# Supplement to "Modeling time varying risk of natural resource assets: Implications of climate change"

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## Appendix A: Leroux–Martin water portfolio model

Leroux and Martin (2016) specify a water portfolio model where an optimal allocation of water assets and future water consumption are determined by maximizing an intertemporal discounted utility function subject to a set of constraints that represent the flows of the alternative water assets. Let $x(t)$ represent water consumption, and $w_1(t)$ and $w_2(t)$ the cost-adjusted shares from two risky water assets given by reservoir inflows and rainwater harvesting, respectively, with the property that the third water asset is determined from the adding-up constraint $w_3(t) = 1 - w_1(t) - w_2(t)$. In the analysis, the third water asset is taken as desalinated water, which is assumed to be risk-free as providing water from this source is perfectly reliable.

The aim of the social planner is to choose $x(t)$ and the water portfolio shares $w_1(t)$ and $w_2(t)$, to maximize the following intertemporal utility function:

$$\max_{x(t),w_1,w_2} E\int_0^\infty \left[e^{(\xi-\delta)t}\frac{x(t)^{1-\gamma}}{1-\gamma}\right]dt, \tag{S1}$$

subject to the following constraints:

$$dS_1(t) = \mu_1\,dt + \sigma_1\,dz_1(t), \tag{S2}$$

$$dS_2(t) = \mu_2\,dt + \sigma_2\,dz_2(t), \tag{S3}$$

$$dS_3(t) = \mu_3\,dt, \tag{S4}$$

$$dW(t) = \left(a_1(t)w_1(t) + a_2(t)w_2(t) + \frac{\mu_3}{S_3(t)} + \frac{c_3}{p}\lambda_3\mu_3\right)W(t)\,dt - x(t)\,dt$$

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$$+\left(\frac{\sigma_1}{S_1(t)}+\frac{c_1}{p}\lambda_1\sigma_1\right)w_1(t)W(t)\,dz_1(t)$$
$$+\left(\frac{\sigma_2}{S_2(t)}+\frac{c_2}{p}\lambda_2\sigma_2\right)w_2(t)W(t)\,dz_2(t), \quad \text{(S5)}$$

and the initial condition for total cost-adjusted water stock $W(0)=W_0$.[1] The parameter $\gamma$ is the relative risk aversion parameter, $\xi$ is the population growth rate and $\delta$ is the discount rate, with $\delta > \xi$. The constraints in equations (S2) to (S4) represent the flow equations for the three water assets, $dS_i(t)$, $i=1,2,3$. Reservoir inflows and rainwater harvesting are assumed to have Brownian motions with respective means $\mu_1$ and $\mu_2$, and respective variances $\sigma_1^2$ and $\sigma_2^2$. Both of these assets are sensitive to random changes in climatic conditions with the property $dz_i(t)\sim N(0,dt)$. Variations in climatic conditions also affect reservoir inflows and rainwater harvesting jointly, resulting in the flows from these two water assets being correlated with parameter $\rho$, such that $dz_1(t)\,dz_2(t)=\rho\,dt$. In contrast, desalinated water is treated as risk-free as water flows from this source are independent of climatic conditions with the average flow given by $\mu_3$. The final constraint given by (S5) is the equation governing the flows in the total cost-adjusted water stock with $\lambda_i = p(K_i+O_i-O_i'S_i)/(K_i+O_i)^2$, $i=1,2,3$, where $p$ is the price of water, $K_i$ are capital costs, $O_i$ are operating costs, and $O_i'=dO_i/dS_i$. Finally, the $a_1(t)$ and $a_2(t)$ terms are given by

$$a_1(t)=\frac{\mu_1}{S_1(t)}-\frac{\mu_3}{S_3(t)}+\frac{c_1\lambda_1}{p}\left(\mu_1+\frac{\sigma_1^2}{S_1(t)}\right)-\frac{c_3\lambda_3\mu_3}{p}, \quad \text{(S6)}$$

$$a_2(t)=\frac{\mu_2}{S_2(t)}-\frac{\mu_3}{S_3(t)}+\frac{c_2\lambda_2}{p}\left(\mu_2+\frac{\sigma_2^2}{S_2(t)}\right)-\frac{c_3\lambda_3\mu_3}{p}. \quad \text{(S7)}$$

The optimal solution of (S1) to (S5) is derived from solving the dynamic programming problem (see Kamien and Schwartz (1981))

$$(\delta-\xi)V=\max_{x(t),w_1,w_2}\left\{\frac{x(t)^{1-\gamma}}{1-\gamma}\right.$$
$$+\left[\left(a_1(t)w_1(t)+a_2(t)w_2(t)+\frac{\mu_3}{S_3(t)}+\mu_3\lambda_3\frac{c_3}{p}\right)W(t)-x(t)\right]V_W$$
$$+\left[\frac{1}{2}\left(\frac{\sigma_1}{S_1(t)}+\sigma_1\lambda_1\frac{c_1}{p}\right)^2w_1^2+\frac{1}{2}\left(\frac{\sigma_2}{S_2(t)}+\sigma_2\lambda_2\frac{c_2}{p}\right)^2w_2^2\right.$$
$$\left.\left.+\left(\frac{1}{S_1(t)}+\lambda_1\frac{c_1}{p}\right)\left(\frac{1}{S_2(t)}+\lambda_2\frac{c_2}{p}\right)\rho\sigma_1\sigma_2w_1(t)w_2(t)W(t)^2V_{WW}\right]\right\}, \quad \text{(S8)}$$

where $V$ is the time-invariant indirect utility function with respective first and second derivatives $V_W$ and $V_{WW}$. The optimal solution of the water portfolio shares follow Leroux and Martin (2016) and are given in equations (29) to (31) in the main text.

[1] In neoclassical growth models with population growth, utility is typically expressed in terms of per capita consumption. Adopting this formulation here would not affect the optimal share equations as they are independent of the effective discount rate.

## Appendix B: Water portfolios under forecast climate change with fixed parameter estimates

Table S1. Simulated water portfolio results based on a 20-year forecast horizon where sampling is from the combined lower and upper tails of the residual distribution with the cutoff points given in the first column. The number of bootstraps is 100,000. Reported are the average annual shares computed from optimal monthly water portfolios based on equations (29) to (34) evaluated using the parameter values as per Table 6 and RV-DCC point estimates reported in columns 2 and 4 of Table 5 in the main text. *Desal Use* refers to the percentage of years in which desalination optimally contributes to the total cost-adjusted water stock. *Costs* report the average annual total supply cost of the optimal water portfolios.

| | | Asset Shares[a] | | |
|---|---|---|---|---|
| Sampling | Statistics | Reservoir | Rainwater | Desal. |
| | Mean | 0.88 | 0.07 | 0.05 |
| | Median | 0.96 | 0.04 | 0.00 |
| All | SD | 0.22 | 0.08 | 0.16 |
| | Desal. Use (% of yrs) | | | 12.96 |
| | Costs ($bn p.a.) | | | 1.67 |
| | Mean | 0.86 | 0.08 | 0.06 |
| | Median | 0.96 | 0.04 | 0.00 |
| 0.48 | SD | 0.24 | 0.08 | 0.17 |
| | Desal. Use (% of yrs) | | | 16.03 |
| | Costs ($bn p.a.) | | | 1.73 |
| | Mean | 0.83 | 0.09 | 0.08 |
| | Median | 0.95 | 0.05 | 0.00 |
| 0.45 | SD | 0.27 | 0.09 | 0.19 |
| | Desal. Use (% of yrs) | | | 19.97 |
| | Costs ($bn p.a.) | | | 1.81 |
| | Mean | 0.76 | 0.11 | 0.13 |
| | Median | 0.93 | 0.07 | 0.00 |
| 0.40 | SD | 0.31 | 0.09 | 0.23 |
| | Desal. Use (% of yrs) | | | 29.57 |
| | Costs ($bn p.a.) | | | 2.02 |
| | Mean | 0.67 | 0.14 | 0.19 |
| | Median | 0.87 | 0.13 | 0.00 |
| 0.35 | SD | 0.35 | 0.10 | 0.27 |
| | Desal. Use (% of yrs) | | | 45.03 |
| | Costs ($bn p.a.) | | | 2.34 |

[a]Based on a desalination flow of $\mu_3 = 0.4$, and stocks of $S_1 = 0.65 \times 1290$ and $S_2 = 0.75 \times 0.217$, which represent the average stocks over the last 25 years of the data. A nonnegativity restriction is imposed on some shares.

Table S2. Simulated water portfolio results based on a 20-year forecast horizon where sampling is from the residuals of the Millennium Drought. The number of bootstraps is 100,000. Reported are the average annual shares computed from optimal monthly water portfolios based on equations (29) to (34) evaluated using the parameter values as per Table 6 and the RV-DCC point estimates reported in columns 2 and 4 of Table 5 in the main text. *Desal Use* refers to the percentage of years in which desalination optimally contributes to the total cost-adjusted water stock. *Costs* report the average annual total supply cost of the optimal water portfolios.

| | Asset Share[a] | | |
|---|---|---|---|
| Statistics | Reservoir | Rainwater | Desalinated |
| Mean | 0.81 | 0.09 | 0.10 |
| Median | 0.95 | 0.05 | 0.00 |
| SD | 0.28 | 0.08 | 0.21 |
| Desal. Use % of yrs | | | 21.94 |
| Costs ($bn p.a.) | | | 1.83 |

[a]Based on a desalination flow of $\mu_3 = 0.5$, and stocks of $S_1 = 0.50 \times 1290$ and $S_2 = 0.60 \times 0.217$, which represent the average stocks during the Millennium Drought. A nonnegativity restriction is imposed on some shares.

## References

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