# SUPPLEMENT TO "THE COSTS OF ENVIRONMENTAL REGULATION IN A CONCENTRATED INDUSTRY"
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## APPENDIX A: Identification

THERE ARE TWO SETS of parameters in the model: those that are estimable without appeal to a dynamic model, and those that depend on the continuation value. The former category includes the demand curve and production costs, while the latter encompasses the costs of investment and divestment, along with the distributions of fixed costs of investment, divestment, and exit.

The demand curve is nonparametrically identified under much weaker monotonicity and exclusion restrictions than imposed by the linear functional form in equation (10).[1] The parameters of the production function are identified by functional form.[2] The solution to the capacity-constrained Cournot game is unique, as the best-response curves are downward-sloping in their rivals' production. As the residual demand curve facing an individual firm moves in and out, it traces out the marginal cost of production. As mentioned earlier, the fixed cost of production would be identified when the firm chooses not to produce anything in a given period. However, in the present data sample, all firms produce in all periods, so this parameter must be normalized to zero.

With regard to the dynamic parameters of the model, I provide a novel constructive approach to showing identification of two-step estimators, and demonstrate that the necessary and sufficient identification conditions are met in the present model. I also show how to estimate and identify parameters of unknown distributions in the underlying dynamic game, such as the distribution of fixed adjustment costs, which extends the class of models previously considered in the literature. The identification conditions are easy to verify, and apply to a wide class of dynamic games.

The approach to identifying the dynamic parameters is constructive. With policy functions for firms in hand, the econometrician can construct the ex ante value function for firm $i$ at state $\bar{s}$:

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[1]See Newey and Powell (2003) and references therein for a general treatment of identification and estimation in nonparametric instrumental variables models.

[2]Functional form is a sufficient but not necessary condition for the identification of these parameters; given the availability of firm-specific cost shifters (capacity), these parameters are identified under more general conditions.

$$(A.1) \quad W_i(\bar{s}) = E_{s|\sigma,\bar{s}}\left[\sum_{t=0}^{\infty}\beta^t\pi_i(\sigma(s_t))\right]$$

$$= E_{s|\sigma,\bar{s}}\left[\sum_{t=0}^{\infty}\beta^t(1 \quad \theta)\cdot\begin{pmatrix}\bar{\pi}_i(s_t)\\\zeta_i(\sigma(s_t))\end{pmatrix}\right],$$

where the last equality follows by the linearity of the unknown parameters in the payoff function defined by equation (5), and $\zeta_i(\sigma(s_t))$ is a vector of expected actions undertaken as state $s_t$, such as investment. The notation $E_{s|\sigma,\bar{s}}$ represents the integration over all possible paths of the state space in the future, conditional on the policy function, $\sigma$, which contains the probabilities of discrete choices and the levels of continuous choices. For discrete choices, these probabilities reflect optimal cutoff thresholds in the firm's private shock for undertaking a given discrete action. Optimality in equilibrium demands that no firm finds it payoff-increasing to make changes to these thresholds or levels.[3] This implies that the derivative of the ex ante value function with respect to the $j$th aspect (either the level of an action or the probabilities of undertaking two or more actions) of $\sigma(s)$ at a single point in the state space, $\hat{s}$, is

$$(A.2) \quad \frac{\partial W_i(\bar{s})}{\partial\sigma_{ij}(\hat{s})} = \theta\cdot E_{s|\sigma,\bar{s}}\sum_{t=0}^{\infty}\left[\beta^t\frac{\partial\zeta_i(\sigma(\hat{s}_t))}{\partial\sigma_{ij}(\hat{s}_t)}\right]$$

$$+ E_{s|\hat{\sigma},\bar{s}}\sum_{t=0}^{\infty}\beta^t\left[(1 \quad \theta)\cdot\begin{pmatrix}\bar{\pi}_i(s_t)\\\zeta_i(\sigma(s_t))\end{pmatrix}\right] = 0.$$

There are two terms in the derivative: the change in the value function accruing to changes in the profit function, holding the distribution over states constant, and the change in the value function accruing from changes in the distribution over states expected to be visited in the future, holding per-period payoffs constant. Equation (A.2) neatly summarizes the opposing marginal costs and benefits that firms face when making optimal decisions. For example, firms weigh the marginal cost of investment against the marginal increase in product market profits when making optimal investment decisions.

Since the unknown parameters enter linearly, one can group terms such that

$$(A.3) \quad -E_{s|\hat{\sigma},\bar{s}}\sum_{t=0}^{\infty}\beta^t\bar{\pi}_i(s_t)$$

$$= \theta\cdot\left[E_{s|\sigma,\bar{s}}\sum_{t=0}^{\infty}\left[\beta^t\frac{\partial\zeta_i(\sigma(\hat{s}_t))}{\partial\sigma_{ij}(\hat{s}_t)}\right] + E_{s|\hat{\sigma},\bar{s}}\sum_{t=0}^{\infty}\beta^t\zeta_i(\sigma(s_t))\right]$$

[3]To an outside observer, deviations to the optimal threshold change the probability that a firm undertakes an action.

or equivalently,

$$(A.4) \qquad y(\bar{s}, \sigma_{ij}(\hat{s})) = \theta \cdot x(\bar{s}, \sigma_{ij}(\hat{s})).$$

One can evaluate equation (A.4) at $k = \dim(\theta)$ different states for the same perturbation, several different perturbations at the same state, or some mix of the two. In any case, one can then stack the resulting set of equations into a vector, $Y$, and the corresponding elements on the right-hand side into a matrix, $X$, resulting in

$$(A.5) \qquad Y = \theta \cdot X.$$

The identification of $\theta$ then follows from the standard uniqueness conditions for a solution to ordinary least squares: as long as $X$ has full rank, then $\theta$ is identified.

It remains to show that the estimated truncated fixed cost functions, $\widetilde{\gamma}_{1i}(p_i)$, $\widetilde{\gamma}_{4i}(p_d)$, and $\widetilde{\phi}_i(p_e)$, identify their associated fixed cost distributions. It is necessary and sufficient to establish that the distribution function is one-to-one with the truncated fixed cost function. I illustrate the identification arguments with the case of fixed costs of investment. First, I make the following support assumption:

ASSUMPTION A.1: *There exists a set of states s such that* (a) $p_d(s) = 0$ *for all* $p_i(s) \in (0, 1)$ *and* (b) $p_i(s) = 0$ *for all* $p_d(s) \in (0, 1)$.

Assumption A.1 is a support assumption on the equilibrium probabilities. Analogous assumptions have been used in the games literature to simplify multiple-factor inference problems into a single-factor problem. For example, in Tamer (2002), similar support conditions allow simultaneous entry games to be simplified into single-agent decision problems.[4] Here, the assumption allows the econometrician to invert the probability of investment onto the distribution of fixed investment costs, without having to worry about the convolution of divestment costs.[5]

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[4]Tamer requires payoff shifters to go to infinity to drive the equilibrium probability of one player to zero for an action; this allows the econometrician to look at the relationship between covariates and outcomes for the other player in isolation. Assumption A.1 has the same flavor: it assumes that there exist states of the world where the econometrician observes the probability of either investment or divestment as being equal to zero.

[5]This assumption requires zero probabilities, which are technically violated in the present application due to unbounded support on the errors of the targets and bands; there is always an infinitesimally small probability of having a firm receive an arbitrarily large shock, which would induce either investment or divestment. Practically speaking, however, this is not a concern since I have verified that the computer is incapable of resolving the infinitesimal positive probability of this occurrence from zero. From the perspective of the estimator, you would obtain exactly the same results using either true zeros or the arbitrarily tiny probabilities implied by the estimated investment policy function.

For clarity of notation, denote the value of making an investment, divestment, and doing nothing as

$$V_i^+(s; \gamma_{i1}) = \max_{x_i^* > 0} \left[ -\gamma_{i1} - \gamma_2 x_i^* - \gamma_3 x_i^{*2} \right.$$

$$\left. + \beta \int E_{\varepsilon_i} V_i(s'; \sigma(s'), \theta, \varepsilon_i) \, dP(s_i + x^*, s'_{-i}; s, \sigma(s)) \right],$$

$$V_i^-(s; \gamma_{i4}) = \max_{x_i^* < 0} \left[ -\gamma_{i4} - \gamma_5 x_i^* - \gamma_6 x_i^{*2} \right.$$

$$\left. + \beta \int E_{\varepsilon_i} V_i(s'; \sigma(s'), \theta, \varepsilon_i) \, dP(s_i + x^*, s'_{-i}; s, \sigma(s)) \right],$$

and

$$V_i^0(s) = \beta \int E_{\varepsilon_i} V_i(s'; \sigma(s'), \theta, \varepsilon_i) \, dP(s_i, s'_{-i}; s, \sigma(s)).$$

The probability that a firm invests is equal to the joint probability

$$(A.6) \qquad p_i(s) = \Pr(V_i^+(s; \gamma_{i1}) > V_i^0(s), V_i^+(s; \gamma_{i1}) > V_i^-(s; \gamma_{i4})).$$

This probability depends on the continuation values for investment, divestment, and doing nothing; the draw of fixed costs of investment; and, critically for identification, also the draws of fixed costs of divestment and scrap values. Assumption A.1 simplifies this problem by ensuring that there exists a part of the state space where the probability of investment is positive while the probability of divestment is approximately zero, which implies $\Pr(V_i^+(s; \gamma_{i1}) > V_i^-(s; \gamma_{i4})) = 1$. The probability of observing investment is simplified:

$$(A.7) \qquad p_i(s) = \Pr(V_i^+(s; \gamma_{i1}) > V_i^0(s), V_i^+(s; \gamma_{i1}) > V_i^-(s; \gamma_{i4}))$$

$$(A.8) \qquad \approx \Pr(V_i^+(s; \gamma_{i1}) > V_i^0(s)),$$

where the second line follows from the assumption that the distribution of fixed investment costs is independent of the distribution of fixed costs of divestment. Letting $d(s)$ represent the direct and opportunity costs of investment, we can relate this probability to the distribution of fixed investment costs:

$$(A.9) \qquad p_i(s) = \Pr(\gamma_1 \leq d(s)) = F_\gamma(d(s)).$$

Define the inverse of the distribution function as

$$(A.10) \qquad F_\gamma^{-1}(p_i(s)) = \inf\{x \in R : p_i(s) \leq F_\gamma(x)\}.$$

If $F$ is strictly increasing, $F^{-1}$ is unique; otherwise, it is the smallest value $x$ such that the inequality is satisfied. In either case, knowledge of the inverse function fully characterizes the distribution function. By the definition of conditional expected value,

$$(A.11) \quad \widetilde{\gamma}_{1i}(p_i(s)) = E\big(\gamma_1 | \gamma_1 \leq F_\gamma^{-1}(p_i(s))\big) = \frac{1}{p_i(s)} \int_{-\infty}^{F_\gamma^{-1}(p_i(s))} x f_\gamma(x)\, dx.$$

Multiplying both sides by $p_i(s)$ and differentiating with respect to $p_i(s)$ results in

$$(A.12) \quad \frac{d}{dp_i(s)} \widetilde{\gamma}_{1i}(p_i(s)) p_i(s) = F_\gamma^{-1}(p_i(s)) f_\gamma\big(F_\gamma^{-1}(p_i(s))\big) \frac{dF_\gamma^{-1}(p_i(s))}{dp_i(s)}.$$

Applying the definition of a derivative of an inverse function,

$$(A.13) \quad \frac{dF_\gamma^{-1}(p_i(s))}{dp_i(s)} = \frac{1}{f_\gamma(F_\gamma^{-1}(p_i(s)))},$$

and substituting into equation (A.12) obtains

$$(A.14) \quad \frac{d\widetilde{\gamma}_{1i}(p_i(s)) p_i(s)}{dp_i(s)} = F_\gamma^{-1}(p_i(s)),$$

which establishes that the inverse distribution function is one-to-one in $\widetilde{\gamma}_{1i}(p_i(s)) p_i(s)$. The desired identification result follows from the fact that the inverse distribution function completely characterizes the distribution function. The distribution function can be completely nonparametrically recovered by allowing the degree of the sieve estimator to grow as the sample size goes to infinity.[6] It is possible to show the identification of the distributions of divestment and sunk exit costs in an analogous fashion.

The identification of the distribution of sunk entry costs is analogous to identification of a single-agent probit. Restating equation (29),

$$(A.15) \quad \Pr(\text{entry}; s) = \Pr(\kappa_i + \gamma_{1i} \leq EV^e(s)) = \Phi(EV^e(s); \mu_\kappa + \mu_\gamma, \sigma_\kappa^2 + \sigma_\gamma^2),$$

where $\mu_\kappa$ and $\sigma_\kappa^2$ are the mean and variance of the distribution of entry costs, which is distributed normally with CDF $\Phi$. The terms $\mu_\gamma$ and $\sigma_\gamma^2$ represent the random fixed costs of investment; they enter as indicated since the sum of two normally distributed variables is also distributed normally with mean and variance equal to the sum of their respective components. The distribution of fixed costs of investment is known, as discussed above. The probability of entry

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[6]See Chen (2006) for details.

is known perfectly, and is a continuous function of the state variables, while the expected value of entering the market, $EV^e(s)$, is fully known from the behavior of incumbent firms. Identification requires that there exist two states, $s$ and $s'$, such that $EV^e(s) \neq EV^e(s')$, which would be satisfied, for example, by considering the entry of a monopolist into two markets with differing levels of demand.

The present paper meets the requirements for identification. It is straightforward to check the rank condition on $X$ in equation (A.5) for a set of deviations required to identify the structural and reduced-form parameters in equation (17); intuitively, nonlinearity in the per-period payoff function traces out these parameters. The truncated expected values are also one-to-one in their underlying distributions, as there are several combinations of observed states where the probability of investment varies while the probability of divestment and exit asymptote to zero. As divestment is highly unlikely at all states, the distribution of exit costs can be recovered by then examining states where the probability of exit is positive.

## REFERENCES

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