

SUPPLEMENT TO “UNEMPLOYMENT AND BUSINESS CYCLES”:
TECHNICAL APPENDIX
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APPENDIX A: THE ROLE OF WAGE INERTIA IN THE RESULTS

IN THE MAIN TEXT we emphasize the role of wage inertia in helping our model account for labor market dynamics. The role of wage inertia in labor market dynamics is the subject of some controversy in the literature. For example, Hall (2005), Shimer (2005), and Hall and Milgrom (2008) argue that wage inertia is important. In contrast, Hagedorn and Manovskii (2008) and Ljungqvist and Sargent (2015) challenge that view. In this appendix we clarify the relationship between our findings and the literature.

In the first two subsections we focus on the steady state response of labor market tightness to a change in steady state labor productivity, $\eta_{l,\vartheta}$. We develop a decomposition of $\eta_{l,\vartheta}$ that isolates the role of wage inertia. In the first subsection we compare a model with extreme wage inertia (i.e., a constant wage) with a Nash bargaining model. In the second subsection we assess the role of wage inertia in the Nash and AOB models. We find that the value of $\eta_{l,\vartheta}$ is lower in the Nash model than it is in the AOB and constant wage models. Our decomposition indicates that this finding reflects the effects of wage inertia.

The third section considers the relationship between steady state and dynamic analyses. There we show that steady state analysis can be very misleading for the dynamics of models like ours. There is no good substitute for analyzing dynamic impulse response functions in such models.

A.1. *The Potential for Wage Inertia to Resolve the Volatility Puzzle*

To understand the role of wage inertia it is useful to consider the steady state of our model. The latter is characterized by a particular recursive structure. The capital–labor ratio and ϑ are determined by equations of the model that do not involve the labor market. Given ϑ , the steady state value of l is determined by the equations describing the labor market.

The free entry condition and the bargaining equation, play a central role in the equilibrium conditions of the model. Making use of the relationship between the vacancy filling probability, Q , and market tightness, Γ , given by

$$Q = \sigma_m \Gamma^{-\sigma},$$

[Correction added on 4 March 2021, after first online publication: The copyright line was updated.]

the steady state version of the free entry condition is

$$(A.1) \quad \frac{s}{\sigma_m} \Gamma^\sigma + \kappa = \frac{\vartheta - w}{1 - \rho\beta}.$$

Here, s denotes the cost of posting a vacancy, κ denotes the fixed cost of bargaining with a worker, w denotes the wage rate, ρ denotes the match survival rate, and β denotes the representative household's discount factor. Also, σ_m and σ denote the parameters of the matching function. The standard case in the literature is $\kappa = 0$. We also consider that case in our numerical experiments, for robustness.¹

We denote the elasticity of market tightness with respect to ϑ by

$$\eta_{\Gamma, \vartheta} = \frac{d \log \Gamma}{d \log \vartheta},$$

where the derivative holds all model parameters fixed. As in the literature, we use $\eta_{\Gamma, \vartheta}$ as our measure of labor market volatility. It is easy to see that (A.1) implies

$$(A.2) \quad \eta_{\Gamma, \vartheta} = \frac{1}{\sigma} \frac{\vartheta}{\vartheta - w - \kappa(1 - \rho\beta)} \left[1 - \frac{dw}{d\vartheta} \right].$$

Here, we define

$$(A.3) \quad \textit{profit rate} = \frac{\vartheta - w - \kappa(1 - \rho\beta)}{\vartheta}.$$

Expression (A.2) decomposes labor market volatility into a component that reflects the profit rate and a component that is a function of wage inertia, $dw/d\vartheta$. The greater is wage inertia, that is, the smaller is $dw/d\vartheta$, the bigger is $\eta_{\Gamma, \vartheta}$. The intuition is simple. When the wage rate is more inertial, then firms receive a greater share of the rent associated with vacancies after a rise in technology, ϑ . As a result, the more inertial is the wage, the greater is the incentive of the firm to post vacancies in the wake of an increase in ϑ . This increased incentive leads to a greater increase in market tightness.

The degree of wage inertia is determined by the bargaining relationship between firms and workers. We consider three models of that relationship. In the constant wage model, w is simply a constant, \tilde{w} . We require

$$D \leq \tilde{w} \leq \vartheta,$$

¹The free entry condition that holds on a steady state growth path, after scaling by Φ_t (defined in the main text). The values of s and κ in (A.1) actually correspond to the steady state values of $s\Omega_t/\Phi_t$ and $\kappa\Omega_t/\Phi_t$, respectively. Similarly, γ and D below actually correspond to the steady state values of $\gamma\Omega_t/\Phi_t$ and $D\Omega_t/\Phi_t$, respectively. Here, Ω_t is defined in the main text.

so that the firm and worker each have an incentive to produce. Let $\eta_{\Gamma, \vartheta}^{\text{constant } w}$ denote labor market volatility in the constant wage model. In this case, $\eta_{\Gamma, \vartheta}^{\text{constant } w}$ is (A.2) with $dw/d\vartheta = 0$ and with w replaced by the constant wage rate, \tilde{w} :

$$(A.4) \quad \eta_{\Gamma, \vartheta}^{\text{constant } w} = \frac{1}{\sigma} \frac{\vartheta}{\vartheta - \tilde{w} - \kappa(1 - \rho\beta)}.$$

The other two models are the Nash and AOB models, respectively. The bargaining relationship in those models is characterized by the sharing rule. Recall that S , J , and f denote the surplus of an employed worker, the value of an employed worker to a firm, and the job finding rate, respectively. Substituting

$$(A.5) \quad S = \frac{w - D}{1 - \beta\rho(1 - f)}, \quad J = \frac{\vartheta - w}{1 - \rho\beta}, \quad f = \sigma_m \Gamma^{1-\sigma},$$

into the sharing rule we obtain an expression that only involves w and Γ . Totally differentiating that expression and using (A.2), we solve for $\eta_{\Gamma, \vartheta}$ to obtain

$$(A.6) \quad \eta_{\Gamma, \vartheta} = Y \frac{\vartheta}{\vartheta - D - \tau_\kappa \kappa - \tau_\gamma \gamma}.$$

Here Y , τ_κ , and τ_γ are functions of ρ , β , σ , f , and the bargaining parameters:

$$\begin{aligned} \psi &\equiv \frac{\rho\beta f + \sigma(1 - \rho\beta)(1 + \beta_1)}{\rho\beta f + (1 - \rho\beta)(1 + \beta_1)}, \\ Y &= \frac{\beta_1 + \beta_3(1 - \rho\beta(1 - f))}{\psi a}, \\ \tau_\kappa &\equiv \frac{(1 + \beta_1)(1 - \rho\beta) + \beta\rho f + \frac{\rho\beta f(\sigma - 1)}{\psi}}{a}, \\ \tau_\gamma &\equiv \frac{\left[1 - \beta\rho(1 - f) + \frac{\rho\beta f(\sigma - 1)}{\psi}\right] \beta_2}{a}, \\ a &= \beta_1 + \left(1 - \beta\rho(1 - f) + \frac{\rho\beta f(\sigma - 1)}{\psi}\right) \beta_3. \end{aligned}$$

In the case of AOB, $\beta_i = \alpha_{i+1}/\alpha_1$, for $i = 1, 2, 3$ and

$$\begin{aligned} \alpha_1 &= 1 - \delta + (1 - \delta)^M, \\ \alpha_2 &= 1 - (1 - \delta)^M, \\ \alpha_3 &= \alpha_2 \frac{1 - \delta}{\delta} - \alpha_1, \end{aligned}$$

$$\alpha_4 = \frac{1 - \delta}{2 - \delta} \frac{\alpha_2}{M} + 1 - \alpha_2.$$

Following Ljungqvist and Sargent (2015) (LS), we define

$$\text{fundamental surplus fraction} = \frac{\vartheta - D - \tau_\kappa \kappa - \tau_\gamma \gamma}{\vartheta}.$$

To evaluate (A.6), we need a value for the endogenous variable, f . So when we use (A.6) to evaluate $\eta_{\Gamma, \vartheta}$ for alternative values of a parameter like D or η , then we implicitly adjust the values of other parameters (σ_m and s) to keep f unchanged.

In the case of Nash bargaining, (A.6) reduces to

$$(A.7) \quad \eta_{\Gamma, \vartheta}^{\text{Nash}} = Y \frac{\vartheta}{\vartheta - D - \tau_\kappa \kappa},$$

$$Y = \frac{\eta \rho \beta f + (1 - \rho \beta)}{\eta \rho \beta f + \sigma(1 - \rho \beta)},$$

$$\tau_\kappa = \frac{1 - \rho \beta + \frac{\eta \beta \rho f}{\eta \rho \beta f + \sigma(1 - \rho \beta)} [\eta \sigma \rho \beta f + (2\sigma - 1)(1 - \rho \beta)]}{1 - \eta},$$

$$\tau_\gamma = 0.$$

Below we show that $\eta_{\Gamma, \vartheta}^{\text{constant } w}$ can be orders of magnitude larger than $\eta_{\Gamma, \vartheta}^{\text{Nash}}$. It is easy to see this in the case, $\kappa = 0$. Observe that

$$(A.8) \quad 1 \leq Y \leq \frac{1}{\sigma}, \quad Y \text{ strictly decreasing in } \eta.$$

Comparing (A.4) with (A.7) and taking into account (A.8) we conclude:

PROPOSITION 1: *If $\kappa = 0$, then $\eta_{\Gamma, \vartheta}^{\text{constant } w} \geq \eta_{\Gamma, \vartheta}^{\text{Nash}}$ with the inequality strict when $\eta > 0$.*

The intuition behind the proposition is straightforward. In the Nash model, w rises in response to an increase in ϑ , diverting some of the rents associated with vacancies away from the firm. As a result, the firm has less incentive to post vacancies and this prevents a substantial rise in labor market tightness. As discussed above, this diversion of resources does not happen in the constant wage model.

To assess the quantitative difference between $\eta_{\Gamma, \vartheta}^{\text{constant } w}$ and $\eta_{\Gamma, \vartheta}^{\text{Nash}}$, we consider the version of the Nash model that we estimated subjected to the restriction $D/w = 0.39$. We refer to this model as the *restricted Nash model*. At the

posterior mode, the relevant model parameter values are

$$\begin{aligned} \sigma_m = 0.6544, \quad \sigma = 0.6598, \quad \beta = 0.9968, \quad \rho = 0.9, \quad \eta = 0.9028, \\ D = 0.3315, \quad \kappa = 0.0831, \quad s = 1.763 \times 10^{-4}, \quad \vartheta = 0.8585. \end{aligned}$$

These values imply

$$f = 0.6321, \quad q = 0.7, \quad \frac{\kappa}{\frac{s}{q} + \kappa} = 0.997, \quad w = 0.8499$$

and

$$\text{profit rate} = 3.018 \times 10^{-5} (0.00998), \quad Y = 1.060 (1.060).$$

Numbers in parentheses correspond to the case $\kappa = 0$. In this case, we hold constant the values of ρ , β , ϑ , σ , and η , and adjust σ_m , D , and c to keep the values of q , f , and D/w unchanged.

Evaluating (A.6) for the restricted Nash model, we obtain

$$\eta_{\Gamma, \vartheta}^{\text{Nash}} = 5.710 (1.73).$$

In sharp contrast, when we evaluate (A.2) for the constant wage model and set \tilde{w} to the steady state value of w in the restricted Nash model just discussed (i.e., $\tilde{w} = 0.8499$), we obtain

$$\eta_{\Gamma, \vartheta}^{\text{constant } w} = 50, 215.1 (151.79).$$

Clearly, wage inertia has the potential to increase labor market volatility by orders of magnitude.

Our results may appear to contradict existing claims in the literature, which argue that wage inertia has at best only a marginal impact on labor market volatility. See, for example, Hagedorn and Manovskii (2008) and LS. The former reach this conclusion by using the restricted Nash model and implementing wage inertia by reducing the bargaining power of labor, η . For intuition about why a reduction in η leads to an increase in wage inertia, note that as the bargaining power of the worker goes to zero, the wage converges to the worker's outside option, D . In that case, $dw/d\vartheta = 0$. When wage inertia is increased in this way, the impact on labor market volatility is relatively small. To see why, consider equation (A.7). The consensus in the literature is that σ is roughly $1/2$, so that the upper bound on Y is roughly 2. When $\kappa = 0$ the surplus fraction is not a function of η . So in this case a reduction in η can at most double $\eta_{\Gamma, \vartheta}$. That is, raising wage inertia can at most raise volatility from 1.73 to roughly 3.4, a value that is much smaller than what authors in this literature consider to be empirically relevant (see Shimer (2005)). If $\kappa > 0$ and

$\sigma \geq 1/2$, then τ_κ is increasing in η (see (A.7)). So in the empirically relevant case of $\kappa > 0$, an increase in wage inertia due to a reduction in η leads to an even smaller rise in $\eta_{r,\vartheta}$.

The previous experiment does not establish that wage inertia is irrelevant. The reason is that the experiment changes two things at once: wage inertia and the level of the wage rate. According to (A.2), the first effect *increases* $\eta_{r,\vartheta}$ and the second effect *decreases* $\eta_{r,\vartheta}$. So the experiment convolves two forces and does not reveal what the effects of wage inertia per se are. The experiment that we conduct with the constant wage model focuses exclusively on wage inertia. Our results indicate that the impact of wage inertia is potentially enormous.

A.2. *The Role of Wage Inertia in the Transition From the Restricted Nash to the AOB Model*

We now investigate the role of wage inertia in the AOB model. Again, we focus only on steady states. At the posterior mode of our parameter estimates, the bargaining parameters are

$$M = 60, \quad \delta = 0.00219, \quad \gamma = 0.0074538.$$

The other model parameters relevant for the steady state calculations are

$$\begin{aligned} \sigma_m &= 0.6623, & \sigma &= 0.542, & \beta &= 0.9968, & \rho &= 0.9, & \eta &= 0.9028, \\ D &= 0.3654, & \kappa &= 0.0605, & s &= 0.0040, & \vartheta &= 0.8646. \end{aligned}$$

These values imply

$$f = 0.6321, \quad q = 0.7, \quad \frac{\kappa}{\frac{s}{q} + \kappa} = 0.9137, \quad w = 0.8578$$

and

$$\text{profit rate} = 6.7993 \times 10^{-4} (0.00788), \quad Y = 1.652 (1.060).$$

Then

$$\eta_{r,\vartheta}^{\text{AOB}} = 22.400 (8.906).$$

Evidently, going from the restricted Nash model to the AOB model raises $\eta_{r,\vartheta}$ by roughly factor 4, from 5.7 to 22.4. When $\kappa = 0$, $\eta_{r,\vartheta}$ rises by a factor of 5.

We now decompose the rise in $\eta_{r,\vartheta}$ into the profit rate and wage inertia components. Taking the ratio of (A.2) for the AOB restricted Nash model, we

obtain

$$\begin{aligned}
 3.923 &= \frac{\eta_{\Gamma, \vartheta}^{\text{AOB}}}{\eta_{\Gamma, \vartheta}^{\text{restricted Nash}}} \\
 &= \frac{\frac{1}{\sigma^{\text{AOB}}}}{\frac{1}{\sigma^{\text{restricted Nash}}}} \times \frac{\overbrace{\left(\frac{\vartheta}{\vartheta - w - \kappa(1 - \rho\beta)} \right)^{\text{AOB}}}^{\text{profit rate component}}}{\overbrace{\left(\frac{\vartheta}{\vartheta - w - \kappa(1 - \rho\beta)} \right)^{\text{restricted Nash}}}^{\text{profit rate component}}} \\
 &\quad \times \frac{\overbrace{\left[1 - \frac{dw}{d\vartheta} \right]^{\text{AOB}}}^{\text{wage inertia component}}}{\overbrace{\left[1 - \frac{dw}{d\vartheta} \right]^{\text{restricted Nash}}}^{\text{wage inertia component}}} \\
 &= 1.217 \times 0.044 \times 72.593.
 \end{aligned}$$

Here, the superscript refers to the relevant model.² Notice that the wage inertia channel alone would have resulted in an increase in $\eta_{\Gamma, \vartheta}$ by a factor of roughly 73. We conclude that wage inertia plays by far the biggest role in accounting for the high value of $\eta_{\Gamma, \vartheta}^{\text{AOB}}$ relative to $\eta_{\Gamma, \vartheta}^{\text{restricted Nash}}$.

A.3. *The Role of Wage Inertia in the Transition From the Restricted Nash to the Simple Wage Rule Model*

We now investigate the role of wage inertia in the simple wage rule model. At the posterior mode of our parameter estimates, the bargaining parameters are

$$M = 60, \quad \delta = 0.00219, \quad \gamma = 0.0074538.$$

The other model parameters relevant for the steady state calculations are

$$\begin{aligned}
 \sigma &= 0.523, \quad \kappa = 0.05832, \quad \vartheta = 0.8927, \quad \tilde{w} = 0.8860, \\
 \text{profit rate} &= 7.8401 \times 10^{-4}.
 \end{aligned}$$

Then

$$\eta_{\Gamma, \vartheta}^{\text{simple wage}} = 2437.$$

²To compute $dw/d\vartheta$ we first compute $\eta_{\Gamma, \vartheta}$ using (A.6) and then we back out $dw/d\vartheta$ using (A.2).

Evidently, going from the restricted Nash model to the simple wage rule model raises $\eta_{r,\vartheta}$ by a very large amount. A key question is whether this reflects the profit rate component or the wage inertia component. Using the same decomposition as in the previous subsection, we obtain

$$\begin{aligned}
 426.854 &= \frac{\eta_{r,\vartheta}^{\text{simple wage}}}{\eta_{r,\vartheta}^{\text{estimated Nash}}} \\
 &= \frac{\frac{1}{\sigma^{\text{simple wage}}}}{\frac{1}{\sigma^{\text{estimated Nash}}}} \times \frac{\overbrace{\left(\frac{\vartheta}{\vartheta - w - \kappa(1 - \rho\beta)}\right)^{\text{simple wage}}}_{\text{profits channel}}}{\left(\frac{\vartheta}{\vartheta - w - \kappa(1 - \rho\beta)}\right)^{\text{estimated Nash}}} \\
 &\quad \times \frac{\overbrace{1^{\text{simple wage}}}_{\text{wage inertia}}}{\left[1 - \frac{dw}{d\vartheta}\right]^{\text{estimated Nash}}} \\
 &= 1.261 \times 0.038 \times 8794.051.
 \end{aligned}$$

Clearly, the wage inertia plays an overwhelmingly important role.

A.4. *Steady State versus Dynamic Considerations*

In this section we show that steady calculations of the sort described above can be deeply misleading about the dynamic response of a model to a persistent shock. This phenomenon occurs in models that include features like investment adjustment costs, the cost of capital utilization, and the degree of stickiness in prices.³ The parameters that govern these features have a important impact on dynamics. But those parameters do not appear in the equilibrium conditions that characterize steady state. No steady state elasticity calculation can uncover the effects of these features. Other features that can cause steady state calculations to be misleading are those that lead to the presence of state variables. Examples of state variables that occur in many models include lagged consumption, lagged investment, and capital. Real business cycle models have capital as a state variable. When such a model satisfies balanced growth, then steady state employment is not a function of steady state technology. But it is well known that employment does respond along the dynamic adjustment path in the wake of a persistent technology shock.

³Price stickiness literally has no impact on steady state when there is price indexation. Without price indexation we have found that the steady state effects of price stickiness are very small.

A simple modification of our model can be used to demonstrate the limitations of steady state analyses. Suppose that the equilibrium wage rate is given by

$$(A.9) \quad w_t = \phi \vartheta_t - \gamma(\vartheta_t - \vartheta), \quad \phi > 0, 0 < \gamma < \phi.$$

The dynamics of w_t depend on γ . But that parameter has no impact on the steady state value of w_t . Our dynamic model consists of a version of (A.1) in which Γ , w , and ϑ have time subscripts. For simplicity, we assume $\kappa = 0$. In addition, we assume

$$(A.10) \quad \vartheta_t = (1 - \nu)\vartheta + \nu\vartheta_{t-1} + \varepsilon_t.$$

Here ε_t is uncorrelated over time and with ϑ_{t-s} , $s > 0$. The value of a worker to a firm, J_t , is

$$(A.11) \quad J_t = \vartheta_t - \phi \vartheta_t - \gamma(\vartheta_t - \vartheta) + \beta \rho E_t J_{t+1}.$$

Here, we have substituted out for w_t using (A.9).

We find the equilibrium of this model as follows. First, we identify a stochastic process for J_t that satisfies (A.10) and (A.11). Consider the process

$$(A.12) \quad J_t = \delta_0 + \delta_1 \vartheta_t,$$

where δ_0 and δ_1 are undetermined coefficients. To satisfy (A.10) and (A.11), δ_0 and δ_1 must satisfy

$$\delta_1 = \frac{1 - \phi + \gamma}{1 - \beta \rho \nu}, \quad \delta_0 = \frac{\beta \rho (1 - \nu) \delta_1 - \gamma}{1 - \beta \rho} \vartheta.$$

Second, we substitute out for J_t from (A.12) into (A.1), to obtain

$$\frac{c}{\sigma_m} \Gamma_t^\sigma = \delta_0 + \delta_1 \vartheta_t.$$

The period t impact of an innovation to technology, ε_t , on $\log \Gamma_t$ is (after log linearization)

$$\begin{aligned} \bar{\eta}_{\Gamma, \vartheta} &= \frac{1}{\sigma} \frac{1 - \phi + \gamma}{\frac{1 - \phi}{1 - \beta \rho} (1 - \beta \rho \nu)} \\ &\simeq \frac{1}{\sigma} \frac{1 - \phi + \gamma}{1 - \phi} \end{aligned}$$

for ν close to unity.

Now consider the comparative steady state analysis. Using (A.2), we obtain,

$$\eta_{\Gamma, \vartheta} = \frac{1}{\sigma}.$$

It follows that

$$\bar{\eta}_{\Gamma, \vartheta} = \eta_{\Gamma, \vartheta} \left[1 + \frac{\gamma}{1 - \phi} \right].$$

Note that $\bar{\eta}_{\Gamma, \vartheta} > \eta_{\Gamma, \vartheta}$, since $\gamma > 0$. Indeed, by making ϕ sufficiently close to unity, $\bar{\eta}_{\Gamma, \vartheta}$ can be made arbitrarily large, even though $\eta_{\Gamma, \vartheta}$ is always simply $1/\sigma$.

In the previous example, the more inertia there is in wages, that is, the larger is γ , the higher is the contemporaneous impact of a technology shock on labor market tightness. Clearly, in this example comparative steady state analysis is very misleading about the dynamic effects of a persistent shock to technology.

APPENDIX B: SOLUTION TO THE AOB BARGAINING PROBLEM

We develop an analytic expression relating the equilibrium wage rate to economy-wide variables taken as given by firms and workers when bargaining.

It is useful to re-state the indifference conditions for the worker and the firm given in the main text:

$$\begin{aligned} & w_{j,t} + \tilde{w}_t^p + A_t \\ &= \delta \left[\frac{M-j+1}{M} D + \tilde{U}_t \right] + (1-\delta) \left[\frac{D}{M} + w_{j+1,t} + \tilde{w}_t^p + A_t \right] \\ & \quad \text{for } j = 1, 3, \dots, M-1, \\ & \frac{M-j+1}{M} \vartheta_t + \tilde{\vartheta}_t^p - (w_{j,t} + \tilde{w}_t^p) \\ &= (1-\delta) \left[-\gamma + \frac{M-j}{M} \vartheta_t + \tilde{\vartheta}_t^p - (w_{j+1,t} + \tilde{w}_t^p) \right] \\ & \quad \text{for } j = 2, 4, \dots, M-2, \\ & \frac{\vartheta_t}{M} + \tilde{\vartheta}_t^p - (w_{j,t} + \tilde{w}_t^p) = 0 \quad \text{for } j = M. \end{aligned}$$

Rewrite the previous expressions and abbreviate variables taken as given during the wage bargaining:

$$w_{j,t} + \tilde{w}_t^p = \underbrace{\frac{D}{M} + \delta(U_t - A_t)}_a - \underbrace{\frac{\delta D}{M} j}_{c_j} + (1 - \delta)(w_{j+1,t} + \tilde{w}_t^p)$$

$$\text{for } j = 1, 3, 5, \dots, M - 1,$$

$$w_{j,t} + \tilde{w}_t^p = \underbrace{\frac{\vartheta_t}{M} + \delta\vartheta_t^p + (1 - \delta)\gamma}_b - \underbrace{\frac{\delta\vartheta_t}{M} j}_{d_j} + (1 - \delta)(w_{j+1,t} + \tilde{w}_t^p)$$

$$\text{for } j = 2, 4, 5, \dots, M - 2,$$

$$w_{j,t} + \tilde{w}_t^p = \left(\frac{1 - M}{M}\right)\vartheta_t + \vartheta_t^p \quad \text{for } j = M,$$

or, in short,

$$w_{j,t} + \tilde{w}_t^p = a - c_j + (1 - \delta)(w_{j+1,t} + \tilde{w}_t^p)$$

$$\text{for } j = 1, 3, 5, \dots, M - 1,$$

$$w_{j,t} + \tilde{w}_t^p = b - d_j + (1 - \delta)(w_{j+1,t} + \tilde{w}_t^p)$$

$$\text{for } j = 2, 4, 5, \dots, M - 2.$$

Write out

$$w_t^p = w_{1,t} + \tilde{w}_t^p = a - c_1 + (1 - \delta)(w_{2,t} + \tilde{w}_t^p),$$

$$w_{2,t} + \tilde{w}_t^p = b - d_2 + (1 - \delta)(w_{3,t} + \tilde{w}_t^p),$$

$$w_{3,t} + \tilde{w}_t^p = a - c_3 + (1 - \delta)(w_{4,t} + \tilde{w}_t^p),$$

$$w_{4,t} + \tilde{w}_t^p = b - d_4 + (1 - \delta)(w_{5,t} + \tilde{w}_t^p),$$

...

$$w_{M-1,t} + \tilde{w}_t^p = a - c_{M-1} + (1 - \delta)(w_{M,t} + \tilde{w}_t^p).$$

Substituting several times results in the pattern

$$\begin{aligned} w_t^p &= a + (1 - \delta)^2 a + (1 - \delta)^4 a + (1 - \delta)^6 a \\ &\quad + (1 - \delta)b + (1 - \delta)^3 b + (1 - \delta)^5 b \\ &\quad - c_1 - (1 - \delta)^2 c_3 - (1 - \delta)^4 c_5 - (1 - \delta)^6 c_7 \\ &\quad - (1 - \delta)d_2 - (1 - \delta)^3 d_4 - (1 - \delta)^5 d_6 \\ &\quad + (1 - \delta)^7 (w_{8,t} + \tilde{w}_t^p). \end{aligned}$$

Rearranging yields

$$\begin{aligned}
w_t^p &= a + (1 - \delta)^2 a + (1 - \delta)^4 a + (1 - \delta)^6 a + \dots + (1 - \delta)^{M-2} a \\
&\quad + (1 - \delta)b + (1 - \delta)^3 b + (1 - \delta)^5 b + \dots + (1 - \delta)^{M-3} b \\
&\quad - c_1 - (1 - \delta)^2 c_3 - (1 - \delta)^4 c_5 \\
&\quad - (1 - \delta)^6 c_7 - \dots - (1 - \delta)^{M-2} c_{M-1} \\
&\quad - (1 - \delta)d_2 - (1 - \delta)^3 d_4 \\
&\quad - (1 - \delta)^5 d_6 - \dots - (1 - \delta)^{M-3} d_{M-2} \\
&\quad + (1 - \delta)^{M-1} (w_{M,t} + \tilde{w}_t^p)
\end{aligned}$$

or, equivalently,

$$\begin{aligned}
\text{(A.13)} \quad w_t^p &= a[1 + (1 - \delta)^2 + (1 - \delta)^4 + (1 - \delta)^6 + \dots + (1 - \delta)^{M-2}] \\
&\quad + b(1 - \delta)[1 + (1 - \delta)^2 + (1 - \delta)^4 + (1 - \delta)^6 + \dots \\
&\quad + (1 - \delta)^{M-4}] \\
&\quad - c_1 - (1 - \delta)^2 c_3 - (1 - \delta)^4 c_5 \\
&\quad - (1 - \delta)^6 c_7 - \dots - (1 - \delta)^{M-2} c_{M-1} \\
&\quad - (1 - \delta)d_2 - (1 - \delta)^3 d_4 \\
&\quad - (1 - \delta)^5 d_6 - \dots - (1 - \delta)^{M-3} d_{M-2} \\
&\quad + (1 - \delta)^{M-1} \left[\left(\frac{1 - M}{M} \right) \vartheta_t + \vartheta_t^p \right].
\end{aligned}$$

Note that

$$\begin{aligned}
S &= 1 + x + x^2 + x^3 + \dots + x^n, \\
xS &= x + x^2 + x^3 + \dots + x^n + x^{n+1}.
\end{aligned}$$

Subtracting and rearranging yields

$$S = \frac{1 - x^{n+1}}{1 - x},$$

so that

$$\text{(A.14)} \quad 1 + x + x^2 + x^3 + \dots + x^n = \frac{1 - x^{n+1}}{1 - x}.$$

Using (A.14), we can write the square brackets multiplying a and b in (A.13) as

$$\begin{aligned} & \left[1 + \underbrace{(1-\delta)^2}_x + \underbrace{(1-\delta)^4}_{x^2} + \underbrace{(1-\delta)^6}_{x^3} + \cdots + \underbrace{(1-\delta)^{M-2}}_{x^{(M-2)/2}} \right] \\ &= \frac{1 - (1-\delta)^M}{1 - (1-\delta)^2} \end{aligned}$$

and

$$\begin{aligned} & \left[1 + \underbrace{(1-\delta)^2}_x + \underbrace{(1-\delta)^4}_{x^2} + \underbrace{(1-\delta)^6}_{x^3} + \cdots + \underbrace{(1-\delta)^{M-4}}_{x^{(M-4)/2}} \right] \\ &= \frac{1 - (1-\delta)^{M-2}}{1 - (1-\delta)^2}. \end{aligned}$$

Hence,

$$\begin{aligned} \text{(A.15)} \quad w_t^p &= \frac{1 - (1-\delta)^M}{1 - (1-\delta)^2} a + b(1-\delta) \frac{1 - (1-\delta)^{M-2}}{1 - (1-\delta)^2} \\ &\quad - [c_1 + (1-\delta)^2 c_3 + (1-\delta)^4 c_5 + \cdots + (1-\delta)^{M-2} c_{M-1}] \\ &\quad - (1-\delta) [d_2 + (1-\delta)^2 d_4 \\ &\quad + (1-\delta)^4 d_6 + \cdots + (1-\delta)^{M-4} d_{M-2}] \\ &\quad + (1-\delta)^{M-1} \left[\left(\frac{1-M}{M} \right) \vartheta_t + \vartheta_t^p \right]. \end{aligned}$$

The square bracket in the last line in (A.15) can be written as

$$\begin{aligned} & [d_2 + (1-\delta)^2 d_4 + (1-\delta)^4 d_6 + \cdots + (1-\delta)^{M-4} d_{M-2}] \\ &= 2 \frac{\delta \vartheta_t}{M} \left[1 + (1-\delta)^2 2 + (1-\delta)^4 3 + \cdots + (1-\delta)^{M-4} \frac{(M-2)}{2} \right]. \end{aligned}$$

Note that differentiating both sides of

$$1 + x + x^2 + x^3 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

yields

$$1 + 2x + 3x^2 + \cdots + nx^{n-1} = \frac{-(n+1)x^n(1-x) + (1-x^{n+1})}{(1-x)^2}.$$

Hence, the square bracket in the last line in (A.15) can be expressed more compactly as

$$\begin{aligned} & \left[1 + \underbrace{(1-\delta)^2}_x 2 + \underbrace{(1-\delta)^4}_{x^2} 3 + \underbrace{(1-\delta)^6}_{x^3} 4 + \cdots + \underbrace{(1-\delta)^{M-4}}_{x^{(M-4)/2}} \underbrace{\frac{(M-2)}{2}}_n \right] \\ &= \frac{(1 - (1-\delta)^M) - \frac{M}{2}(1-\delta)^{(M-2)}(1 - (1-\delta)^2)}{(1 - (1-\delta)^2)^2}. \end{aligned}$$

Finally, the terms involving c in (A.15) can be rewritten as

$$\begin{aligned} & [c_1 + (1-\delta)^2 c_3 + (1-\delta)^4 c_5 + (1-\delta)^6 c_7 + \cdots + (1-\delta)^{M-2} c_{M-1}] \\ &= \frac{\delta D}{M} [1 + (1-\delta)^2 3 + (1-\delta)^4 5 \\ & \quad + (1-\delta)^6 7 + \cdots + (1-\delta)^{M-2} (M-1)] \\ &= \frac{\delta D}{M} 2 [1/2 + (1-\delta)^2 2 + (1-\delta)^4 3 \\ & \quad + (1-\delta)^6 4 + \cdots + (1-\delta)^{M-2} M/2] \\ & \quad - \frac{\delta D}{M} [1 + (1-\delta)^2 + (1-\delta)^4 \\ & \quad + (1-\delta)^6 + \cdots + (1-\delta)^{M-2}] + \frac{\delta D}{M} \\ &= 2 \frac{\delta D}{M} \frac{(1 - (1-\delta)^{M+2}) - \left(1 + \frac{M}{2}\right) (1-\delta)^M (1 - (1-\delta)^2)}{(1 - (1-\delta)^2)^2} \\ & \quad - \frac{\delta D}{M} \frac{1 - (1-\delta)^M}{1 - (1-\delta)^2}. \end{aligned}$$

Pulling everything together, we can write (A.15) as

$$\begin{aligned} w_t^p &= \frac{1 - (1-\delta)^M}{1 - (1-\delta)^2} \left[\frac{D}{M} + \delta(U_t - A_t) \right] \\ & \quad + (1-\delta) \frac{1 - (1-\delta)^{M-2}}{1 - (1-\delta)^2} \left[\frac{\vartheta_t}{M} + \delta \vartheta_t^p + (1-\delta)\gamma \right] \end{aligned}$$

$$\begin{aligned}
& -2 \frac{\delta D}{M} \frac{(1 - (1 - \delta)^{M+2}) - \left(1 + \frac{M}{2}\right)(1 - \delta)^M(1 - (1 - \delta)^2)}{(1 - (1 - \delta)^2)^2} \\
& + \frac{\delta D}{M} \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2} \\
& - (1 - \delta) 2 \frac{\delta \vartheta_t}{M} \frac{(1 - (1 - \delta)^M) - \frac{M}{2}(1 - \delta)^{(M-2)}(1 - (1 - \delta)^2)}{(1 - (1 - \delta)^2)^2} \\
& + (1 - \delta)^{M-1} \left[\left(\frac{1 - M}{M} \right) \vartheta_t + \vartheta_t^p \right].
\end{aligned}$$

Collecting terms gives

$$\begin{aligned}
w_t^p &= \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2} \delta (U_t - A_t) \\
& + \left[(1 - \delta) \delta \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2} + (1 - \delta)^{M-1} \right] \vartheta_t^p \\
& + (1 - \delta) \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2} (1 - \delta) \gamma \\
& + \left[(1 + \delta) \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2} \right. \\
& \quad \left. - 2\delta \frac{(1 - (1 - \delta)^{M+2}) - \left(1 + \frac{M}{2}\right)(1 - \delta)^M(1 - (1 - \delta)^2)}{(1 - (1 - \delta)^2)^2} \right] \frac{D}{M} \\
& + \left[(1 - \delta) \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2} + (1 - \delta)^{M-1} (1 - M) \right. \\
& \quad \left. - (1 - \delta) 2\delta \frac{(1 - (1 - \delta)^M) - \frac{M}{2}(1 - \delta)^{(M-2)}(1 - (1 - \delta)^2)}{(1 - (1 - \delta)^2)^2} \right] \\
& \times \frac{\vartheta_t}{M}.
\end{aligned}$$

Simplifying using straightforward algebra yields

$$\begin{aligned} (2 - \delta)w_t^p &= (1 - (1 - \delta)^M)(U_t - A_t) + (1 - \delta + (1 - \delta)^M)\vartheta_t^p \\ &\quad + \frac{1}{\delta}((1 - \delta)^2 - (1 - \delta)^M)\gamma \\ &\quad + \frac{(1 - \delta)^M(1 - \delta - (2 - \delta)M) - (1 - \delta)}{2 - \delta} \left[\frac{\vartheta_t}{M} - \frac{D}{M} \right]. \end{aligned}$$

After some further rewriting, we can express the previous expression as the alternating offer bargaining sharing rule

$$(\alpha_1 + \alpha_2)w_t^p = \alpha_1\vartheta_t^p + \alpha_2(U_t - A_t) + \alpha_3\gamma - \alpha_4(\vartheta_t - D),$$

where

$$\begin{aligned} \alpha_1 &= 1 - \delta + (1 - \delta)^M, \\ \alpha_2 &= 1 - (1 - \delta)^M, \\ \alpha_3 &= \alpha_2 \frac{1 - \delta}{\delta} - \alpha_1, \\ \alpha_4 &= \frac{1 - \delta}{2 - \delta} \frac{\alpha_2}{M} + 1 - \alpha_2. \end{aligned}$$

Note that $\alpha_1, \dots, \alpha_4 > 0$. Alternatively, we can write the alternating offer bargaining sharing rule in terms of the variables

$$\alpha_1 J_t = \alpha_2(V_t - U_t) - \alpha_3\gamma + \alpha_4(\vartheta_t - D).$$

Finally, notice that for $M \rightarrow \infty$, the sharing rule becomes

$$J_t = \frac{1}{1 - \delta} \left[V_t - U_t - \frac{(1 - \delta)^2}{\delta} \gamma \right].$$

APPENDIX C: MEDIUM-SIZED DSGE MODEL

Here, we list the dynamic equilibrium equations for the medium-sized DSGE model with alternating offer bargaining.

C.1. *Medium-Sized Model: Scaled Dynamic Equations*

$$\begin{aligned} \text{Cons. FOC (1): } \psi_t &= \zeta_t^c (c_t - bc_{t-1}/\mu_t)^{-1} \\ &\quad - \beta b E_t \zeta_{t+1}^c (c_{t+1}\mu_{t+1} - bc_t)^{-1} \end{aligned}$$

Bond. FOC (2): $\psi_t = \beta E_t \psi_{t+1} R_t / (\pi_{t+1} \mu_{t+1})$

Invest. FOC (3): $1 = p_{k',t} Y_t [1 - \tilde{S}_t - \tilde{S}'_t \mu_t \mu_{\psi,t} i_t / i_{t-1}]$
 $+ \beta E_t \psi_{t+1} / \psi_t p_{k',t+1} Y_{t+1} \tilde{S}'_{t+1} (i_{t+1} / i_t)^2 \mu_{\psi,t+1} \mu_{t+1}$

Capital FOC (4): $\psi_t = \beta E_t \psi_{t+1} r_{t+1}^k / (\pi_{t+1} \mu_{t+1})$

LOM capital (5): $\bar{k}_t = (1 - \delta^k) / (\mu_t \mu_{\psi,t}) \bar{k}_{t-1} + Y_t (1 - \tilde{S}) i_t$

Cost. minim. (6): $0 = a'(u_t^k) u_t^k \bar{k}_{t-1} / (\mu_{\psi,t} \mu_t)$
 $- \alpha / (1 - \alpha) \vartheta_t [v^f R_t + 1 - v^f] l_t$

Production (7): $y_t = \hat{p}_t^{\lambda / (\lambda - 1)} [\epsilon_t (u_t^k \bar{k}_{t-1} / (\mu_t \mu_{\psi,t}))^\alpha l_t^{1-\alpha} - n_{\phi,t} \phi]$

Resources (8): $y_t = n_{g,t} g + c_t + i_t + a(u_t^k) \bar{k}_{t-1} / (\mu_{\psi,t} \mu_t)$
 $+ n_{s,t} s Q_t^{-1} x_t l_{t-1} + n_{\kappa,t} \kappa x_t l_{t-1}$

Taylor rule (9): $\ln(R_t / R) = \rho_R \ln(R_{t-1} / R)$
 $+ (1 - \rho_R) [r_\pi \ln(\pi_t / \pi) + r_y \ln(\mathcal{Y}_t / \mathcal{Y})]$
 $+ \sigma_R \epsilon_{R,t} / 400$

Pricing 1 (10): $F_t = \psi_t y_t + \beta \xi E_t (\tilde{\pi}_{t+1} / \pi_{t+1})^{1/(1-\lambda)} F_{t+1}$

Pricing 2 (11): $K_t = \lambda \psi_t y_t m c_t + \beta \xi E_t (\tilde{\pi}_{t+1} / \pi_{t+1})^{\lambda/(1-\lambda)} K_{t+1}$

Pricing 3 (12): $(1 - \xi)(K_t / F_t)^{1/(1-\lambda)} = 1 - \xi (\tilde{\pi}_t / \pi_t)^{1/(1-\lambda)}$

Price disp. (13): $\hat{p}_t^{\lambda/(1-\lambda)} = (1 - \xi)^{1-\lambda} [1 - \xi (\tilde{\pi}_t / \pi_t)^{1/(1-\lambda)}]^\lambda$
 $+ \xi [\tilde{\pi}_t / \pi_t \hat{p}_{t-1}]^{\lambda/(1-\lambda)}$

PV wages (14): $w_t^p = w_t + \rho \beta E_t \psi_{t+1} / \psi_t w_{t+1}^p$

PV revenue (15): $\vartheta_t^p = \vartheta_t + \rho \beta E_t \psi_{t+1} / \psi_t \vartheta_{t+1}^p$

Free entry (16): $n_{s,t} s = Q_t (J_t - n_{\kappa,t} \kappa)$

Firm value (17): $J_t = \vartheta_t^p - w_t^p$

Work value (18): $V_t = w_t^p + A_t$

Cont. value (19): $A_t = (1 - \rho) \beta E_t \psi_{t+1} / \psi_t [f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1}]$
 $+ \rho \beta E_t \psi_{t+1} / \psi_t A_{t+1}$

Unemp. value (20): $U_t = n_{D,t} D$
 $+ \beta E_t \psi_{t+1} / \psi_t [f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1}]$

Sharing rule (21): $\alpha_1 J_t = \alpha_2 (V_t - U_t) - \alpha_3 n_{\gamma,t} \gamma + \alpha_4 (\vartheta_t - n_{D,t} D)$

Real GDP (22): $\mathcal{Y}_t = n_{g,t} g_t + c_t + i_t$

Unemp. rate (23): $u_t = 1 - l_t$

Finding rate (24): $f_t = x_t l_{t-1} / (1 - \rho l_{t-1})$

Matching (25): $x_t l_{t-1} = \sigma_m (1 - \rho l_{t-1})^\sigma (v_t^{\text{tot}})^{1-\sigma}$

Vacancies (26): $v_t^{\text{tot}} = v_t l_{t-1}$

Filling rate (27): $Q_t = x_t / v_t$

LOM empl. (28): $l_t = (\rho + x_t) l_{t-1}$

Comp. tech. (29): $\ln \mu_t = \alpha / (1 - \alpha) \ln \mu_{\psi,t} + \ln \mu_{z,t}$

Neutr. tech. (30): $\ln \mu_{z,t} = (1 - \rho_{\mu_z}) \ln \mu_z + \rho_{\mu_z} \ln \mu_{z,t-1}$
 $+ \sigma_{\mu_z} \varepsilon_{\mu_z,t} / 100$

Invest. tech. (31): $\ln \mu_{\psi,t} = (1 - \rho_{\mu_\psi}) \ln \mu_\psi + \rho_{\mu_\psi} \ln \mu_{\psi,t-1}$
 $+ \sigma_{\mu_\psi} \varepsilon_{\mu_\psi,t} / 100$

Tech. diffus. (32): $n_{i,t} = n_{i,t-1}^{1-\theta_i} \mu_t^{-1}$ for $i \in \{\phi, \kappa, \gamma, g, D, s\}$,
 where $n_{i,t} = \Omega_{i,t} / \Phi_t$

Check: 32 equations in the following 32 endogenous unknowns:

$$\psi_t \ c_t \ R_t \ \pi_t \ p_{k',t} \ i_t \ u_t^k \ \bar{k}_t \ \vartheta_t \ l_t \ y_t \ \dot{p}_t \ x_t \ F_t \ K_t \ S_t \ U_t \ J_t$$

$$w_t \ v_t \ Q_t \ f_t \ u_t \ \mathcal{Y}_t \ \vartheta_t^p \ w_t^p \ A_t \ v_t^{\text{tot}} \ n_{i,t} \ \mu_t \ \mu_{\psi,t} \ \mu_{z,t}.$$

In the above 32 equations, it is useful to define several abbreviated variables that are functions of the 32 endogenous variables. In particular,

Cap. util. cost (33): $a(u_t^k) = 0.5 \sigma_b \sigma_a (u_t^k)^2 + \sigma_b (1 - \sigma_a) u_t^k$
 $+ \sigma_b (\sigma_a / 2 - 1)$

Cap. util. deriv. (34): $a'(u_t^k) = \sigma_b \sigma_a u_t^k + \sigma_b (1 - \sigma_a)$

Invest. adj. cost (35): $\tilde{S}_t = 0.5 \exp[\sqrt{\tilde{S}''}(\mu_t \mu_{\psi,t} \dot{i}_t / i_{t-1} - \mu \cdot \mu_\psi)]$
 $+ 0.5 \exp[-\sqrt{\tilde{S}''}(\mu_t \mu_{\psi,t} \dot{i}_t / i_{t-1} - \mu \cdot \mu_\psi)]$
 $- 1$

Invest. adj. deriv. (36): $\tilde{S}'_t = 0.5 \sqrt{\tilde{S}''} \exp[\sqrt{\tilde{S}''}(\mu_t \mu_{\psi,t} \dot{i}_t / i_{t-1} - \mu \cdot \mu_\psi)]$
 $- 0.5 \sqrt{\tilde{S}''}$
 $\times \exp[-\sqrt{\tilde{S}''}(\mu_t \mu_{\psi,t} \dot{i}_t / i_{t-1} - \mu \cdot \mu_\psi)]$

$$\begin{aligned} \text{Capital return (37): } r_t^k &= \pi_t / (\mu_{\psi,t} p_{k',t-1}) \\ &\quad \times (u_t^k a'(u_t^k) - a(u_t^k) + (1 - \delta_k) p_{k',t}) \end{aligned}$$

$$\begin{aligned} \text{Marginal cost (38): } mc_t &= \tau_t (\mu_{\psi,t} \mu_t)^\alpha \vartheta_t [\nu^f R_t + 1 - \nu^f] (u_t^k \bar{k}_{t-1} / l_t)^{-\alpha} \\ &\quad / (\epsilon_t (1 - \alpha)) \end{aligned}$$

$$\text{Price indexation (39): } \tilde{\pi}_t = \pi_{t-1}^{\kappa^f} \pi^{1-\kappa^f - \varkappa^f} \tilde{\pi}^{\varkappa^f}$$

We adopt $\kappa^f = 0$ and $\varkappa^f = \tilde{\pi} = 1$, which corresponds to the case of no indexation of prices. We set $\nu^f = 1$, which corresponds to the working capital specification in the main text. The variables ζ_t^c , Y_t , ϵ_t , and τ_t are exogenous and set equal to 1 for all t . In addition, we set $g_t = g$ for all t .

Also, the case of Nash sharing can be obtained by replacing the alternating offer sharing rule (21) with the equation

$$\text{Nash sharing (21') : } V_t - U_t = \eta[V_t - U_t + J_t].$$

C.2. Medium-Sized Model: Steady State

IMPOSE $u^k = 1$, solve (37) for σ_b later

$$(33): a(1) = 0$$

$$(29): \mu_z = \mu / (\mu_\psi)^{\alpha/(1-\alpha)}$$

$$(32): n_i = \mu^{-1/\vartheta_i}$$

$$(30): \varepsilon_{\mu_z} = 0$$

$$(31): \varepsilon_{\mu_\psi} = 0$$

$$(36): \tilde{S} = 0$$

$$(37): \tilde{S}' = 0$$

IMPOSE u , solve (21) or (21') for γ or η later

$$(23): l = 1 - u$$

IMPOSE π , “drop” equation (9), i.e., $R = R$

$$(2): R = \pi \mu / \beta$$

$$(3): p_{k'} = 1$$

$$(4): r^k = \pi \mu / \beta$$

$$(37): \sigma_b = R^k \mu_\psi p_{k'} / \pi - (1 - \delta_k) p_{k'}$$

$$(34): a'(1) = \sigma_b$$

$$(40): \tilde{\pi}_t = \pi^{\kappa^f} \pi^{1-\kappa^f - \lambda^f} \tilde{\pi}^{\lambda^f}$$

$$(10)-(12): mc = \frac{1}{\lambda} \frac{1 - \beta \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}}{1 - \beta \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}} \left[\frac{1 - \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}}{1 - \xi} \right]^{1-\lambda}$$

$$(13): \hat{p} = \left[\frac{1 - \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}}{1 - \xi} \right]^{1-\lambda} / \left[\frac{1 - \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}}{1 - \xi} \right]^{(1-\lambda)/\lambda}$$

$$(6) \& (39): \bar{k}/l = [\alpha(\mu_\psi \mu)^{1-\alpha} mc / \sigma_b / \tau]^{1/(1-\alpha)}$$

$$\rightarrow \bar{k} = \bar{k}/l \cdot l$$

$$(39): \vartheta = \frac{(1-\alpha)mc}{\tau(\mu_\psi \mu)^\alpha [v^f R + 1 - v^f]} (\bar{k}/l)^\alpha$$

Steady state profits are

$$Prof = Py - MC(y + n_\phi \phi) \quad \text{solve for } n_\phi \phi$$

$$n_\phi \phi = \left(\frac{1 - mc}{mc} \right) y - \frac{Prof/P}{mc}$$

substitute in (7) and rewrite

$$y = \frac{mc}{(\hat{p}^{\lambda/(1-\lambda)} - 1)mc + 1 - \frac{Prof}{Py}} (\bar{k}/l / (\mu \cdot \mu_\psi))^\alpha l$$

$$\text{for given } \frac{Prof}{Py}$$

$$(7): \phi = [(\bar{k}/l / (\mu \cdot \mu_\psi))^\alpha l - y \hat{p}^{\lambda/(1-\lambda)}] / n_\phi$$

$$(5): i = [1 - (1 - \delta_k) / (\mu \cdot \mu_\psi)] \bar{k}$$

Assume g equals share η_g of y and recruiting/search cost equal

share $\eta_h + \eta_s$ of y

$$(8): c = (1 - \eta_g - \eta_s - \eta_h) y - i$$

for some given $\eta_g, \eta_s, \eta_h \rightarrow g = \eta_g y / n_g$

$$(1): \psi = (c - bc/\mu)^{-1} - \beta b(c\mu - bc)^{-1}$$

$$(22): \mathcal{Y} = n_g g + c + i$$

$$(11): K = \frac{\lambda \cdot \psi \cdot y \cdot mc}{1 - \beta \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}}$$

$$(10): F = \frac{\psi \cdot y}{1 - \beta \xi (\tilde{\pi}/\pi)^{1/(1-\lambda)}}$$

$$(28): x = 1 - \rho$$

$$(24): f = xl/(1 - \rho l)$$

IMPOSE Q , solve (25) for σ_m

$$(27): v = \frac{x}{Q}$$

$$(26): v^{\text{tot}} = v \cdot l$$

$$(25): \sigma_m = xl(1 - \rho l)^{-\sigma} (v^{\text{tot}})^{\sigma-1}$$

given η_h and η_s calculate κ and s

$$\rightarrow \kappa = \eta_h/(x)/l \cdot y/n_\kappa$$

$$\rightarrow s = \eta_s/(Q^{-1}x)/l \cdot y/n_s$$

$$(16): J = n_s s Q^{-1} + n_\kappa \kappa$$

$$(15): \vartheta^p = \frac{\vartheta}{1 - \rho\beta}$$

$$(17): w^p = \vartheta^p - J$$

$$(14): w = w^p(1 - \rho\beta)$$

$$(18)-(20): V - U = \left(w^p - \frac{\frac{n_D D}{w} w}{1 - \beta\rho} \right) / \left(\frac{1 - (1-f)\beta\rho}{1 - \beta\rho} \right)$$

where $\frac{n_D D}{w}$ is the estimated replacement ratio

$$(20): U = \frac{\frac{n_D D}{w} w + \beta f(V - U)}{1 - \beta}$$

$$\rightarrow V = V - U + U$$

$$(18): A = V - w^p$$

$$(21): \gamma = \left(\alpha_2(V - U) - \alpha_1 J + \alpha_4 \left(\vartheta - \frac{n_D D}{w} w \right) \right) / (n_\gamma \cdot \alpha_3)$$

$$(21'): \eta = \frac{V - U}{V - U + J}$$

C.3. *Medium-Sized Sticky Wage Model: Scaled Dynamic Equations*

Cons. FOC (1): $\psi_t = \zeta_t^c (c_t - bc_{t-1}/\mu_t)^{-1}$

$$- \beta b E_t \zeta_{t+1}^c (c_{t+1} \mu_{t+1} - bc_t)^{-1}$$

Bond. FOC (2): $\psi_t = \beta E_t \psi_{t+1} R_t / (\pi_{t+1} \mu_{t+1})$

Invest. FOC (3): $1 = p_{k',t} Y_t [1 - \tilde{S}_t - \tilde{S}'_t \mu_t \mu_{\psi,t} i_t / i_{t-1}]$

$$+ \beta E_t \psi_{t+1} / \psi_t p_{k',t+1} Y_{t+1} \tilde{S}'_{t+1} (i_{t+1} / i_t)^2 \mu_{\psi,t+1} \mu_{t+1}$$

Capital FOC (4): $\psi_t = \beta E_t \psi_{t+1} R_{t+1}^k / (\pi_{t+1} \mu_{t+1})$

LOM capital (5): $\bar{k}_t = (1 - \delta^k) / (\mu_t \mu_{\psi,t}) \bar{k}_{t-1} + Y_t (1 - \tilde{S}) i_t$

Cost. minim. (6): $0 = a'(u_t^k) u_t^k \bar{k}_{t-1} / (\mu_{\psi,t} \mu_t)$

$$- \alpha / (1 - \alpha) w_t [\nu^f R_t + 1 - \nu^f] \dot{w}_t^{\lambda w / (\lambda w - 1)} l_t$$

Production (7): $y_t = \hat{p}_t^{\lambda / (\lambda - 1)}$

$$\times [\epsilon_t (u_t^k \bar{k}_{t-1} / (\mu_t \mu_{\psi,t}))^\alpha (\dot{w}_t^{\lambda w / (\lambda w - 1)} l_t)^{1 - \alpha} - n_{\phi,t} \phi]$$

Resources (8): $y_t = n_{g,t} g_t + c_t + i_t + a(u_t^k) \bar{k}_{t-1} / (\mu_{\psi,t} \mu_t)$

Taylor rule (9): $\ln(R_t/R) = \rho_R \ln(R_{t-1}/R)$

$$+ (1 - \rho_R) [r_\pi \ln(\pi_t/\pi) + r_y \ln(\mathcal{Y}_t/\mathcal{Y})]$$

$$+ \sigma_R \varepsilon_{R,t} / 400$$

Pricing 1 (10): $F_t = \psi_t y_t + \beta \xi E_t (\tilde{\pi}_{t+1} / \pi_{t+1})^{1/(1-\lambda)} F_{t+1}$

Pricing 2 (11): $K_t = \lambda \psi_t y_t m c_t + \beta \xi E_t (\tilde{\pi}_{t+1} / \pi_{t+1})^{\lambda/(1-\lambda)} K_{t+1}$

Pricing 3 (12): $(1 - \xi)(K_t/F_t)^{1/(1-\lambda)} = 1 - \xi(\tilde{\pi}_t/\pi_t)^{1/(1-\lambda)}$

Price disp. (13): $\hat{p}_t^{\lambda/(1-\lambda)} = (1 - \xi)^{1-\lambda} [1 - \xi(\tilde{\pi}_t/\pi_t)^{1/(1-\lambda)}]^\lambda$

$$+ \xi[\tilde{\pi}_t/\pi_t \hat{p}_{t-1}]^{\lambda/(1-\lambda)}$$

Wage disp. (14): $\dot{w}_t^{\lambda w / (1-\lambda w)} = (1 - \xi_w)^{1-\lambda w}$

$$\times (1 - \xi_w (\tilde{\pi}_{w,t} / \pi_{w,t})^{1/(1-\lambda w)})^{\lambda w}$$

$$+ \xi_w [\tilde{\pi}_{w,t} / \pi_{w,t} \dot{w}_{t-1}]^{\lambda w / (1-\lambda w)}$$

$$\begin{aligned} \text{Wage setting 1 (15): } F_{w,t} &= \psi_t / \lambda_w \dot{w}_t^{\lambda_w / (\lambda_w - 1)} l_t \\ &+ \beta \xi_w E_t (w_{t+1} / w_t) (\tilde{\pi}_{w,t+1} / \pi_{w,t+1})^{1/(1-\lambda_w)} \\ &\times F_{w,t+1} \end{aligned}$$

$$\begin{aligned} \text{Wage setting 2 (16): } K_{w,t} &= (\dot{w}_t^{\lambda_w / (\lambda_w - 1)} l_t)^{1+\sigma_L} \\ &+ \beta \xi_w E_t (\tilde{\pi}_{w,t+1} / \pi_{w,t+1})^{\lambda_w(1+\sigma_L)/(1-\lambda_w)} \\ &\times K_{w,t+1} \end{aligned}$$

$$\begin{aligned} \text{Wage setting 3 (17): } 1 - \xi_w (\tilde{\pi}_{w,t} / \pi_{w,t})^{1/(1-\lambda_w)} \\ = (1 - \xi_w) (A \cdot K_{w,t} / (w_t F_{w,t}))^{1/(1-\lambda_w(1+\sigma_L))} \end{aligned}$$

$$\text{Wage inflation (18): } \pi_{w,t} = w_t \mu_t \pi_t / w_{t-1}$$

$$\text{Real GDP (19): } \mathcal{Y}_t = n_{g,t} g_t + c_t + i_t$$

$$\text{Comp. tech. (20): } \ln \mu_t = \alpha / (1 - \alpha) \ln \mu_{\psi,t} + \ln \mu_{z,t}$$

$$\begin{aligned} \text{Neutr. tech. (21): } \ln \mu_{z,t} &= (1 - \rho_{\mu_z}) \ln \mu_z + \rho_{\mu_z} \ln \mu_{z,t-1} \\ &+ \sigma_{\mu_z} \varepsilon_{\mu_z,t} / 100 \end{aligned}$$

$$\begin{aligned} \text{Invest. tech. (22): } \ln \mu_{\psi,t} &= (1 - \rho_{\mu_{\psi}}) \ln \mu_{\psi} + \rho_{\mu_{\psi}} \ln \mu_{\psi,t-1} \\ &+ \sigma_{\mu_{\psi}} \varepsilon_{\mu_{\psi},t} / 100 \end{aligned}$$

$$\text{Tech. diffus. (23): } n_{i,t} = n_{i,t-1}^{1-\theta_i} \mu_t^{-1} \quad \text{for } i \in \{\phi, g\},$$

$$\text{where } n_{i,t} = \Omega_{i,t} / \Phi_t$$

Check: 23 equations in the following 23 endogenous unknowns:

$$\psi_t \ c_t \ R_t \ \pi_t \ p_{k',t} \ i_t \ u_t^k \ \bar{k}_t \ l_t \ y_t \ \hat{p}_t \ F_t \ K_t \ w_t \ \dot{w}_t \ F_{w,t}$$

$$K_{w,t} \ \pi_{w,t} \ \mathcal{Y}_t \ \mu_t \ \mu_{z,t} \ \mu_{\psi,t} \ n_{i,t}.$$

In the above 23 equations, it is useful to define several abbreviated variables that are functions of the 23 endogenous variables. In particular,

$$\begin{aligned} \text{Cap. util. cost. (24): } a(u_t^k) &= 0.5 \sigma_b \sigma_a (u_t^k)^2 + \sigma_b (1 - \sigma_a) u_t^k \\ &+ \sigma_b ((\sigma_a / 2) - 1) \end{aligned}$$

$$\text{Cap. util. deriv. (25): } a'(u_t^k) = \sigma_b \sigma_a u_t^k + \sigma_b (1 - \sigma_a)$$

$$\begin{aligned} \text{Invest. adj. cost (26): } \tilde{S}_t &= 0.5 \exp[\sqrt{\tilde{S}''}(\mu_t \mu_{\Psi,t} i_t / i_{t-1} - \mu \cdot \mu_{\Psi})] \\ &\quad + 0.5 \exp[-\sqrt{\tilde{S}''}(\mu_t \mu_{\Psi,t} i_t / i_{t-1} - \mu \cdot \mu_{\Psi})] \\ &\quad - 1 \end{aligned}$$

$$\begin{aligned} \text{Inv. adj. deriv. (27): } \tilde{S}'_t &= 0.5 \sqrt{\tilde{S}''} \exp[\sqrt{\tilde{S}''}(\mu_t \mu_{\Psi,t} i_t / i_{t-1} - \mu \cdot \mu_{\Psi})] \\ &\quad - 0.5 \sqrt{\tilde{S}''} \\ &\quad \times \exp[-\sqrt{\tilde{S}''}(\mu_t \mu_{\Psi,t} i_t / i_{t-1} - \mu \cdot \mu_{\Psi})] \end{aligned}$$

$$\begin{aligned} \text{Capital return (28): } R_t^k &= \pi_t / (\mu_{\Psi,t} p_{k',t-1}) \\ &\quad \times (u_t^k a'(u_t^k) - a(u_t^k) + (1 - \delta^k) p_{k',t}) \end{aligned}$$

$$\begin{aligned} \text{Marginal cost (29): } mc_t &= \tau_t (\mu_{\Psi,t} \mu_t)^\alpha w_t [\nu^f R_t + 1 - \nu^f] \\ &\quad \times (u_t^k \bar{k}_{t-1} / (\hat{w}_t^{\lambda_w / (\lambda_w - 1)} l_t))^{-\alpha} / (\epsilon_t (1 - \alpha)) \end{aligned}$$

$$\text{Price indexation (30): } \tilde{\pi}_t = \pi_{t-1}^{\kappa^f} \pi^{1 - \kappa^f - \varkappa^f} \check{\pi}^{\varkappa^f}$$

$$\text{Wage indexation (31): } \tilde{\pi}_{w,t} = \pi_{t-1}^{\kappa^w} \pi^{1 - \kappa^w - \varkappa^w} \check{\pi}^{\varkappa^w} \mu^{\theta^w}$$

In the baseline specification, we set $\kappa^f = 0$ and $\varkappa^f = \check{\pi} = 1$, which corresponds to the case of no indexation of prices. Likewise, we set $\kappa^w = 0$, $\varkappa^w = \check{\pi} = 1$, and $\theta^w = 0$, which results in no wage indexation. We set $\nu^f = 1$, which corresponds to the working capital specification in the main text. The variables ζ_t^c , Y_t , ϵ_t , and τ_t are exogenous and set equal to 1 for all t . In addition, we set $g_t = g$ for all t .

C.4. Medium-Sized Sticky Wage Model: Steady State

IMPOSE $u^k = 1$, solve (28) for σ_b later

$$(24): a(1) = 0$$

$$(20): \mu_z = \mu / (\mu_{\Psi})^{\alpha/(1-\alpha)}$$

$$(23): n_i = \mu^{-1/\delta_i}$$

$$(21): \varepsilon_{\mu_z} = 0$$

$$(22): \varepsilon_{\mu_{\Psi}} = 0$$

$$(26): \tilde{S} = 0$$

$$(27): \tilde{S}' = 0$$

IMPOSE π , “drop” equation (9), i.e., $R = R$

$$(2): R = \pi\mu/\beta$$

$$(3): p_{k'} = 1$$

$$(4): R^k = \pi\mu/\beta$$

$$(28): \sigma_b = R^k \mu_\Psi p_{k'}/\pi - (1 - \delta^k) p_{k'}$$

$$(25): a'(1) = \sigma_b$$

$$(30): \tilde{\pi}_t = \pi^{\kappa^f} \pi^{1-\kappa^f-\kappa^f} \tilde{\pi}^{\kappa^f}$$

$$(10)-(12): mc = \frac{1}{\lambda} \frac{1 - \beta \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}}{1 - \beta \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}} \left[\frac{1 - \xi (\tilde{\pi}/\pi)^{1/(1-\lambda)}}{1 - \xi} \right]^{1-\lambda}$$

$$(13): \dot{p} = \left[\frac{1 - \xi (\tilde{\pi}/\pi)^{1/(1-\lambda)}}{1 - \xi} \right]^{1-\lambda} / \left[\frac{1 - \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}}{1 - \xi} \right]^{(1-\lambda)/\lambda}$$

$$(6) \& (29): kl = \bar{k}/(\dot{w}^{\lambda_w/(\lambda_w-1)} l) = [\alpha(\mu_\Psi \mu)^{1-\alpha} mc/\sigma_b/\tau]^{1/(1-\alpha)}$$

$$(18): \pi_w = \mu\pi$$

$$(31): \tilde{\pi}_{w,t} = \pi_{t-1}^{\kappa^w} \pi^{1-\kappa^w-\kappa^w} \tilde{\pi}^{\kappa^w} \mu^{\theta^w}$$

$$(14): \dot{w} = \left(\frac{1 - \xi_w (\tilde{\pi}_w/\pi_w)^{1/(1-\lambda_w)}}{1 - \xi_w} \right)^{1-\lambda_w} / \left(\frac{1 - \xi_w (\tilde{\pi}_w/\pi_w)^{\lambda_w/(1-\lambda_w)}}{1 - \xi_w} \right)^{(1-\lambda_w)/\lambda_w}$$

$$(6) \& (29): kl = \bar{k}/(\dot{w}^{\lambda_w/(\lambda_w-1)} l) = [\alpha(\mu_\Psi \mu)^{1-\alpha} mc/\sigma_b/\tau]^{1/(1-\alpha)}$$

$$(29): w = \frac{(1-\alpha)mc}{\tau(\mu_\Psi \mu)^\alpha [\nu^f R + 1 - \nu^f]} (kl)^\alpha$$

Steady state profits are

$$Prof = Py - MC(y + n_\phi \phi) \quad \text{solve for } n_\phi \phi$$

$$n_\phi \phi = \left(\frac{1 - mc}{mc} \right) y - \frac{Prof/P}{mc}$$

substitute in (7) and rewrite

$$y = \frac{mc}{(\dot{p}^{\lambda/(1-\lambda)} - 1)mc + 1 - \frac{Prof}{Py}} (kl/(\mu \cdot \mu_\Psi))^\alpha \dot{w}^{\lambda_w/(\lambda_w-1)} l$$

for given $\frac{Prof}{Py}$

IMPOSE l , solve (17) or A later

$$\rightarrow \bar{k} = kl \cdot \dot{w}^{\lambda_w/(\lambda_w-1)} l$$

$$(7): \phi = [(kl/(\mu \cdot \mu_\psi))^\alpha \dot{w}^{\lambda_w/(\lambda_w-1)} l - y \bar{p}^{\lambda/(1-\lambda)}] / n_\phi$$

$$(5): i = [1 - (1 - \delta^k)/(\mu \cdot \mu_\psi)] \bar{k}$$

Assume g equals share η_g of y

$$(8): c = (1 - \eta_g)y - i \quad \text{for some given } \eta_g \rightarrow g = \eta_g y / n_g$$

$$(1): \psi = (c - bc/\mu)^{-1} - \beta b(c\mu - bc)^{-1}$$

$$(19): \mathcal{Y} = n_g g + c + i$$

$$(11): K = \frac{\lambda \cdot \psi \cdot y \cdot mc}{1 - \beta \xi (\tilde{\pi}/\pi)^{\lambda/(1-\lambda)}}$$

$$(10): F = \frac{\psi \cdot y}{1 - \beta \xi (\tilde{\pi}/\pi)^{1/(1-\lambda)}}$$

$$(16): K_w = \frac{(\dot{w}^{\lambda_w/(\lambda_w-1)} l)^{1+\sigma_L}}{1 - \beta \xi_w (\tilde{\pi}_w/\pi_w)^{\lambda_w(1+\sigma_L)/(1-\lambda_w)}}$$

$$(15): F_w = \frac{\psi/\lambda_w \dot{w}^{\lambda_w/(\lambda_w-1)} l}{1 - \beta \xi_w (\tilde{\pi}_w/\pi_w)^{1/(1-\lambda_w)}}$$

$$(17): A = \left[\frac{1 - \xi_w (\tilde{\pi}_w/\pi_w)^{1/(1-\lambda_w)}}{1 - \xi_w} \right]^{1-\lambda_w(1+\sigma_L)} w F_w / K_w$$

APPENDIX D: IMPULSE RESPONSES: SENSITIVITY ANALYSIS

Finally, to get a sense of which features of the data help to identify the bargaining parameters, (δ, γ) , and the parameters governing the matching technology, (σ, s, κ) , we proceeded as follows. We recomputed the impulse response functions for the estimated AOB model, perturbing each parameter one at a time. Figures A.1–A.3 show the results. We found that the impulse responses to the monetary policy shock are the most sensitive to the perturbations. This result suggests that most of the information about these parameters comes from the monetary policy impulse responses. The response of inflation, real wages, the job finding rate, and, to a lesser extent, the unemployment rate and GDP, are particularly sensitive to perturbations in δ , γ , s , and κ . The response of vacancies to a monetary policy shock is very sensitive to a perturbation in σ .

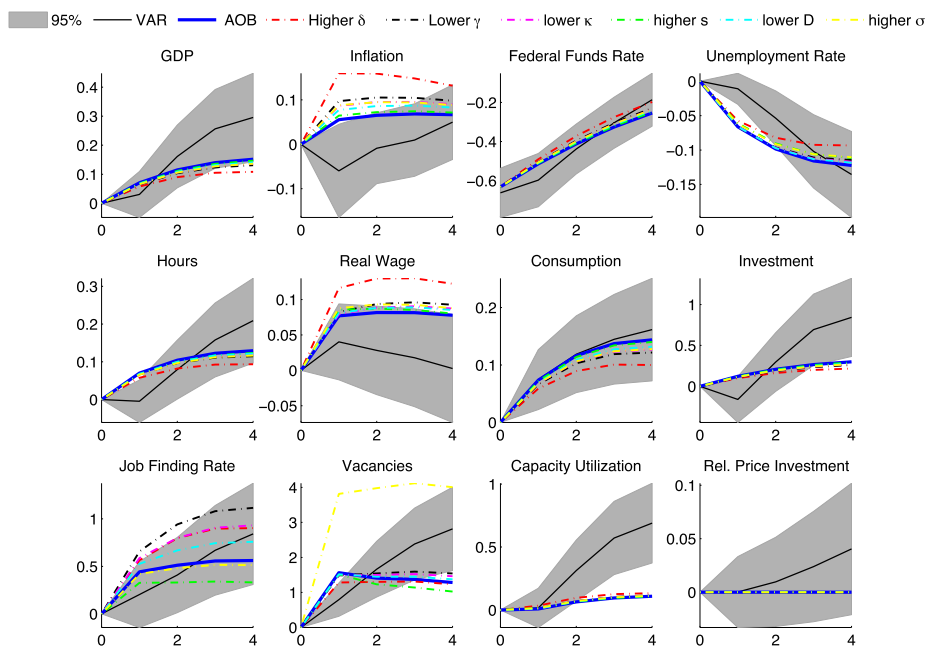


FIGURE A.1.—Impulse responses to a monetary policy shock.

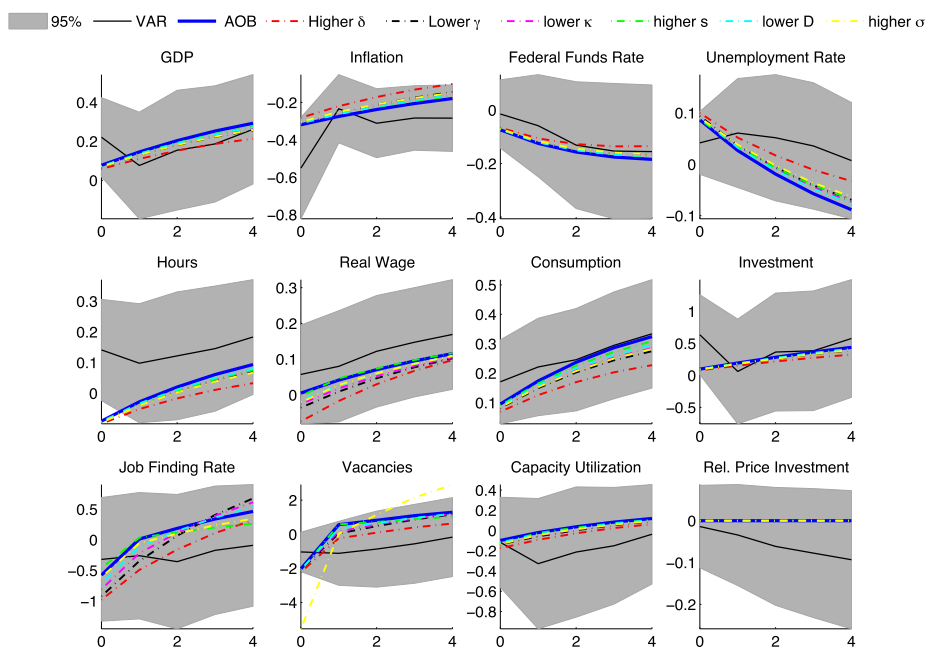


FIGURE A.2.—Impulse responses to a neutral technology shock.

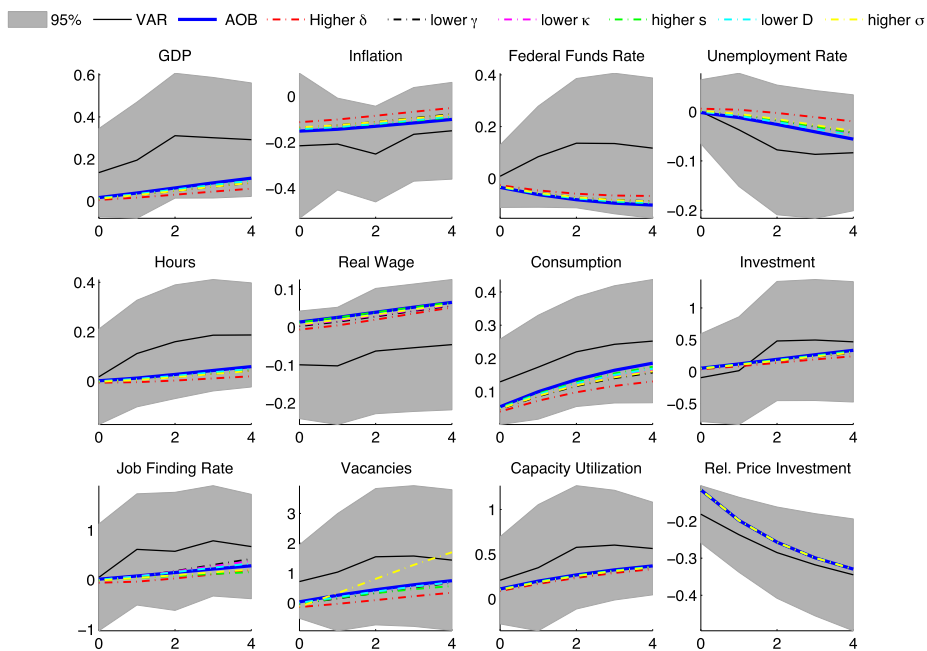


FIGURE A.3.—Impulse responses to an investment specific technology shock.

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