Communication Technology Advance and Consequences: Using Two-sided Search Model*

Takahiro Moriya†

25th August 2024

Abstract

Does communication technology advance, such as online dating sites and social networking services, really make us happier? In this paper, I construct a non-stationary two-sided search market equilibrium model to analyze the quantitative effects of the communication technology advance on individuals' marital behavior and social welfare. In the model, I include cohabitation as well as marriage as an individual choice, and provide a new identification argument for separately identifying parameters that have been considered important but difficult to identify, with new proof of the existence of the non-stationary market equilibrium. Using the National Longitudinal Study of the High School Class of 1972 and the National Longitudinal Survey of Youth 1997, I quantify the effects of the communication technology advance on society and reveal which types of individuals benefit from it.

Keywords: Marriage, divorce, cohabitation, two-sided search model, market equilibrium, online dating, structural estimation.

JEL: C57, C62, C68, C78, D1, D58, I31

1 Introduction

Communication technology has significantly changed our lives during the past decades. People use texting services, social networking services, and even online dating sites on their phone. Certainly, people's communication styles have changed a lot from the past. For example, as discussed in Rosenfeld et al. (2019), today, most couples meet online.[1] Just as Amazon.com has made shopping easier, finding and communicating with a

*I am extremely indebted to Steven Stern for invaluable guidance and support. I am grateful to Yiyi Zhou, Robert Millard and Hugo Benitez-Silva. I would like to thank Christopher Flinn, Yuichiro Kamada, Hidenori Takahashi, Victor Aguirregabiria, Ken Urai, Mihai Manea, Hülya Eraslan, Yifan Gong, Masato Nishiwaki, David Wiczer, Hyunjae Kang, Takanori Adachi, Gabriel Mihalache, Sofia Ruiz-Palazuelos, Yuya Sasaki, Sandro Brusco, Tilman Borgers, Brennan Platt, Hiro Ishise, Dongya Koh and also immensely thank Ting Liu and Pradeep Dubey. I would like to thank participants of North America Econometric Society Summer Meeting 2024, International Game Theory Center at Stony Brook 2024, SWET, Sophia, Kyoto, Osaka and Kobe University. All errors are my own.

†Stony Brook University

[1]There are some structural empirical studies focusing on online dating, for example, Hitsch et al. (2010).

potential partner has also become easier. Just as you open Amazon.com and hit the buy button when you have a free moment, you swipe and hit the like button on the profile pictures of potential mates on your phone.

What kind of changes have happened due to the ease of communication brought about by the technology advance at equilibrium? How would people react in this situation in terms of their relationship formulation? In this research, I present a non-stationary two-sided search market equilibrium model for analysing marital relationship formation including cohabitation. Using the model, I quantify the effects of the communication technology advance on relationship formation patterns and social welfare.

Due to the communication technology advance, ways of communication have changed. The change makes it easier to contact with and meet with potential partners, for example, using text message services, social networking services and online dating sites, and it enlarges a possible partner choice set. It seems better for all at first glance. If we have more choices, we probably reach a higher level of welfare in total as a society itself, which might be impossible to achieve when we have less choices (see, for example, Mas-Colell et al. (1995)). However, even though we can guess, theoretically to some extent, the social welfare goes up in total, there are certain types of people suffering disadvantages, and it is also important to quantify those disadvantages.

Theoretically, the technology advance could be thought as what facilitates competition either because it brings bigger choice sets to individuals, or because it reduces a search cost and information friction in the economy. In either case, the situation of the marriage market becomes closer to a sort of competitive equilibrium situation, which might cause more inequality in a sense.[2] Therefore, I could also reframe this research as evaluating quantitatively the efficiency and inequality trade-off caused by the communication technology advance at equilibrium.[3]

In addition to evaluating the impact of the communication technology advance as whole, it is also worthwhile to understand through which paths the society is affected by the technology change; for example, exogenous individuals' preference change or stocks of individuals change. Depending on the paths, implications of the model and, therefore, policy implications would change. We need to use data to get the sense of what is actually happening through which paths. In this research, specifically, by using data, I try to reveal which parts of my marital behavioral model change by the advent of communication technology advance, and measure changes of social welfare.

In this research, the main contributions are the following two aspects: The first one is to quantitively analyze the complicated impacts of the communication technology advance on cohabitation and marriage patterns, and on individual and social welfare, as I have just mentioned. Additionally, this research provides another contribution: In the search-matching literature, identifying parameters of the model is laborious (see, for example, Flinn and Heckman (1982)). Suppose we have observed more matches between men and women with particular characteristics (for example, a highly educated man and a highly educated woman). This might happen because (i) they like each other (preference), (ii) there are more opportunities for them to meet (meeting technology), (iii) there are just larger single stocks of them (stocks of singles issue) and/or (iv) a variance of

[2]Remember that, typically, market completeness only ensures the efficient allocation, but it is ambiguous whether the allocation at equilibrium is good or bad from a fairness perspective (for example, Debreu (1959)). Depending on initial endowments, an economy might reach an unfair allocation even if it is still efficient.

[3]Because I employ a search model framework, at the same time, I need to pay attention to Hosios-like inefficiency argument with externalities (Hosios (1990)). It says that an optimal search friction level exists which leads to an efficient allocation in the economy.

a stochastic part of a match value is large.[4] In the structural marriage matching literature, typically, we can observe only data about "Who matches with whom" and "How long they have matched." This is a typical issue in a structural estimation for marriage matching literature, and this is what I overcome in this paper.

In many cases, for identification, we rely on arbitrary specific functional forms on a meeting probability in search-matching marriage models, or just ignore the possible sources from a model (for example, Flabbi and Moro (2012)).[5] However, according to complicated interactions caused by the technology advance, in this research, it seems a bit problematic to justify assuming a particular parametric functional forms to each primitive in the model beforehand. Needless to say, ignoring a possible path is also problematic. So, I include all of the above four possibilities into the model, and show a possible way to identify parameters without fully relying on functional form assumptions on some of key primitives in the model, while using a market equilibrium concept. Throughout the model framework I employ in this research, I can identify the above four factors. This identification method is widely applicable to other search studies also. Also related to the second contribution, I provide a set of sufficient conditions for the existence of equilibrium under the non-stationarity and limited commitment assumptions. Proving the sufficient conditions is worthwhile on its own.

I use the National Longitudinal Study of High School Class of 1972 (NLS 72) and the National Longitudinal Survey of Youth 1997 (NLSY 97) as two different cohorts. The NLS 72 is assumed to represent the cohort under the situation before the communication technology advance occurs, and the NLSY 97 is assumed to represent the cohort under the situation after the communication technology advance occurs. In addition to detailed demographic information, both data sets have relationship type information, including cohabitation, from early ages of respondents. Compared with other national representative surveys, the two data sets are unique in that they contain detailed retrospective marital history information.

I estimate a dynamic discrete choice model with three alternatives: single, cohabitation, and marriage. In my market equilibrium model setting, stocks of individuals and meeting probabilities are also equilibrium objects. Individuals optimally decide their action, single, cohabitation, or marriage, throughout their lifetime, taking into account the endogenously decided aggregate stocks of individuals and meeting probabilities. The structural parameters are estimated through indirect inference. In matching moments, I use a *consistency belief* (rational expectations) condition associated with dynamics of equilibrium stocks, which acts as new sources of identification for parameters of meeting probabilities. It shows up, as a new set of moments conditions, because my model is a market equilibrium model.

After estimating the model, I conduct several counterfactual experiments to examine the change of social welfare. As counterfactual experiments, I evaluate the welfare effects by the communication technology advance and decompose through which paths the marital behaviors change, while taking into account *equilibrium effects*. My estimation results show that, in the NLSY 97 cohort, people enjoy more efficient meeting technology on average. Therefore, people with particular characteristics who enjoy more efficient meeting technology in the NLS 72, for example, highly educated people, lose their comparative advantage. First, my counterfactual experiment reveals that the type of people are worse off at equilibrium in the end. Second, a series of counterfactual experiments shows a dominant factor which changes the marital behaviors during the NLS 72 and the

[4]See, for example, Weizman's Pandora box argument (Weizman (1979)).

[5]The idea is the same as a wage distribution in job search models: We typically assume, for example, a normal or log-normal distribution in advance (Flinn and Heckman (1982) and Flinn (2006)).

NLSY 97 is not the change of the meeting technology itself, but the changes of the distribution of the stochastic part of the match value, which is also considered caused by the communication technology advance.

To underscore the impact of the communication technology advance on marital patterns, I abstract away from several real-life aspects within the model. The primary goal is not to fully capture all social changes over the two cohorts, but rather to sharply emphasize the significance of the communication technology advance, particularly in meeting technology aspects. Other factors like wage mechanisms or bargaining structures are not explicitly modeled as endogenous. However, the inclusion of varying model primitives, such as match surplus, separation costs and exogenous non-structural probabilities of childbearing, attempts to indirectly address the issue caused by abstracting away other aspects.

Roughly speaking, this study can be characterized from the following three perspectives: (a) I employ a non-stationary market equilibrium model. (b) There is search friction in the economy. (c) I take cohabitation into account as well as marriage. In these respects, this study differs from previous studies. I introduce some of the previous works related to my research in turn.

(a): The recent marriage search models developed by, for example, Akın and Platt (2016), Shepard (2019), Goussé et al. (2017) and Beauchamp et al. (2018) are most closely related. The main differences are that they do not consider cohabitation. Also, they use stationarity of the economy to construct the model. Cohabitation has considerably increased in the US and seems to have the same importance as marriage. Without taking it into account, the results would be somehow misleading. This is because is one of big changes brought by technology advance is the advent of online dating sites. However, marriage might not be directly affected by the new style of dating using online dating sites. By modeling cohabitation, we are able to indirectly capture the effects of online dating sites, which might not be captured by modeling marriage only. We can think that cohabitation is closer to dating. It causes bias in the estimates. I accommodate the non-stationarity of the economy in this research to explicitly control the difference of the stocks of singles.

(b): My model is related to the matching studies with the large market and perfect information assumption, for example, Chiappori (1988) and Choo and Siow (2006) in labor economics and several recent industrial organization studies, for example, Lee (2020). The large market with a perfect information approach makes identification and estimation considerably easy and tractable. However, the large market with perfect information assumption where there is no search friction is still controversial. Incorporating the meeting probability into the model, which is conceptually equivalent to the search friction, is key to this research. So, in this sense, my research is closer to studies with a search friction, for example, Shepard (2019) and Goussé et al. (2017).

(c): The model employed here relates to the existing literature that treats cohabitation as well as marriage, for example, Brien et al. (2006) and Blasutto (2023). Brien et al. (2006) neatly treat cohabitation together with marriage with the learning structure, while focusing only on women's sides. The main difference between Brien et al. (2006), Blasutto (2023) and this research is that I explicitly employ a market equilibrium setting to capture the strategic interaction aspect and the marriage market dynamics.[6]

[6]Note that, only after employing a market equilibrium framework, the stocks of individuals show up as an important aspect in the model. The stocks should not play any role in a partial equilibrium framework. This point is a key for the identification argument in this research. I will discuss in more detail in Section 6.

1.1 Roadmap

Section 2 documents the trend of relationship formulation in the US. Section 3 describes the basic environment of the model. Section 4 describes how a player decides their optimal behavior under the game rule I assume. In Section 5, I discuss the equilibrium concept employed in this paper. Section 6 explains the identification strategy. In Section 7, I provide the description of data sets I use. Section 8 provides the estimation method to estimate the model primitives. Section 9 provides the estimation results. In Section 10, I introduce several counterfactual experiments. Section 11 concludes.

2 Empirical trend about relationship formation

One of the most striking changes US society has experienced over the last 40 years is the change in ways of communication and dating styles. People did not even have a cell phone 40 years ago. So, if a person wanted to communicate with or ask for a date from someone, he/she needed to do it in person. Even in a relationship, mostly, daily communication within a couple was done only through home phones. However, gradually, our lifestyles have changed alongside the advent of new technology. People started using messaging services that made communication among people much easier. Today, with the advent of the internet, almost everyone uses their own smartphone with several messaging services, social networking services, and even online dating apps.[7] This phenomenon has definitely induced further changes with respect to relationship formation.

Along with the advent of new technology, one surprising fact we have seen associated with relationship formation is that today, most couples meet their partners online. In the past, the typical ways of meeting a partner were within their network, for example, through friends of friends, their colleagues, or at a bar near where they lived. In this sense, their choice set was totally restricted. However, the rise of the Internet has allowed individuals to use it even in choosing their partners.

As shown in Figure 1, for example, Rosenfeld et al. (2019) document that the percentage of couples who met online had risen from 0 percent before 1995 to about 22 percent in 2009, and has increased even further to 39 percent by 2017.[8] Figure 1 provides solid evidence of how much our society accepts online dating. It also indicates that we now have access to numerous potential partners simply by swiping photos on our phones. This phenomenon could lead to changes in cohabitation and marriage patterns.

3 Model

The model builds on the work of, for example, Manea (2017), Drewianka (2006), Beauchamp et al. (2018), Saez (2009), Shaphard (2019), Akın and Platt (2016) and Brien et al. (2006). In this research, I present a finite horizon non-stationary two-sided search market equilibrium model for analysing marital relationship formulation in-

[7] According to a study from Pew Research Center (2024), 95 percent of U.S. people have their own smartphone in 2023.

[8] The paper uses data from the How Couples Meet and Stay Together (HCMST) survey. The HCMST is a nationally representative longitudinal survey of adults in the US with a spouse or partner, conducted in 2009 and 2017. In each wave, the sample answers a "how did you meet"question. It is retrospective because the question can only be asked about relationships that have already formed. See, Rosenfeld et al. (2019) for more detailed information. Note that this is information about flow changes *not about stock changes*. However, still, we can say that it indicates changes of dating styles.

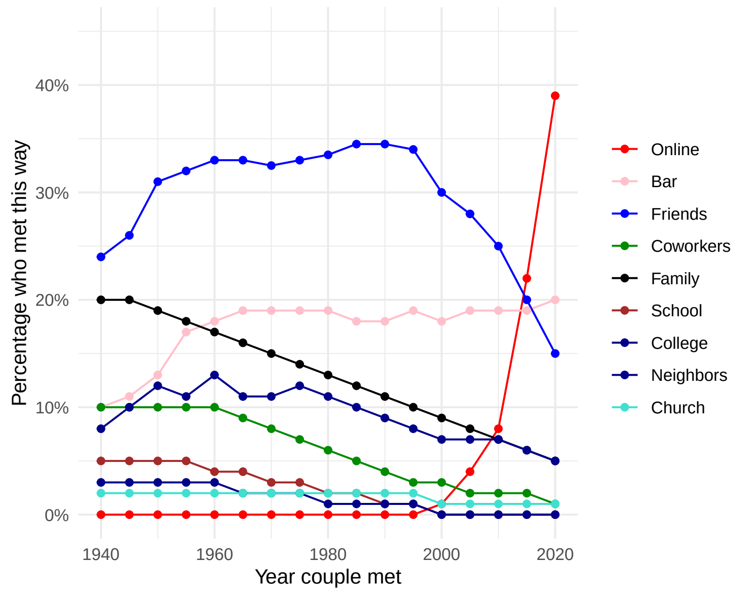

Figure 1: How coupes met

cluding cohabitation. The environment of the game is theoretically described in the following section, while its detailed empirical specification is provided in Section 8.

3.1 Environment

The economy consists of continuum of men and women with observable finite discrete types. The type of an individual is specified by various dimensions. A man's type is indexed by $i \in \mathcal{I} = \{1, 2, ..., I\}$, and a woman's type is indexed by $j \in \mathcal{J} = \{1, 2, ..., J\}$. The discrete type consists of time-variant and time-invariant characteristics. For example, $\mathcal{I}$ and $\mathcal{J}$ include race and education. I explain the specific types in more detail later in Section 8.

I confine my attention to a non-stationary economy, where situations an individual faces change through time periods: Parameters individual faces change through time periods and stocks of individuals in the economy change through time periods. For the discussion under a non-stationary assumption, I explicitly introduce an additional dimension, $t = \{1, 2, .., T\}$, which implies a time period or age.

Also, the technology exposure level is denoted as $\kappa \in \mathcal{K} = \{1, 2, ..., K\}$. It represents, for example, whether people can use texting services, social networking services, or whether there are online dating sites in the economy. This technology index κ is determined when an individual enters into the marriage market. It does not evolve during the game. The whole economy is assumed to be completely divided into sub-economies based on κ. This corresponds to segregated sub-marriage markets for individuals.[9] This point is explained in more detail in Section 5.

[9]Note that each marriage market is also defined, for example, by race. I will mention how to define each segregated sub-marriage market in more detail later in Section 8.

Let $\Lambda_{it\kappa}^{Sm}$ be the stock of type i single men with κ at time t, $\Lambda_{it\kappa}^{Sw}$ be for women, $\Lambda_{ijt\kappa}^{C}$ be the stock of cohabitating couples by a type i man and a type j woman with κ at time t, and $\Lambda_{ijt\kappa}^{M}$ be the stock for married couples. I assume a bounded support for each stock. Let $\mathbf{\Lambda}_t$ be a vector of stocks of individuals in the economy given time t, $\mathbf{\Lambda}_t = \{\Lambda_{it\kappa}^{Sm}, \Lambda_{jt\kappa}^{Sw}, \Lambda_{ijt\kappa}^{C}, \Lambda_{ijt\kappa}^{M}\}_{ij\kappa}^{IJK}$, and assume that it is defined on a subspace of the topological vector space, $\mathbb{R}_+^{\mathbb{N}}$. Let $\mathbf{\Lambda}$ be a vector of stocks of individuals in the economy, $\mathbf{\Lambda} = \{\Lambda_{it\kappa}^{Sm}, \Lambda_{jt\kappa}^{Sw}, \Lambda_{ijt\kappa}^{C}, \Lambda_{ijt\kappa}^{M}\}_{ijt\kappa}^{IJTK}$, and asuume it is defined on a subspace of the topological vector space, $\mathbb{R}_+^{\mathbb{N}}$. The stocks of individuals, $\mathbf{\Lambda}$, are determined endogenously in the model, which I discuss in more detail in Section 5.

3.2 Decision timing, game type and information assumption

The decision timing, game type and information structure are specified next. Let $\alpha_{ijt\kappa}^{m}(\mathbf{\Lambda}_t) \in [0,1]$ be a type i man's probability of meeting a type j woman under a level of technology κ at time t. Similarly, let $\alpha_{ijt\kappa}^{w}(\mathbf{\Lambda}_t) \in [0,1]$ be a type j woman's probability of meeting a type i man under a level of technology κ at time t. A meeting probability is a continuous function mapped from stocks of individuals in the economy at time t, $\mathbf{\Lambda}_t$.

At the beginning of each period, all of the following happen: A type i man and a type j woman meet based on their meeting probability if they are single. If they meet, they draw a flow match value (match surplus), $s_{ijt\kappa} \in \mathbb{R}$, from the conditional distribution $F_{s|ijt\kappa}$ conditional on observable types, i, j, t and κ, which is introduced in more detail later. If they are already matched, they redraw a new match value at every period.

Utility flows from a match are perfectly transferable with side payments within the couple. After meeting, singles divide the value of their match under a Nash bargaining procedure based on their bargaining weight, $\phi \in [0,1]$, which is denoted as a woman's bargaining weight. They jointly decide whether to remain single, cohabit or get married. If they already are matched, they jointly decide whether to continue being in the same relationship or dissolve at every decision time (limited commitment). If a single does not meet with anyone, the individual stays single in the time period. Note that separation is endogenous and occurs when the value of a match is too low for a couple to remain mutually beneficial. Namely, under the limited commitment setting, a couple commits to shares within their match at the time of cohabitation and marriage with an option to renegotiate if their situation changes. All individuals play this until they reach the terminal time period, T.

I assume that an individual has perfect information only about his potential partner after meeting. However, an individual is assumed to know about his/her potential partners the individual has not met yet in the economy with information friction (search friction). It means player knows distributions of others in the economy.[10]

There are many theoretical studies and several empirical studies about the marriage market using non-cooperative game frameworks also (for example, Wong (2003), Friedberg and Stern (2014), Del Boca and Flinn (2012) and Del Boca and Flinn (2014)). However, in this research, I employ a generalized Nash bargaining approach, which is frequently used in cooperative games. Under a Nash bargaining framework, side payments between players can be incorporated into the model, which makes each player's decision process more straightforward. I return to this point in more detail in Section 4.

Another possibility is to include a directed search aspect to take into account an individual's choice about,

[10]So, the model is not the Shapley-Shubik cooperative matching framework (Shapley and Shubik 1971), where an individual can access every potential partner in the economy with perfect information.

for example, whether he or she uses online dating sites. First, mathematically, it is not such difficult to include the aspect into my fixed point argument shown in Section 5.[11] Second, however, I do not have the information about an individual's use of a particular dating platform. Third, first of all, the stage, the stage when an individual decides whether to use a particular platform, is redundant. What matters in making a marital decision is not a potential partner is actually using, for example, online dating sites, but *the potential partner could use the platform, if she or he want to.*

3.3 Flow match value

Let $d_{ijt\kappa} = 1, 2$, or 3 be a mutually exclusive choice at time t by a decision unit denoted by i, j, t and κ: $d_{ijt\kappa} = 1$ represents staying single, $d_{ijt\kappa} = 2$ represents cohabiting, and $d_{ijt\kappa} = 3$ represents getting married.[12]

Let $u^m_{ijt\kappa} + u^w_{ijt\kappa} \in \mathbb{R}$ be a deterministic part of the flow match value conditioned on observables, i, j, t and κ. It represents the deterministic part of the flow match value caused by a match itself between an i man and a j woman under κ at time t. Note that κ in the subscript of $u^m_{ijt\kappa} + u^w_{ijt\kappa}$ captures an additional benefit/loss caused by a level of technology κ. We can think of the effect of technology on a match itself in the following way: People would, *on average*, match with more/less compatible partners due to technology advance, even conditioned on the same observables, i, j and t. There is a marriage bonus, $\mathbb{M}_{ijt\kappa} \in \mathbb{R}$, which a couple by a type i man and a type j woman gets under κ at time t when they select marriage, $d_{ijt\kappa} = 3$. There are separation costs included in this research. A couple incurs separation costs when going back to single from a match. Let $\mathbb{C}^C_{ijt\kappa} \in \mathbb{R}$ be a cohabitation separation cost between a type i man and a type j woman at time t with κ. Similarly, let $\mathbb{C}^M_{ijt\kappa} \in \mathbb{R}$ be a divorce cost between i and j at time t with κ. Due to the cooperative game setting, we do not have to explicitly mention how to split separation costs when they separate, if we focus only on relationship dynamics. This is because, in a typical cooperative game setting with perfectly transferable utilities, a couple decides their action based only on their total match value. I describe the dynamics in more detail in Section 3.4.

I suppress the notation for an individual in the following discussion. Given a man i and a woman j with their state variables, t and κ, the flow match value, $s_{ijt\kappa} \in \mathbb{R}$, is

$$
\begin{aligned}
s_{ijt\kappa} = {} & u^m_{ijt\kappa} + u^w_{ijt\kappa} + \mathbb{M}_{ijt\kappa}\mathbb{I}[d_{ijt\kappa} = 3] + (\mu^{Cm}_\kappa + \mu^{Cw}_\kappa)\mathbb{I}[d_{ijt\kappa} = 2] + (\mu^{Mm}_\kappa + \mu^{Mw}_\kappa)\mathbb{I}[d_{ijt\kappa} = 3] + \epsilon_{ijt} \\
& - \mathbb{C}^C_{ijt\kappa}\mathbb{I}[d_{ijt-1\kappa} = 2, d_{ijt\kappa} = 1] - \mathbb{C}^M_{ijt\kappa}\mathbb{I}[d_{ijt-1\kappa} = 3, d_{ijt\kappa} = 1],
\end{aligned}
\tag{1}
$$

where $\mathbb{I}[\cdot]$ is an indicator function. In equation (1), $\epsilon_{ijt} \in \mathcal{E} \subseteq \mathbb{R}$ is a stochastic part of the flow match value, which is match-specific. The stochastic component, ϵ_{ijt}, represents a match value shock independent of technology after conditioning on observables i, j, t and κ. I assume that, after a match, it starts exhibiting serial correlation. I provide a detailed specification of the dynamics of ϵ_{ijt} in Section 8. Additionally, there are two persistent unobserved heterogeneity terms: Let $\mu^{Cm}_\kappa \sim iidN(0, \sigma^2_{\mu^C}(\kappa))$ and $\mu^{Mm}_\kappa \sim iidN(0, \sigma^2_{\mu^M}(\kappa))$ be

[11] This is because the stage of deciding whether he or she enters into an online dating market is just one additional layer of a his or her sub-game theoretically.

[12] Note that, about a "decision unit," if a single does not meet with anyone, the individual cannot *jointly* decide and needs to stay single. For example, if an i man does not meet with any woman ($j = \emptyset$), then $d_{i\emptyset\kappa t} = 1$. Otherwise, a couple is assumed to jointly decide as in typical cooperative game literature.

cohabitation-specific unobserved heterogeneity and marriage-specific unobserved heterogeneity for a man, and $\mu_\kappa^{Cw} \sim iidN(0, \sigma^2_{\mu^C}(\kappa))$ and $\mu_j^M \sim iidN(0, \sigma^2_{\mu^M}(\kappa))$ for a woman, which an individual draws when the individual enters into the marriage game.[13] They are observed by a couple but not by econometricians.

Note that the technology effect on the stochastic part of the flow match value is captured by allowing the variances, $\sigma^2_{\mu^C}(\kappa)$ and $\sigma^2_{\mu^M}(\kappa)$, to differ depending on technology level κ. Intuitively, even conditional on the same observables, i, j, t and t, an individual might match with an extremely good/bad partner based on the technology level κ.

3.4 Value functions

In the following sections, I suppress the notation κ for simpler notation.[14] With the above flow match value, I can write down the value functions of single, cohabitation and marriage in the game. Let $U_{it}^{Sm} \in \mathbb{R}$ be a type i man's value function of staying single under at time t. Similarly, let $U_{jt}^{Sw} \in \mathbb{R}$ be for a j woman. Denote $W_{ijt}^{C}(\epsilon_{ijt}) \in \mathbb{R}$ as a match value caused by cohabitation between a type i man and a type j woman at time t with a realization of the stochastic part of the flow match value, ϵ_{ijt}. Similarly, denote $W_{ijt}^{M}(\epsilon_{ijt}) \in \mathbb{R}$ for marriage.[15] Let $i' \in \mathcal{I}$ and $j' \in \mathcal{J}$ be the next period's type of a man and the next period's type of a woman respectively.

I denote a sum of the amounts of marriage surpluses caused by marriage between a type i man and a type j woman at time t as $Z_{ijt}^{SM}(\epsilon_t) \in \mathbb{R}$. Namely,

$$Z_{ijt}^{SM}(\epsilon_{ijt}) = W_{ijt}^{M}(\epsilon_{ijt}) - U_{it}^{Sm} - U_{jt}^{Sw}. \tag{2}$$

Similarly, a sum of the amounts of cohabitation surpluses, $Z_{ijt}^{SC}(\epsilon_t) \in \mathbb{R}$, is denoted as

$$Z_{ijt}^{SC}(\epsilon_{ijt}) = W_{ijt}^{C}(\epsilon_{ijt}) - U_{it}^{Sm} - U_{jt}^{Sw}. \tag{3}$$

Let $\phi \in [0, 1]$ be a woman's bargaining weight. Let $\varsigma \in (0, 1)$ be a discount factor. Let the flow utility of being single be normalized to 0. The value functions for being single, cohabiting and getting married are

$$\begin{aligned} U_{it}^{Sm} =& \varsigma \sum_j \mathbb{E}_{t+1|t}[\alpha_{i'jt+1}^{m}(\mathbf{\Lambda}_{t+1}) \max\{U_{i't+1}^{Sm}, U_{i't+1}^{Sm} + (1-\phi) Z_{i'jt+1}^{SC}(\epsilon_{i'jt+1}), U_{i't+1}^{Sm} + (1-\phi) Z_{i'jt+1}^{SM}(\epsilon_{i'jt+1})\} \\ &+ \varsigma(1 - \sum_j \alpha_{i'jt+1}^{m}(\mathbf{\Lambda}_{t+1})) U_{i't+1}^{Sm}]; \end{aligned} \tag{4}$$

[13] *Ideally*, we can and *should* say an individual meet with a partner with opposite sex with different values of cohabitation and marriage specific unobserved heterogeneity in each meeting. If we have a data set where an individual get matched with several different partners in his whole life, we can set up this ideal and reasonable situation because the distribution of the unobserved heterogeneity terms are identifiable, even if we assume a new potential partner has different values of cohabitation and marriage specific unobserved heterogeneity in each meeting. However, for a practical purpose, I impose the assumption that an individual only meets with a partner with the same values of unobserved heterogeneity terms.

[14] As I described in Section 3.1, κ is fixed when an individual enters into the game. Therefore, it does not play a role when we consider the dynamics of the game.

[15] Note that, for simple notation, I deliberately represent the value functions without the notion of μ^C and μ^M.

$$
\begin{aligned}
U^{Sw}_{jt} =& \varsigma \sum_i \mathbb{E}_{t+1|t}[\alpha^w_{ij't+1}(\mathbf{\Lambda}_{t+1}) \max\{U^{Sw}_{j't+1}, U^{Sw}_{j't+1} + \phi Z^{SC}_{ij't+1}(\epsilon_{t+1}), U^{Sw}_{j't+1} + \phi Z^{SM}_{ij't+1}(\epsilon_{ij't+1})\} \\
&+ \varsigma(1 - \sum_i \alpha^w_{ij't+1}(\mathbf{\Lambda}_{t+1}))U^{Sw}_{j't+1}];
\end{aligned} \tag{5}
$$

$$
W^C_{ijt}(\epsilon_{ijt}) = s_{ijt} + \varsigma \mathbb{E}_{t+1|t} \max\{U^{Sm}_{i't+1} + U^{Sw}_{j't+1}, W^C_{i'j't+1}(\epsilon_{i'j't+1}), W^M_{i'j't+1}(\epsilon_{i'j't+1})\}; \tag{6}
$$

$$
W^M_{ijt}(\epsilon_{ijt}) = s_{ijt} + \varsigma \mathbb{E}_{t+1|t} \max\{U^{Sm}_{i't+1} + U^{Sw}_{j't+1}, W^M_{i'j't+1}(\epsilon_{i'j't+1})\}. \tag{7}
$$

In equations (4) and (5), the surplus division occurs through a generalized Nash bargaining over potential gains represented by spouses' value functions. I assume that the bargaining weight, ϕ, is the same in each bargaining case, cohabitation and marriage.[16] [17] Individuals in the economy are assumed to have rational expectations on stocks of individuals and meeting probabilities in the future marriage market. Following typical search literature with a continuum of agents, in equation (4) and (5), we can treat the future meeting probabilities as exogenous given for each individual.[18] The duration dependence, for example, caused by the increased household tightness or the investment between spouses, is captured through the evolution of ϵ_t in s_{ijt} and the change in the length of a relationship which is captured by changes in i and j. Because the law of motion for ϵ_{ijt} exhibits serial correlation, I need to take the conditional expectation of $\epsilon_{i'j't+1}$ conditional on its previous realization, ϵ_{ijt} after a match. The separation costs in s_{ijt}, $\mathbb{C}^C_{ijt}$ and $\mathbb{C}^M_{ijt}$, ensure a long-term relationship somewhat. I include them for the model to explain some typical household behaviours that require long-term commitment.[19] Separation is endogenous and occurs when the value of a match is too low for a couple to remain mutually beneficial. When a couple separates, they move to single. Note that, in equation (7), transitions from marriage to cohabitation are assumed not to happen.[20] In the model, the ideal of the search cost is captured by the discount factor.[21]

[16]Note that, in this research, I take ϕ as exogenously given. The value of the bargaining weight ϕ decides the distribution of assets or resources between spouses after their match. Therefore, a different value of ϕ might lead to a big difference in their lifetime utility. However, in focusing only on the dynamics of the marital formulation/dissolution, the value of ϕ does not matter by the assumption of the cooperative game with transferable utilities employed in this research. I describe this point in more detail in Section 4. I give 0.5 to ϕ. This seems reasonable because the parameter of the bargaining weight conceptually corresponds to *the probability of each player offering how to divide their match surplus* when we reinterpret the cooperative game setting as an infinite horizon non-cooperative sequential bargaining game (Rubinstein and Wolinsky (1995)). It is reasonable to assume each spouse's offering opportunity is the same ($\phi = 0.5$). However, this might be an issue because my research question emphasizes *social welfare*.

[17]Note that I can identify ϕ by using a "happy variable" in the spirit of, for example, Byrne et al. (2009) and Friedberg and Stern (2014). Of course, we can think of ϕ as an endogenous equilibrium object also. However, this extension is beyond the scope of this research.

[18]An individual needs to take expectations on the aggregate future stocks and meeting probabilities only. A player does not need to take expectations on other players' actions in the economy. The aggregate stocks of individuals and meeting probabilities work as a kind of sufficient statistics for each individual in making their decision. See, for example, Stokey and Lucas (1989) and Dubey and Kaneko (1984) about exogenous treatment of endogenous whole-market-related objects under general/market equilibrium frameworks.

[19]For example, a typical household behaviour is specializing in housework. Imagine that one of spouses is willing to specialize in housework. This situation might happen only under an environment with reliable commitment about their future. It gives an incentive for him/her to specialize in housework. This is because such specialization might result in giving up not only his/her current wage and the accumulation of the work-related skill but also his/her future career that would bring higher wage.

[20]We can say this without loss of generality because I can model the separation cost from marriage happens in going single *as well as going back to cohabitation*. Then, the transitions from marriage to cohabitation do not happen.

[21]Note that implicitly the periods left for a searcher is also one of search costs (see, for example, Rust and Hall (2003)).

3.5 Law of motions of stocks

Because my model is a market equilibrium model, to close the model, I need to describe the law of motions associated with the aggregate stocks, which are endogenous objects. Let $\boldsymbol{D}_t$ be a vector of mappings associated with the law of motion of aggregate stocks at time t, which is decided by the model in a complicated way. The law of motions of the stocks in this economy is described as, at t,

$$\boldsymbol{\Lambda}_{t+1} = \boldsymbol{D}_t \cdot \boldsymbol{\Lambda}_t. \tag{8}$$

4 Decision process

Under a transferable utilities with side payments between spouses setting, we can simplify the decision process of a decision unit after meeting and during a match. Namely, we focus only on the sum of the match surpluses of a couple.

In this section, I explain why the sum of the amounts of the match surpluses plays an important role as a decision criteria under a Nash bargaining framework setting even with three-alternatives with a market equilibrium, single, cohabitation and marriage, which is considered in this research.[22]

4.1 Relationship decision

Remember that, as I described in Section 3.4, I denote a sum of the amounts of marriage surpluses caused by a transition from single to marriage between a type i man and a type j woman at time t as $Z^{SM}_{ijt}(\epsilon_{ijt})$. Similarly, the sum of cohabitation surpluses is denoted as $Z^{SC}_{ijt}(\epsilon_{ijt})$.

For convenience of explanation for the case with three-alternatives, I first focus on the commonly used two-alternatives case; deciding single or marriage. As in standard search-matching-bargaining studies (see, Shimer and Smith (2000)), after meeting, a type i man and a type j woman compare their profit from getting married with their value of remaining single under a generalized Nash bargaining framework. Namely, with a woman's bargaining weight, ϕ, a type i man and a type j woman compare

$$U^{Sm}_{it} + (1-\phi) Z^{SM}_{ijt}(\epsilon_{ijt}) \lesseqgtr U^{Sm}_{it}; \tag{9}$$

$$U^{Sw}_{jt} + \phi Z^{SM}_{ijt}(\epsilon_{ijt}) \lesseqgtr U^{Sw}_{jt}. \tag{10}$$

The left-hand side of equation (9) represents the (potential) gain that a type i man would get from marriage with a type j woman through the generalized Nash bargaining framework. On the other hand, the right-hand side of equation (9) means the value that a type i man would get if he selects to remain single at time t. I can similarly describe a type j woman's situation , which is equation (10).

By modifying equations (9) and (10), for a type i man and a type j woman, their critical value (net value) of

[22] See, for example, Shimer and Smith (2000) for a basic search-matching-bargaining model with the two-alternatives, single and marriage.

whether they decide to get married is described as

$$(1-\phi)Z_{ijt}^{SM}(\epsilon_{ijt}) \lesseqqgtr 0; \tag{11}$$

$$\phi Z_{ijt}^{SM}(\epsilon_{ijt}) \lesseqqgtr 0. \tag{12}$$

Here, I assume that the woman's bargaining weight, ϕ, is $\phi \in [0,1]$. If $Z_{ijt}^{SM}(\epsilon_{ijt}) \geq 0$, both agree on marriage because, for both, their net value of their match, $(1-\phi)Z_{ijt}^{SM}(\epsilon_{ijt})$ and $\phi Z_{ijt}^{SM}(\epsilon_{ijt})$, are positive. Therefore, if $Z_{ijt}^{SM}(\epsilon_{ijt}) \geq 0$, getting married is better than remaining single for both of them. In this sense, the sum of the marriage surpluses between spouses, $Z_{ijt}^{SM}(\epsilon_{ijt})$, determines their single to marriage transition. See, for example, Shimer and Smith (2000) for a more formal discussion.

Now, focus on the three-alternatives case; Single, cohabitation and marriage. In this case, a couple compares the sum of each match surpluses, $Z_{ijt}^{SM}(\epsilon_{ijt})$ and $Z_{ijt}^{SC}(\epsilon_{ijt})$, as follows. As in the marriage surplus case, the net value of cohabitation from single for a type i man and a type j woman at time t is defined as $(1-\phi)Z_{ijt}^{SC}(\epsilon_{ijt})$ and $\phi Z_{ijt}^{SC}(\epsilon_{ijt})$ respectively.

With the three alternatives, a couple compares the net values of cohabitation with the net values of marriage presented above. If $Z_{ijt}^{SC}(\epsilon_{ijt}) \geq Z_{ijt}^{SM}(\epsilon_{ijt})$, a type i man and a type j woman jointly decide to cohabit. This is because, in the case,

$$U_{it}^{Sm} + (1-\phi)Z_{ijt}^{SC}(\epsilon_{ijt}) \geq U_{it}^{Sm} + (1-\phi)Z_{ijt}^{SM}(\epsilon_{ijt}) \tag{13}$$

for a type i man, and

$$U_{it}^{Sw} + (1-\phi)Z_{ijt}^{SC}(\epsilon_{ijt}) \geq U_{it}^{Sw} + (1-\phi)Z_{ijt}^{SM}(\epsilon_{ijt}) \tag{14}$$

for a type j woman hold.[23] Therefore, they both agree on cohabitation from single rather than on marriage. On the other hand, if $Z_{ijt}^{SC}(\epsilon_{ijt}) \leq Z_{ijt}^{SM}(\epsilon_{ijt})$, they move to marriage from single. Accordingly, only the amounts, $Z_{ijt}^{SC}(\epsilon_{ijt})$ and $Z_{ijt}^{SM}(\epsilon_{ijt})$, matter, when they decide their decision.

4.2 Existence and uniqueness of reservation match values

Given the decision process provided above, I move to properties of reservation match values. I first give definitions of the reservation match values in the game. Then, I provide a single-crossing property of the value functions and a proof of existence of the reservation match values.[24]

Let $\boldsymbol{\epsilon}^*$ be a vector of reservation match values, $\boldsymbol{\epsilon}^* = \{\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*Si}, \epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS}\}_{ijt}^{IJT}$, where ϵ_{ijt}^{*SC} is a reservation match value such that $Z_{ijt}^{SC}(\epsilon_{ijt}^{*SC}) = 0$. Namely, ϵ_{ijt}^{*SC} is a reservation match value with which a couple indexed by i and j at time t is indifferent between staying single and moving to cohabitating. Similarly, ϵ_{ijt}^{*SM} is defined for single to marriage. Let ϵ_{ijt}^{*Si} be a reservation match value such that $Z_{ijt}^{SC}(\epsilon_{ijt}^{*Si}) = Z_{ijt}^{SM}(\epsilon_{ijt}^{*Si})$. Namely, ϵ_{ijt}^{*Si} is a reservation match value with which a couple indexed by i and j at time t is indifferent between moving cohabitation or marriage from single. Hereafter, I refer to ϵ_{ijt}^{*Si} as a *single-crossing point*. Let ϵ_{ijt}^{*CS} be a reservation

[23]Two important assumptions to conclude this are that the bargaining weights for cohabitation and marriage are assumed to be exogenously given and they are the same between cohabitation and marriage. We can extend this setting to a more general one. For example, we can think of the bargaining weights as an endogenous equilibrium object. However, this extension is beyond the scope of this research.

[24]The existence of the reservation values and their uniqueness immediately follow from the argument of the existence of the single-crossing point.

match value such that $Z^{CS}_{ijt}(\epsilon^{*CS}_{ijt}) = 0$. It means that ϵ^{*CS}_{ijt} is a reservation match value with which a couple indexed by i and j at time t is indifferent between staying in cohabitation and going back to single. Similarly, ϵ^{*MS}_{ijt} is defined from marriage to single.

In the following, I propose two propositions: First, there exists the *single-crossing point*, ϵ^{*Si}_{ijt}. Second, there exists the reservation match values, $\epsilon^{*SC}_{ijt}, \epsilon^{*SM}_{ijt}, \epsilon^{*CS}_{ijt}, \epsilon^{*MS}_{ijt}$ in a finite range, $[\underline{\epsilon}, \bar{\epsilon}]$, and they are unique.

Proposition 1. *Under the two-sided market equilibrium setting, with* $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) > 0$ *and* $\mathbb{C}^{M}_{ijt} > \mathbb{C}^{C}_{ijt} > 0$ *and* $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) < \varsigma(\mathbb{C}^{M}_{i'j't+1} - \mathbb{C}^{C}_{i'j't+1})$ *for all, i, j and t, there exists a unique* ϵ^{*Si}_{ijt}.

Proposition 2. *The reservation values,* $\epsilon^{*SC}_{ijt}, \epsilon^{*SM}_{ijt}, \epsilon^{*CS}_{ijt}, \epsilon^{*MS}_{ijt} \in [\underline{\epsilon}, \bar{\epsilon}]$ *for all, i, j and t exist and are unique.*

Proof. Proof of Propositions 1 and 2: The proof proceeds in the following way: I first prove that $W^{M}_{ijt}(\epsilon_{ijt}) - W^{C}_{ijt}(\epsilon_{ijt})$ is increasing in ϵ_{ijt} for all t.[25] Second, under the sufficient conditions, I prove that the case where both functions, $W^{M}_{ijt}(\epsilon_{ijt})$ and $W^{C}_{ijt}(\epsilon_{ijt})$, are parallel or overlap with measure 0.

Focus on the last period T, $W^{M}_{ijT}(\epsilon_{ijT}) - W^{C}_{ij)T}(\epsilon_{ijT^*})$ is written as

$$W^{M}_{ijT}(\epsilon_{ijT}) - W^{C}_{ijT}(\epsilon_{ijT}) = \mathbb{M}_{ijT} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}).$$

Because the right-hand side of the equation is independent of ϵ_{ijT}, so it is increasing in ϵ_{ijT} , but not *strictly* increasing in ϵ_{ijT}. The last period's continuation value is assumed to be 0.

Going back one time period before, $W^{M}_{ijT-1}(\epsilon_{ijT-1}) - W^{C}_{ijT-1}(\epsilon_{ijT-1})$ is written as

$$\begin{aligned} W^{M}_{ijT-1}(\epsilon_{ijT-1}) - W^{C}_{ijT-1}(\epsilon_{ijT-1}) &= \mathbb{M}_{ijT-1} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) \\ &+ \varsigma \mathbb{E} \max_{T|T-1} \{U^{Sm}_{i'T} + U^{Sw}_{j'T} - \mathbb{C}^{M}_{i'j'T}, W^{M}_{i'j'T}(\epsilon_{i'j'T})\} \\ &- \varsigma \mathbb{E} \max_{T|T-1} \{U^{Sm}_{i'T} + U^{Sw}_{j'T} - \mathbb{C}^{C}_{i'j'T}, W^{C}_{i'j'T}(\epsilon_{i'j'T}), W^{M}_{i'j'T}(\epsilon_{i'j'T})\}. \end{aligned}$$

This is further modified to

$$\begin{aligned} W^{M}_{ijT-1}(\epsilon_{ijT-1}) - W^{C}_{ijT-1}(\epsilon_{ijT-1}) &= \mathbb{M}_{ijT-1} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) \\ &+ \varsigma \mathbb{E}_{T|T-1}[\max\{U^{Sm}_{i'T} + U^{Sw}_{j'T} - \mathbb{C}^{M}_{i'j'T}, , W^{M}_{i'j'T}(\epsilon_{i'j'T})\} \\ &- \max\{U^{Sm}_{i'T} + U^{Sw}_{j'T} - \mathbb{C}^{C}_{i'j'T}, W^{C}_{i'j'T}(\epsilon_{i'j'T}), W^{M}_{i'j'T}(\epsilon_{i'j'T})\}]. \end{aligned}$$

The above $[\max\{\cdot\} - \max\{\cdot\}]$ part is rewritten with division into cases, depending on a realization of $\epsilon_{i'j'T}$. Accordingly, the overall $[\max\{\cdot\} - \max\{\cdot\}]$ part can be rewritten in one of the following cases:

- 1. $[W^{M}_{i'j'T}(\epsilon_{i'j'T}) - (U^{Sm}_{i'T} + U^{Sw}_{j'T} - \mathbb{C}^{C}_{i'j'T})]$,
- 2. $[W^{M}_{i'j'T}(\epsilon_{i'j'T}) - W^{C}_{i'j'T}(\epsilon_{i'j'T})]$,

[25] Note that the difference between *strictly* increasing and increasing is the following: Let $f(x) \in \mathbb{R}$ be a function mapped by $x \in \mathbb{R}$, and let a be a strictly positive value, $a \in \mathbb{R}_{++}$. The statement, $f(x)$ is *strictly* increasing in x, means that, $\forall a$ and x, $f(x+a) > f(x)$. The statement, $f(x)$ is increasing in x, means, $\forall a$ and x, $f(x+a) \geq f(x)$.

- 3. 0,
- 4. $[U^{Sm}_{i'T} + U^{Sw}_{j'T} - \mathbb{C}^{C}_{i'j'T} - (U^{Sm}_{i'T} + U^{Sw}_{j'T} - \mathbb{C}^{M}_{i'j'T})]$.

Note that, under the region of ϵ_{ijT} where going back to single from marriage is better than staying in marriage, going back to single from cohabitation is better than staying in cohabitation or moving to marriage. This is because we assume that a marriage bonus is $\mathbb{M}_{ijT-1} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) > 0$ for all t, and that the separation cost structure is $\mathbb{C}^{M}_{ijt} > \mathbb{C}^{C}_{ijt} > 0$ for all t. It follows $\epsilon^{*MS}_{ijt} < \epsilon^{*CS}_{ijt}$. This is why we need to consider only the above 4 cases in total when considering all possible cases of $[\max\{\cdot\} - \max\{\cdot\}]$.

The property of increasing in ϵ_{ijt} is preserved by the following operations; maximization, integration over an increasing transition function, which is equivalent to say the law of motion of $\epsilon_{i'j't+1}$ based on a realization of ϵ_{ijt} is first-order stochastically dominant in ϵ_{ijt}, and summation (Milgrom and Shannon (1994) and Hopenhayan and Prescott (1992)).

So, we can ensure that $W^{M}_{ijT-1}(\epsilon_{ijT-1}) - W^{C}_{ijT-1}(\epsilon_{ijT-1})$ is also increasing in ϵ_{ijT-1}. This is because, in either case (case 1 - 4), we can say $\mathbb{E}[\max\{\cdot\} - \max\{\cdot\}]$ part is increasing in ϵ_{ijT-1}: The integrant is increasing in $\epsilon_{i'j'T}$ (case 1 and 2), a constant value (case 3) and independent of $\epsilon_{i'j'T}$(case 4). Note that, again, the flow part, $\mathbb{M}_{ijT-1} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw})$, is independent of $\epsilon_{i'j'T1}$, and, therefore, it is also increasing in $\epsilon_{i'j'T}$. So, overall, $W^{M}_{ijT-1}(\epsilon_{ijT-1}) - W^{C}_{ijT-1}(\epsilon_{ijT-1})$ is increasing in ϵ_{ijT-1}. I do the same procedure backward till $t = 1$. So, for all t, $W^{M}_{ijt}(\epsilon_{ijt}) - W^{C}_{ijt}(\epsilon_{ijt})$ is increasing in ϵ_{ijt}.

The increasing property of $W^{M}_{ij(t}(\epsilon_{ijt}) - W^{C}_{ijt}(\epsilon_{ijt})$ in ϵ_{ijt} cannot get rid of the case where two functions, $W^{M}_{ijt}(\epsilon_{ijt})$ and $W^{C}_{ijt}(\epsilon_{ijt})$ are parallel or perfectly overlap. In the following, I show that I can get rid of the cases under the sufficient condition in Proposition 1.

I first give a condition ensuring that, at far right part of ϵ_{ijt} support, $W^{M}_{ijt}(\epsilon_{ijt}) > W^{C}_{ijt}(\epsilon_{ijt})$ holds. Second, I provide a condition ensuring that, at far left part of ϵ_{ijt} support, $W^{M}_{ijt}(\epsilon_{ijt}) < W^{C}_{ijt}(\epsilon_{ijt})$ holds. Then, I show that they are actually the sufficient condition for the single-crossing property.

Remember that the law of motion of $\epsilon_{i'j't+1}$ based on a realization of ϵ_{ijt} is first-order stochastically dominant in ϵ_{ijt}. So, when $\epsilon_{ijt} \to +\infty$, the probability of $\epsilon_{i'j't+1}$ becoming below than any value of $\epsilon_{i'j't+1}$, $\bar{\epsilon}_{i'j't+1}$, converges to 0, $f(\bar{\epsilon}_{i'j't+1}|\epsilon_{ijt}) \to 0$. Therefore, as long as $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) > 0$, at far right part of ϵ_{ijt} support (equivalent to say $\epsilon_{ijt} \to +\infty$), $W^{M}_{ijt}(\epsilon_{ijt})$ is, at least, $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw})$ greater W^{C}_{ijt}. This is because, at the extreme case ($\epsilon_{ijt} \to +\infty$), the probability of drawing a negative value of $\epsilon_{i'j't+1}$ small enough that going back to single is the best choice converges to 0. It is equivalent to say that, at the extreme case, a couple only enjoys a marriage bonus without their risk of separations in the next period.[26]

On the other hand, let us focus on another extreme case, $\epsilon_{ijt} \to -\infty$. As long as $\mathbb{M}_{ijT} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) < \varsigma(\mathbb{C}^{M}_{i'j't+1} - \mathbb{C}^{C}_{i'j't+1})$ holds, at far left part of ϵ_{ijt} support, $W^{M}_{ijt}(\epsilon_{ijt}) < W^{C}_{ijt}(\epsilon_{ijt})$ holds. This is from the similar logic: When $\epsilon_{ijt} \to -\infty$, a probability of drawing a relatively high value of $\epsilon_{i'j't+1}$ converges to 0. So, the probability of going back to single at time $t+1$ converges to 1 when $\epsilon_{ijt} \to -\infty$. So, a couple with the extreme law ϵ_{ijt} will incur separation costs with probability 1 at time $t+1$, even if the couple enjoys a marriage bonus at time t. A couple thinks about a trade-off between a current marriage bonus and the next period's bigger separation cost. Therefore, As long as $\mathbb{M}_{ijT} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) < \varsigma(\mathbb{C}^{M}_{i'j't+1} - \mathbb{C}^{C}_{i'j't+1})$ holds,

[26]Note that, mathematically, it is enough to focus on the extreme case ($\epsilon_{ijt} \to +\infty$) to check whether it is a sufficient condition.

at the extreme ($\epsilon_{ijt} \to -\infty$), $W^M_{ijt}(\epsilon_{ijt}) < W^C_{ijt}(\epsilon_{ijt})$ holds.

Now, we focus on the middle part of the support of ϵ_{ijt}. At the region, it might be the case, mathematically, the parallel situation happens or the two curves might overlap perfectly. It is equivalent to say multiple values (actually, infinite number) of ϵ_{ijt} make $W^M_{ijt}(\epsilon_{ijt}) - W^C_{ijt}(\epsilon_{ijt}) = 0$ hold.

Define ϵ^0_{ijt} such that $W^M_{ijt}(\epsilon^0_{ijt}) - W^C_{ijt}(\epsilon^0_{ijt}) = 0$. Focus on $\mathbb{E}[\max\{\cdot\} - \max\{\cdot\}]$ part, which is the same as above (case 1-4) for all $t < T$. In case 1 and 2, there are no multiple values of ϵ^0_{ijt} happening. This is because the integrant is strictly increasing in $\epsilon_{i'j't+1}$ in case 1. In case 2, the integrant is increasing in $\epsilon_{i'j't+1}$, but the transition function is strictly increasing in ϵ_{ijt}. Therefore, in either case (case 1 and 2), the continuation part is strictly increasing in ϵ_{ijt}.[27] So, $W^M_{ijt}(\epsilon_{ijt}) - W^C_{ijt}(\epsilon_{ijt})$ itself is strictly increasing in ϵ_{ijt}. In case 3, the integrant is a constant value. So, there are no multiple values of ϵ^0_{ijt}. This is because, in case 3, we need to satisfy

$$0 = \mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) + \int 0 f(\epsilon_{i'j't+1}|\epsilon^0_{ijt}) d\epsilon_{i'j't+1}.$$

However, with measure 1, there is no value of ϵ^0_{ijt} which satisfies the equation because μ^{Mm}, μ^{Mw}, μ^{Cm} and μ^{Cw} are exogenously drawn from a continuous distribution. Therefore, the probability of $\mathbb{M}_{ijT} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}))$ becoming any particular value is measure 0.[28] In case 4, the logic is the same as case 3. Therefore, we can have a unique single-crossing point.

The proof of Proposition 2 directly follows from the proof of Proposition 1. □

I provide how a couple decides their martial choice based on the a joint decision structure.

Theorem 1. *Under the situation where Proposition 1 holds, a couple decision $d_{ijt} = \{1, 2, 3\}$ shows the property of increasing in ϵ_{ijt} under a market equilibrium setting.*

Proof. This directly comes from the existence of the single-crossing point. □

This theorem makes the numerical computation significantly easier. The roots of the two value functions for cohabitation and marriage should be unique. Otherwise, we cannot use any numerical algorithm to find the reservation values.

5 Equilibrium

In this section, I characterize the equilibrium concept in this research. I first provide the formal definition of the equilibrium. Next, I give a proof of the existence of the equilibrium.

[27] Mathematically, it is not a correct statement. In case 2, the integrant might be just a constant along with ϵ_{ijt} because the integrant is just increasing in ϵ_{ijt}. In this case, the basic structure is the same as the following case 3 below.

[28] Note that in case 3, the integrant is 0. So, the integration part does not matter. However, even though the integrant is any constant value, the mathematical structure is the same because the integration part changes to a constant value. Therefore, the discussion does not change.

5.1 Technology-dependent sub-economies setting

The whole economy is assumed to be completely divided into several non-stationary sub-economies, depending on their level of technology, κ. Namely, there are multiple segregated non-stationary economies existing simultaneously at different technology levels κ, and each sub-economy has its own equilibrium.

Associated with the segregated technology-dependent sub-economies, two important assumptions are also imposed: An individual's technology level, κ, does not evolve during the dynamic marriage game, and people who live in different κ worlds cannot meet with each other.[29] At first glance, restricting possible meetings seems somehow a strong assumption. It is true that, theoretically, the model does not have to restrict possible meetings between singles to happen only in the same technology κ world. Neither, do we need to divide the whole economy into its sub-economies. However, restricting possible meetings is not a such strong assumption in this research. This is because, eventually, I compare the cohort without the communication technology advance (NLS 72), which corresponds to $\kappa = 1$, and the cohort with the communication technology advance (NLYS 97), which corresponds to $\kappa = 2$. Although I mention in more detail in Section 8, the two groups are separated by a significant amount of time. So, it seems reasonable to put the restriction because people in different cohorts rarely match with each other. I will give more details about how to construct the marriage market in Section 8. [30]

5.2 Requirements, definition and existence of equilibrium

The equilibrium conditions consist of two requirements; an *optimality condition* and a *consistency belief condition*. The *optimality condition* requires, at equilibrium, each individual behaves optimally given their perception (*belief*) about the future dynamics of the marriage market (stocks of individuals in the economy and meeting probabilities). The *consistency belief condition* requires, at equilibrium, their given perception about the dynamics of the marriage market should match with the actual aggregate dynamics of the marriage market derived by aggregating each individuals' decision in the economy.

I propose the formal definition of the market equilibrium in the model, and state that, under my construction of the model, the equilibrium exists.

Definition 1. *Denote* $\boldsymbol{\epsilon}^{**} = \{\epsilon_{ijt}^{**SC}, \epsilon_{ijt}^{**SM}, \epsilon_{ijt}^{**Si8}, \epsilon_{ijt}^{**CM}, \epsilon_{ijt}^{**CS}, \epsilon_{ijt}^{**MS}\}_{ijt}^{IJT}$ *as a vector of equilibrium reservation match values,* $\boldsymbol{\Lambda}^{**} = \{\Lambda_{it}^{**Sm}, \Lambda_{jt}^{**Sw}, \Lambda_{ijt}^{**C}, \Lambda_{ijt}^{**M}\}_{ijt}^{IJT}$ *as a vector of equilibrium stocks of individuals in the economy and* $\boldsymbol{\alpha}^{**} = \{\alpha_{ijt}^{m^{**}}, \alpha_{ijt}^{w^{**}}\}_{ijt}^{IJT}$ *as a vector of equilibrium meeting probabilities.*

Definition 2. *A non-stationary market equilibrium is defined by a triple* $(\boldsymbol{\epsilon}^{**}, \boldsymbol{\Lambda}^{**}, \boldsymbol{\alpha}^{**})$ *such that:*

- *Each individual optimizes their behavior, given their own perception about future stocks of individuals and meeting probabilities (Optimality condition);*
- *The given perception of the stocks of individuals and meeting probabilities are consistent with the actual aggregate dynamics of the economy (Consistency belief condition).*

[29] Accordingly, we can think of κ in the following way: The technology level index, κ, is not about the attribute that an individual has, but rather about the segregated world that the individual enters into.

[30] Empirically, as Shepherd (2019) and Beauchamp et al. (2018) restrict possible meetings between players for computational tractability, I also need to put some restrictions eventually.

Theorem 2. *There exists the market equilibrium.*

Proof. Now, I prove the existence of the market equilibrium defined above. Accordingly, first, I introduce the main mathematical objects which are used in the proof although some of them have been already provided in previous sections.

Let $\mathbf{\Lambda} = \{\Lambda^{Sm}_{it}, \Lambda^{Sw}_{jt}, \Lambda^{C}_{ijt}, \Lambda^{M}_{ijt}\}^{IJT}_{ijt}$ be a vector of stocks of individuals in the economy. Each stock is assumed to be located in a closed and bounded region between 0 and $\overline{\Lambda}$, which is denoted as an upper bound. Let $\mathcal{M}$ be a convex Euclidean product space for stocks of individuals in the economy, $\mathcal{M} = \{\mathbf{\Lambda}\}$. The space, $\mathcal{M}$, is compact as well because, by Tychonoff's theorem, a finite product of compact spaces is compact. Next, let $\boldsymbol{\alpha} = \{\alpha^{m}_{ijt}, \alpha^{w}_{ijt}\}^{IJT}_{ijt}$ be a vector of meeting probabilities. Each meeting probability is bounded between 0 and 1. Let $\mathcal{P}$ be a convex and compact Euclidean product space for meeting probabilities, $\mathcal{P} = \{\boldsymbol{\alpha}\}$.[31] Moreover, let $\boldsymbol{U} = \{U^{Sm}_{it}, U^{Sw}_{jt}\}^{IJT}_{ijt}$ be a vector of value functions for single. Define the space for $\boldsymbol{U}$ as $\mathcal{U} = \{\boldsymbol{U}\}$. Let $\boldsymbol{W} = \{W^{C}_{ijt}(\epsilon), W^{M}_{ijt}(\epsilon)\}$ be a vector of value functions for a match, where ϵ is a shorthand notation for a realization of ϵ_{ijt}. Define a functional space $\mathcal{W}$ which is for continuously differentiable functions, $\mathcal{W} = \{\boldsymbol{W}\}$. [32] Each value of a value function is assumed to be located in a closed and bounded region. This is because I assume that every parameter set is compact, and, by the structure of the value functions defined in Section 3.4, each value function is continuously mapped from the compact parameter spaces. As a result, each value function is defined also on a compact set. Let $\boldsymbol{V} = \{\boldsymbol{U}, \boldsymbol{W}\} \in \mathcal{C}^1$, and let $\mathcal{V}$ be a measurable convex and compact functional space for value functions, $\mathcal{V} = \{\boldsymbol{V}\}$. Furthermore, let $\boldsymbol{\epsilon^*} = \{\epsilon^{*SC}_{ijt}, \epsilon^{*SM}_{ijt}, \epsilon^{*Si}_{ijt}, \epsilon^{*CS}_{ijt}, \epsilon^{*MS}_{ijt}\}^{IJT}_{ijt}$ be a vector of reservation match values. I can say that each reservation match value is located in a closed and bounded region between $\underline{\epsilon}$ and $\overline{\epsilon}$, which is a lower bound and an upper bound respectively. This is because, as I have proved in Section 4.2, each reservation match value does not go to $-\infty$ or $+\infty$. Let $\mathcal{E}^*$ be a convex and compact Euclidean product space for reservation match values, $\mathcal{E}^* = \{\boldsymbol{\epsilon^*}\}$. Finally, let $\boldsymbol{\tau} = \{\tau^{SS(m)}_{ijt}, \tau^{SS(w)}_{ijt}, \tau^{SC(m)}_{ijt}, \tau^{SC(w)}_{ijt}, \tau^{SM(m)}_{ijt}, \tau^{SM(w)}_{ijt}, \tau^{CS}_{ijt\epsilon}, \tau^{CC}_{ijt\epsilon}, \tau^{Si}_{ijt\epsilon}, \tau^{MS}_{ijt\epsilon}, \tau^{MC}_{ijt\epsilon}, \tau^{MM}_{ijt\epsilon}\}$ be a vector of transition probabilities, where ϵ is a shorthand notation for a realization of ϵ_{ijt-1}. Each transition probability is located in a closed and bounded region between 0 and 1. Let $\mathcal{T}$ be a convex and compact Euclidean product space for marital status transition probabilities, $\mathcal{T} = \{\boldsymbol{\tau}\}$.

I construct a mapping $\Phi : \mathcal{M} \to \mathcal{M}$ composing the paths of the whole endogenous interactions of the model: The map Φ is decomposed as,

$$\mathcal{M} \overbrace{\to}^{\varpi} \mathcal{P} \overbrace{\to}^{\iota} \mathcal{V} \overbrace{\to}^{\psi} \mathcal{E}^* \overbrace{\to}^{\xi} \mathcal{T} \overbrace{\to}^{\varrho} \mathcal{M}. \tag{15}$$

These mappings are specified as:

- $\varpi : \mathcal{M} \to \mathcal{P}$ describes a mapping from stocks of individuals to meeting probabilities. Given a structure about a meeting probability as I mentioned in Section 8, it is assumed to be a continuous and bounded function in $\mathbf{\Lambda}$.

[31] Its compactness comes also from Tychonoff's theorem.

[32] The values of value functions and the reservation values, which will be introduced below, depend also on $\mu^C_i, \mu^M_i, \mu^C_j$ and μ^M_j. However, for simple notation, I suppress the notation of them in this section.

- $\iota : \mathcal{P} \rightarrow \mathcal{V}$ describes a mapping from meeting probabilities to the space for the value functions. Each 7 value function is uniquely mapped given a set of meeting probabilities. The structure of a value function described in Section 3.4 ensures that value functions continually changes in $\boldsymbol{\alpha}$, and therefore the space $\mathcal{V}$ is compact.

- $\psi : \mathcal{V} \rightarrow \mathcal{E}^*$ describes a mapping from value functions to reservation match values. Note that, again, this ψ is also a function. This is because, remember, given $\boldsymbol{V} \in \mathcal{V}$, the reservation match values, $\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*Si}, \epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS}$, are uniquely decided, as I have proven in Section 4.2. From the Implicit Function Theorem, $\boldsymbol{\epsilon}^*$ is continuous in $\boldsymbol{V}$.

- $\xi : \mathcal{P} \times \mathcal{E}^* \rightarrow \mathcal{T}$ denotes a mapping from meeting probabilities and reservation match values to transition probabilities. This is a continuous function in $\boldsymbol{\alpha}$ and $\boldsymbol{\epsilon}^*$ given a structure of a transition probability described in Section 6.

- $\varrho : \mathcal{T} \times \mathcal{M} \rightarrow \mathcal{M}$ describes a function from transition probabilities and the stocks of individuals to stocks of individuals in the economy. This mapping, conceptually, corresponds to a mapping from *stocks of individuals (belief)* to *stocks of individuals (actual)*.

By Tychonoff's theorem, the space $\mathcal{M}$ is compact. It is also convex. The function $\varpi(\boldsymbol{\Lambda})$ is continuous in $\boldsymbol{\Lambda}$, the function $\iota(\boldsymbol{\alpha})$ is continuous in $\boldsymbol{\alpha}$, the function $\psi(\boldsymbol{V})$ is continuous in $\boldsymbol{V}$, the function $\xi(\boldsymbol{\alpha}, \boldsymbol{\epsilon}^*)$ is continuous in $\boldsymbol{\alpha}$ and $\boldsymbol{\epsilon}^*$. The function $\varrho(\boldsymbol{\tau}, \boldsymbol{\Lambda})$ is continuous in $\boldsymbol{\tau}$ and $\boldsymbol{\Lambda}$. In sum, the whole mapping Φ is a function and continuous in $\boldsymbol{\Lambda}$. So, by Brouwer's fixed point theorem, there exists $\boldsymbol{\Lambda}^{**}$, which is a vector of equilibrium stocks satisfying the equilibrium conditions. Therefore, there also exist $\boldsymbol{\alpha}^{**} = \varpi(\boldsymbol{\Lambda}^{**})$, $\boldsymbol{V}^{**} = \iota(\boldsymbol{\alpha}^{**})$, $\boldsymbol{\epsilon}^{**} = \psi(\boldsymbol{V}^{**})$ and $\boldsymbol{\tau}^{**} = \xi(\boldsymbol{\alpha}^{**}, \boldsymbol{\epsilon}^{**})$. □

5.3 Uniqueness

It is possible that, given a set of parameter values, the model has multiple equilibrium, meaning multiple equilibrium stocks, and, therefore, multiple equilibrium reservation values and multiple equilibrium meeting probabilities. Although I cannot give mathematical proof of the uniqueness of the equilibrium, given a set of parameter values, numerically I have checked that the equilibrium stocks converge to the same values with different initial starting values of stocks. Theoretically multiple equilibrium would be possible given a set of parameter values, however, I can say that empirically it is unimportant.[33]

6 Identification

In this section, I explain my identification strategy and how it works in detail. The identification argument uses the similar idea to Friedberg and Stern (2014). Friedberg and Stern (2014) conceptually divide their identification argument into parts to provide intuitively clear identification sources for each parameter to readers.

[33] Proving theoretically the uniqueness of the equilibrium with this level of generality is obviously a future direction we need to pursue.

Before jumping into the identification argument, remember that the main *exogenous* primitives in the model, which are represented as a function with exogenous parameters, are

$$\{\{s_{ijt}, \mathbb{M}_{ijt}, \mathbb{C}^{M}_{ijt}, \mathbb{C}^{C}_{ijt}\}^{IJT}_{ijt}, F_{\epsilon}, F_{\tilde{\epsilon}|\epsilon}, F_{\mu^C,\mu^M}, \varsigma, \phi\}, \tag{16}$$

where s_{ijt} is the deterministic part of the flow match value which a type i man and a type j woman jointly get at time t, a set of model primitives associated with the marriage bonus, the divorce cost and the separation cost from cohabitation are denoted as $\mathbb{M}_{ijt}$, $\mathbb{C}^{M}_{ijt}$ and $\mathbb{C}^{C}_{ijt}$ respectively. The distribution of the stochastic part in the flow match value is denoted as F_{ϵ} for an initial draw, and $F_{\tilde{\epsilon}|\epsilon}$ is for the law of motion for ϵ_{ijt}. The joint distribution associated with cohabitation- and marriage-specific unobserved heterogeneity are denoted as F_{μ^C,μ^M}. Denote an exogenously-given discount factor by ς and a woman's bargaining weight by ϕ.

Let $\boldsymbol{\beta}^{s}$ be a vector of the parameters associated with the flow match value, $\boldsymbol{\beta}^{M}$ be a vector of the parameters associated with the marriage bonus, $\boldsymbol{\beta}^{C^C}$ be a vector of parameters associated with the cohabitation separation cost and $\boldsymbol{\beta}^{C^M}$ be a vector of the exogenous parameters associated with the marriage cost. We can rewrite the set of the primitives (1) more explicitly, with $\boldsymbol{\beta}^{s}$, $\boldsymbol{\beta}^{M}$, $\boldsymbol{\beta}^{C^C}$ and $\boldsymbol{\beta}^{C^M}$, as

$$\{\{s_{ijt}(\boldsymbol{\beta}^{s}), \mathbb{M}_{ijt}(\boldsymbol{\beta}^{M}), \mathbb{C}^{M}_{ijt}(\boldsymbol{\beta}^{C^M}), \mathbb{C}^{C}_{ijt}(\boldsymbol{\beta}^{C^C})\}^{IJT}_{ijt}, F_{\epsilon}, F_{\tilde{\epsilon}|\epsilon}, F_{\mu^C,\mu^M}, \varsigma, \phi\}. \tag{17}$$

Additionally, the meeting probabilities are also model primitives *endogenously* decided in the model, which are represented as a function mapped from exogenous parameters, $\boldsymbol{\beta}^{\alpha^m}$ and $\boldsymbol{\beta}^{\alpha^w}$. Namely,

$$\{\alpha^{m}_{ijt}(\boldsymbol{\beta}^{\alpha^m}), \alpha^{w}_{ijt}(\boldsymbol{\beta}^{\alpha^w})\}^{IJT}_{ijt}. \tag{18}$$

Let Ω be the parameter space in the model, and it is an union of the sets,

$$\Omega = \{\boldsymbol{\beta}^{s}, \boldsymbol{\beta}^{M}, \boldsymbol{\beta}^{C^M}, \boldsymbol{\beta}^{C^C}, F_{\epsilon}, F_{\tilde{\epsilon}|\epsilon}, F_{\mu^C,\mu^M}, \varsigma, \phi\} \cup \{\boldsymbol{\beta}^{\alpha^m}, \boldsymbol{\beta}^{\alpha^w}\}. \tag{19}$$

For the following argument, it is also important to remember, again, that each equilibrium reservation match value is represented as a function of all parameters in the economy (equilibrium meeting probabilities also). A vector of equilibrium reservation values, $\boldsymbol{\epsilon}^{**} = \{\epsilon^{**SC}_{ijt}, \epsilon^{**SM}_{ijt}, \epsilon^{**Si}_{ijt}, \epsilon^{**CS}_{ijt}, \epsilon^{**MS}_{ijt}\}^{IJT}_{ijt}$ is explicitly represented as $\boldsymbol{\epsilon}^{**}(\Omega) = \{\epsilon^{**SC}_{ijt}(\Omega), \epsilon^{**SM}_{ijt}(\Omega), \epsilon^{**Si}_{ijt}(\Omega), \epsilon^{**CS}_{ijt}(\Omega), \epsilon^{**MS}_{ijt}(\Omega)\}^{IJT}_{ijt}$ in this section.

6.1 Parameters associated with mating preference, s_{ijt}, and marriage bonus, $\mathbb{M}_{ijt}$

From this subsection, I start to provide sources of identification for each parameter. In this subsection, I explain how to separately identify the parameters associated with the mating preference and the marriage bonus while treating all of the other parameters as hypothetically fixed.

Assume the existence of equilibrium reservation match values and an equilibrium single-crossing point, and assume to satisfy the conditions for the coexistence of cohabitation and marriage discussed in Section 4.2.[34] I can neatly divide the support of ϵ_{ijt} into three parts with the thresholds, the equilibrium reservation

[34] Under some values of the parameter vector and realizations of persistent unobserved heterogeneity terms, there would be some cases

match value from single to cohabitation, ϵ_{ijt}^{**SC}, and the equilibrium single-crossing point, ϵ_{ijt}^{**Si}.

Let $h_{ijt}^{SM(m)}$ be a type i man's hazard rate out of being single to being married with a type j woman at time t. It is given by

$$h_{ijt}^{SM(m)} = \alpha_{ijt}^{m^{**}}(1 - F_{\epsilon}(\epsilon_{ijt}^{**Si})). \tag{20}$$

Similarly, the single to cohabitation hazard is

$$h_{ijt}^{SC(m)} = \alpha_{ijt}^{m^{**}}\left[F_{\epsilon}(\epsilon_{ijt}^{**Si}) - F_{\epsilon}(\epsilon_{ijt}^{**SC})\right]. \tag{21}$$

Equations (20) and (21) are originally with the unknown parameters, Ω. However, at this point of discussion, ϵ_{ijt}^{**SC} and ϵ_{ijt}^{**Si} are assumed hypothetically represented by a function of the following unknown parameters associated with the mating preference and marriage bonus, $\boldsymbol{\beta}^s$ and $\boldsymbol{\beta}^M$ only.

So, in the end, at this point of discussion, only the parameters associated with the mating preference, $\boldsymbol{\beta}^s$, and marriage bonus, $\boldsymbol{\beta}^M$, are left. Accordingly, I can rewrite equations (20) and (21) as

$$h_{ij(c_t)\kappa t}^{SM(m)} = \alpha_{ijt}^{m^{**}}(1 - F_{\epsilon}(\epsilon_{ijt}^{**Si}(\{\boldsymbol{\beta}^s, \boldsymbol{\beta}^M\})), \tag{22}$$

$$h_{ijt}^{SC(m)} = \alpha_{ijt}^{m^{**}}\left[F_{\epsilon}(\epsilon_{ijt}^{**Si}(\{\boldsymbol{\beta}^s, \boldsymbol{\beta}^M\})\}) - F_{\epsilon}(\epsilon_{ijt}^{**SC}(\{\boldsymbol{\beta}^s, \boldsymbol{\beta}^M)\}))\right]. \tag{23}$$

First, remember that the equilibrium meeting probability, $\alpha_{ijt}^{m^{**}}$, is treated as given at this point of argument. I have $2 \times IJT$ equations because I get two equations (22) and (23) for each i, j and t. Note that we can use a woman side's information in the same way. In sum, the parameters associated with the mating preference and the marriage bonus are mainly identified by matching the sample moments of $h_{ijt}^{SC(m)}$ and $h_{ijt}^{SM(m)}$, $\hat{h}_{ijt}^{SC(m)}$ and $\hat{h}_{ijt}^{SM(m)}$ for men, and $h_{ijt}^{SC(w)}$ and $h_{ijt}^{SM(w)}$, $\hat{h}_{ijt}^{SC(w)}$ and $\hat{h}_{ijt}^{SM(w)}$ for women to the corresponding theoretical moments. We can put it in a traditional way of explanation of identification: The *ceteris paribus* effect by changing the explanatory variable associated with $\boldsymbol{\beta}^s$ and $\boldsymbol{\beta}^M$ on $\hat{h}_{ijt}^{SC(w)}$ and $\hat{h}_{ijt}^{SM(w)}$ mainly identify $\boldsymbol{\beta}^s$ and $\boldsymbol{\beta}^M$.

6.2 Parameters associated with cohabitation separation cost, $\mathbb{C}_{ijt}^C$, and divorce cost, $\mathbb{C}_{ijt}^M$

Next, I move to the discussion of how to pin down the parameters associated with separation costs, $\mathbb{C}_{ijt}^C$ and $\mathbb{C}_{ijt}^M$. As in the previous procedure where I have focused only on the parameters associated with the mating preference and the marriage bonus, I hypothetically fix parameters other than the parameters associated with $\mathbb{C}_{ijt}^C$ and $\mathbb{C}_{ijt}^M$. To identify the parameters associated with the separation costs, I use different moments, a sample cohabitation to single hazard rate and a sample marriage to single hazard rate.

6.2.1 Parameters associated with cohabitation separation cost

Let h_{ijt}^{CS} be a hazard rate out of cohabitation by a type i man and a type j woman to single at time t. Remember ϵ_{ijt}^{**CS} is an equilibrium reservation match value with which a couple is indifferent between continuing to cohabit and returning to single.

where, for some men and women, cohabitation is not their best choice. However, in the following identification argument, I assume the ideal situation where the single-crossing condition and the coexistence condition are ensured and the number of observations goes infinity.

The hazard rate is represented as, using the notion of $\boldsymbol{\beta}^{C^C}$,

$$h_{ijt}^{CS} = \int_{\epsilon_{t-1}} F_{\epsilon|\epsilon_{t-1}} \left(\epsilon_{ijt}^{**CS}(\{\boldsymbol{\beta}^{C^C}\}) \right) dF_{\epsilon_{t-1}}. \tag{24}$$

In equation (9), I emphasize that the equilibrium reservation match values, ϵ_{ijt}^{**CS}, depends on the parameters associated with the cohabitation separation cost, $\boldsymbol{\beta}^{C^C}$. Remember there is serial correlation in the law of motion of ϵ_{ijt}. Therefore, in taking the integral, I take into account the previous draw, ϵ_{ijt-1}, which is written as ϵ_{t-1} in equation (9) for shorter notation. The parameters, $\boldsymbol{\beta}^{C^C}$, is mainly identified by matching the sample moment constructed from the data, $\hat{h}_{ijt}^{CS}$, to the corresponding theoretical moments in the same way as the parameters associated with the mating preference and the marriage bonus case.

6.2.2 Parameter associated with divorce cost

Similarly, let h_{ijt}^{MS} be a hazard rate out of marriage by a type i man and a type j woman to single at time t. Similarly, define ϵ_{ijt}^{**MS} as an equilibrium reservation match value with which a couple is indifferent between remaining married and returning to single. The hazard rate is

$$h_{ijt}^{MS} = \int_{\epsilon_{t-1}} F_{\epsilon|\epsilon_{t-1}} \left(\epsilon_{ijt}^{**MS}(\{\boldsymbol{\beta}^{C^M}\}) \right) dF_{\epsilon_{t-1}}. \tag{25}$$

I emphasize that ϵ_{ijt}^{**MS} depends on $\boldsymbol{\beta}^{C^M}$. It is mainly identified by matching the sample moment constructed from the data, $\hat{h}_{ijt}^{MS}$, to the corresponding theoretical moment in the same way as before.

6.3 Parameters associated with second moments

The variance, covariance and the law of motion terms, F_{ϵ}, F_{μ^C,μ^M} and $F_{\tilde{\epsilon}|\epsilon}$, are identified through second sample moments calculated by generalized residuals. Note that, for example, in a standard ordinary least squares model, we can easily calculate its residuals to construct its second sample moments. However, in a discrete choice model, it is not so straightforward to calculate its generalized residuals due to the unobservable latent variable. See, for example, Gourieroux et al. (1987), Goeree (2008) and Friedberg and Stern (2014) for more detail about how to construct them.

6.4 Parameters associated with meeting probability, α_{ijt}^m and α_{ijt}^w

In the last step, I pin down the parameters associated with α_{ijt}^m and α_{ijt}^w. So far, they have been taken as given. They are identified by using the equilibrium condition, the *consistency belief* condition, which we need to satisfy at equilibrium discussed in Section 5. Namely, the law of motions for the stocks in the economy mainly backs up the meeting probabilities. Let $\boldsymbol{D}_t^{**}$ be a vector of equilibrium operators associated with the law of motion for equilibrium stocks at time t, which is complicated functions of the model primitives. We have, given t,

$$\hat{\boldsymbol{\Lambda}}_{t+1}^{**} = \boldsymbol{D}_t^{**}(\Omega)\hat{\boldsymbol{\Lambda}}_t^{**}. \tag{26}$$

I treat the observed stocks as equilibrium stocks, $\hat{\mathbf{\Lambda}}_t^{**}$, assuming that the economy is in equilibrium. I have explicitly proven the existence of the equilibrium stocks in the economy.[35] It is, theoretically, ensured that there are the equilibrium stocks satisfying equation (26). Therefore, we can use the equations as a set of new moments. The same as the previous arguments, I emphasize the dependence of the meeting technology parameters, $\boldsymbol{\beta}^{\alpha^m}, \boldsymbol{\beta}^{\alpha^w}$, and rewrite it,

$$\hat{\mathbf{\Lambda}}_{t+1}^{**} = \boldsymbol{D}_t^{**}(\{\boldsymbol{\beta}^{\alpha^m}, \boldsymbol{\beta}^{\alpha^w}\})\hat{\mathbf{\Lambda}}_t^{**}. \quad (27)$$

The set of moments mainly identifies $\boldsymbol{\beta}^{\alpha^m}$ and $\boldsymbol{\beta}^{\alpha^w}$. A point which should be emphasized is that *market thickness* affects the meeting probabilities through an aggregate matching function but does not affect other model primitives. See, Section 8.4 for more detail.[36]

7 Data

In this research, I use the two data sets, the National Longitudinal Study of High School Class of 1972 (NLS 72) and the National Longitudinal Survey of Youth 1997 (NLSY 97). The NLS 72 is assumed to represent the cohort under the situation before the communication technology advance occurs, and the NLSY 97 is assumed to represent the cohort under the situation after the communication technology advance occurs. In addition to detailed demographic information, both data sets have relationship type information, including cohabitation, from early ages of respondents. Compared with other national representative surveys, the two data sets are unique in that they contain detailed retrospective marital history information during sample periods. Capturing shorter spells of cohabitation than marriage, I track individuals' marital status transitions every six months.

In the NLS 72, around 22,000 students are first interviewed when they are leaving a high school in the spring of 1972 with follow-up interviews in 1973, 1974, 1976 and 1979, and, for a limited group, 1986. I use the subset of the whole sample, which answers the 1986 follow-up interview. As Brien et al. (2006) and Lillard et al. (1995) mention, the limited group of the whole sample, which answers the 1986 interview, does not represent the whole population composition. Therefore, I give appropriate weights on the observations to adjust a choice-based sampling problem (see, Manski and Lerman, 1977, Hellerstein and Imbens, 1999, and Nevo, 2003). As used in Weiss and Willis (1997), I use the weights discussed in Tourangeau (1987).

In the NLYS 97, 8,984 men and women born between 1980 and 1984 are interviewed between 1997 and 2019. Respondents are interviewed annually between 1997 and 2011, then biennially until 2019. I remove individuals serving in the military as well as individuals for whom I cannot identify demographic characteristics during the sample periods.

Particularly, on each cohort, I focus on the data associated with martial status transitions of an individual, conditioning on an individual's gender, race, educational level as well as a partner's educational level if matched, whether there is a child from a previous relationship, whether there is a child from a current re-

[35] Note that, *theoretically*, the equality needs to hold exactly. However, in constructing moments associated with the consistency belief condition, I add a sampling error to make the moment conditions work properly in estimation.

[36] To strengthen the identification argument, I also include *exclusion restrictions*: To separately identify the parameters associated with the preference (match surplus) and those of the meeting technology, there are explanatory variables which only belong to either of the two parameter specifications. See, Section 8.3, for more detailed discussion.

lationship, match durations, and age which is assumed equivalent to time effects.[37] For detailed empirical specification for how the marriage market with the observable data is formed, see, Section 8.

The original NLS 72 sample consists of 22,650 men and women, and the original NLSY 97 consists of 8,984 men and women. In this research, I restrict my sample to black and white respondents. I exclude individuals who serve in the military because their marital behavior differs from that of the rest of population. In addition, I remove individuals for whom I cannot identify demographic characteristics during the sample periods. Since my main objective in this research is to investigate individual marital dynamics, I remove individuals who miss or refuses to answer questions associated with their marital history during the survey periods. Even though there are several imputation methods to deal with missing data issues (for example, Lavy, et al. (1998), Keane and Sauer (2010), and Van der Klaauw and Wolpin (2008)), I remove them for simplification of the analysis. This is partly justified because I still have enough observations left for estimation, 210,630 person-period observations in the NLS 72 and 190,440 person-period observations in the NLSY 97.

7.1 Detailed missing data table

Tables 1-2 show the detailed sample selection criteria in this research. Originally, I have 22,650 sample in the NLS72. I restrict respondents who answer the fifth follow-up interview, 12,840. I further restrict sample left to black and white individuals, 10,790. I throw away sample who I cannot identify their basic demographic variables, for example, race and gender, 10,720. I drop individuals who I cannot identify their marital history, 10,400. Because a partner's information is important in this research, I drop respondents who do not report their partner's eduction level or whether their partner has children from their previous relationship, 9,920. I drop sample who serves the military, 9,160. Lastly, I confine sample who has not experienced cohabitation before age 18, 7,410.[38]

On the other hand, in the NLSY 97, originally, I have 8,984 sample. I restrict sample to black and white individuals, 7,622. I throw away sample who I cannot identify their basic demographic variables, 6,766. For the NLSY 97 sample, even if a respondent misses an annual interview about his/her relationship, I impute the missing answer consistent with his/her answer for the periods before and after the period.[39] For a partner's education level, not a few responses do not annually report the information. In this research, as long as a partner has the same unique id, I fill in the missing partner's information during the corresponding spells with the partner's lowest level of education.[40] I drop sample who serves the military, 6348.[41]

[37] The NLS 72 is basically a single age cohort in which every sample has the same age. So, the effects of respondents' age and the calendar time on marital behaviors cannot be separately identified. I basically interprets the date of cohabitation/marriage as a measure of age at cohabitation/marriage.

[38] This is because of the choice-based sampling nature of the fifth follow-up interview in the NLS 72. There are more sample who has cohabited before age 18, which does not represents the whole population tendency. As Kaplan (2012) does, I need to check how estimates differ using two alternative samples: One that contains individuals who have cohabited before age 18 and one without the individuals.

[39] For example, suppose an individual misses or refuses to answer an annual interview. However, if he reports that he cohabits with the part with the same unique id for periods before and after the period. I regard he also cohabits with the same person during the missing period. I can only do this imputation for the NLSY 97 sample because, for the NLSY 97, the sampling period is annual. On the other hand, the NLS 72 only ask the retrospective questions once at 1986.

[40] As Guvenen and Smith Jr (2014) do, I need to check the robustness of this imputing filling-in method on the estimates.

[41] Note that, as Fiorini and Keane (2011) do, I need to check whether my attrition, missing data and sample selection criteria lead to a sample selection issue with respect to the original data sets: I run a probit model where the dependent variable is equal to 1 if the respondent is in my sample and zero otherwise. The explanatory variables are reported demographic characteristics in the original data sets. I can implement a statistically significant test, for example, by seeing the values of the coefficients. Detailed discussion is in Appendix.

Table 1: Sample selection criteria(NLS 72)

Selection criteria	Observation size
Whole Population	22,650
Fifth follow-up	12,840
Black or white	10,790
With basic demographic variables	10,720
With marital history	10,400
With partner's information	9,920
Not military	9,160
Cohabitation starts after age 18	7,410
Number of observations	7,410

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

Table 2: Sample selection criteria (NLSY 97)

Selection criteria	Observation size
Whole Population	8,984
Black or white	7,622
With basic demographic variables	6,776
With marital history	6,776
With partner's information	6,776
Not military	6,348
Number of observations	6,348

7.2 Descriptive statistics of date-0 distribution and marital status dynamics

For a non-stationary model, the date-0 distribution of individuals in the economy is also an exogenous model primitive.[42] Tables 3-6 show the composition of individuals in the economy, which comes from the NLS 72 and the NLSY 97 with proper sampling weights. Note that the information is represented as stocks because I use a special case of a Pissarides' style matching function whose arguments are stocks not proportions, as discussed in Section 8.4.[43]

Table 3: Population Statistics aged 14-18 in 1980 (NLS 72)

Exogenous characteristics	Number/Percentage
Whole Population	12,716 (in thousands)
Women	50 %
Black	11 %
Without High School Degree	26 %
High School Degree	52 %
Above College	21 %

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

Table 4: Population Statistics aged 14-18 in 2000 (NLSY 97)

Exogenous characteristics	Number/Percentage
Whole Population	16,131 (in thousands)
Women	50 %
Black	12 %
Without High School Degree	13 %
High School Degree	57 %
Above College	29 %

Describing the descriptive statistics and emphasizing remarkable changes is quite important in this research: I want to quantify how much of the changes in marital patterns come from the changes of the composition of the economy (Tables 3-6) or the changes of the model primitives.

In terms of the exogenous characteristics shown in Tables 3-6, one remarkable change is that, in the NLSY 97

[42] In a non-stationary model, the initial (time 0) distribution is also an exogenous model primitive: If we observe the difference in equilibrium outputs of the two cohorts, it might come just from the difference of the initial distributions, not from the difference of the other model primitives. However, in my model setting, I can overcome this issue by controlling the stocks in estimation. See, Section 8.4 for the empirical specification of meeting probabilities.

[43] Not only the ratio but also the amount of stocks matters because my matching function uses stocks as its arguments.

Table 5: Educational attainment by gender and race (NLS 72)

Male	White	Black
Without High School Degree	10.9 %	25.3 %
High School Degree	62.3 %	64.2 %
College or above	26.8 %	10.5 %
Female	White	Black
Without High School Degree	10.8 %	21.7 %
High School Degree	66 %	65.9 %
College or above	23.2 %	12.4 %

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

Table 6: Educational attainment by gender and race (NLSY 97)

Male	White	Black
Without High School Degree	7.9 %	8.8 %
High School Degree	60.7 %	77.7 %
College or above	31.4 %	13.5 %
Female	White	Black
Without High School Degree	5.5 %	13.4 %
High School Degree	57 %	66.6 %
College or above	37.5 %	20 %

cohort, a smaller proportion is without high school degree. The percentage is 26% in the NLS 72 and it reduces to 13% in the NLSY 97. Other than the educational attainment difference, a remarkable change is the total population, which changes from 12,716 thousands to 16,131 thousands. The other exogenous characteristics stay similar during these two cohorts.

Next, we show simple descriptive statistics of marital dynamics to point out that there quantitative differences exist at an aggregate level before and after the communication technology advance: Certainly, we can observe, to non-negligible extent, their marital behavior differs during the two cohorts, which is shown in Figures 2 - 9. Aggregate stocks evolve in a different way in each cohort. One of notable points I need to mention is that, compared with the NLS 72 cohort, people in the NLSY 97 cohort get married less and cohabit more. As shown in Figure 5, during the sample periods of the NLS 72, the proportion of the stock of cohabitation is at most 10%. For people in the NLSY 97, cohabitation is a more popular choice. When the sample in the NLS 72 reaches age 32, which is 1986 as shown in Figure **??**NLS 72 Stock of marriage actual, more than half of them get married (over 60 percent). In the NLSY 97 cohort, the proportion of the stock of marriage is much lower than in the NLS 72 cohort throughout the sampling period.

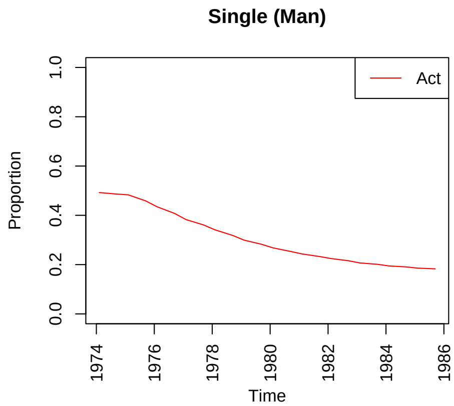

Figure 2: Stock of single men (NLS 72)

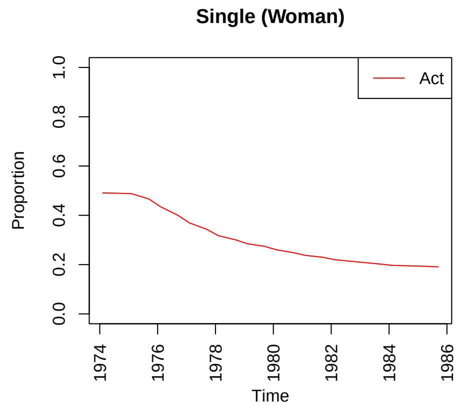

Figure 3: Stock of single women (NLS 72)

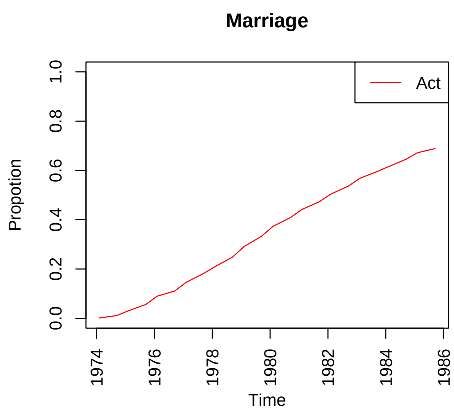

Figure 4: NLS 72 Stock of marriage actual

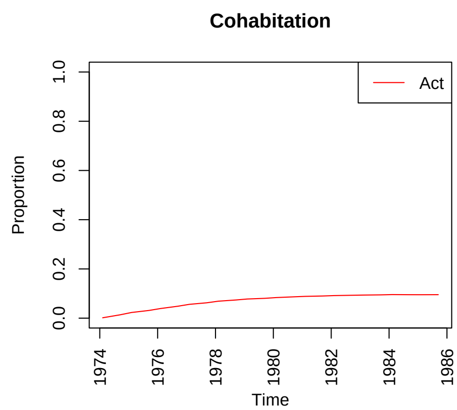

Figure 5: Stock of Cohabitation (NLS 72)

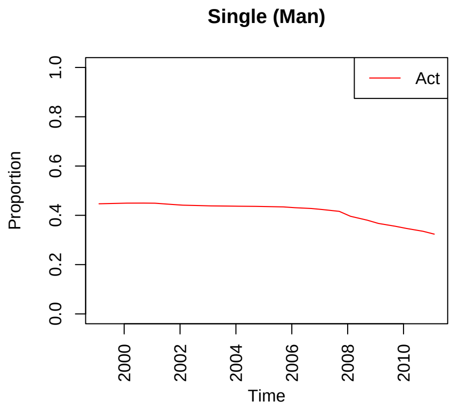

Figure 6: Stock of single men (NLSY 97)

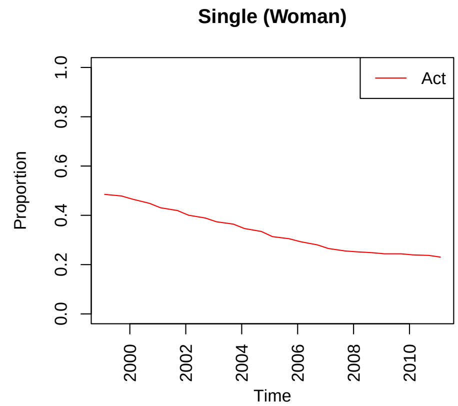

Figure 7: Stock of single women (NLSY 97)

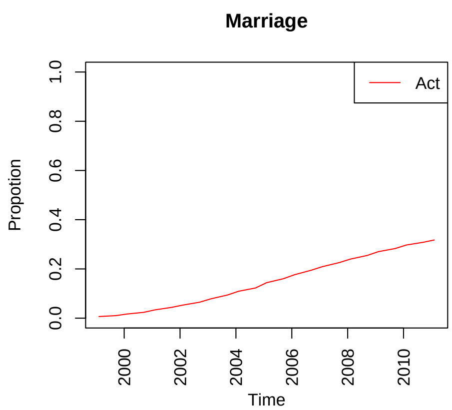

Figure 8: Stock of marriage (NLSY 97)

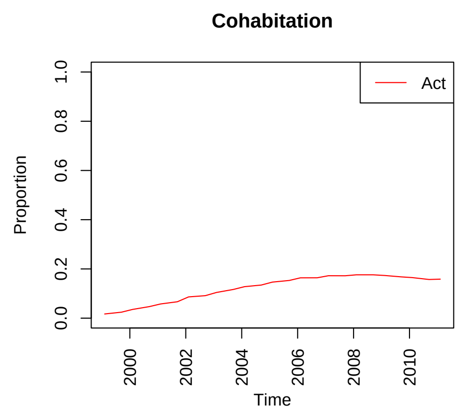

Figure 9: Stock of Cohabitation (NLSY 97)

7.3 Non-structural analysis of two cohorts difference

In addition to the changes in the descriptive patterns of the aggregate exogenous characteristics and endogenous marital behavior, I measure the distance of the two date sets, using auxiliary statistics, to get more sense of the importance of the technology advance: If the two data sets can be considered as close measured by a certain norm, for example, Euclidean norm or Manhattan distance, it implies that there are no changes during the two cohorts. I use the coefficients of the following non-structural model as a set of auxiliary statistics.[44]

Consider a type i man and a type j woman at time t under technology level κ represented by a vector of demographic and endogenous state variables, $X^m_{it\kappa}$ and $X^w_{jt\kappa}$ with $X_\kappa = [X^m_{it\kappa}, X^w_{jt\kappa}]$. The vector of variables, X_κ, includes gender, race, educational level, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age which is assumed equivalent to time effects.

Let $y^S_{t\kappa} = \{1, 2, 3\}$ be a dependent variable representing that a *single* individual in selects single, $y^S_{t\kappa} = 1$,

[44] We do not have to have a correctly-specified model. If two data sets are considered as the same, auxiliary statistics, for example, regression coefficients are the same across the regressions.

cohabitation, $y^S_{t\kappa} = 2$, and marriage, $y^S_{t\kappa} = 3$ at time t under technology κ. Let $y^C_{t\kappa} = \{1, 2, 3\}$ be a dependent variable representing that a *cohabiting* individual selects single, $y^C_{t\kappa} = 1$, cohabitation, $y^C_{t\kappa} = 2$, and marriage, $y^C_{t\kappa} = 3$ at time t under κ. Similarly, $y^M_{t\kappa}$ is defined for transitions from *marriage*.

Then, the non-structural ordered linear probability models are

$$y^S_{t\kappa} = \boldsymbol{X_\kappa} \gamma^{\boldsymbol{S}}_{\boldsymbol{\kappa}} + e^S; \tag{28}$$

$$y^C_{t\kappa} = \boldsymbol{X_\kappa} \gamma^{\boldsymbol{C}}_{\boldsymbol{\kappa}} + e^C; \tag{29}$$

$$y^M_{t\kappa} = \boldsymbol{X_\kappa} \gamma^{\boldsymbol{M}}_{\boldsymbol{\kappa}} + e^M; \tag{30}$$

where $\gamma^{\boldsymbol{S}}_{\boldsymbol{\kappa}}, \gamma^{\boldsymbol{C}}_{\boldsymbol{\kappa}}$ and $\gamma^{\boldsymbol{M}}_{\boldsymbol{\kappa}}$ are vectors of coefficients for the non-structural ordered linear probability models about a transition from single, cohabitation and marriage under technology level κ respectively, and e^S, e^C and e^M are specified as $e^S, e^C, e^M \sim iidN(0, 1)$.

The difference (distance) of the two cohorts is summarised by using the auxiliary statistics in Tables 7-9. I implement a F-statistics test where the null hypothesis is the coefficients of those of the NLSY 97 cohort is the same as the NLS 72 cohort. In the case of the transition from single, the F-statistic is 32.8. I reject the hypothesis. In the case of the transition from cohabitation, the F-statistic is 36.7. I reject the hypothesis. In the case of the transition from marriage, the F-statistic is 1.77. The value is smaller compared with the F-statistics for the transition from single and the F-statistics for the transition from cohabitation. This is reasonable because, in each cohort, the non-structural data feature of married couple for each cohort shows relatively similar data pattern.[45]

Table 7: Coefficients associated with Transition from single

Parameter	NLS 72[a]	NLSY 97
Constant	1.005	0.013
Age (time)	0.006	0.000
Age spline $\leq$ 5 years	0.003	-0.002
Age spline $\geq$ 10 years	-0.008	-0.009
Female	0.000	-0.000
Respondent education	0.139	0.170
Partner education	0.139	0.171
Education difference	-0.140	-0.174
Kid from previous relationship	0.104	0.002
Black	-0.005	-0.003

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[45]The non-structural data summary also indicates that married couple in the NLSY 97 cohort are more willing to get back to single. This is because the estimate of the constant term in the NLSY 97, 1.905, is smaller than that of the NLS 72 cohort, 3.141 in Table 9. I will discuss this point in interpreting the estimates of divorce costs in Section 9.

Table 8: Coefficients associated with Transition from cohabitation

Parameter	NLS 72[a]	NLSY 97
Constant	2.893	0.931
Age (time)	0.009	-0.002
Age spline $\leq$ 5 years	0.004	0.002
Age spline $\geq$ 10 years	0.000	0.000
Female	-0.020	0.000
Respondent education	0.125	0.002
Partner education	0.123	0.004
Education difference	-0.136	-0.008
Match duration	-0.051	0.000
Match duration spline $\leq$ 2 years	-0.005	0.000
Black	0.003	-0.005

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

Table 9: Coefficients associated with Transition from marriage

Parameter	NLS 72 [a]	NLSY 97
Constant	3.141	1.905
Age (time)	0.000	-0.001
Age spline $\leq$ 5 years	0.000	0.000
Age spline $\geq$ 10 years	0.000	0.000
Female	-0.238	-0.001
Respondent education	-0.192	0.014
Partner education	-0.196	0.018
Education difference	0.194	-0.110
Kid from previous relationship	-0.005	-0.007
Kid from current relationship	0.006	0.027
Match duration	0.000	-0.001
Match duration spline $\leq$ 2 years	0.000	0.004
Black	0.000	0.000

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

8 Estimation

In this section, I explain the estimation process. A brief description of the estimation process is the following: Given a set of the parameters, Ω, there are equilibrium reservation values in this game as proven in Section 5. Therefore, through the equilibrium reservation values calculated, I calculate the objective function, which will be introduced below. I keep doing the above procedure with different values of the parameters until the numerically calculated objective function reaches its minimum. Particulary, I look for a set of values of the parameters in the parameter space which minimizes the objective function with the idea of indirect inference, which will be introduced briefly below.

In this research, the terminal period for all individuals, T, is assumed to be 45. I treat the state of an individual at time T as an absorbing state, which means that, after the period, an individual does not change their marital status, and I give an *arbitrary* value to it.[46]

So far, particular functional forms for model primitives have not been explicitly specified. I need to specify the functional forms of the model primitives before I start solving the model. So, in the end of this section, I give a detailed specification of the economy for an estimation purpose. Meanwhile, assume that it is appropriately specified according to the theoretical requirements discussed in Section 5.

8.1 Indirect Inference

I use indirect inference in estimation (Gourieroux et al. (1993)). The underlying concept of the indirect inference involves three steps: first, selecting a set of moments, which is also called auxiliary statistics, that captures the characteristics of the real data (observed moments), second, simulating the structural economic model and calculating simulated moments several times with different values of parameters, and, third, picking parameter values such that the simulated moments closely replicate the observed moments. When the number of observations becomes infinite, the indirect inference gives us consistent estimates of the parameters (Gourieroux et al. (1993)).

Following previous studies with indirect inference, I select a set of auxiliary statistics which are easy to compute and are able to capture a variety of patterns in the data. Also, the set of moments should be informative for the underlying structural model.[47] We evaluate the distance between the two data sets using the Euclidian distance constructed by the following auxiliary statistics.

As a part of auxiliary statistics, I focus on a simple non-structural ordered linear probability model for a single-single, single-cohabitation and single-marriage transition. Similarly, I also focus on a transition from cohabitation and marriage using a non-structural ordered linear probability model. They are introduced in more detail below.

Consider a type i man and a type j woman at time t represented by a vector of demographic and endogenous state variables, $\boldsymbol{X}^m_{it}$ and $\boldsymbol{X}^w_{jt}$ with $\boldsymbol{X} = [\boldsymbol{X}^m_{it}, \boldsymbol{X}^w_{jt}]$. The vector of variables, $\boldsymbol{X}$, includes gender, race, educational level, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age which is assumed equivalent to time effects. Let $y^S_t = \{1, 2, 3\}$ be a dependent variable representing that a *single* individual selects single, $y^S_t = 1$, cohabitation, $y^S_t = 2$, and marriage, $y^S_t = 3$, at time t. Let $y^C_t = \{1, 2, 3\}$ be a dependent variable representing that a *cohabiting* individual selects single, $y^C_t = 1$, cohabitation, $y^C_t = 2$, and marriage, $y^C_t = 3$, at time t. Similarly, y^M_t is defined for

[46] I use a set of moments to match for 30 time periods. As Wolpin (1992) points out, if we have a discount factor in a model, the information about utilities difference of the distant future state is not so important in deciding a *current* decision. As shown in Appendix, the estimation results are quite robust. In addition to

[47] As, for example, Collard-Wexler (2013) and van der Klaauw and Wolpin (2008) say, auxiliary statistics do not need to have any interpretation. They are just required to describe the characteristics of data as much as possible. As Hall and Rust (2020) mention, a variety of moments can be used as a possible set of auxiliary statistics (first, second, third and fourth moments, covariance and quantiles, etc.). In general, it does not matter which auxiliary statistics I use as long as I can properly measure the distance between the simulated and actual data through the "lens" of the statistics (for example, see, Alan (2006) and Gourieroux et al. (1993)), as long as the moments are related to the underlying structural model (*invertibility*).

transitions from *marriage*. Then, the non-structural ordered linear probability models are

$$y_t^S = \boldsymbol{X}\boldsymbol{\gamma^S} + e^S; \tag{31}$$

$$y_t^C = \boldsymbol{X}\boldsymbol{\gamma^C} + e^C; \tag{32}$$

$$y_t^M = \boldsymbol{X}\boldsymbol{\gamma^M} + e^M; \tag{33}$$

where $\boldsymbol{\gamma^S}, \boldsymbol{\gamma^C}$ and $\boldsymbol{\gamma^M}$ are vectors of coefficients for the non-structural ordered linear probability models about a transition from single, cohabitation and marriage respectively, and e^S, e^C and e^M are specified as $e^S, e^C, e^M \sim N(0,1)$.

Particularly, as a set of auxiliary statistics, I focus on the partial derivative of relationship status transitions of individuals with respective to a set of explanatory variables, which are captured by the coefficients of the linear probability models, $\boldsymbol{\gamma^S}, \boldsymbol{\gamma^C}$ and $\boldsymbol{\gamma^M}$.

I also calculate the covariance matrix of vectors of an individual's relationship choice as a part of auxiliary statistics. The set of auxiliary statistics associated with the covariance matrix are denoted as $\boldsymbol{\vartheta}$.

Aggregate stocks of single individuals, cohabitating individuals and individuals in marriage also work as a part of auxiliary statistics. I denote $\boldsymbol{q^S}, \boldsymbol{q^C}$ and $\boldsymbol{q^M}$ as vectors of auxiliary statistics associated with aggregate stocks of individuals of single, cohabitation and marriage at equilibrium respectively. Let Ξ be a set of auxiliary statistics. Particularly,

$$\Xi = \{\boldsymbol{\gamma^S}, \boldsymbol{\gamma^C}, \boldsymbol{\gamma^M}, \boldsymbol{q^S}, \boldsymbol{q^C}, \boldsymbol{q^M}, \boldsymbol{\vartheta}\}.$$

Table 10: Auxiliary statistics

#	Auxiliary statistics
1	Coefficients of ordinal linear probability of transitions from single to cohabitation/marriage conditional on gender, race, educational level, partner's educational level, children from previous relationship, age and age spline modification
2	Coefficients of ordinal linear probability of transitions from cohabitation to single/marriage conditional on gender, race, educational level, partner's educational level, children from previous relationship, age, age spline modification, children from current relationship, duration and duration spline modification
3	Coefficients of ordinal linear probability of transitions from marriage to single conditional on gender, race, educational level, partner's educational level, children from previous relationship, age, age spline modification, children from current relationship, duration and duration spline modification
4	Across-person variation in length of single
5	Across-person variation in length of cohabitation
6	Across-person variation in length of marriage
7	Across-person variation in length of a match (cohabitation or marriage)
8	Across-person correlation between lengths of cohabitation and marriage
9	Calculated Stocks (stock of single, cohabitation and marriage individuals)

Table 10 summarizes the auxiliary statistics used in this research to identify the parameters.

8.2 Objective function

With the set of auxiliary statistics, I can construct the objective function I need to minimize. Let Ω be a vector of the structural parameters. Let R be the number of simulations. Let $\hat{\Xi}(\Omega)$ be a vector of auxiliary statistics calculated by using R simulated data sets with Ω. Let $\bar{\Xi}$ be a vector of auxiliary statistics calculated by the actual data. Let Γ be a weighting matrix. Let $\hat{\Omega}$ be a vector of estimates of the structural parameters, and it is written as

$$\hat{\Omega} = \arg\min_{\Omega}[[\hat{\Xi}(\Omega) - \bar{\Xi}]'\Gamma[\hat{\Xi}(\Omega) - \bar{\Xi}]]. \tag{34}$$

Following much of the literature (for example, Altonji and Segal (1996)), I do not use the optimal weighting matrix. Instead, I use a *diagonal* weighting matrix for the weighting matrix, Γ, to adjust scales of each moment. See Appendix in more detail.

In estimation, the equilibrium consistency belief condition is not explicitly imposed because of extremely large computational burdens, as in Seitz (2009) and Beauchamp et al. (2018). I assume that the economy is already on the equilibrium mapped by the underlying structural parameters. Therefore, I regard that the observed stocks in the data as the equilibrium stocks in the economy. In solving an individual dynamic decision process, all players are assumed to have the consistent belief already. However, as I discuss in Section 10, when I do counterfactual experiments with different values of parameters, I iterate the fix-point algorithm and derive the corresponding equilibrium stocks.[48]

In making the simulated data, I perform one-step-ahead simulation as follows: Given a set of values of the structural parameters, I solve the dynamic programming problem. Using the information of an individual's observable initial exogenous characteristics and drawing the permanent unobserved heterogeneity terms, I simulate an individual decision by drawing a flow match value shock, and update the individual's state variables using the decision rules driven by the dynamic programming. I keep doing this procedure until the terminal period.

In this research, I employ a moment based estimation method (indirect inference), which creates non-smoothness of the objective function on the parameter space, given the finite number of simulated draws. In this research, I set $R = 450$ which makes the number of simulated draws at each iteration approximately 100 million. This number is sufficient to consider the objective function as smooth enough on the parameter space (see, for example, Pakes (1986)). In this research, I use a derivative-based optimization method, accommodating the possibility of my objective function is non-smooth (Lewis and Oberton (2013)).

8.3 Empirical Specification

Before estimating the model, I need to give specific functional forms to the model primitives described in Section 3. Also, I impose some practical restrictions on dynamics of explanatory variables and marriage markets feasible for players. In this section, I provide parsimonious functional forms of the model primitives and some restrictions imposed on dynamics of a part of explanatory variables and marriage markets in more detail. Particular focuses of this section are the following model primitives; flow match value, $s_{ijt\kappa}$, marriage bonus,

[48] Given parameter values, I need to recalculate a fixed-point algorithm so that players' belief converge to the actual aggregate dynamics of stocks.

$\mathbb{M}_{ijt\kappa}$, cohabitation separation cost, $\mathbb{C}^C_{ijt\kappa}$, divorce cost, $\mathbb{C}^M_{ijt\kappa}$, meeting probability, $\alpha^m_{ijt\kappa}$ and $\alpha^w_{ijt\kappa}$, childbearing probability, $P^b_{t\kappa}$, distributions associated with cohabitation and marriage unobserved heterogeneity terms and the stochastic part of a match value, F_{μ^C,μ^M}, F_ϵ and $F_{\tilde{\epsilon}|\epsilon}$. I also introduce restrictions on dynamics of stocks of children, evaluation of match durations and possible marriage markets.

8.3.1 Flow match value, marriage bonus, separation costs and unobserved terms

Most of the model primitives are assumed to be approximated by a function of a linear index in parameters. Consider an observable type for an i man and an observable type for a j woman at time t represented by a vector of demographic and endogenous state variables, $\boldsymbol{X}^m_{it}$ and $\boldsymbol{X}^w_{jt}$, respectively, and denote $\boldsymbol{X} = [\boldsymbol{X}^m_{it}, \boldsymbol{X}^w_{jt}]$. The matrix of the variables, $\boldsymbol{X}$, includes gender, race, educational level, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age which is assumed equivalent to time effects.

Let $\boldsymbol{\beta}^m_\kappa$ and $\boldsymbol{\beta}^w_\kappa$ be a vector of coefficients for the flow match value associated with a man's observable type and a woman's observable type, and denote $\boldsymbol{\beta}_\kappa = [\boldsymbol{\beta}^m_\kappa, \boldsymbol{\beta}^w_\kappa]$. The variable κ emphasizes that the parameter values change depending on the communication technology level.

The match surplus between a type i man and a type j woman at time t under κ presented in equation (1) in Section 3 is specified as [49]

$$\begin{aligned} s_{ijt\kappa} = {} & \boldsymbol{X}\boldsymbol{\beta}_\kappa + \mathbb{M}_{ijt\kappa}\mathbb{I}[d_{ijt} = 3] + (\mu^{Cm}_\kappa + \mu^{Cw}_\kappa)\mathbb{I}[d_{ijt} = 2] + (\mu^{Mm}_\kappa + \mu^{Mw}_\kappa)\mathbb{I}[d_{ijt} = 3] + \epsilon_{ijt\kappa} \\ & - \mathbb{C}^C_{ijt\kappa}\mathbb{I}[d_{ijt-1} = 2, d_{ijt} = 1] - \mathbb{C}^M_{ijt\kappa}\mathbb{I}[d_{ijt-1} = 3, d_{ijt} = 1], \end{aligned}$$

where, letting $\boldsymbol{m}^m_\kappa$ and $\boldsymbol{m}^w_\kappa$ be a vector of marriage bonus coefficients associated with a man's type and a woman's type with $\boldsymbol{m}_\kappa = [\boldsymbol{m}^m_\kappa, \boldsymbol{m}^w_\kappa]$, $\mathbb{M}_{ijt\kappa}$ is specified as

$$\mathbb{M}_{ijt\kappa} = \boldsymbol{X}\boldsymbol{m}_\kappa, \tag{35}$$

and, the costs, $\mathbb{C}^C_{ijt\kappa}$ and $\mathbb{C}^M_{ijt\kappa}$, are specified in a similar fashion as,

$$\mathbb{C}^C_{ijt\kappa} = \boldsymbol{X}\boldsymbol{c}^C_\kappa; \tag{36}$$

$$\mathbb{C}^M_{ijt\kappa} = \boldsymbol{X}\boldsymbol{c}^M_\kappa, \tag{37}$$

and

$$\epsilon_{ijt\kappa} \sim N(0, \sigma^2_f(\kappa)) \text{ if a couple first meets;} \tag{38}$$

$$\epsilon_{ijt\kappa} = \rho_\kappa \epsilon_{ijt-1\kappa} + \eta_t \text{ after a match;}$$

$$\eta_t \sim iidN(0,1), \tag{39}$$

[49] In the following discussion, I suppress a notation which represents an individual.

and μ_κ^{Cm} and μ_κ^{Cw} are cohabitation-specific unobserved heterogeneity terms for a man and a woman respectively. Their variance depends on the technology level κ. Similarly, μ_κ^{Mm} and $\mu_\kappa^{Mw} \sim iidN(0, \sigma^2_{\mu^M}(\kappa))$ are marriage-specific unobserved heterogeneity terms for a man and a woman respectively. Let $\boldsymbol{\mu}_\kappa^m = [\mu_\kappa^{Cm}, \mu_\kappa^{Mm}]$ and $\boldsymbol{\mu}_\kappa^w = [\mu_\kappa^{Cw}, \mu_\kappa^{Mw}]$ with their joint distribution

$$\boldsymbol{\mu}_\kappa^m \sim iidN(0, \boldsymbol{\Gamma}_{\mu(\kappa)}); \tag{40}$$

$$\boldsymbol{\mu}_\kappa^w \sim iidN(0, \boldsymbol{\Gamma}_{\mu(\kappa)}). \tag{41}$$

For identification of $\boldsymbol{\Gamma}_{\mu(\kappa)}$, I impose the restriction on the realizations of cohabitation and marriage specific unobserved terms, $\mu_\kappa^{Cm} = \mu_\kappa^{Cw}$ and $\mu_\kappa^{Mm} = \mu_\kappa^{Mw}$. Because I assume the continuous support for $\mu_\kappa^{Cm}, \mu_\kappa^{Cw}, \mu_\kappa^{Cm}$ and μ_κ^{Cm}, for numerically solving the model recursively, I discretize each support with 3 grid points, and interpolate each support with a B-spline method. See, Appendix for the computation in this research.

8.4 Meeting probability

In this subsection, I provide function specifications for the meeting probabilities. Let $\Lambda_{t\kappa}^{Sm}$ and $\Lambda_{t\kappa}^{Sw}$ be an aggregate stock of single men and an aggregate stock of single women at time t under κ in the economy respectively, and $\Lambda_{t\kappa}^{Sm} = \sum_i^I \Lambda_{it\kappa}^{Sm}$ and $\Lambda_{t\kappa}^{Sw} = \sum_j^J \Lambda_{jt\kappa}^{Sw}$. Let $M_{t\kappa}$ be the total number of meetings happening at time t under κ. Let $z(\cdot)$ be an aggregate matching function, and $M_{t\kappa} = z(\Lambda_{t\kappa}^{Sm}, \Lambda_{t\kappa}^{Sw})$ (See, for example, Petrongolo and Pissarides (2001)).[50] Let $\alpha_{ijt\kappa}^m$ be a meeting probability of a type i man of meeting a type j woman at time t under κ from a man's side and $\alpha_{ijt\kappa}^w$ be a meeting probability from a woman's side. Let $\delta_{ijt\kappa}(\cdot)$ be a meeting efficiency between a type i man and a type j woman at time t under κ. The meeting efficiency is eventually mapped from a vector of observable characteristics for a type i man at time t, $\boldsymbol{X}_{it}$, and a vector of observable characteristics for a type j woman at time t, $\boldsymbol{X}_{jt}$, as I will show soon. Let $\boldsymbol{\beta}_\kappa^m$ and $\boldsymbol{\beta}_\kappa^w$ be a vector of coefficients for the meeting efficiency associated with a man's observable type and a woman's observable type, and denote $\boldsymbol{\beta}_\kappa^{\boldsymbol{\alpha}} = [\boldsymbol{\beta}_\kappa^{\alpha^m}, \boldsymbol{\beta}_\kappa^{\alpha^w}]$. I specify the meeting efficiency as

$$\delta_{ijt\kappa}(\boldsymbol{X}) = \frac{exp(\boldsymbol{X}\boldsymbol{\beta}_\kappa^{\boldsymbol{\alpha}})}{1 + exp(\boldsymbol{X}\boldsymbol{\beta}_\kappa^{\boldsymbol{\alpha}})}.$$

Then, the meeting probability, $\alpha_{ijt\kappa}^m$ and $\alpha_{ijt\kappa}^w$, is specified as

$$\alpha_{ijt\kappa}^m = \left[\lambda \delta_{ijt\kappa}(\boldsymbol{X}) z(\Lambda_{t\kappa}^{Sm}, \Lambda_{t\kappa}^{Sw}) \frac{\Lambda_{it\kappa}^{Sm}}{\Lambda_{t\kappa}^{Sm}} \frac{\Lambda_{jt\kappa}^{Sw}}{\Lambda_{t\kappa}^{Sw}} \right] / \Lambda_{it\kappa}^{Sm}; \tag{42}$$

$$\alpha_{ijt\kappa}^w = \left[\lambda \delta_{ijt\kappa}(\boldsymbol{X}) z(\Lambda_{t\kappa}^{Sm}, \Lambda_{t\kappa}^{Sw}) \frac{\Lambda_{it\kappa}^{Sm}}{\Lambda_{t\kappa}^{Sm}} \frac{\Lambda_{jt\kappa}^{Sw}}{\Lambda_{t\kappa}^{Sw}} \right] / \Lambda_{jt\kappa}^{Sw}, \tag{43}$$

where λ is an exogenously-decided scaling parameter to make the upper bound of $\delta_{ijt\kappa}(\boldsymbol{X})$, which is 1, to λ, and, at the same time, to ensure $\alpha_{ijt\kappa}^m$ and $\alpha_{ijt\kappa}^w$ are less than 1 (for example, see, Guvenen and Rendall (2015)).

[50] The aggregate matching function, $z(\Lambda_{t\kappa}^{Sm}, \Lambda_{t\kappa}^{Sw})$, should have the aggregate stocks, $\Lambda_{t\kappa}^{Sm}$ and $\Lambda_{t\kappa}^{Sw}$, as its arguments to capture aggregate externalities on meetings caused by actions of other types. See, for example, Petrongolo and Pissarides (2001).

The specific reason why I explicitly model $\delta_{ijt\kappa}(\cdot)$ is the following: It would be the case that, even if the stocks are the same, an individual with a certain type tends to meet with an individual with a certain type more or less. To capture this, I explicitly include $\delta_{ijt\kappa}(\cdot)$. Following much of previous studies, the aggregate matching technology, $z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa})$, is assumed to have the functional form, $z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa}) = 1/\lambda \cdot \Lambda^{Sm^{0.5}}_{t\kappa} \Lambda^{Sw^{0.5}}_{t\kappa}$.[51]

Remember that, because I assume the non-stationarity of the economy, I can explicitly include the effects of the stocks on the meeting probabilities and, therefore, marital behaviors. If I assume a stationarity of the economy used commonly in previous studies, I cannot control the effects of changes of stocks.

8.4.1 Childbearing probability

A part of uncertainty of the model also arises from the imperfect control women have over childbirth. Let $\boldsymbol{p}^m_\kappa$ and $\boldsymbol{p}^w_\kappa$ be a vector of childbearing coefficients associated with a man's type and a woman's type, and denote $\boldsymbol{p}^{\boldsymbol{b}}_\kappa = [\boldsymbol{p}^m_\kappa, \boldsymbol{p}^w_\kappa]$. The probability of giving a birth at time $t+1$, $P^b_{t\kappa}$ is specified as,

$$P^b_{t\kappa} = \frac{exp^{\boldsymbol{X}\boldsymbol{p}^b_\kappa}}{1 + exp^{\boldsymbol{X}\boldsymbol{p}^b_\kappa}}. \tag{44}$$

The parameters of this function are estimated outside of the main structural model to reduce the computational cost and treated as given in the estimation of the main structural estimation. It implicitly indicates that how childbearing probabilities is treated as given by players.

8.5 Law of motion of explanatory variables

8.5.1 Children

Women are assumed to inherit children from their current marriage, L^c_t, and from their previous relationship, L^{pr}_t. A child born to single and from dissolved cohabitation and marriage are treated as a child from a past relationship. I denote $b_{t\kappa}$ as whether to have a birth at time t. So, $b_{t\kappa} = 1$ with probability $P^b_{t\kappa}$. The law of motion of the stock of children evolves as,

$$L^{pr}_{t+1} = \begin{cases} L^{pr}_t + b_{t\kappa} & \text{if } d_{ijt} = 2,3 \ and \ d_{ijt-1} = 1 \\ L^{pr}_t + L^c_t + b_{t\kappa} & \text{if } d_{ijt} = 1, \end{cases} \tag{45}$$

[51] A point about the role of λ should be noted. The specification typically used in matching literature with a Pissarides' style matching function, $z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa}) = \Lambda^{Sm^{0.5}}_{t\kappa} \Lambda^{Sw^{0.5}}_{t\kappa}$, does not work in this research. This is because, as shown in Section 7, the ratio of women in the economy is 0.5 in both cohorts. It means, first, that $\Lambda^{Sm}_{t\kappa} = \Lambda^{Sw}_{t\kappa} \equiv \Lambda^S_{t\kappa}$. Second, the specification, $z(\Lambda^S_{t\kappa}, \Lambda^S_{t\kappa}) = \Lambda^{S^{0.5}}_{t\kappa} \Lambda^{S^{0.5}}_{t\kappa}$ implies the number of total matches, z, is also $\Lambda^S_{t\kappa}$. This conceptually means there is no friction mechanism in the aggregate matching box, $z(\cdot)$, because we can get the same amount of output as the amounts of the inputs. *A meeting probability per person is 1 both from a man's side and a woman's side.* The role of λ is dealing with this trivial situation by making a friction. At the same time, in my research, I can deal with the situation where a certain type of a man and a woman meet more efficiently. There is $\delta_{ijt\kappa}(\cdot)$ for this reason, and λ in front of $\delta_{ijt\kappa}(\cdot)$ makes possible upper bound of a meeting probability 1, even if we make a friction through the aggregate matching box. Eventually, both λs are canceled out, but distinguishing how each λ works is conceptually important. Note that, as seen in the aggregate matching technology specification, which is constant return to scale, what is only relevant is *ratios* of stocks not *stocks* themselves (Petronglo and Pissarides (2001), and Rogerson et al. (2005)).

and

$$L^c_{t+1} = \begin{cases} L^c_t + b_{t\kappa} & \text{if } d_{ijt} = 2,3 \ and \ d_{ijt-1} = 2,3 \\ 0 & \text{if } d_{ijt} = 1. \end{cases} \tag{46}$$

Note that the above law of motion implies that if a couple gets separated at time t, their children from the match become the stock of children from a previous relationship upon entering time $t+1$. If a birth happens at time t and the individual is single, the child becomes the stocks of the previous children even if the individual gets matched at the end of time t.

8.5.2 Match Duration evolution

The state space for the dynamic programming of the model is too large without imposing some restrictions on evolutions of some part of state variables. Particularly, I put an upper bound for the evolution of the match durations at time t, $duration_t$. It evolves following the following law of motion,

$$duration_{t+1} = \begin{cases} 8 & \text{if } duration_t = 8 \ and \ d_{ijt} = 2,3 \\ duration_t + 1 & \text{if } duration_t \leq 7 \ and \ d_{ijt} = 2,3 \\ 0 & \text{if } d_{ijt} = 1. \end{cases} \tag{47}$$

Note that the implication of this limit is not that individuals cannot experience the duration effects more than 8 sampling periods. It means that the extra years beyond do not have any marginal effects on individual behavior.

8.6 Marriage market

An individual's marriage market is assumed limited to same race individuals, individuals with the similar age and individuals in the same technology level cohort. The same race marriage market assumption is motivated by computational burden and the low rates of interracial cohabitation and marriage. The similar age assumption is motivated by the fact that almost all individuals get matched with an individual within the similar age range. As well, I cannot separately identify the age and calendar time effects as I mentioned in Section 7.[52]

9 Estimation results

I take the following two steps to estimate the structural model: First, I estimate the non-structural childbirth probability function outside of the main structural model to reduce the computational cost. Then, taking the estimates of the childbirth probability function as given, I estimate the main structural model. The estim-

[52]The NLS 72 is basically a single age cohort in which every sample has the same age. So, the effects of respondents' age and the calendar time on marital behaviors cannot be separately identified. On the other hand, the NLSY 97 has the variation on the respondents' age at a given time. However, I assume that the effects on marital behaviors from the age difference are not significant. This assumption is partly justified because the variation of ages is small in the NLSY 97 cohort as I wrote above. So, it is not such strange to assume similar preference on marital behaviors within respondents within the age variations. Practically, with respect to computation issues as well, I do not want to deal with age and the calendar time as different state variables.

ates are estimated under the assumption that the discount factor is 0.9.[53] The age and duration variables in the following tables are measure in 6-month time periods. In this section, I discuss the parameter estimates, especially, highlighting those associated with the meeting technology.

9.1 Estimates of outside of main structural model

Tables 11 and 12 represent the estimates of the non-structural childbirth probability function, which is parsimoniously specified, and their standard errors.[54] Table 11 gives the coefficients of the logistic childbearing probability function when a woman is single. As expected, with more education, a woman has a smaller probability of giving birth. If an individual's race is black, she is more likely to give birth. Table 12 shows the coefficients of the logistic childbearing probability function when a woman is matched with a man. Overall, if women are matched rather than single, the probability of giving birth is higher. The results in Tables 11 and 12 are consistent with Seitz (2006) and Sheran (2007).

Table 11: Parameters associated with the logistic function for childbirth (single woman)

#	Parameter	NLS 72	NLSY 97 [a]
1	Constant	-3.878	-3.822
		(0.226)	(0.167)
2	Age (time)	-0.036	0.060
		(0.012)	(0.008)
3	Age spline $\leq$ 5 years (school effect)	0.028	0.010
		(0.020)	(0.015)
4	Age spline $\geq$ 10	-0.087	-0.045
		(0.013)	(0.006)
5	Education	-0.596	-0.834
		(0.067)	(0.054)
6	Black	2.108	1.334
		(0.111)	(0.081)

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

9.2 Estimates of main structural model

Tables 13- 18 show the main structural parameter estimates, the estimates of the standard errors with a bootstrap procedure. First, Table 13 provides the coefficient estimates of a flow value of cohabitation relative to being single. Table 14 represents coefficients for marriage bonus.[55] The set of the estimates in Table 13 is in-

[53] It means the annual discount factor is 0.81. When we change the discount factor to a different value, the estimates change. As mentioned in Magnac and Thesmar (2002), I cannot *non-parametrically* identify the discount factor without imposing assumptions.

[54] I can specify the functional form in a more flexible way with more explanatory variables. However, this part is not my main focus of this research.

[55] I can have symmetric explanatory variables between the cohabitation flow value and the marriage bonus specifications. Compared with Brien et al. (2006), they consider the utilities explicitly as a cohabitation flow value and a marriage flow value. Under the interpretation, their model is a multinomial (ordered) choice model. Therefore, it is more natural to have symmetric explanatory variables. I interpret the marriage flow value as an additional marriage bonus. It means an additional utility a couple gets by selecting marriage

Table 12: Parameters associated with the logistic function for childbirth (matched woman)

#	Parameter	NLS 72	NLSY 97 [a]
1	Constant	-2.691	-1.284
		(0.155)	(0.175)
2	Age (time)	0.042	-0.064
		(0.008)	(0.009)
3	Age spline ≤ 5 years (school effect)	0.024	-0.004
		(0.010)	(0.013)
4	Age spline ≥ 10	-0.006	0.018
		(0.003)	(0.004)
5	Education	-0.060	-0.148
		(0.035)	(0.037)
6	Education partner	-0.012	0.011
		(0.015)	(0.008)
7	Match duration	-0.036	0.026
		(0.006)	(0.002)
8	Match duration spline ≤ 2	-0.143	-0.122
		(0.024)	(0.034)
9	Kid from previous relationship	-1.371	0.523
		(0.131)	(0.072)
10	Black	0.180	-0.080
		(0.081)	(0.064)

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

terpreted as an incremental value by cohabitation by changing its corresponding explanatory variable. For example, the coefficient on *Black*, which is -0.087, means that, holding other variables constant, black people get less utility from cohabitation, compared with white people. The interpretation of the coefficient estimates in Table 14 is as follows: If a couple selects marriage, they can get their corresponding marriage bonus part in addition to the corresponding cohabitation flow match value based on their characteristics.[56]

These estimation results are caused by choosing parameter values to match data features. For example, a result that black people get a lower utility flow both from marriage and cohabitation than white people is reflected that the transition rates from single to marriage and cohabitation of black people are lower than white people after controlling for other observable characteristics. This is consistent with Seitz (2006), Sheran (2007), Keane and Wolpin (2010) and Beauchamp et al. (2018).

(See, for example, Drewianka (2006)). There is no reason why I am forced to employ symmetric explanation variables used in a typical multinomial choices model setting. The estimates of the parameters do depend on parameterizations. As explicitly mentioned in Keane and Wolpin (2001), by the nature of structural estimation, functional specification is arbitrary such that we typically decide our functional forms after iterative specification search based on the fit of the model. Note that, I can non-parametrically identify the parameters. Therefore, as long as I can get relatively good fit, specification does not essentially matter. Of course, as Berkovec and Stern (1991), Keane and Wolpin (2010), where there are around 200 parameters, do, I can include more explanatory variables with their corresponding parameters. However, specifications and getting *better* fit is not my main focus.

[56]For example, in the NLS 72 cohort, if a black couple selects cohabitation, they get -2.294 as their constant part and -0.087 for their match. However, if they select marriage, they get additionally 0.190 as their constant part for their match and -0.038 caused by the *Black* term. Therefore, their flow match value of marriage is the sum of these two terms, $-2.294 + 0.190$. As well, they get -0.089 (*Black*) for cohabitation and -0.038 for marriage. In sum, a black married couple gets $-2.294 + 0.190 + (-0.087 - 0.038)$, if we ignore other characteristics.

Table 13: Parameters associated with cohabitation flow match value

#	Parameter	NLS 72 [a]	NLSY 97 [b]
1	Constant	-2.294*	-3.824*
2	Age (time)	-0.366*	-0.091*
3	Age spline ≤ 5 years (school effect)	-1.048*	-2.287*
4	Age spline ≥ 10	0.269*	0.052*
5	Man education (High school)	0.033*	0.048*
6	Man education (College degree)	0.015*	0.321*
7	Woman education (High school)	0.074*	0.024*
8	Woman education (College degree)	-0.011*	-0.036*
9	Education difference	-0.653*	-0.660*
10	Kid from previous relationship	-0.283*	-0.688*
11	Kid from current relationship	0.756*	0.742*
12	Match duration	0.482*	0.113*
13	Match duration spline ≤ 2	-1.532*	-0.104*
14	Black	-0.087*	-0.035*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)
[b] $*$ represents that the standard error is less than 10^{-3}.

Next, consider the estimates of the effects of age on the flow match values are negative ($\sharp 2-4$ and $\sharp 16-18$), but, with the spline modification after ten years, the negative effects are slightly mitigated. Eventually, with the separation costs discussion below, it reflects the positive relationship observed in the data between marriage (cohabitation) and age.

In both cohabitation and marriage, the structural model estimates indicate positive effects of duration of a match. Marriage has a stronger positive effect than cohabitation ($\sharp 12-13$ and $\sharp 24-25$). The positive effects partly reflect more investment or accumulation of match-specific capital for the couple in marriage than in cohabitation. The results of the sign of duration effects are consistent with Brien et al, (2006). However, Sheran (2007) and Beauchamp et al. (2018). (This might be a sign of weak identification for separating age and duration effects with the finite number of observations)

We see that women with more education get lower flow match utility in marriage ($\sharp 8$ and $\sharp 21-22$). This may occur because there might be better employment opportunities for singles or they just prefer being single. It directly affects the value of their outside options of being single. Remember that I do not include a mechanism for how an individual wage is decided. Nor do I include wages as a part of state variables. I assume that the level of education can partly capture the difference of wages. However, note that Eckstein et al. (2019) point out that married women earn 18% more than single women, which might not be true in this research. This is because, in the NLSY 97 cohort, the education effects are more negative. They discuss that controlling changing labor market opportunities and a mother's education matters. The difference of the estimates between Eckstein et al. (2019) and mine comes partly from the fact that I do not control for them.

Thinking that getting higher education is disadvantage for women is misleading ($\sharp 8$ and $\sharp 21-22$). As I will show below, women with higher education get more chances to meet their potential partner as pointed in Eckstein et al. (2019) and and Ge (2011).

Table 14: Parameters associated with marriage bonus

#	Parameter	NLS 72[a]	NLSY97 [b]
15	Constant	0.190*	-0.113*
16	Age (time)	-0.029*	-0.005*
17	Age spline ≤ 5	0.010*	-2.073*
18	Age spline ≥ 10	0.011*	0.005*
19	Man education (High school)	0.108*	0.115*
20	Man education (College degree)	-0.416*	-0.451*
21	Woman education (High school)	-0.040*	-0.135*
22	Woman education (College degree)	-0.277*	-0.334*
23	Education difference	-0.335*	-0.341*
24	Match duration	0.627*	0.1440*
25	Match duration ≤ 2 spline	-0.235*	-0.003*
26	Black	-0.038*	-0.001*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

The estimates indicate that couples with children are less likely to separate and male divorcees are more likely to get remarried than female, as other empirical studies suggests (Guvenen and Rendall (2015)): The estimates of having a kid within the relationship causes a positive effect (♯10), but having a kid outside of the relationship causes a negative effect on the relationship (♯11). This might be because, if people are assumed to gain happiness from emotional satisfaction (Becker (1973)), having children increases emotional attachment and satisfaction between the biological father and mother.However, if children are stepchildren, we do not expect to have an increase in emotional attachment; rather it reduces. This might be because children from previous relationship would be a potential source of conflict within the new relationship, as pointed out in Beauchamp et al. (2018). As written in Section 8, it is assumed that a custodial parent is the mother after their separation. So, the negative coefficient of the existence of a kid from previous relationship indicates that a woman is less likely to get remarried if she has a kid. The magnitude of children from a current relationship and from a previous relationship for a marriage bonus is not strong. This is partly because they affect when a couple forms their relationship either cohabitation or marriage.

Compared with previous research with cohabitation, as shown in Brien et al. (2006), constant terms for the flow values for cohabitation and marriage are negative (♯1 and ♯15). However, the value (magnitude) of the estimates in this research are larger than those in Brien et al. (2006). One possible reason for this might be that this research explicitly includes meeting probabilities which are strictly less than 1. This mechanism causes the flow value of single to further go down. If observed data patterns are similar between Brien et al. (2006) and this research, to explain the similar data pattern, the flow value of a match should also go down.

When we look at the NLS 72 cohort, the value for the separation costs (♯27 and ♯30) are similar to those of Brien et al. (2006). The divorce cost is higher than the cohabitation separation cost. However, when we focus on the NLSY 97 cohort, the difference of the two separation costs is smaller. This comes from the observed data pattern: In the NLSY 97 cohort, more married couples get divorced and go back to single. As discussed in Matouschek and Rasul (2008), the lower divorce cost induce higher turnover of relationship. This is also consistent with the data pattens we observe.

Table 15: Parameters associated with cohabitation separation cost

#	Parameter	NLS72 [a]	NLSY 97 [b]
27	Constant	2.568*	2.547*
28	Existence of kid	0.351*	0.347*
29	Black	0.203*	0.104*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

Table 16: Parameters associated with marriage separation cost

#	Parameter	NLS 72[a]	NLSY 97 [b]
30	Constant	4.681*	2.885*
31	Existence of kid	0.560*	0.568*
32	Black	0.216*	0.070*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

Table 17: Parameters associated with second moments

#	Parameter	NLS 72 [a]	NLSY 97 [b]
33	ρ (Coef AR1)	0.706*	0.697*
34	Standard deviation of cohabitation unobserved heterogeneity	1.559*	1.678*
35	Standard deviation of marriage unobserved heterogeneity	2.345*	1.704*
36	Covariance of cohabitation and marriage unobserved heterogeneity	1.524*	1.328*
37	Standard deviation of of match value when single	2.021*	2.013*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

Table 17 provides the parameters associated with second moments. Compared with Brien et al. (2006), where the specification of variance of the stochastic part of a match value of a first meet is given as $\frac{1}{1-\rho^2}$. Instead, I estimate the term from the data without the restriction (see, the empirical specification described in Section 8.3). The standard deviation of the stochastic part of the flow match value when a couple first meets, which is around 2 in both cohorts (♯37).

Compared with Keane and Wolpin (2010) which reports putting an unobserved type to a marriage utility specification is redundant, the unobserved heterogenous types are important in both cohabitation and marriage (♯34 − 35). This may be because, in this research, I do not control, for example, detailed labor market information or tax system changes for computational reason. As a result, the information may be captured by the cohabitation and marriage specific unobserved heterogeneity parts.

The standard errors are calculated by 20 nonparametric block bootstrap replications, using individual level clusters (Kaplan (2012), and Baum-Snow and Pavan (2012)). I re-estimate the parameters with bootstrapped sample and do the same procedure 20 times. The standard errors are quite small partly because the number of individuals, $N = 6348$, and the time periods, which is 30, used in estimation are quite large.[57]

Table 18: Parameters associated with meeting technology

#	Parameter	NLS 72 [a]	NLSY 97 [b]
38	Constant	0.653*	2.04*
39	Age (time)	-0.012*	-0.013*
40	Age spline $\leq$ 5 years (school effect)	0.688*	0.684*
41	Age spline $\geq$ 10	-0.019*	-0.001*
42	Man education (High school)	0.225*	0.121*
43	Man education (College degree)	0.728*	0.429*
44	Woman education (High school)	0.429*	0.322*
45	Woman education (College degree)	0.672*	0.577*
46	Education difference	-0.147*	-0.341*
47	Black	0.203*	0.204*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

9.3 Meeting technology estimates

Table 18 shows the estimates of the parameters associated with the meeting technology. Remember that we allow the meeting probabilities of individuals to differ from what is implied by purely random meeting models. The meeting probabilities are specified as a product of population frictions (frictions associated with stocks of singles) times a scaling factor depending on a man's and woman's characteristics which is called *meeting efficiency*.

Remember that the specification of the meeting efficiency parameter and the meeting probabilities is

$$\delta_{ijt\kappa}(\boldsymbol{X}) = \frac{exp(\boldsymbol{X}\boldsymbol{\beta}^{\boldsymbol{\alpha}}_{\boldsymbol{\kappa}})}{1+exp(\boldsymbol{X}\boldsymbol{\beta}^{\boldsymbol{\alpha}}_{\boldsymbol{\kappa}})};$$

$$\alpha^{m}_{ijt\kappa} = \left[\lambda\delta_{ijt\kappa}(\boldsymbol{X})z(\Lambda^{Sm}_{t\kappa},\Lambda^{Sw}_{t\kappa})\frac{\Lambda^{Sm}_{it\kappa}}{\Lambda^{Sm}_{t\kappa}}\frac{\Lambda^{Sw}_{jt\kappa}}{\Lambda^{Sw}_{t\kappa}}\right]/\Lambda^{Sm}_{it\kappa}; \tag{48}$$

$$\alpha^{w}_{ijt\kappa} = \left[\lambda\delta_{ijt\kappa}(\boldsymbol{X})z(\Lambda^{Sm}_{t\kappa},\Lambda^{Sw}_{t\kappa})\frac{\Lambda^{Sm}_{it\kappa}}{\Lambda^{Sm}_{t\kappa}}\frac{\Lambda^{Sw}_{jt\kappa}}{\Lambda^{Sw}_{t\kappa}}\right]/\Lambda^{Sw}_{jt\kappa}. \tag{49}$$

Given the underlying logit functional form for meeting efficiency parameters I gave in Section 8, I briefly provide how to interpret Table 18: Holding other things constant, black people are more efficient meeters compared with white people in Table 18 (♯47). If individuals are in a school, they have more opportunity to meet. After 10 years, people suffer a bit less meeting opportunities (♯41).

One of main focuses of this research is how the meeting technology changes during the two cohorts. Remarkably, there is a huge increase in the constant term during the two cohorts (♯38). It means, *on average*, people in the NLSY 97 have more opportunities to meet their potential partner. However, there is less premium of getting higher education associated with getting more meeting opportunities ($\sharp 42-45$).

A point to be emphasized, while it is also interesting, is the following: In interpreting the meeting techno-

[57] Suppose there is no bootstrap error, the variance of estimates by indirect inference is $\mathcal{O}(\frac{1}{\sharp\text{Observations}})$ (Gourieroux et al. (1993)).

logy parameter estimates, we cannot immediately conclude that the bigger constant value in the NLSY 97 is beneficial for people in the NLSY 97. Remember that we need to focus on what happens at equilibrium. Therefore, we need to focus rather on the behavioral marital patterns' changes driven by all factors in the model taking into account *equilibrium effects*, not focus only on each coefficient. It might be the case that people or some types of people in the NLSY 97 might be worse off in the end at equilibrium.[58] It is further discussed in the following counterfactual and policy implication section.

9.4 Goodness of fit

Figures 10 - 17 compare the simulated and actual proportions of stocks of individuals of single, cohabitation and marriage in the NLS 72 cohort and the NLSY 97 cohort, as a function of a year. My model predicts the change of each proportion relatively well. Some parts of the simulated stocks have kink points. They would happen because my functional specification would have non-smoothness at the places with the spline modification. I would get better fit by abandoning the parsimonious specification and employing more sophisticated and flexible functional specification with more explanatory variables.

[58] Note that, somehow, we can guess this conclusion *before running experiences discussed below in Section 10*: Some type of people lose their comparative (relative) advantage associated with the meeting technology because, in the NLSY 97 cohort, everyone enjoys more efficient meeting technology (♯38). The comparative advantage some type of people enjoy in the NLS 72 cohort, for example, people who have high education, is lost in a relative sense. So these type of people may end up with worse equilibrium in the NLSY 97. Even if the model employed is not exactly the same as mine, but we can see the similar conjecture more theoretically in Balasko and Shell (1981) and empirically in Shepard (2017).

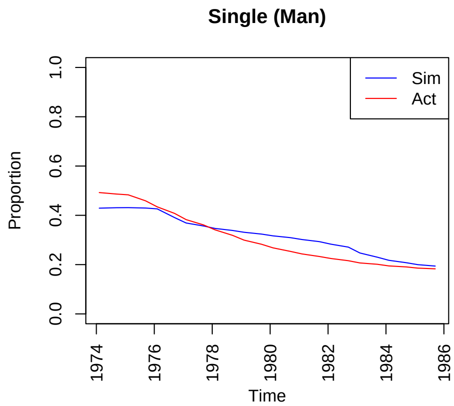

Figure 10: Stock of single men (NLS 72)

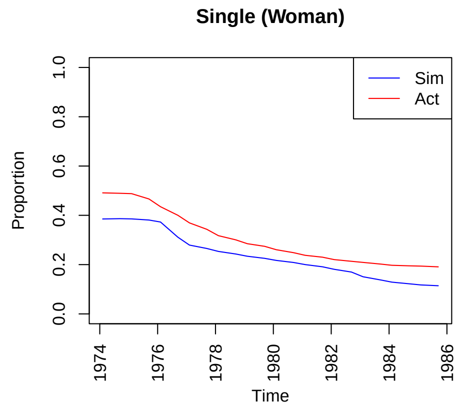

Figure 11: Stock of single women (NLS 72)

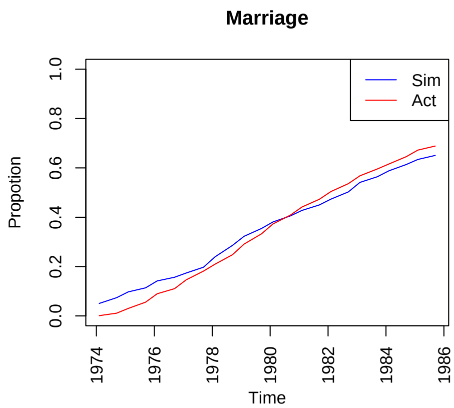

Figure 12: Stock of marriage (NLS 72)

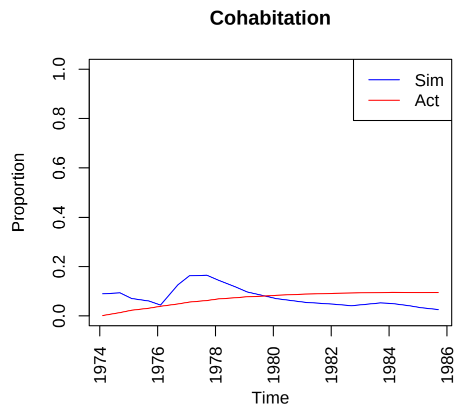

Figure 13: Stock of Cohabitation (NLS 72)

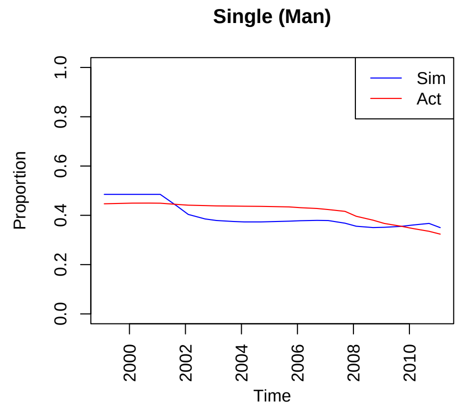

Figure 14: Stock of single men (NLSY 97)

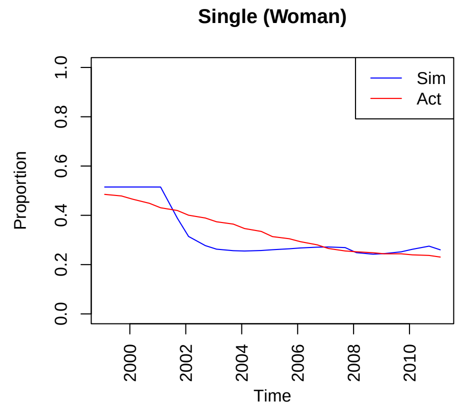

Figure 15: Stock of single women (NLSY 97)

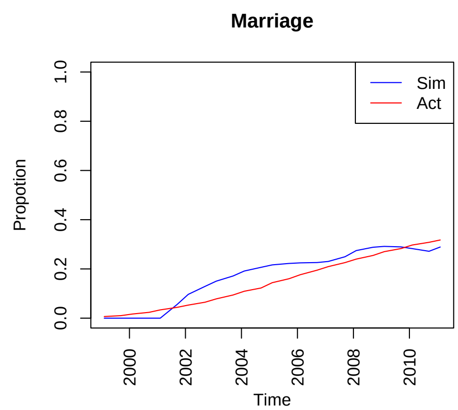

Figure 16: Stock of marriage (NLSY 97)

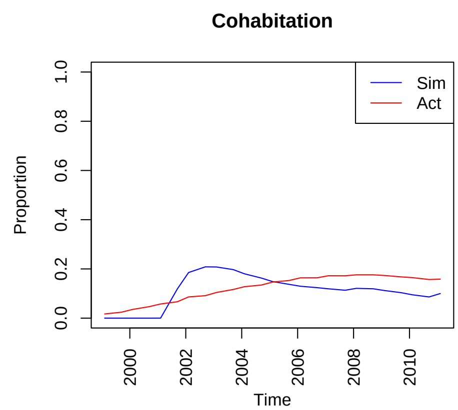

Figure 17: Stock of Cohabitation (NLSY 97)

10 Counterfactuals

Based on the parameter estimates above, the natural question we want to ask is "Does the change of the communication technology truly change the marital behaviors and welfare?" In other words, "How much can the change in the marital behaviors be explained only by the change in the communication technology?" In this section, I try to answer the question.

The main benefit of getting the estimates of the structural parameters is that they are assumed to represent the true underlying model primitives if the model is correctly specified. These parameter estimates are assumed invariant to change in policy environments. The estimates fit the data relatively well shown in Section 9.4 and, therefore, they seem credible to use the counterfactual experiments discussed below.

Guideline

The structural approach taken in this research enables us to evaluate the welfare implication of the change of the communication technology advances. I conduct three types of experiments: First, I evaluate how much the welfare changes during the two cohorts. This is done by comparing the amount of welfare of the NLS 72 cohort and that of the NLSY 97 cohort. Second, I decompose through which paths the marital behaviors' change happens during the NLS 72 cohort and the NLSY 97 cohort. I implement several counterfactuals by changing an exogenous environment to an alternative environment.[59] I perform a series of the counterfactuals by isolating the effects of each particular channel. I use these counterfactual experiments to evaluate the contribution of specific parameters on the observed marital patterns taking into account *equilibrium effects*. I can see how much of the marital behaviors in the NLSY 97 cohort are left to be explained, based on the NLS 72 parameters. Third, I check whether the technology advance is beneficial or not. Focusing on the NLSY 97 cohort, I change the parameters of interest, the parameters associated with the technology advance, which is the meeting technology parameters and the second moments parameters of the NLSY 97 cohort, to its corresponding parameter of the NLS 72 cohort, and everything else remains the same as the parameters estimated by the NLSY 97 data. Then, I simulate the model. This is exactly "What would happen when the way of meetings goes back to 1970s, holding other things the same as 2010s?"

Before implementing counterfactuals, note that some of parameters are estimated outside of the main structural model, such as the childbearing probabilities. I assume that the parameters of the childbearing probabilities are invariant in the following experiments. Following most of bargaining literature, I keep assuming that the bargaining parameter of a woman is take as given, 0.5, and it is also policy invariant.[60]

10.1 Experiment 1: Total welfare comparison

I put an equal weight to each individual in the economy. I calculate the average life time welfare for individuals in both cohorts. The procedure to generate the lifetime welfare begins by fixing values of all structural parameters of interest. For each cohort, the NLS 72 and the NLSY 97, around one million sample histories were generated by simulation. Each sample history is given a value by summing up the discounted flow value of all marital choices till the terminal period. It is -0.633 in the NLS 72 and -0.701 in the NLSY 97 cohort. The results are basically consistent with the data we observe. Remember that almost all individuals stay single during their relatively earlier stage of their lives, and the flow value of single is normalized 0. Any deviation from being single has less effects on his lifetime welfare after several time periods because there is the discount factor.[61]

Specially, focus on a particular type of people of interest. As I mentioned above, in the NLSY 97, people enjoy more efficient meeting technology on average. Therefore, it suggests that highly educated people lose

[59] Under each experiment below with different values of parameters, I recalculate the equilibrium stocks based on the parameter values, using a fixed point algorithm: In Section 5.2, I proved the existence of the equilibrium. This fixed point arugment guarantees to use a direct method of solving the equilibrium; the method of successive approximation (fixed point iteration), originally proposed by Rust (1994). I iteratively solve the model with a guess of stocks and get the stocks calculated by the model until the guess and calculated stocks are close enough.

[60] Note that they are non-negligible limitations. Especially, the fixed bargaining weight might be a strong assumption. This is, first, because, ideally, it should be also an endogenous object in the model, and, second, because, if so, the bargaining weight should change accordingly with the changes of the environment. This part is the part we need to further extend in the future.

[61] In this research, I set the discount factor as 0.9 for each period. The annual discount factor is 0.81. Note that the results depend on the discount factor, the terminal value of the value functions and the bargaining weight. I check how much the results are robust depending on functional form assumptions, a value of the discount factor, a terminal value of the value functions and a bargaining weight. See, Appendix in more detail.

their relative advantage associated with the meeting technology. Table 19 shows the amount of welfare change of highly educated people with their college degree. An individual with their collage degree reduces their welfare by $\frac{-0.042+0.036}{0.036} \cdot 100 = -16.6\%$ on average. The magnitude of the change is bigger than than the average life time welfare $\frac{-0.701+0.633}{0.633} \cdot 100 = -11.0\%$

Table 19: Welfare comparison: Individual and Collage graduate

	NLS 72	NLSY 97
Total welfare	−0.633	−0.701
Collage graduate welfare	−0.036	−0.042

I now turn to a consideration of of dispersion in lifetime welfare outcomes, which is shown in Figures 18 and 19. Interestingly, these two distributions do not significantly differ, even if their marital patters are considerably different shown in Figure 2 - 9. One possible explanation is the following: At their early stage of their sampling periods, their marital behaviors are not so different between the NLS 72 cohort and the NLSY 97 cohort. Even there are significant changes at their late stage, the discount factor mitigates the lifetime welfare difference caused by their different marital behaviors.

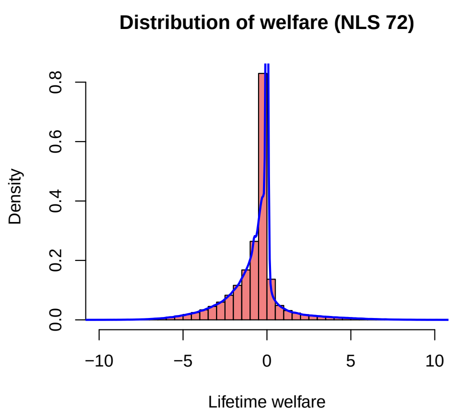

Figure 18: Welfare distribution NLS 72

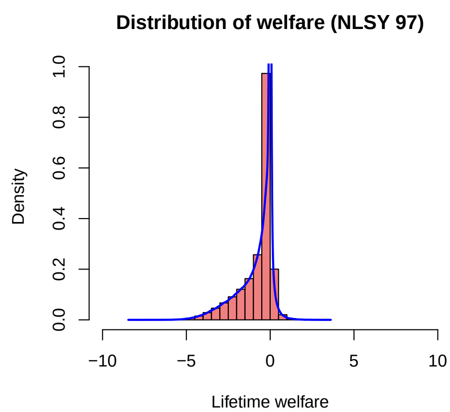

Figure 19: Welfare distribution NLSY 97

10.2 Experiment 2: Equilibrium impact and decomposition

In this section, I compare the simulated equilibrium marital patterns calculated by the NLSY 97 model with that calculated from the NLS 72 setting with one-by-one modification. First, we re-calculate the NLS 72 equilibrium marital patterns using the NLSY 97's date-0 individual distribution, while all other things remain the same as the NLS 72 setting. By doing this, we can know how much the change of the compositions of the economy contributes the change of marital behaviors during two cohorts. Then, I start allowing a part of the parameters to vary in the model: First, I change the meeting technology and second moments part from those of the NLS 72 estimates to those of the NLSY 97 estimates. We know how much the advance in the technology affects an individual's marital behavior. To isolate the effects of the changes and evaluate each contribution, I change the parameters one-by-one. My goal is to know which paths contribute crucially to the marital behavior changes during the NLS 72 and NLSY 97 cohorts.

10.2.1 Stage 1: Exogenous date-0 distribution change, Stage 2: Meeting technology change and Stage 3: Second moments change

Figure 20-23 shows the difference between the simulated marital behavior in the NLSY 97 cohort and the simulated marital behavior in the NLS 72 cohort with the initial distribution, the meet technology and the second moments modification respectively.[62]

The interesting point I emphasize is that, in the date-0 and the meeting technology experiments, there is less deviations from the original NLS 72 marital pattern, which makes sense. Either the change of the date-0 distribution or the meeting technology induces the change in meeting probabilities. Suppose, Stage 1 and/or 2 experiments induce meeting probabilities to go up. There are two directions possible for people in the economy as their response. People are more likely to meet each other, which induces more matches, but, at the same time, the value of single also goes up because their future meeting probabilities go up. The two effects cancel each other out, resulting in a relatively small overall effect of the marital pattern. We can see similar discussion in different contexts in Santos and Weiss (2016) and Blasutto (2024).[63]

The second moments change is more important to determine the marital behaviors' change. This part is less paid attention to empirically in previous literature. However, the results with the larger changes are theoretically consistent with Weizman pandora's box argument (Weizman (1979)) in the sense that the changes of the second moments induce the changes of optimal stopping rule.

[62] I replace the childbearing probabilities also with those of the NLSY 97 cohort.

[63] This also indicates weak identification of parameters associated with meeting probabilities because the change of them does not bring big changes as a result because the two effects are cancelling out each other.

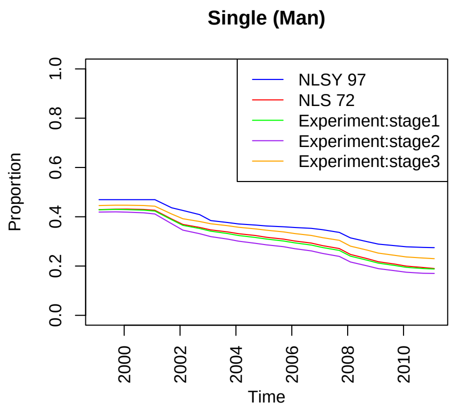

Figure 20: Stock of single men (Stage 1, 2 and 3)

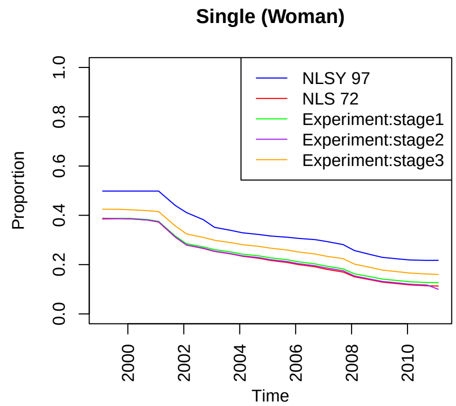

Figure 21: Stock of single women (Stage 1, 2 and 3)

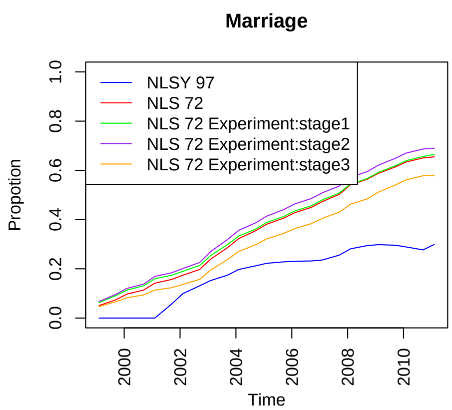

Figure 22: Stock of marriage (Stage 1, 2 and 3)

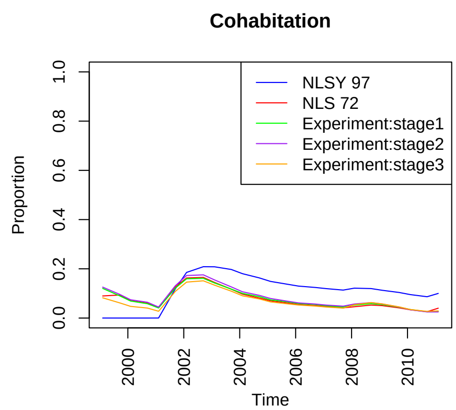

Figure 23: Stock of Cohabitation (Stage 1, 2 and 3)

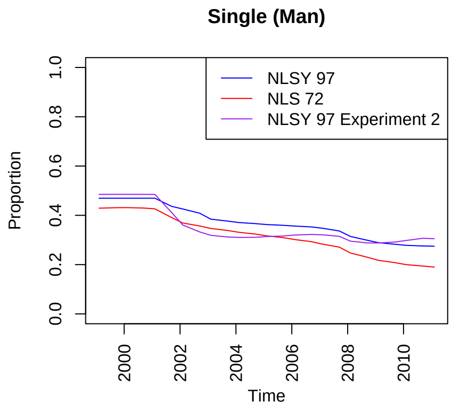

Figure 24: Stock of single men (Experiment 3)

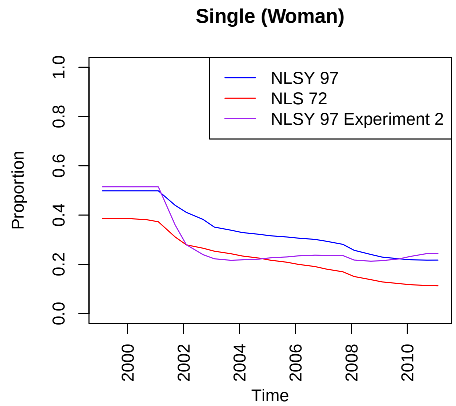

Figure 25: Stock of single women (Experiment 3)

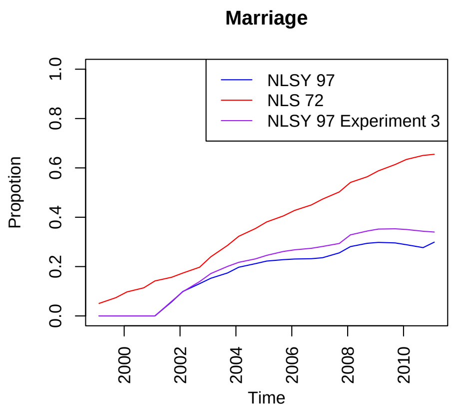

Figure 26: Stock of marriage (Experiment 3)

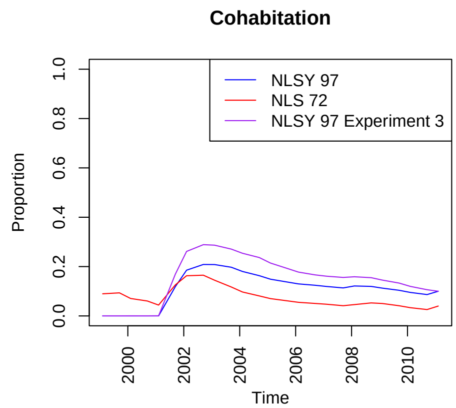

Figure 27: Stock of Cohabitation (Experiment 3)

The decomposition results indicate that the determinant factors affecting the marital behaviors during the NLS 72 cohort and the NLSY 97 cohort are rather the change of the preference (cohabitation match surplus and marriage bonus) and the separation costs.

10.3 Experiment 3: Is the change of the meeting technology beneficial ?

In this section, by using the structural estimates, I examine whether the technology advance is beneficial or not. First, I show how marital behavior changes when only the parameters associated with the technology advance (the meeting technology parameters and the second moments parameters) get back to the NLS 72 situation and everything else remains the same as the NLSY 97 situation. Figures 24-27 show the changes in the marital behaviors. Table 20 provides the individual welfare on average under Experiment 3. Table 20 provides the individual welfare on average under Experiment 3. Note that the series of the experiments shows the inefficiency which the NLSY 97 people suffer does not come from the Hosios-related search friction argument (Hosios (1990)). This is because, as I showed above, the change of the meeting technology does not have huge impacts on marital behaviors. Rather, the inefficiency comes from the change of the second moments.

Table 20: Welfare comparison: NLSY 97 vs Experiment 3

	NLSY 97	Experiment 3
Total welfare	-0.701	-0.962

11 Conclusion

This research provides a non-stationary two sided search market equilibrium model, which can be widely applicable to other contexts also. Using the model, I evaluate the quantitative effects of the communication technology advance on marital behaviors including cohabitation.

I estimate the structural model with the two data sets, NLS 72 and the NLSY 97. The NLS 72 is assumed to represent the cohort before the communication technology advance and the NLSY 97 is assumed to represent the cohort after the communication technology advance. The estimation results reveal that people in the NLSY 97 cohort enjoy more efficient meeting technology.

I conduct several experiments. The series of experiments reveals that people in the NLSY 97 cohort is worse off on average at equilibrium. A determinant factor driving the marital behavior changes is not the meeting technology change but the change of the second moments associated with the stochastic part of the match surplus. I also show that, if we get back to the NLS 72 situation where our potential partner is more restricted, we would be worse off.

There are potential extensions for future research: First, theoretically providing uniqueness of the equilibrium is a big step. Second, we can consider the bargaining weight also as an endogenous equilibrium object. In addition, identifying the bargaining weight from data is another way to go. I leave these issues for future research.

References

- Akın, Ş. N., Platt, B. C. (2016). Accounting for age in marital search decisions. European Economic Review, 85, 245-271.
- Alan, S. (2006). Entry costs and stock market participation over the life cycle. Review of Economic Dynamics, 9(4), 588-611.
- Altonji, J. G., Segal, L. M. (1996). Small-sample bias in GMM estimation of covariance structures. Journal of Business Economic Statistics, 14(3), 353-366.
- Balasko, Y., Shell, K. (1981). The overlapping-generations model. II. The case of pure exchange with money. Journal of Economic Theory, 24(1), 112-142.
- Baum-Snow, N., Pavan, R. (2012). Understanding the city size wage gap. The Review of economic studies, 79(1), 88-127.
- Beauchamp, A., Sanzenbacher, G., Seitz, S., Skira, M. M. (2018). Single moms and deadbeat dads: The role of earnings, marriage market conditions, and preference heterogeneity. International Economic Review, 59(1), 191-232.
- Becker, G. S. (1973). A theory of marriage: Part I. Journal of Political economy, 81(4), 813-846.
- Berkovec, J., Stern, S. (1991). Job exit behavior of older men. Econometrica: Journal of the Econometric Society, 189-210.
- Blasutto, F. (2023). Cohabitation VS. Marriage: Mating Strategies by Education in the Usa. Journal of the European Economic Association, jvad065.
- Brien, M. J., Lillard, L. A., Stern, S. (2006). Cohabitation, marriage, and divorce in a model of match quality. International Economic Review, 47(2), 451-494.
- Byrne, D., Goeree, M. S., Hiedemann, B., Stern, S. (2009). Formal home health care, informal care, and family decision making. International Economic Review, 50(4), 1205-1242.
- Chiappori, P. A. (1988). Rational household labor supply. Econometrica: Journal of the Econometric Society, 63-90.
- Choo, E., Siow, A. (2006). Who marries whom and why. Journal of political Economy, 114(1), 175-201.
- Collard-Wexler, A. (2013). Demand fluctuations in the ready-mix concrete industry. Econometrica, 81(3), 1003-1037.
- Debreu, Gerard, Theory of Value. Yale University Press, New Haven, CT, 1959.
- Del Boca, D., Flinn, C. (2012). Endogenous household interaction. Journal of Econometrics, 166(1), 49-65.

- Del Boca, D., Flinn, C. J. (2014). Household behavior and the marriage market. Journal of Economic Theory, 150, 515-550.
- Drewianka, S. (2006). A generalized model of commitment. Mathematical Social Sciences, 52(3), 233-251.
- Dubey, P., Kaneko, M. (1984). Information patterns and Nash equilibria in extensive games: 1. Mathematical Social Sciences, 8(2), 111-139.
- Eckstein, Z., Keane, M., Lifshitz, O. (2019). Career and family decisions: Cohorts born 1935–1975. Econometrica, 87(1), 217-253.
- Fiorini, M., Keane, M. P. (2014). How the allocation of children's time affects cognitive and noncognitive development. Journal of Labor Economics, 32(4), 787-836.
- Flabbi, L., Leonardi, M. (2010). Sources of earnings inequality: Estimates from an on-the-job search model of the US labor market. European Economic Review, 54(6), 832-854.
- Flabbi, L., Moro, A. (2012). The effect of job flexibility on female labor market outcomes: Estimates from a search and bargaining model. Journal of Econometrics, 168(1), 81-95.
- Flinn, C. J. (2006). Minimum wage effects on labor market outcomes under search, matching, and endogenous contact rates. Econometrica, 74(4), 1013-1062.
- Flinn, C., Heckman, J. (1982). New methods for analyzing structural models of labor force dynamics. Journal of Econometrics, 18(1), 115-168.
- Friedberg, L., Stern, S. (2014). Marriage, divorce, and asymmetric information. International Economic Review, 55(4), 1155-1199.
- Ge, S. (2011). Women's college decisions: how much does marriage matter?. Journal of Labor Economics, 29(4), 773-818.
- Goeree, M. S. (2008). Limited information and advertising in the US personal computer industry. Econometrica, 76(5), 1017-1074.
- Gourieroux, C., Monfort, A., Renault, E. (1993). Indirect inference. Journal of applied econometrics, 8(S1), S85-S118.
- Gourieroux, C., Monfort, A., Renault, E., Trognon, A. (1987). Generalised residuals. Journal of econometrics, 34(1-2), 5-32.
- Goussé, M., Jacquemet, N., Robin, J. M. (2017). Marriage, labor supply, and home production. Econometrica, 85(6), 1873-1919.
- Guvenen, F., Rendall, M. (2015). Women's emancipation through education: A macroeconomic analysis. Review of Economic Dynamics, 18(4), 931-956.

- Guvenen, F., Smith, A. A. (2014). Inferring labor income risk and partial insurance from economic choices. Econometrica, 82(6), 2085-2129.
- Hall, G., Rust, J. (2021). Estimation of endogenously sampled time series: The case of commodity price speculation in the steel market. Journal of econometrics, 222(1), 219-243.
- Hellerstein, J. K., Imbens, G. W. (1999). Imposing moment restrictions from auxiliary data by weighting. Review of Economics and Statistics, 81(1), 1-14.
- Hitsch, G. J., Hortaçsu, A., Ariely, D. (2010). Matching and sorting in online dating. American Economic Review, 100(1), 130-163.
- Hopenhayn, H. A., Prescott, E. C. (1992). Stochastic monotonicity and stationary distributions for dynamic economies. Econometrica: Journal of the Econometric Society, 1387-1406.
- Hosios, A., (1990). On the efficiency of matching and related models of search and unemployment, Review of economic studies, 57, 279-98.
- Kaplan, G. (2012). Inequality and the life cycle. Quantitative Economics, 3(3), 471-525.
- Keane, M. P., Sauer, R. M. (2010). A computationally practical simulation estimation algorithm for dynamic panel data models with unobserved endogenous state variables. International Economic Review, 51(4), 925-958.
- Keane, M. P., Wolpin, K. I. (2001). The effect of parental transfers and borrowing constraints on educational attainment. International Economic Review, 42(4), 1051-1103.
- Keane, M. P., Wolpin, K. I. (2010). The role of labor and marriage markets, preference heterogeneity, and the welfare system in the life cycle decisions of black, hispanic, and white women. International Economic Review, 51(3), 851-892.
- Lavy, V., Palumbo, M., Stern, S. (1998). Simulation of multinomial probit probabilities and imputation of missing data. Advances in Econometrics, 13, 145-180.
- Lee, J. (2020). Estimating the benefits and costs of forming business partnerships. The RAND Journal of Economics, 51(2), 531-562.
- Lewis, A. S., Overton, M. L. (2013). Nonsmooth optimization via quasi-Newton methods. Mathematical Programming, 141, 135-163.
- Lillard, L. A., Brien, M. J., Waite, L. J. (1995). Premarital cohabitation and subsequent marital dissolution: A matter of self-selection?. Demography, 32(3), 437-457.
- Magnac, T., Thesmar, D. (2002). Identifying dynamic discrete decision processes. Econometrica, 70(2), 801-816.
- Manea, M. (2017). Bargaining in dynamic markets. Games and Economic Behavior, 104, 59-77.

- Manski, C. F., Lerman, S. R. (1977). The estimation of choice probabilities from choice based samples. Econometrica: Journal of the Econometric Society, 1977-1988.
- Mas-Colell, A., Whinston, M. D., Green, J. R. (1995). Microeconomic theory (Vol. 1). New York: Oxford university press.
- Matouschek, N., Rasul, I. (2008). The economics of the marriage contract: Theories and evidence. The Journal of Law and Economics, 51(1), 59-110.
- Milgrom, P., Shannon, C. (1994). Monotone comparative statics. Econometrica: Journal of the Econometric Society, 157-180
- Nevo, A. (2003). Using weights to adjust for sample selection when auxiliary information is available. Journal of Business Economic Statistics, 21(1), 43-52.
- Pakes, A. (1984). Patents as options: Some estimates of the value of holding European patent stocks (No. w1340). National Bureau of Economic Research.
- Petrongolo, B., Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. Journal of Economic literature, 39(2), 390-431.
- Pew Research Center. (2007). Mobile Fact Sheet. https://www.pewresearch.org/internet/fact-sheet/mobile/
- Rogerson, R., Shimer, R., Wright, R. (2005). Search-theoretic models of the labor market: A survey. Journal of economic literature, 43(4), 959-988.
- Rosenfeld, M. J., Thomas, R. J., Hausen, S. (2019). Disintermediating your friends: How online dating in the United States displaces other ways of meeting. Proceedings of the National Academy of Sciences, 116(36), 17753-17758.
- Rubinstein, A., Wolinsky, A. (1985). Equilibrium in a market with sequential bargaining. Econometrica: Journal of the Econometric Society, 1133-1150.
- Rust, J., Hall, G. (2003). Middlemen versus market makers: A theory of competitive exchange. Journal of Political Economy, 111(2), 353-403.
- Santos, C., Weiss, D. (2016). "Why not settle down already?" a quantitative analysis of the delay in marriage. International Economic Review, 57(2), 425-452.
- Seitz, S. (2009). Accounting for racial differences in marriage and employment. Journal of Labor Economics, 27(3), 385-437.
- Shapley, L. S., Shubik, M. (1971). The assignment game I: The core. International Journal of game theory, 1, 111-130.
- Shephard, A. (2017). Equilibrium search and tax credit reform. International Economic Review, 58(4), 1047-1088.

- Shephard, A. (2019). Marriage market dynamics, gender, and the age gap.
- Sheran, M. (2007). The career and family choices of women: A dynamic analysis of labor force participation, schooling, marriage, and fertility decisions. Review of Economic Dynamics, 10(3), 367-399.
- Shimer, R., Smith, L. (2000). Assortative matching and search. Econometrica, 68(2), 343-369.
- Stokey, N. L. (1989). Recursive methods in economic dynamics. Harvard University Press.
- Tourangeau, R. (1987). The National Longitudinal Study of the High School Class of 1972 (NLS-72) Fifth Follow-Up (1986) Data File User's Manual. Contractor Report.
- Van der Klaauw, W., Wolpin, K. I. (2008). Social security and the retirement and savings behavior of low-income households. Journal of econometrics, 145(1-2), 21-42.
- Weitzman, M.L., (1979). Optimal search for the best alternative. Econometrica 47 (3), 641–654.
- Wolpin, K. I. (1992). The determinants of black-white differences in early employment careers: Search, layoffs, quits, and endogenous wage growth. Journal of Political Economy, 100(3), 535-560.
- Wong, L. Y. (2003). Structural estimation of marriage models. Journal of Labor Economics, 21(3), 699-727.