In the middle range, however, the removal of the asset tax raises the after-tax return on assets. This achieves some of the saving motives that the asset subsidy is supposed to provide. Therefore, the required optimal subsidy is smaller. Overall, however, two key features of asset subsidies are robust to this variation. They are large and they are progressive.

The result regarding optimal earnings taxes is less robust. The left panel in Figure S18 shows the optimal marginal taxes on earnings, and it compares them to those in the main

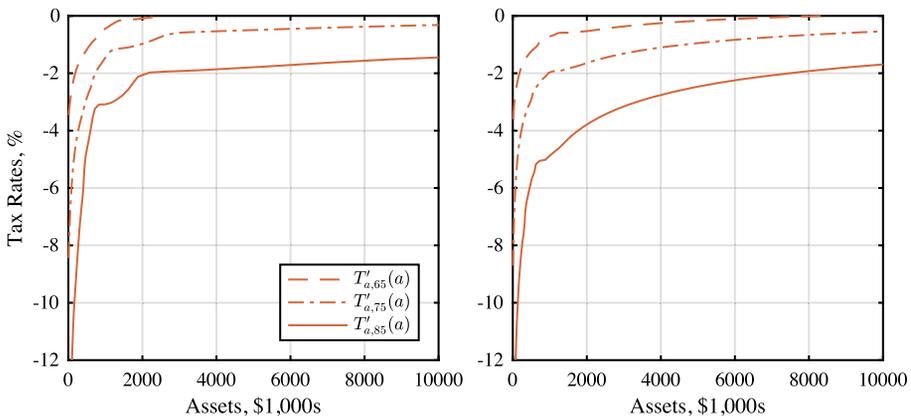


FIGURE S17.—Optimal asset tax functions in the model with a flat (status quo) asset tax. The left panel is optimal marginal taxes in the model with a flat (status quo) asset tax. The right panel shows the same for the benchmark model.

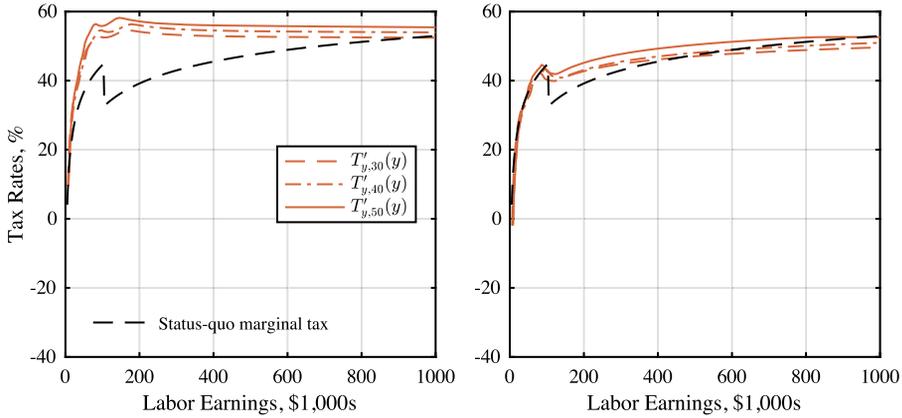


FIGURE S18.—Optimal labor income tax functions in the model with a flat (status quo) asset tax. The left panel is optimal marginal taxes in the model with a flat (status quo) asset tax. The right panel shows the same for the benchmark model.

exercise. The optimal marginal tax rates on earnings are much higher in this exercise, except at the bottom and the very top of the income distribution. Note that except for a fraction of people at the bottom, all individuals pay asset taxes under the status quo. An optimal reform removes this tax and instead subsidizes asset accumulation at older ages, which raises consumption at older ages. In order to reduce the cost of delivering the status quo welfare, optimal policies need to reduce consumption at younger ages. The only instrument the government has to achieve this is the earnings tax. Therefore, earnings taxes must increase. Note, however, that, in this model, removing the asset tax generates huge efficiency gains. Table SX shows the effect on aggregates is quite large. Specifically, the bottom two rows demonstrate a massive decline in cost measures, in terms of both present discounted value of net resources allocated to a cohort and flow consumption.

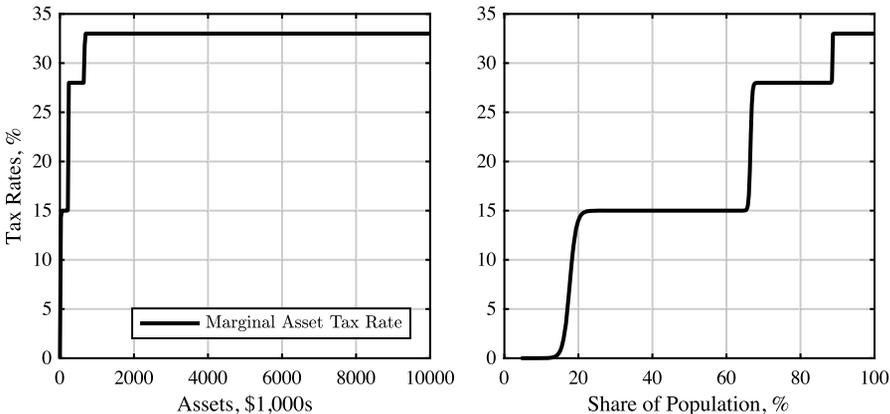


FIGURE S19.—Nonlinear (status quo) asset tax. The left panel shows the marginal asset tax rates on asset income as a function of assets. The right panel plots these tax rates against the fraction of the population who pays these tax rates.

TABLE SX
 AGGREGATE EFFECTS OF OPTIMAL POLICIES—WITH FLAT (STATUS QUO) ASSET TAX^a

	Current U.S.	Continue	Optimal reform		
	(1)	(2)	(3)	(4)	(5)
Factor prices					
Interest rate (%)	5.58	4.64	5.58	4.38	3.79
Wage	1	1.08	1	1.10	1.16
Values relative to GDP					
Consumption	0.69	0.69	0.61	0.66	0.66
Capital	4.00	4.41	5.36	4.54	4.87
Tax revenue (total)	0.32	0.36	0.21	0.25	0.25
Earnings tax	0.20	0.20	0.15	0.19	0.17
Consumption tax	0.04	0.07	0.03	0.04	0.07
Capital (corporate) tax	0.08	0.09	0.03	0.02	0.02
Transfers	0.16	0.19	0.13	0.13	0.11
To retirees	0.08	0.12	0.03	0.02	0.03
To workers	0.08	0.07	0.03	0.04	0.03
Asset subsidies	0.00	0.00	0.07	0.07	0.05
Change (%) (relative to status quo)					
GDP	–	–2.13	25.72	7.49	3.95
Consumption	–	–2.39	11.35	2.75	0.01
Capital	–	7.96	68.35	22.07	26.52
Labor input	–	–9.24	–7.10	–2.54	–10.40
PDV of net resources	–	–	–24.01	–61.26	–35.24
Consumption equivalence			4.04	10.30	8.13

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of the U.S. status quo policies (with the consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal reform policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal reform policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal reform policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

In summary, the qualitative implication of the policy regarding the optimality of asset subsidies is robust. What changes, however, is that implementing these subsidies (while maintaining the status quo welfare) implies a massive reform of earnings taxes. This reform, however, goes in the opposite direction of the most proposed tax reforms in that it implies higher tax rates for those with middle to upper middle income, while leaving the same tax rates for those at the bottom and the very top of the income distribution.

S2.6.2. *Progressive Tax on Asset Income*

Here, we repeat the previous exercise in a setting where the status quo asset tax is not flat but progressive. For this exercise, we keep the corporate income tax rate at 8.2 percent. However, we assume households face a progressive tax schedule on their asset income. We calibrate this tax schedule using [Poterba and Samwick \(2003\)](#) calculations of the effective tax rates on household portfolios. More precisely, we assume households face an incremental tax schedule with tax rates of 0, 15, 28, and 33 percent on their asset income. We choose the increment thresholds so that the distribution of tax rates in the model matches those computed by [Poterba and Samwick \(2003\)](#) for 1989 (Figure 1 in

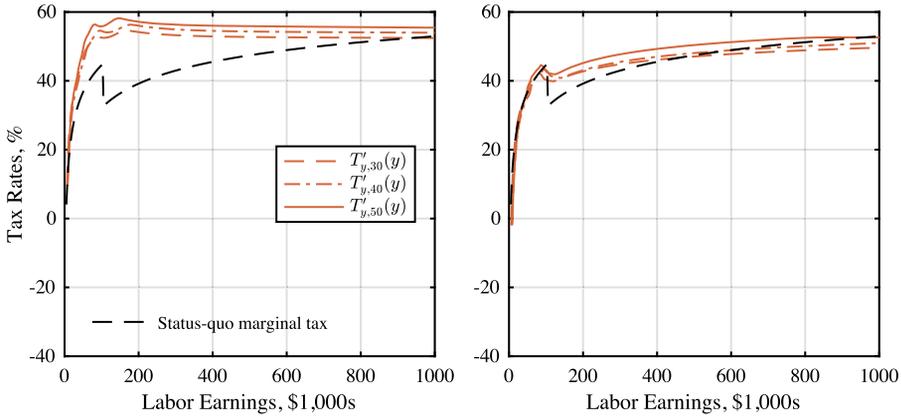


FIGURE S20.—Optimal labor income tax functions in a model with a progressive (status quo) asset tax. The left panel is optimal marginal taxes in the model with a progressive (status quo) asset tax. The right panel shows the same for the benchmark model.

their paper).⁵ Figure S19 shows the asset tax function (left panel) and the cross-sectional distribution of tax rates (right panel).

Even if, in this exercise, we use a progressive asset tax schedule, the results regarding optimal asset subsidies and earnings taxes remain the same as those of the model with a flat savings tax. Figure S20 shows the optimal earnings taxes (left panel) and those in the main exercise. Figure S21 shows the optimal asset subsidy in this setup (left panel) and that of the main exercise. Table SXI shows the aggregate implications.

Overall, as in the case with a flat asset tax, the optimal reform yields massive efficiency gains. To achieve these efficiency gains, optimal policies require implementing large progressive subsidies. However, unlike in our main exercise, to be able to implement those

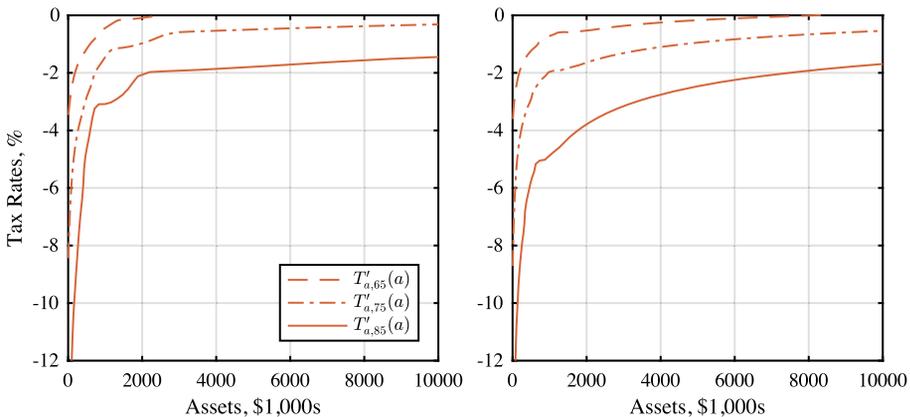


FIGURE S21.—Optimal asset tax functions in the model with a progressive (status quo) asset tax. The left panel is optimal marginal taxes in the model with a progressive (status quo) asset tax. The right panel shows the same for the benchmark model.

⁵The calculations by [Poterba and Samwick \(2003\)](#) ignore the implicit subsidies present in individual retirement accounts as well as the mortgage subsidies and the ability of rich individuals to work around savings taxes. We think this is one of the main reasons that reduce asset taxes for the whole population.

TABLE SXI
 AGGREGATE EFFECTS OF OPTIMAL POLICIES—WITH PROGRESSIVE (STATUS QUO) ASSET TAX^a

	Current U.S.	Continue	Optimal reform		
	(1)	(2)	(3)	(4)	(5)
Factor prices					
Interest rate (%)	5.58	4.63	5.58	4.44	3.39
Wage	1	1.16	1	1.10	1.15
Values relative to GDP					
Consumption	0.69	0.69	0.61	0.66	0.66
Capital	4.00	4.42	5.32	4.51	4.84
Tax revenue (total)	0.31	0.34	0.21	0.24	0.25
Earnings tax	0.20	0.20	0.15	0.18	0.16
Consumption tax	0.04	0.07	0.03	0.04	0.07
Capital (corporate) tax	0.08	0.07	0.03	0.02	0.02
Transfers	0.15	0.19	0.12	0.12	0.11
To retirees	0.08	0.12	0.03	0.02	0.03
To workers	0.07	0.06	0.05	0.03	0.03
Asset subsidies	0.00	0.00	0.07	0.07	0.06
Change (%) (relative to status quo)					
GDP	–	–1.99	25.44	7.27	3.77
Consumption	–	–2.29	11.49	2.65	0.11
Capital	–	8.20	66.80	20.97	25.44
Labor input	–	–10.32	–6.43	–2.21	–10.32
PDV of net resources	–	–	–20.48	–54.77	–31.20
Consumption equivalence			3.61	9.65	7.45

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of the U.S. status quo policies (with the consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal reform policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal reform policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal reform policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

subsidies and achieve the cost savings, the earnings tax rates must rise for those with a middle to upper income.

S3. CONSUMPTION PROFILES

To demonstrate where the efficiency gains (cost savings) come from, we plot the sample consumption and earnings profiles in Figure S22. The left panel shows the 25th, 50th, and 75th percentile of consumption at each age under both status quo policies and optimal reform policies. The right panel displays the same moments for earnings (before taxes and transfers) under both policies.

Note first that the status quo consumption profiles are hump shaped. This is consistent with evidence on consumption over life cycle as documented by [Gourinchas and Parker \(2002\)](#) and [Fernández-Villaverde and Krueger \(2007\)](#) (among others). However, these profiles are steeper and peak later in life relative to the estimated profiles in CEX data. This is mainly because matching the capital-output ratio in our model requires a higher value for the discount factor. This leads to steeper profiles and delaying the peak, especially for higher-ability groups (who also have lower mortality).

Consumption under the optimal reform policies closely follows the status quo consumption at younger ages. It is only at older ages that the two values of consumption differ.

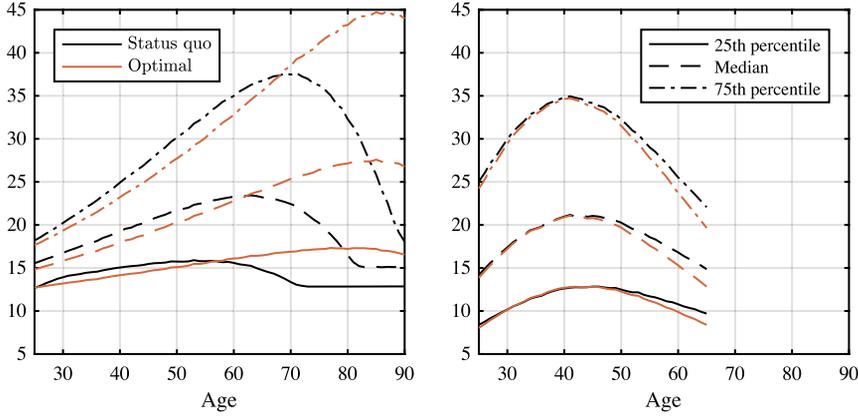


FIGURE S22.—The left panel is the 25th, 50th, and 75th percentile of consumption at each age under the status quo (black) and optimal reform policies (red). The right panel shows the same moments for earnings (before tax and transfers).

This is when mortality becomes large (particularly for poorer types) and providing annuitization is valuable. Note that this increase in consumption in old ages is compensated by a reduction in consumption when individuals are young. When evaluating the present discounted value of consumption for a cohort, this young-age consumption is less discounted. Therefore, this reduction in consumption in young age is the main reason the consumption cost of delivering the status quo welfare goes down. Moreover, as we see in the right panel of Figure S22, there is little change in earnings. Therefore, the present value of consumption next to labor income falls.

S4. MODEL WITH BEQUEST MOTIVES

In this section, we describe the details of the model with bequest motives:⁶

$$P_j(\theta) = \prod_{s=0}^j p_s(\theta).$$

Individuals have preferences over consumption and leisure, a joy-of-giving bequests motive, and a time (and state) separable utility function

$$\sum_{j=0}^J \beta(\theta)^j P_j(\theta) [u(c_j(\theta)) - v(l_j(\theta)) + \beta(1 - p_{j+1}(\theta))w(b_{j+1}(\theta))], \quad (S7)$$

where $\beta(\theta)$ is the subjective discount factor, $P_j(\theta)$ is probability of survival to age t , and $1 - p_{j+1}(\theta)$ is mortality rate at the end of age j . An individual who is alive at age j enjoys leaving bequest b_{j+1} if he or she dies at the end of period j . As in the main model, we assume there is no market for survival contingent assets (i.e., annuities and life insurance policies). Therefore, there are both voluntary and accidental bequests in the model.

The government uses nonlinear taxes on earnings from supplying labor, including the Social Security tax, while we assume that there is a linear tax on capital income and consumption. The government uses the revenue from taxation to finance transfers to workers

⁶Extending this setup to one that includes both bequest motives and exogenous out-of-pocket medical expenditure is straightforward.

and Social Security payments to retirees. While transfers are assumed to be equal for all individuals, Social Security benefits are not and depend on an individual's lifetime income. The difference with the benchmark model is that we allow for the government to tax the assets of the deceased in a distortionary fashion. We call this a bequest tax and denote it by $T_{b,j}^s(\cdot)$ to distinguish it from the asset tax or subsidy that an individual pays or receives upon survival $T_{a,j}^s(\cdot)$. The superscript s stands for status quo.

Given the above market structure and government policies, each individual faces a sequence of budget constraints of the following form:

$$(1 + \tau_c)c_j + a_{j+1} = (w\varphi_j l_j - T_{y,j}^s(w\varphi_j l_j) + \text{Tr}_j^s)\mathbf{1}[t < R] \\ + (1 + r)a_j - T_{a,j}^s((1 + r)a_j) + S_j^s(Y_j)\mathbf{1}[j \geq R] + B_j, \quad (\text{S8})$$

$$b_{j+1} = (1 + r)a_{j+1} - T_{b,j}^s((1 + r)a_{j+1}), \quad (\text{S9})$$

where $T_{y,j}^s(\cdot)$ is the income tax function on earnings from labor, Tr_j^s are transfers to working individuals, S_j^s is the retirement benefit from the government. We assume that bequests are collected and distributed as a lump-sum transfer B to the entire population. The dependence of retirement benefits on lifetime earnings is captured in \mathcal{E} , which is given by

$$\mathcal{E} = \frac{1}{R+1} \sum_{j=0}^R w\varphi_j l_j.$$

The rest of the model (government budget constraint, equilibrium conditions, etc.) is identical to the one described in the main text. Note that even though we allow for bequest motives, we abstract from direct intergenerational transfers. Allowing for it introduces analytical and computational complications to the optimal policy exercise, and is outside the scope of this paper.

S4.1. *Optimal Policies*

The set of policies that we allow for in our optimal reform are very similar to those described in the main text. The only addition is that we allow for a nonlinear tax on bequest.

Planning Problem

Our planning problem maximizes the revenue from delivering a steady-state allocation subject to the implementability constraint (S12) and a minimum utility requirement given by

$$\max \int \sum_{j=0}^J \frac{P_j(\theta)}{(1+r)^j} \left[w\varphi_j(\theta)l_j(\theta) - c_j(\theta) - \frac{1 - p_{j+1}(\theta)}{1+r} b_{j+1}(\theta) \right] dH(\theta) \quad (\text{S10})$$

subject to

$$U'(\theta) = \sum_{j=0}^J \beta(\theta)^j P_j(\theta) \frac{\varphi_j'(\theta)}{\varphi_j(\theta)} \psi l_j(\theta)^\gamma \quad (\text{S11})$$

$$+ \sum_{j=0}^J \beta(\theta)^j P_j(\theta) \left(\frac{P'_j(\theta)}{P_j(\theta)} + j \frac{\beta'(\theta)}{\beta(\theta)} \right) \left[u(c_j(\theta)) - \psi \frac{l_j(\theta)^\gamma}{\gamma} \right] \quad (\text{S12})$$

$$+ \sum_{j=0}^J \beta(\theta)^{j+1} [P_j(\theta) - P_{j+1}(\theta)] \\ \times \left(\frac{P'_j(\theta)}{P_j(\theta)} - \frac{p'_{j+1}(\theta)}{1 - p_{j+1}(\theta)} + (j+1) \frac{\beta'(\theta)}{\beta(\theta)} \right) w(b_{j+1}(\theta)), \quad (\text{S13})$$

$$U(\theta) \geq W_s(\theta). \quad (\text{S14})$$

The above optimal allocations can be used to construct optimal policies. However, the mapping from allocations to policies is less straightforward. This is because, in our implementation, markets are incomplete. Nevertheless, we are able to construct the tax schedules, as described in (S8) and (S9), for this incomplete market economy in the following lemma (in what follows, we adopt the following notation to avoid clutter: $u_{c,j}(\theta) \equiv u'(c_j(\theta))$, $v_{l,j}(\theta) \equiv v'(l_j(\theta))$, and $w_{b,j}(\theta) \equiv w'(b_j(\theta))$):

LEMMA S2: Consider an allocation $\{c_j(\theta), l_j(\theta), b_j(\theta)\}$ that satisfies the implementability constraint (S12) such that $b'_j(\theta) > 0$, $(\varphi_j(\theta)l'_j(\theta))' > 0$, and

$$\sum_{s=j}^j \beta^s P_s(\theta) [u_{c,s}(\theta)c'_s(\theta) + \beta(1 - p_{s+1}(\theta))w_{b,s+1}(\theta)b'_{s+1}(\theta) - v_{l,s}(\theta)(\varphi_s(\theta)l'_s(\theta))'] > 0.$$

Then tax and transfer functions $T_{a,j}(\cdot)$, $T_{b,j}(\cdot)$, $T_{y,j}(\cdot)$, S_j together with asset holdings $a_j(\theta)$ exist so that the allocations $\{c_j(\theta), l_j(\theta), b_j(\theta), a_j(\theta)\}$ satisfy the budget constraints (S8) and (S9) and the first-order conditions associated with the individual optimization.

PROOF: We start by writing the first-order conditions for the maximization problem above for an individual of type θ :

$$1 - T'_{y,j}(\varphi_j(\theta)l'_j(\theta)) = \frac{v'(l_j(\theta))}{\varphi_j(\theta)u_{c,j}(\theta)}, \quad (\text{S15})$$

$$u_{c_j} = \beta(1+r)[p_{j+1}(1 - T'_{a,j+1})u_{c_{j+1}} + (1 - p_{j+1})(1 - T'_{b,j+1})w_{b,j+1}]. \quad (\text{S16})$$

Equation (S15) is the individual intratemporal optimality condition, and equation (S16) is the individual Euler equation.

We can use equation (S15) to back out the optimal marginal taxes on labor earnings at each age. This is possible because the efficient allocations of consumption and hours come directly from solving the planning problem. Thus, the earnings taxes can simply be defined by integrating over the implied marginal rate in (S15). This is well-defined since output in each age is increasing in θ .

The calculation of optimal asset taxes is not straightforward. More importantly, the level of assets a cannot be pinned down independent from the marginal taxes $T'_{a,j+1}$ and $T'_{b,j+1}$. Therefore, we assume that asset holdings of the lowest type are 0 for all ages. This implies that in the equilibrium which decentralizes an incentive compatible allocation, the poorest individual is hand-to-mouth in all ages. Given this restriction, we can use the following procedure to find the optimal asset taxes.

We can combine equations (S15) and (S16) together with (S8) and (S9) and use extensive algebra to show that the derivative of asset holdings with respect to θ , a'_j , satisfies

$$a'_j(\theta) = \frac{1}{u_{c,j}(\theta)} \sum_{s=j}^T \beta^{s-j} \frac{P_s(\theta)}{P_j(\theta)} \left[u_{c,s}(\theta) c'_s(\theta) + \beta(1 - p_{s+1}(\theta)) w_{b,s+1}(\theta) b'_{s+1}(\theta) - v_{l,s}(\theta) (\varphi_s(\theta) l_s(\theta))' \right].$$

Since, by assumption, $a_j(\underline{\theta}) = 0$, the above determines the level of asset holdings at each age and for each type. Additionally, taxes on bequests must satisfy

$$b_j(\theta) = (1 + r)a_j(\theta) - T_{b,j}((1 + r)a_j(\theta)). \quad (\text{S17})$$

Since $a_j(\theta)$ and $b_j(\theta)$ are determined in the optimal allocation, the above formula determines the bequests taxes.

Finally, using (S17) and the Euler equation (S16), we must have

$$1 - T'_{a,j+1} = \frac{u_{c_j}}{\beta(1 + r)p_{j+1}u_{c_{j+1}}} - \frac{1 - p_{j+1}}{p_{j+1}} \frac{w_{b,j+1}}{u_{c_{j+1}}} \frac{b'_{j+1}}{(1 + r)a'_{j+1}}. \quad (\text{S18})$$

The above formula determines the marginal tax rate on asset holdings, and since $a'_j > 0$, a well-defined tax function on asset holdings must exist. This completes the construction. *Q.E.D.*

Unfortunately, we cannot derive a closed form formula for optimal taxes. However, our implementation procedure provides a guideline on how to numerically compute the optimal tax functions.

Note that monotonicity constraints in Lemma S2 are necessary for the existence of the tax function. While we have no way of theoretically checking that they are satisfied, our numerical simulations always involve a check that ensures that they are. Needless to say, in all our simulations, the monotonicity constraint is satisfied.

Finally, it is worth noting that in the model with bequests, the degree of market incompleteness in the presence of risk-free assets depends on the strength of the bequest motive. In particular, when individuals put a high valuation on bequests relative to assets upon survival, a risk-free asset comes very close to implementing efficient allocations. As a result, the strength of the subsidy depends on the strength of the bequest motive. In general, we can write (S18) as

$$T'_{a,j+1} = 1 - \frac{1}{p_{j+1}} + \frac{1}{p_{j+1}} \left(\frac{p'_{j+1}(\theta)}{p_{j+1}(\theta)} + \frac{\beta'(\theta)}{\beta(\theta)} \right) \frac{\tau_{l,j}(\theta)}{1 - \tau_{l,j}(\theta)} \frac{\varphi_j(\theta)}{\varphi'_j(\theta)} \left(\frac{1 + \varepsilon_{F,j}(\theta)}{\varepsilon_{F,j}(\theta)} \right) + \frac{1 - p_{j+1}}{p_{j+1}} \frac{w_{b,j+1}}{u_{c_{j+1}}} \frac{b'_{j+1}}{(1 + r)a'_{j+1}}.$$

The above formula highlights the role of bequests in affecting the optimal savings taxes. The first term is the standard terms associated with market incompleteness. The second term is the redistributive motives discussed in Section 2. The last term is related to the strength of the bequest motive. For example, when there is no mortality heterogeneity

and bequest motives are sufficiently strong, this term becomes close to $\frac{1-p_{j+1}}{p_{j+1}}$, which then cancels out the market incompleteness effect.

When bequests are luxury goods—see our calibration of the function $w(b)$ below—it is efficient for low-income individuals not to leave any bequests. In this case, $b'_{j+1}(\theta) = 0$, and the above becomes similar to equation (4).

S4.2. Calibration

Calibration of the model with bequests follows the baseline calibration whenever possible. For status quo policies, we assume that there are no bequest taxes in the status quo model.⁷ All other policies are the same as in the main model.

Bequest motives are captured by the following utility function:

$$w(b) = \chi \frac{(b + \bar{B})^{1-\sigma}}{\sigma}.$$

Parameter χ determines the strength of the bequest motive, while \bar{B} reflects the extent to which bequests are luxury goods. If $\bar{B} > 0$, the marginal utility of bequests is bounded. At the same time, the marginal utility of large bequests declines more slowly than the marginal utility of consumption. As a result, richer individuals have stronger motives to leave bequests.⁸ We follow [De Nardi \(2004\)](#) and choose the value for parameter \bar{B} to match the fraction of the deceased individuals who leave no bequest. We assume the risk aversion parameter $\sigma = 2$. The strength of bequest χ is chosen to match the bequest to wealth ratio of 0.0118, as reported in [Gale and Scholz \(1994\)](#). To calibrate \bar{B} , we use data on the distribution of bequests reported in [Hurd and Smith \(2002\)](#). We choose this parameter so that, in the model, 25 percent leave no bequest. As in the case with medical expenditure, we hold parameter β_1 (gradient of discount factor with respect to ability type θ) at the benchmark level. The calibrated parameters are reported in Table [SXII](#).

S4.2.1. Results

In [Figure S23](#), we show how well the model captures the pattern of dissaving as well as the distribution of bequests. The left panel in [Figure S23](#) shows the distribution of bequests in the model and in the HRS data as reported in [Hurd and Smith \(2002\)](#). The right panel shows the average assets by permanent income quintile in the model and in the data. The model does a reasonable job at capturing dispersion in bequests. It also captures the pattern of asset decumulation, except perhaps at the highest income quintile.

[Figure S24](#) reports the optimal asset taxes in the optimal reform. As mentioned before, since leaving a bequest is an active decision by individuals, an optimal policy requires the introduction of a new instrument, that is, a tax on the assets of the deceased. In other words, an optimal policy has two components of asset taxation. As before, there is a subsidy on the assets of individuals who survive. The idea behind this policy is the same as the one described throughout the paper. We plot these asset subsidies in the left panel of [Figure S24](#). The new policy, tax on bequest, accomplishes two tasks. On one hand, it deters

⁷Bequest and estate taxes affect only a small portion of the richest U.S. tax payers.

⁸The wealth elasticity of realized and anticipated bequests has been estimated to be about 1.3 (see [Auten and Joulfaian \(1996\)](#) and [Hurd and Smith \(2002\)](#)). Among single Americans who were at least 70 years old in 1993 and died before 1995, the 30th percentile of the bequest distribution was just \$2000, the median was \$42,000, and the mean was \$82,000 ([Hurd and Smith \(2002\)](#)).

TABLE SXII
PARAMETERS CALIBRATED USING THE MODEL WITH BEQUEST
MOTIVE

Parameters	Description	Values
β_0	discount factor: level	0.985
ψ	weight on leisure	0.037
χ	strength of bequest motive	50
\bar{B}	bequest utility shifter	600

Targeted Moments	Data	Model
Capital-output ratio	4.00	4.00
Average annual hours	2000	2000
Bequest-wealth ratio	0.0118	0.013
Fraction who leave no bequest	0.25	0.23

poor individuals from leaving bequests. Since a bequest is a luxury good, delivering utils to poor individuals via bequests is not efficient. As the right panel of Figure S24 demonstrates, the bequest is taxed at 100 percent for asset-poor individuals (they instead receive utils via high asset subsidies and, therefore, they have higher consumption while they are alive). Also, for those who are rich enough, leaving bequests is valuable and, therefore, there is a sharp drop in bequest taxes. For these individuals, bequest taxes spread the dead-weight-loss of taxation between the state of survival and death.

Figure S25 shows the optimal earnings taxes. There is no significant difference relative to the previous analysis here. The earnings tax reform continues to be a rather inessential part of the reform.

Finally, Table SXIII reports the aggregate effect of an optimal policy when implemented under the current U.S. demographics. It is important to highlight the large ef-

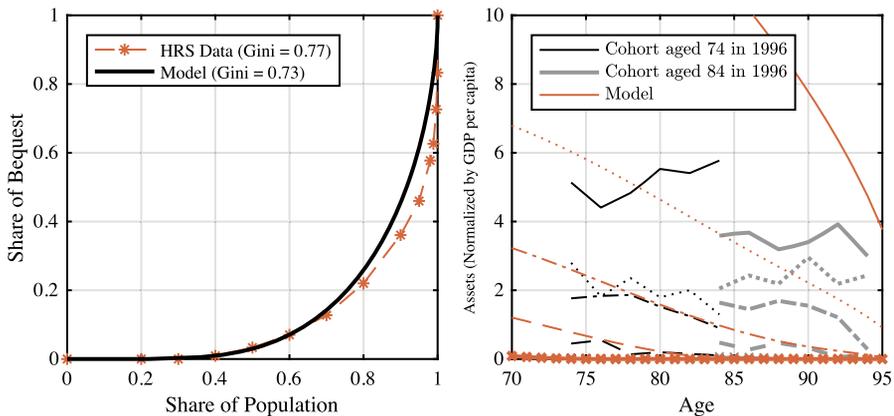


FIGURE S23.—The left panel shows the distribution of bequests left by the deceased in the model (solid line) versus in the data (dashed line). The right panel shows median assets by permanent income quintile in the model (solid line) and in the data (dashed line). Data source: De Nardi, French, and Jones (2010) calculations for AHEAD cohorts who were 74 and 84 years old in 1996.

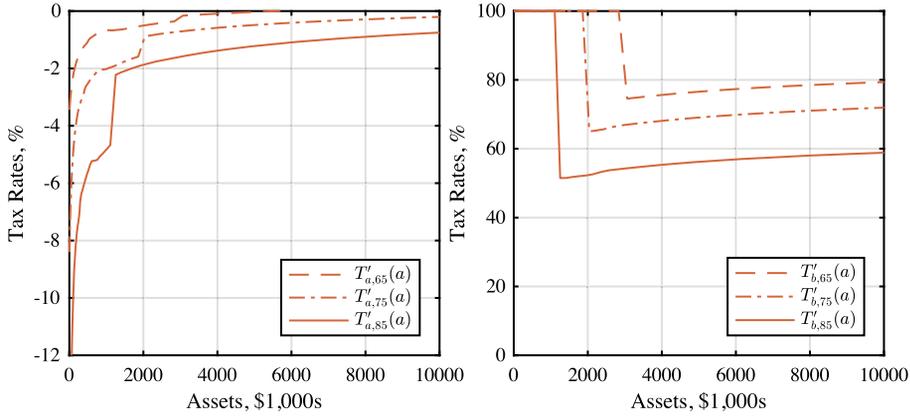


FIGURE S24.—Optimal asset tax functions with out-of-pocket medical expenditures and bequest motives. The left panel shows the optimal marginal asset taxes over all asset levels for surviving individuals at ages 65, 75, and 85. The right panel shows the marginal bequest taxes for the same ages.

efficiency gains resulting from these policies. Under optimal policies, there is a massive efficiency gain from reducing the bequest left by low-income individuals. This reduces the cost of delivering the status quo welfare to the current cohort. The mechanism is the following. Due to market incompleteness (lack of survival contingent assets), poor individuals leave too much bequest. However, this bequest contributes to their lifetime welfare, although at a high cost. Under Pareto optimal reform policies, low-income individuals leave no bequest due to 100 percent bequest tax. Instead they receive status quo welfare entirely through consumption while they are alive (i.e., asset subsidies). This leads to an efficient delivery of status quo welfare and contributes to the large efficiency gains reported in Table SXIII.

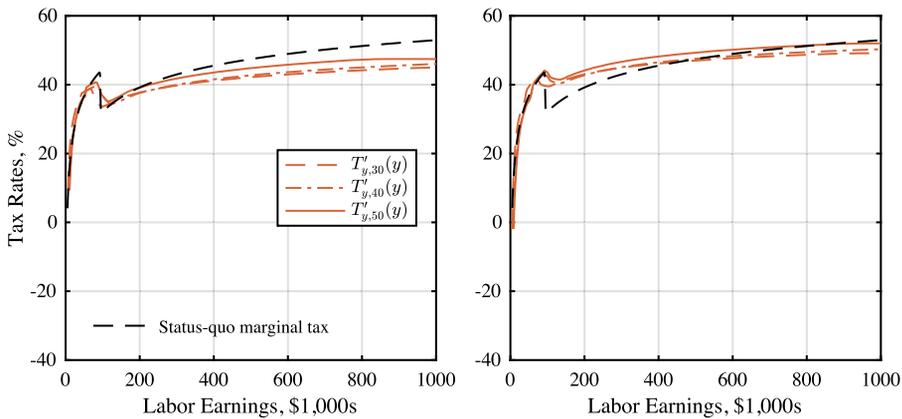


FIGURE S25.—Optimal labor income tax functions with out-of-pocket medical expenditures and bequest motives. The left panel is optimal marginal taxes with out-of-pocket medical expenditures and bequest motives. The right panel shows the same in the benchmark model. The black dashed line is the effective status quo tax schedule.

TABLE SXIII
 AGGREGATE EFFECTS OF OPTIMAL POLICIES—WITH OUT-OF-POCKET MEDICAL EXPENDITURES AND
 BEQUEST MOTIVES^a

	Current U.S.	Optimal reform	
	(1)	(2)	(3)
Factor prices			
Interest rate (%)	4.05	4.05	3.76
Wage	1.00	1.00	1.03
Values relative to GDP			
Consumption	0.69	0.67	0.68
Capital	4.00	4.29	4.17
Tax revenue (total)	0.24	0.33	0.33
Earnings tax	0.12	0.13	0.14
Consumption tax	0.04	0.04	0.04
Bequest tax	0.00	0.07	0.07
Capital (corporate) tax	0.08	0.09	0.08
Transfers	0.15	0.24	0.24
To retirees	0.08	0.06	0.06
To workers	0.07	0.11	0.11
Asset subsidies	0.00	0.07	0.07
Change (%) (relative to status quo)			
GDP	–	3.93	2.25
Consumption	–	1.44	0.83
Capital	–	11.28	6.47
Labor input	–	–2.30	–1.348
PDV of net resources	–	–276.88	–389.29
Consumption equivalence		4.32	6.09

^aColumn (1) is the benchmark calibration to the current U.S. economy. Column (2) is the continuation of the U.S. status quo policies (with the consumption tax adjusted to balance the government's budget constraint). Column (3) is the optimal reform policies with prices and demographics fixed at the current U.S. values. Column (4) is the optimal reform policies with equilibrium prices but fixed demographics (at current U.S. levels). Column (5) is the optimal reform policies with equilibrium prices and future demographics. In columns (3) and (4), the percentage change in PDV is calculated relative to column (1). In column (5), the percentage change in PDV is calculated relative to column (2).

S5. CALIBRATION: CALCULATING U.S. AGGREGATES

In this section, we describe the calculation of capital income share, investment expenditure as share of GDP, government expenditure as share of GDP, debt as share of GDP, and capital to output ratio. We use our calculated values as calibration targets. The main data sources for our calculations are the U.S. National Income and Production Account, the Fixed Asset Tables (compiled by the Bureau of Economic Analysis), and several balance sheet items from the Flow of Funds of the United States (compiled by the Federal Reserve Board of Governors). All the collocations are done using data from year 2000 to 2010.⁹

Income Categories

We closely follow [McGrattan and Prescott \(2017\)](#) and adjust NIPA's measure of income to conform to our theoretical model. These adjustments are mainly subtraction of sales

⁹All calculations are based on the January 26, 2018 release of the data.

taxes and addition of imputed capital service of consumer durables and government capital. Our discussion here is brief. For details, refer to [McGrattan and Prescott \(2017\)](#).

We start with income data from Table 1.10 in the NIPA data. We categorize the compensation of employees and 70 percent of proprietors' income as labor income. The rest of the income is categorized as capital income after the following adjustments. First, we subtract taxes other than property tax from NIPA's measure of "taxes on production and imports." Second, we impute capital services for consumer durables, which we treat as investment, and government capital. The imputed services are estimated to be 4 percent times the current-cost net stock of consumer durable goods and government fixed assets (both of these stocks are reported in BEA's Fixed Asset Tables). In addition, we include depreciation of consumer durables from the Flow of Funds accounts. After these adjustments, the capital income is the sum of corporate profits, 30 percent of proprietors' income, surplus of government enterprises, rents, net income, property taxes, depreciation of capital, and imputed capital services. This sum amounts to 43.5 percent of the adjusted GDP on average between 2000 and 2010. We use this figure as our target for the capital share of income. Table [SXIV](#) shows the breakdown of income and its components relative to GDP. All numbers are averages between 2000 and 2010.

Expenditure Categories

We divide expenditures into three categories: government spending, investment, and consumption. Table [SXV](#) shows each expenditure category relative to GDP. We define "government spending" as the sum of defense expenditure (both consumption and gross investment), general public service, and public order and safety. These add up to about 8 percent of GDP. The rest of government expenditures is treated as either investment or consumption expenditures. The "investment" category consists of NIPA's gross private domestic investment, net export, income from the rest of the world, consumer durable goods (net of imputed sales tax), and government non-defense investment expenditures. Therefore, investment relative to GDP is 23.2 percent. "Consumption" expenditures consist of NIPA's consumption of non-durables and services, imputed capital services of consumer durables and government capital, consumer durable depreciations, and other government consumption expenditures which are included in NIPA but are not included in our measure of "government spending." These are transportation and other economic affairs, housing and community services, health, recreation and culture, education, and welfare, which are mostly services and/or transfers that are close substitutes to private consumption. We assume that these are effectively lump-sum transfers to households.¹⁰

Physical Capital

We present two different approaches to measure the stock of physical capital, one based on NIPA and Fixed Asset Tables and one based on the Flow of Funds accounts. Our first approach closely follows [McGrattan and Prescott \(2017\)](#). We define physical capital as the sum of fixed and private assets, stock of consumer durables, stock of inventories, and land. This approach yields a measure of physical capital that is about 4.07 times GDP (average

¹⁰As [McGrattan and Prescott \(2017\)](#) pointed out, this is consistent with the accounting of the [World Bank \(2014\)](#) that assumes "actual individual consumption comprises all the goods and services that households consume to meet their individual needs . . . whether they are purchased by households or are provided by general government and nonprofit institutions service households" (p. 9).

TABLE SXIV
INCOME CATEGORIES RELATIVE TO GDP, 2000–2010^a

Total income	1.000
Labor income	0.565
Compensation of employees (NIPA 1.10)	0.516
Wages and salary accruals (NIPA 1.10)	0.418
Supplements to wages and salaries (NIPA 1.10)	0.098
70% of proprietors' income with IVA, CCadj (NIPA 1.10)	0.048
Capital income	0.435
Corporate profits with IVA and CCadj (NIPA 1.10)	0.072
30% of proprietors' income with IVA, CCadj (NIPA 1.10)	0.021
Rental income of persons with CCadj (NIPA 1.10)	0.018
Surplus on government enterprises (NIPA 1.10)	-0.000
Net interest and misc. payments, domestic industries (NIPA 1.10)	0.051
Indirect business taxes	0.069
Taxes on production and imports (NIPA 1.10)	0.066
Less: Subsidies (NIPA 1.10)	0.004
Business current transfer payments (NIPA 1.10)	0.007
Less: Sales tax	0.041
Federal excise taxes (NIPA 3.5)	0.005
Federal customs duties (NIPA 3.5)	0.002
State and local sales taxes (NIPA 3.5)	0.028
Motor vehicle licenses (NIPA 3.5)	0.001
Severance taxes (NIPA 3.5)	0.001
Special assessments (NIPA 3.5)	0.000
Other taxes on production and imports (NIPA 3.5)	0.003
Net income, rest of world (NIPA 1.13)	0.007
Consumption of fixed capital (NIPA 1.10)	0.144
Consumer durable depreciation (FOF F.10)	0.058
Imputed capital services	0.038
Consumer durable services	0.012
Government capital services	0.026
Statistical discrepancy (NIPA 1.10)	-0.003

^aData sources are in parentheses. IVA, inventory valuation adjustment; CCadj, capital consumption adjustment; NIPA, national income and product accounts; FoF, the flow of funds of the United States.

between 2000 and 2010). The top panel of Table SXVI lays out the detailed calculations with sources for each subcategory.

Alternatively, we can measure the stock of physical capital as the total sum of all non-financial assets in the Flow of Funds accounts. These include household and nonprofits, non-financial corporates, non-financial non-corporates, and government. This approach results in a stock of physical capital that is 3.97 times GDP. For our calibration, we use a capital to output ratio of 4 as our target, which is a round number that is close to both these measures.

Stock of National Debt

Our measure of government debt includes state and local municipal securities, federal Treasury securities, and federal budget securities. To account for the fact that a portion of this debt is held by government agencies, we subtract government debt held by the Social Security Administration. This results in a debt to adjusted GDP ratio of 0.47. See Table SXVII for details.

TABLE SXV
EXPENDITURE CATEGORIES RELATIVE TO GDP, 2000–2010 ^a

Total expenditure	1.000
Consumption	0.688
Personal consumption expenditures (NIPA 1.1.5)	0.632
Less: Consumer durable goods (NIPA 1.1.5)	0.078
Less: Imputed sales tax, nondurables and services	0.036
Plus: Imputed capital services, durables	0.012
Government consumption expenditures, other (NIPA 3.9.5)	0.064
Plus: Imputed capital services, government capital	0.026
Consumer durable depreciation (FOF F.10)	0.058
Government spending	0.080
Government consumption expenditures, national defense (NIPA 3.9.5)	0.034
Government gross investment, national defense (NIPA 3.9.5)	0.009
General public service (NIPA 3.9.5)	0.016
Public order and safety (NIPA 3.9.5)	0.021
Investment	0.232
Gross private domestic investment (NIPA 1.1.5)	0.164
Consumer durable goods (NIPA 1.1.5)	0.078
Less: Imputed sales tax, durables	0.005
Government gross investment, non-defense (NIPA 3.9.5)	0.029
Net exports of goods and services (NIPA 1.1.5)	−0.040
Net income rest of world (NIPA 1.13)	0.007

^aData sources are in parentheses. NIPA, national income and product accounts; FoF, the flow of funds of the United States.

TABLE SXVI
STOCK OF PHYSICAL CAPITAL (TOP PANEL) AND NON-FINANCIAL
ASSETS (BOTTOM PANEL), AVERAGES RELATIVE TO GDP,
2000–2010^a

Physical capital	4.072
Fixed private assets (FA 1.1)	2.123
Fixed government assets (FA 1.1)	0.637
Consumer durables (FA 1.1)	0.294
Inventories (NIPA 5.8.5B)	0.130
Land	0.887
Households and non-profits (FOF B.101)	0.519
Non-financial corporate (FOF B.103)	0.103
Non-financial non-corporate (FOF B.104)	0.265
Stock of non-financial assets	3.974
Households and non-profits (FOF B.101)	1.727
Non-financial corporate (FOF B.103)	0.989
Non-financial non-corporate (FOF B.104)	0.620
Government (FOF B.1)	0.637

^aData sources are in parentheses. FA, fixed asset tables; NIPA, national income and product accounts; FoF, the flow of funds of the United States.

TABLE SXVII
STOCK OF GOVERNMENT DEBT RELATIVE TO GDP, 2000–2010^a

Government consolidated debt	0.467
State and local municipal securities (FOF L.105)	0.165
Federal Treasury securities (FOF L.105)	0.434
Federal budget agency securities (FOF L.105)	0.002
Less: Government debt held by SSA	0.134

^aData sources are in parentheses. NIPA, national income and product accounts; FoF, the flow of funds of the United States; SSA, Social Security Administration.

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