Supplement to "Sufficient statistics for frictional wage dispersion and growth"

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Appendix B: Description of auxiliary parameters used for estimation

Let w denote log real wages. The auxiliary parameters will be measured in two samples: The main data sample will be the 1979 version of the National Longitudinal Survey of Youth (NLSY79) and we will supplement this with data from the SIPP (see Section 3.1 for a description of the SIPP data set). The NLSY79 consists of a representative sample of individuals born in 1957–1964, where each birth year is sampled equally.

All auxiliary parameters are calculated for workers with between 2–25 years of potential experience (age − years-of-schooling − 6). For the auxiliary parameters, we do not make any age other requirements (i.e., we drop the prime-aged worker selection from Section 3.1).

1. *Transition Rates*: Measured in the NLSY79. Weekly job histories have been constructed for each worker.

   (a) $\Pr(E \to U)$: For the population of workers that are employed in the first week of January in year X: What is the probability that they are observed in displacement in year X?

   (b) $\Pr(E \to E)$: For the population of workers that are employed in the first week of January in year X: What is the probability that they are observed working for a new employer in the first week of January in year $X+1$ without an intervening displacement spell. (Our definition of $E \to U$ and $E \to E$ are mutually exclusive. So $\Pr(E \to U) + \Pr(E \to E) + \Pr(\text{stayer}) = 1$).

   (c) $\Pr(U \to E)$: For the population of workers that are employed in the first week of January in year $X-1$ and then unemployed in the first week of January in year X: What is the probability that they are observed employed in the first week of January in year $X+1$?

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2. *Wage distribution and regression*: Moments on wages are taken from the NLSY79 data set. The wages are measured annually from 1979 through 1994 and bi-annually after that. In order to account for potential bias from the sampling, we weigh wage observations so that each experience year bin has the same weight in calculating the auxiliary parameters based on wages.
   (a) *Mincer wage regression (ζ_0, ζ_1, ζ_2)*: Regress $w_{it} = \zeta_0 + \zeta_1 \exp_{it} + \zeta_2 \exp_{it}^2 + \varepsilon_{it}$.
   (b) σ_w: Standard deviation of wages.
3. *Wage dynamics across JUJ spells*: These auxiliary parameters will be calculated from the SIPP, which is a series of short monthly panels; see Section 3.1 for more details on sample selection.
   (a) $\Pr(w_x^{\text{post}} < w_x^{\text{pre}})$: The fraction earning lower wages after displacement compared to before.
   (b) $\text{corr}(w_x^{\text{pre}}, w_x^{\text{post}})$: Correlation between the wages of the pre- and post-displacement spell jobs.
   (c) $\text{Corr}(w_t, w_{t+0.33} | \text{within match})$: The within-match wage correlation for the SIPP cross-sectional worker sample; see Sections 3.1 and 3.3.

Appendix C: Alternate derivation of frictional wage-growth statistic using Burdett–Mortensen steady-state accounting

Now consider an economy populated with heterogeneous, infinitely-lived workers. As before, let worker heterogeneity be described by discrete types, $x \in \mathcal{X}$. Unemployed workers receive job offers at rate λ_x^u. If a worker accepts a job, they receive job offers at rate λ_x^e while employed. Separations occur at rate δ_x. The wage of a job offer is drawn from a well-behaved job offer distribution function $w_x \sim F_x(w)$.[1] We assume that the wage is an order statistic of the worker's value of a job, hence a worker prefers any job that pays a higher wage. We assume that the economy is in steady state.

We now show that the fraction of workers who suffer a wage loss after displacement identifies $\kappa_x = \lambda_x^e / \delta_x$, independent of the wage-offer distribution. Let $G_x(w)$ be the wage-earned distribution of employed workers of type x. In other words, the cross-sectional distribution of wages of workers of type x. We can relate the probability that a worker suffers a wage loss to the wage-offer and wage-earned distributions

$$\Pr_x\big(w^{\text{post}} < w^{\text{pre}}\big) = \int F_x(w)\, dG_x(w),$$

where w^{pre} and w^{post} are the pre-displacement and post-displacement spell wage, respectively.

[1]The fact that we assume $F_x(w)$ is exogenous here is not restrictive. We follow Hornstein, Krusell, and Violante (2011) in arguing that allowing the wage-offer distribution to be determined in equilibrium has no impact on our results. Their argument can easily be amended to allow for on-the-job search.

We use steady-state flow accounting (à la Burdett and Mortensen (1998)) to find an analytical relationship between $F_x(w)$ and $G_x(w)$. In steady state, we can equate the flow of workers into and out of jobs with wage less than w and the flow of workers into and out of unemployment (u_x),

$$\begin{aligned} \lambda_x^u F_x(w) u_x &= \big(\delta_x + \lambda_x^e\big(1 - F_x(w)\big)\big) G_x(w)(1 - u_x), \\ \lambda_x^u u_x &= \delta_x (1 - u_x). \end{aligned} \tag{C.1}$$

These equations can be used to solve for $F_x(w)$ in terms of $G_x(w)$,

$$F_x(w) = \frac{(1 + \kappa_x) G_x(w)}{1 + \kappa_x G_x(w)}, \tag{C.2}$$

where $\kappa_x = \lambda_x^e / \delta_x$.

The probability of earning a lower wage after displacement is then

$$\begin{aligned} \Pr_x\big(w^{\text{post}} < w^{\text{pre}}\big) &= \int F_x(w)\, dG_x(w) \\ &= \int \frac{(1 + \kappa_x) G_x(w)}{1 + \kappa_x G_x(w)}\, dG_x(w) \\ &= (1 + \kappa_x) \int_0^1 \frac{z}{1 + \kappa_x z}\, dz \\ &= 1 - \frac{(1 + \kappa_x) \ln(1 + \kappa_x) - \kappa_x}{\kappa_x^2}. \end{aligned}$$

The first equality is found by substituting $F_x(w)$ from equation (C.2) and the second equality by doing a change of variables $z = G_x(w)$. Importantly, notice that the fraction of workers earning lower wages after displacement depends only on κ_x and is independent of the wage-offer distribution. This is because the probability $\Pr_x(w^{\text{post}} < w^{\text{pre}})$ depends only on the order statistic of w^{pre} and not the actual value of the wage.

C.1 *Extension: Reallocation shocks*

Our results can be extended to the case where workers are exposed to a reallocation shock. A reallocation shock is a shock where employed workers receive a job offer while employed that they must accept or go into displacement. Adding a reallocation shock λ_x^d, equation (C.1) becomes

$$\begin{aligned} &\lambda_x^u F_x(w) u_x + \lambda_x^d F_x(w)\big[1 - G_x(w)\big](1 - u_x) \\ &\quad = \big(\delta_x + \lambda_x^e\big[1 - F_x(w)\big] + \lambda_x^d\big[1 - F_x(w)\big]\big)(1 - u_x) G_x(w). \end{aligned}$$

Solving for $F_x(w)$,

$$F_x(w) = \frac{(1 + \tilde{\kappa}_x) G_x(w)}{1 + \tilde{\kappa}_x G_x(w)},$$

where $\tilde{\kappa}_x = \frac{\lambda_x^e}{\delta_x + \lambda_x^d}$. In terms of the job wage ladder, a reallocation shock functions in a similar way to a job destruction shock, in that workers lose their search capital.

The probability of a wage loss is similar to the model without reallocation shocks

$$\Pr_x\left(w^{\text{post}} < w^{\text{pre}}\right) = 1 - \frac{(1+\tilde{\kappa}_x)\ln(1+\tilde{\kappa}_x) - \tilde{\kappa}_x}{\tilde{\kappa}_x^2}.$$

Appendix D: Model appendix

D.1 *Value functions*

The value function for a unemployed worker with human capital (α, k) is given by

$$\begin{aligned}\left(\lambda^u + \rho\right)U(\alpha, k) &= u_0(\alpha, k) \\ &\quad + \lambda^u \int W(\alpha, x, k) - U(\alpha, k)\, dF(x),\end{aligned}$$

where $W(\alpha, p, k)$ is the value function of a worker with human capital (α, k) working at a firm with productivity p.

The value function for an α-type employed worker with human capital $k < K$, who is working at a firm with productivity p and nonpecuniary aspect z is given by

$$\begin{aligned}&\left(\lambda^d + \lambda_h + \delta + \rho\right)W(\alpha, p, k, z) \\ &\quad = u_1(\alpha, p, k, z) + \lambda^d \int\int W(\alpha, x, k, y)\, dF(x, y) \\ &\qquad + \lambda^e \int\int \max\left[W(\alpha, p, k, z), W(\alpha, x, k, y)\right] - W(\alpha, p, k, z)\, dF(x, y) \\ &\qquad + \lambda_h W(\alpha, p, k+1, z) \\ &\qquad + \delta U(\alpha, k).\end{aligned}$$

When $k = K$ learning-by-doing human capital does not increase anymore, so $\lambda_h = 0$ in this state.

D.2 *Solution of the model*

We estimate the model by indirect inference, Gourieroux, Monfort, and Renault (1993). In order to do this, we simulate 5,000,000 worker histories from time 0 to time 25 (recall that the model is cast in continuous time with the unit of time being a year). All the workers draw an ability $\alpha \sim \mathrm{N}(\mu, \sigma_\alpha)$ and start out being unemployed. For an unemployed worker, the current spell length is given by T_{λ^u}, where T_{λ^u} is drawn from an exponential distribution with parameter λ^u. In the next spell, he is employed drawing a productivity level, $p \sim \Gamma(\sigma_p)$, and a nonpecuniary level, $z \sim \Gamma(\sigma_z)$. The spell length of the employed worker is given as $T_{\text{Empl.}} = \min(T_\delta, T_{\lambda_1}, T_{\lambda^d}, T_{\lambda_h})$, which are respectively the time until arrival of a job destruction shock, a job offer, an involuntary job offer shock, and a human capital shock. Again, these are all drawn from exponential distributions. If, for example, $T_{\text{Empl.}} = T_\delta$ then the worker becomes unemployed. If $T_{\text{Empl.}} = T_{\lambda_1}$, then the worker makes a JJ transition if the new drawn productivity and nonpecuniary aspect yields a higher utility than the current job. If $T_{\text{Empl.}} = T_{\lambda^d}$, the worker is forced to move to a new firm with a random drawn productivity. If $T_{\text{Empl.}} = T_{\lambda_h}$, the human capital of the worker increases with 1 if $k < K$.

Simulated data. Having simulated the worker histories, the wage is given by $w = \alpha + p + \beta_1 k + \beta_2 k^2 + \epsilon$ with $\epsilon \sim N(0, \sigma_\epsilon)$. We assume that each time we measure the wage we will draw a new measurement error.

Discretization. In the solution, we discretize worker ability, the nonpecuniary aspect, and productivity. We chose the grid as equally spaced points on the CDFs. Thus, the grid size is determined $\frac{1}{I+1}$ where I is the number of intervals. In the simulation $I = 999$, such that the grid points ranges from 0.001 to 0.999. These grid types are mapped into ability, non-pecuniary aspect, and productivity space by the use of the inverse of the CDFs of the normal and and exponential distributions, respectively.

Appendix E: Data appendix

E.1 *Descriptive statistics*

Table E.1. Survey of income and program participation: descriptive statistics.

	JU	JUJ		CS
		Pre-Disp	Post-Disp	
Age	38.79 (8.42)	38.34 (8.25)		38.41 (8.71)
Male	0.553 (0.497)	0.592 (0.492)		0.533 (0.499)
White	0.642 (0.479)	0.667 (0.472)		0.694 (0.461)
HS	0.295 (0.456)	0.290 (0.454)		0.305 (0.460)
Some college	0.350 (0.477)	0.358 (0.480)		0.349 (0.477)
College	0.215 (0.411)	0.226 (0.418)		0.250 (0.433)
Tenure	4.664 (5.715)	4.459 (5.453)	0 –	6.027 (7.068)
Wage	3196.37 (2189.59)	3386.19 (2270.00)	3141.64 (3001.60)	3473.00 (2311.61)
Observations	4241	1914		90,995

Note: The standard deviation is reported in parentheses. The monthly wage is deflated to 2010 dollars using the CPI. JU = job to displacement transition, JUJ = job to displacement to job transition, CS = cross-section of employed workers. The post-displacement wage is measured the first month the worker is observed working at the firm, and hence, the tenure is zero by construction. The SIPP sample includes the 1996, 2001, 2004, and 2008 panels; see Section 3.1 for more details on the sample construction. *Source: Survey of Income and Program Participation.*

Table E.2. Current population survey descriptive statistics: full DWS sample (1984–2016) and DWS-ORG comparison sample (1996–2016).

	Full Sample		DWS-ORG Comparison Sample			
	DWS JUJ			DWS JUJ		
	Pre-Disp	Post-Disp	DWS JU	Pre-Disp	Post-Disp	ORG
Age	37.41		39.28	38.69		39.23
	(8.02)		(8.41)	(8.12)		(8.45)
Male	0.562		0.523	0.544		0.530
	(0.496)		(0.500)	(0.498)		(0.499)
White	0.866		0.827	0.844		0.830
	(0.341)		(0.379)	(0.363)		(0.375)
High school	0.274		0.349	0.353		0.317
	(0.446)		(0.477)	(0.478)		(0.465)
Some college	0.409		0.310	0.279		0.296
	(0.492)		(0.462)	(0.449)		(0.456)
College	0.228		0.248	0.290		0.309
	(0.420)		(0.432)	(0.454)		(0.462)
Tenure	5.200	1.011	5.815	5.576	1.004	6.926
	(5.859)	(0.601)	(6.555)	(6.079)	(0.601)	(6.998)
Wage	779.75	720.20	795.73	838.21	783.99	857.03
	(425.25)	(407.78)	(458.55)	(468.33)	(456.17)	(476.18)
Observations	2241		1886	987		58,100

Note: The standard deviation is reported in parentheses. The weekly wage is deflated to 2010 dollars using the CPI. JU = job to displacement transition, JUJ = job to displacement to job transition. Full Sample includes all CPS-DWS surveys from 1984–2016. The DWS-ORG comparison sample is constructed to compare workers the CPS-DWS and the CPS-ORG. The comparison sample includes the CPS-DWS sample from 1998–2016 and the CPS-ORG surveys from 2 years earlier (1996–2014); See Section 3.1 for more details on the sample construction. *Source: Current Population Survey*.

Appendix F: Additional results and robustness checks

F.1 *Checking identifying assumptions for different subsamples*

TABLE F.1. Comparing pre-displacement wages of displaced workers to the cross-section for different subsamples.

	SIPP		CPS	
Subpopulation	JUJ	JU	JUJ	JU
Men	−0.00972 (0.0139)	−0.0814 (0.0135)	−0.00948 (0.0186)	−0.0466 (0.0203)
Women	0.0326 (0.0160)	−0.0426 (0.0135)	0.0267 (0.0192)	−0.0707 (0.0192)
HS or less	−0.0151 (0.0152)	−0.0498 (0.0133)	−0.00889 (0.0195)	−0.0407 (0.0199)
Some college	0.0107 (0.0176)	−0.0803 (0.0164)	0.0276 (0.0251)	−0.0815 (0.0237)
College graduate	0.0357 (0.0240)	−0.0585 (0.0229)	0.00198 (0.0263)	−0.0595 (0.0329)
Exper $<$ 10 years	0.0323 (0.0242)	−0.0396 (0.0241)	−0.0430 (0.0295)	−0.0414 (0.0352)
Exper 10–20 years	0.0164 (0.0174)	−0.0742 (0.0162)	0.00849 (0.0229)	−0.0624 (0.0241)
Exper 20–30 years	−0.0173 (0.0184)	−0.0638 (0.0167)	0.0287 (0.0236)	−0.0902 (0.0249)
Exper $>$ 30 years	0.00854 (0.0302)	−0.0483 (0.0239)	0.0144 (0.0373)	−0.0116 (0.0331)
Tenure $<$ 1 years	0.0311 (0.0212)	−0.0323 (0.0195)	−0.0349 (0.0299)	−0.0972 (0.0290)
Tenure 1–4 years	0.0132 (0.0170)	−0.0795 (0.0159)	−0.000583 (0.0221)	−0.0391 (0.0241)
Tenure $>$ 4 years	−0.0254 (0.0176)	−0.0728 (0.0154)	0.0323 (0.0206)	−0.0528 (0.0218)
Occ Executive	0.0310 (0.0288)	−0.0595 (0.0279)	0.0213 (0.0328)	−0.169 (0.0391)
Occ Professional	0.0374 (0.0298)	−0.0365 (0.0277)	−0.00469 (0.0382)	−0.0388 (0.0439)
Occ Sales	0.0487 (0.0358)	−0.0670 (0.0345)	−0.0172 (0.0423)	−0.0318 (0.0441)
Occ Admin support	0.0322 (0.0249)	−0.0630 (0.0230)	0.0576 (0.0281)	−0.0644 (0.0321)
Occ Services	−0.0135 (0.0307)	−0.0730 (0.0267)	−0.00826 (0.0461)	−0.0915 (0.0409)
Occ Transportation	−0.00275 (0.0325)	−0.0840 (0.0288)	0.0199 (0.0520)	−0.0194 (0.0453)
Occ Other manual	−0.0302 (0.0164)	−0.0668 (0.0145)	−0.0391 (0.0266)	−0.0474 (0.0262)

Note: Standard errors are in parentheses. This table repeats specification (4) in Table 1 for sex, education, experience, tenure, and occupation subsamples; see the notes in Table 1 for more information.

TABLE F.2. Relationship between unemployment duration and pre-displacement wage for different subsamples.

Subpopulation	Log Wage Coefficient (SIPP)	Log Wage Coefficient (CPS)
Men	4.159	−11.40
	(6.875)	(9.080)
Women	−0.860	−6.858
	(8.948)	(10.95)
HS or less	3.123	0.459
	(9.656)	(11.60)
Some college	−6.382	−4.696
	(9.097)	(11.74)
College graduate	7.642	−30.20
	(11.14)	(13.09)
Exper < 10 years	7.673	0.346
	(13.65)	(13.39)
Exper 10–20 years	−1.824	13.85
	(9.490)	(11.21)
Exper 20–30 years	7.286	−35.60
	(8.818)	(14.05)
Exper > 30 years	−9.506	−12.19
	(20.30)	(28.82)
Tenure < 1 years	9.944	−19.37
	(10.49)	(13.40)
Tenure 1–4 years	−0.195	−3.180
	(8.559)	(10.22)
Tenure > 4 years	−1.788	−9.544
	(10.28)	(12.69)
Occ Executive	7.601	−71.31
	(15.00)	(26.49)
Occ Professional	−25.16	−50.74
	(13.97)	(30.29)
Occ Sales	−9.413	12.17
	(15.42)	(22.50)
Occ Admin support	−11.63	16.28
	(18.95)	(31.91)
Occ Services	34.99	−57.08
	(22.83)	(82.53)
Occ Transportation	0.442	−100.5
	(22.82)	(64.53)
Occ Other manual	8.238	3.528
	(10.42)	(21.61)

Note: Standard errors are in parentheses. This table repeats the linear-log specification (4) in Table 2 for sex, education, experience, tenure, and occupation subsamples; see the notes in Table 2 for more information.

F.2 *Sufficient statistics results for men*

TABLE F.3. Results on wage-dispersion statistic for men.

	N	$\mathrm{corr}(w^{\mathrm{pre}}, w^{\mathrm{post}})$	$\frac{\mathrm{corr}(w^{\mathrm{pre}}, w^{\mathrm{post}})}{\lambda_w^{\mathrm{rel}}}$
Displaced (SIPP)	**1134**	**0.746** **(0.016)**	**0.845** **(0.018)**
High school or less	455	0.612 (0.035)	0.755 (0.043)
Some college	408	0.653 (0.032)	0.780 (0.038)
College graduate	271	0.750 (0.039)	0.855 (0.045)
Plant closure (CPS-DWS)	**1259**	**0.701** **(0.017)**	**0.794** **(0.019)**
High school or less	448	0.617 (0.033)	0.761 (0.041)
Some college	505	0.667 (0.028)	0.797 (0.033)
College graduate	306	0.684 (0.041)	0.781 (0.047)

Note: Standard errors are in parentheses. Samples selection: prime aged (25–54 years old), full-time, private-sector male workers, not working in agriculture or construction, who made a full-time private sector to full-time private sector transition. Displaced workers in the SIPP sample includes male workers who were displaced due to a layoff, employer bankrupcy, or slack work conditions. The CPS-DWS sample includes male workers who were displaced due to a plant closing. λ_w^{rel} is the reliability ratio for measurement error in wages. Education-specific reliability ratios are calculated using the SIPP cross-sectional data set. We estimate λ_w^{rel} to be 0.855, 0.797, 0.818, and 0.850 for the full sample, high school or less, some college, and college graduates, respectively; see Section 3.3 for more details. Standard errors are estimated via 10,000 bootstrap samples. *Sources: SIPP:Survey of Income and Program Participation 1996, 2001, 2003, and 2008 panels. CPS-DWS Current Population Survey—Displaced Workers Survey 1984–2016.*

F.3 *Alternate structural model estimations*

In this section, we perform several robustness checks of the estimated models. First, we estimate versions of the compensating differential model, where firm productivity and nonpecuniary aspects are allowed to be correlated with correlations of −0.17 and −0.5 (Models (1) and (2)). Second, we estimate a version of the model, where the job offer distributions of employed and unemployed differ, which departs from the assumptions needed for the derivation of the statistics (Model (3)). Finally, we estimate a version of the model, which encompass both of the baseline models (Models (4)). Below we will comment on each model. Table F.5 contains the fit of the models to the data. All models perfectly fit the data, which is not surprising given that the model is just identified. Table F.6 contains the estimated parameters. Finally, Tables F.7 and F.8 contain the variance and wage-growth decompositions.[2]

[2]Due to expositional reasons, we do not show the graphs used for the main results, but only replicate the summary results in the tables.

Table F.4. Results on wage-growth statistic for men.

	N	$\hat{\Pr}(\Delta w < 0)$	$\hat{\Pr}_{corr}(\Delta w < 0)$	$\hat{\Pr}_{corr}(n = 1)$	$\hat{\kappa}_{corr}$
Displaced (SIPP)	**1134**	**0.601** **(0.014)**	**0.620** **(0.017)**	[**0.282**, **0.761**] (**0.101**, **0.034**)	**1.078** **(0.224)**
High school or less	455	0.580 (0.023)	0.595 (0.027)	[0.431, 0.810] (0.163, 0.054)	0.778 (0.308)
Some college	408	0.659 (0.023)	0.687 (0.026)	[0, 0.626] (0.159, 0.053)	2.229 (0.613)
College graduate	271	0.550 (0.030)	0.559 (0.036)	[0.646, 0.882] (0.214, 0.071)	0.427 (0.323)
Plant closure (CPS-DWS)	**1259**	**0.581** **(0.014)**	**0.595** **(0.016)**	[**0.428**, **0.809**] (**0.099**, **0.033**)	**0.783** **(0.184)**
High school or less	448	0.623 (0.023)	0.645 (0.026)	[0.132, 0.711] (0.159, 0.053)	1.437 (0.431)
Some college	505	0.590 (0.022)	0.606 (0.026)	[0.361, 0.787] (0.154, 0.051)	0.911 (0.314)
College graduate	306	0.503 (0.029)	0.504 (0.034)	[0.977, 0.992] (0.126, 0.042)	0.024 (0.155)

Note: Standard errors in parentheses. Sample selection: prime aged (25–54 years old), full-time, private-sector male workers, not working in agriculture or construction, who made a full-time private sector to full-time private sector transition. Displaced workers in the SIPP sample includes male workers who were displaced due to a layoff, employer bankruptcy, or slack work conditions. The CPS-DWS sample includes male workers who were displaced due to a plant closing. $\lambda^{rel}_{\Delta w}$ is the average reliability ratio for men and women from Bound and Krueger (1991) ($\lambda^{rel}_{\Delta w} = 0.711$). $\hat{\Pr}_{corr}(n = 1)$ shows the bounds on the fraction of workers receiving zero job offers during the last employment spell after correcting for measurement error. $\hat{\kappa}_{corr}$ is the implied κ after correcting for measurement error; see Section 3.3 for more details. Standard errors are estimated via 10,000 bootstrap samples. *Sources: SIPP:Survey of Income and Program Participation 1996, 2001, 2003, and 2008 panels. CPS-DWS Current Population Survey—Displaced Workers Survey 1984–2016.*

F.3.1 *Correlated productivity and nonpecuniary benefits* We imposed a zero correlation between nonpecuniary benefits and productivity in the baseline compensating differential model. This was taken from Taber and Vejlin (2020). Another recent paper by Hall and Mueller (2018) estimates the correlation to be −0.17.[3] Thus, we have estimated the model imposing this correlation. If the correlation becomes one, then the compensating differential and wage-ladder models will be identical in terms of observed behav-

[3]They assume that offered wages, y, and nonwage values, n, are determined by $n = \eta - \kappa(y - \mu_y)$, where η, y are jointly normal, but independent and μ_y is the mean of y. Thus, the correlation is given by

$$\begin{aligned}\operatorname{corr}(y, n) &= \frac{\operatorname{cov}(n, y)}{\sqrt{\operatorname{var}(y)\operatorname{var}(n)}} \\ &= \frac{\operatorname{cov}\big(\eta - \kappa(y - \mu_y), y\big)}{\sqrt{\operatorname{var}(y)\operatorname{var}\big(\eta - \kappa(y - \mu_y)\big)}} \\ &= \frac{-\kappa \operatorname{var}(y)}{\sqrt{\operatorname{var}(y)\big(\operatorname{var}(\eta) + \kappa^2 \operatorname{var}(y)\big)}}.\end{aligned}$$

Hall and Mueller in Table 2 report $\sigma_y = 0.24$, $\sigma_\eta = 0.34$, and $\kappa = 0.25$. This gives $\operatorname{corr}(y, n) = -0.17$.

TABLE F.5. Auxiliary parameter fit: robustness models.

	CD Model Corr$(p, z) = -0.17$	CD Model Corr$(p, z) = -0.5$	WL Model Diff. Offer Distr.	Joint Model	Data
Model:	(1)	(2)	(3)	(4)	
$\Pr(E \to U)$	0.109	0.109	0.109	0.109	0.109
$\Pr(U \to E)$	0.639	0.639	0.639	0.639	0.639
$\Pr(E \to E)$	0.109	0.109	0.109	0.109	0.109
$\Pr(w_x^{\text{post}} < w_x^{\text{pre}})$	0.570	0.570	0.571	0.570	0.570
σ_w	0.574	0.574	0.574	0.574	0.574
$\text{corr}(w_x^{\text{pre}}, w_x^{\text{post}})$	0.755	0.756	0.755	0.755	0.755
ζ_0	2.291	2.291	2.291	2.291	2.291
ζ_1	0.028	0.028	0.028	0.028	0.028
$\zeta_2 \times 100$	−0.034	−0.034	−0.033	−0.034	−0.034
$\text{Corr}(w_t, w_{t+0.33} \mid \text{within match})$	0.882	0.882	0.882	0.882	0.882

Note: The models are simulated using 5,000,000 worker histories from year 0 to year 25.

ior. Thus, we would expect that more negative correlations could be problematic. We have therefore also estimated a model with a correlation of −0.5.

Looking at the estimated parameters, we find that the only estimate that change is the variance of the nonpecuniary aspect. This is decreasing as the correlation becomes more negative. This is only natural, since a negative correlation implies that workers choose low productive jobs to a higher extend than before, because these now have high nonpecuniary values. Thus, a high variance of the nonpecuniary aspect is no longer needed.

Turning to the estimated impacts on frictional wage growth and wage dispersion, we find no difference between the models with different correlations.

TABLE F.6. Structural parameter estimates: robustness models.

	Baseline WL Model		Baseline CD Model		CD Model Corr$(p, z) = -0.17$		CD Model Corr$(p, z) = -0.5$		WL Model Diff Off Distr		Joint Model	
	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.
Model:					(1)		(2)		(3)		(4)	
δ	0.116	0.002	0.116	0.002	0.116	0.002	0.116	0.002	0.116	0.002	0.116	0.002
λ^u	1.108	0.029	1.109	0.029	1.108	0.029	1.108	0.029	1.109	0.029	1.108	0.029
λ^e	0.173	0.034	0.424	0.010	0.424	0.010	0.424	0.010	0.056	0.010	0.276	0.005
λ^d	0.052	0.010	–	–	–	–	–	–	0.073	0.008	0.026	0.001
σ_α	0.476	0.007	0.475	0.007	0.475	0.007	0.474	0.007	0.468	0.006	0.475	0.007
σ_p	0.194	0.010	0.189	0.009	0.187	0.009	0.183	0.009	0.161	0.013	0.191	0.009
μ	1.284	0.010	1.279	0.010	1.280	0.010	1.282	0.010	1.289	0.009	1.283	0.010
χ_1	0.018	0.003	0.018	0.003	0.018	0.003	0.018	0.003	0.017	0.003	0.018	0.003
$\chi_2 \times 100$	0.017	0.011	0.015	0.012	0.014	0.011	0.015	0.011	0.020	0.012	0.016	0.012
σ_ϵ	0.196	0.003	0.196	0.003	0.196	0.003	0.197	0.003	0.196	0.003	0.196	0.003
σ_z	–	–	0.312	0.064	0.247	0.040	0.182	0.021	–	–	0.209	0.063

Note: Standard errors are computed using the formula in Gourieroux, Monfort, and Renault (1993).

Table F.7. Wage variance decompositions: robustness models.

Model:	CD Model Baseline	WL Model Baseline	CD Model Corr$(p, z) = -0.17$ (1)	CD Model Corr$(p, z) = -0.5$ (2)	WL Model Diff. Offer Distr. (3)	Joint Model (4)
Wage Var	0.331	0.331	0.331	0.331	0.331	0.331
Var(ϵ)	0.039	0.039	0.039	0.039	0.039	0.039
Var(α)	0.225	0.226	0.225	0.224	0.219	0.226
Var(p)	0.044	0.043	0.044	0.045	0.050	0.044
Var$(f(k))$	0.019	0.019	0.019	0.019	0.018	0.019
2 Cov$(p, f(k))$	0.004	0.005	0.004	0.005	0.005	0.005

Note: The table shows a linear decomposition of the wage variance for each model. It decomposes the true variance into measurement error, a worker component (Var(α)), a firm component (Var(p)), a human capital component (Var$(f(k))$), and the covariance between the worker and firm components (2 Cov$(p, f(k))$.

F.3.2 *Joint model* We have also estimated a joint model, which encompass both of the baseline models used previously. We use the fraction of job-to-job transitions that are reported as "voluntary" as a moment to separate the involuntary reallocation rate (λ_d) and importance of the nonpecuniary aspect.[4] We saw in Table 7 that this fraction was high in the baseline compensating differential model compared to previous estimates. We see that the joint model is, in terms of estimates, a convex combination of the two baseline models with λ_d equal to 0.026 (0.052 in baseline) and $\sigma_z = 0.209$ (0.312 in baseline). However, neither the variance nor the wage growth decompositions change.

F.3.3 *Differences in offer distributions* Finally, we have estimated a model in which the offer distributions for employed and unemployed differ.[5] Recall that we showed in Section 2.3 that the offer arrival rate was an upper bound if the job offer distribution of the employed stochastically dominate that of the unemployed. We target the aver-

Table F.8. Decomposition of wage growth at 25 years of experience: robustness models.

Model:	CD Model Baseline	WL Model Baseline	CD Model Corr$(p, z) = -0.17$ (1)	CD Model Corr$(p, z) = -0.5$ (2)	WL Model Diff. Offer Distr. (3)	Joint Model (4)
Total growth	0.455	0.451	0.454	0.453	0.452	0.452
HC growth	0.387	0.381	0.386	0.383	0.374	0.382
Fric. growth	0.068	0.070	0.068	0.070	0.078	0.070

Note: The table decompose total wage growth from labor market entry to 25 years after into human capital wage growth and frictional wage growth. Frictional wage growth is measured by simulating the model setting $\chi_1 = \chi_2 = 0$, while human capital wage growth is the difference in growth between total wage growth and frictional wage growth.

[4]Voluntary job-to-job transitions are defined as transitions where workers change jobs for one of the following three reasons (1) "Quit to take another job," (2) "Unsatisfactory working arrangements," and (3) "Quit for some other reason."

[5]In this version, we assume that workers hit by an involuntary job offer shock draw offers from the unemployed offer distribution.

age unconditional difference in the offered wage estimated by from Faberman, Mueller, Şahin, and Topa (2017), who find that it is 0.362 log-points higher for employed workers compared to unemployed.[6] We implement this by simply adding 0.362 to each job offer drawn by an employed worker. This, of course, increases the wage-growth statistic (fraction of worker experiencing a wage decrease across an unemployment spell). As predicted in Section 2.3, we now estimate a lower job offer arrival rate for employed. To compensate for this, the estimated λ^d is higher. In total, this makes frictional wage dispersion and frictional wage growth slightly more important, but the change is minor.

Based on the above results, we conclude that our estimates of frictional wage growth and frictional wage dispersion are robust to violations of the identifying assumptions examined here.[7]

F.4 *Numerical example of the lower bound on bargaining power in Proposition 2.1*

We showed in Section 2.4 that we could derive a sufficient condition on the bargaining power of the worker such that the inferred κ from equation (6) is an upper bound in a sequential-auction model. Since one of our points is that κ is fairly small, an upper bound is not problematic. Here, we illustrate that the inferred κ from the wage-ladder model is an upper bound for any value of the bargaining power in the sequential-auction model, using the parameters from our estimated model.

The model specification and parameter values used in these simulations are the ones we estimated earlier in Section 4 for the wage-ladder model. We set parameters to zero if they were not part of the Cahuc, Postel-Vinay, and Robin (2006) model.[8] In Figure F.1, we show the relationship between the wage-growth statistic and κ for different values of β.

Recall that the sufficient condition on β for the inferred κ from the wage-ladder model to be a upper bound for the κ in a sequential-auction model is 0.307 using our estimates.[9] However, as is clear from Figure F.1, we find that the implied κ using equation (6) is still an upper bound for the κ implied by the sequential-auction model for all values of β. This illustrates that the derived sufficient condition is indeed loose, at least in the context of our model.

[6] Table 8 in Faberman et al. (2017). It is important to note that the authors use offered and not accepted wages. Also, note that the controlling for observables decreases the difference to 0.194. We chose to use the larger difference to be conservative.

[7] See identifying assumptions in Section 2.

[8] Specifically, we set $\lambda^d = 0$ (no involuntary job-to-job transitions), $\chi_1 = \chi_2 = 0$ (no human capital accumulation), and $\sigma_\epsilon = 0$ (no measurement error).

[9] $\beta > \frac{0.173}{2\times 0.173+0.05+0.116+0.052}$.

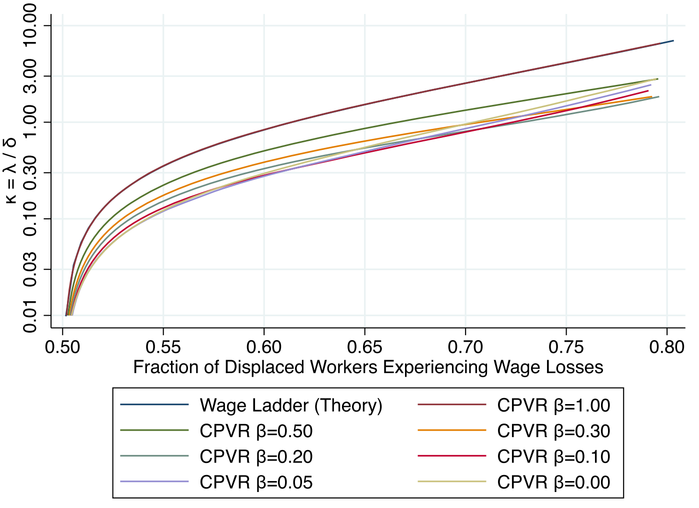

Figure F.1. Alternative models and the frictional wage-growth statistics. Notes: CPVR refers to Cahuc, Postel-Vinay, and Robin (2006) and β is the bargaining power of the worker. The "wage-ladder (theory)" graph in each figure shows the values derived in Section 2.3.

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