Supplement to "Identifying peer achievement spillovers: Implications for desegregation and the achievement gap"

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Appendix

A.1 *Proofs*

Mapping from effort to achievement equilibrium I illustrate how the game in effort maps into a game in achievement. Given that achievement is monotonically increasing in effort, ex ante expected achievement can proxy for effort. Denote the ex ante expected value of achievement ($\tilde{Y}_i$) as

$$\tilde{Y}_i = \tilde{g}(e_i, \mathbf{e}_{-i}; \mathbf{X}, \tilde{\mathbf{K}}) \equiv \int_\Theta g(e_i, \mathbf{e}_{-i}; \mathbf{X}_i, \mathbf{X}_{-i}, \tilde{\mathbf{K}}, \theta_i) f(\theta_i | \mathbf{X}, \tilde{\mathbf{K}})\, d\theta_i.$$

The system that describes the effort for all students in the classroom as a function of ex ante achievement and peer effort is

$$\begin{aligned} e_1 &= \tilde{g}^{-1}(\tilde{Y}_1, e_2, \dots, e_N; \mathbf{X}, \tilde{\mathbf{K}}) \\ &\vdots \\ e_N &= \tilde{g}^{-1}(\tilde{Y}_N, e_1, \dots, e_{N-1}; \mathbf{X}, \tilde{\mathbf{K}}). \end{aligned}$$

I assume that the solution to this system is unique and is captured by the function $G(\cdot)$, that is,

$$e_i = G(\tilde{Y}_i, \tilde{\mathbf{Y}}_{-i}; \mathbf{X}, \tilde{\mathbf{K}}) \quad \text{for } i = 1, \dots, N.$$

The vector of peer effort as a function of the vector of achievement and predetermined variables is

$$\begin{aligned} \mathbf{e}_{-i} &= \bigl(\dots, G(\tilde{Y}_{i-1}, \tilde{\mathbf{Y}}_{-(i-1)}; \mathbf{X}, \tilde{\mathbf{K}}), G(\tilde{Y}_{i+1}, \tilde{\mathbf{Y}}_{-(i+1)}; \mathbf{X}, \tilde{\mathbf{K}}), \dots\bigr) \\ &\equiv \mathbf{G}_{-i}(\tilde{Y}_i, \tilde{\mathbf{Y}}_{-i}; \mathbf{X}, \tilde{\mathbf{K}}). \end{aligned}$$

Therefore, the effort best response can be written as a function of peer achievement, that is,

$$\begin{aligned} e_i^*(\mathbf{e}_{-i}; \mathbf{X}, \tilde{\mathbf{K}}, \mathbf{P}_i) &= e_i^*\bigl(\mathbf{G}_{-i}\bigl(\tilde{Y}_i^*, \tilde{\mathbf{Y}}_{-i}; \mathbf{X}, \tilde{\mathbf{K}}\bigr); \mathbf{X}, \tilde{\mathbf{K}}, \mathbf{P}_i\bigr) \\ &= e_i^*\bigl(\tilde{Y}_i^*, \tilde{\mathbf{Y}}_{-i}; \mathbf{X}, \tilde{\mathbf{K}}, \mathbf{P}_i\bigr). \end{aligned}$$

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Plugging utility-maximizing effort into ex ante expected achievement, we have the achievement best response of a student i to any level of peer achievement $\tilde{\mathbf{Y}}_{-i}$:

$$\tilde{Y}_i^* = g\big(e_i^*\big(\tilde{Y}_i^*, \tilde{\mathbf{Y}}_{-i}; \mathbf{X}, \tilde{\mathbf{K}}, \mathbf{P}_i\big), \mathbf{G}_{-i}\big(\tilde{Y}_i^*, \tilde{\mathbf{Y}}_{-i}; \mathbf{X}, \tilde{\mathbf{K}}\big); \mathbf{X}, \tilde{\mathbf{K}}, \mathbf{P}_i\big).$$

Let $\tilde{q}(\cdot)$ represent an explicit solution for $\tilde{Y}_i^*$ as

$$\tilde{Y}_i^* = \tilde{q}(\tilde{\mathbf{Y}}_{-i}, \mathbf{X}_i, \mathbf{X}_{-i}, \tilde{\mathbf{K}}, \mathbf{P}_i).$$

The ex post achievement realized by i under his best response is

$$Y_i^* = q\big(\tilde{\mathbf{Y}}_{-i}^*, \mathbf{X}_i, \mathbf{X}_{-i}, \tilde{\mathbf{K}}, \mathbf{P}_i, \theta_i\big).$$

Proof of identification of μ Because all results hold conditional on the exogenous characteristics ($\mathbf{X}_i$, $\bar{\mathbf{X}}_{-i}$, $\mathbf{K}$, $\mathbf{P}_i$), dependence on these variables is suppressed. Following the proof in Imbens and Newey (2009),

$$\begin{aligned}
F_{\bar{Y}_{-i}^*|\bar{\mathbf{P}}_{-i}}\big(\bar{Y}_{-i}^*|\bar{\mathbf{P}}_{-i}\big) &\overset{(1)}{=} \Pr\big(\bar{Y}_{-i}^* \le \bar{y}_0|\bar{\mathbf{p}}_0\big) \\
&\overset{(2)}{=} \Pr\big(h(\bar{\mathbf{P}}_{-i}, \mu) \le \bar{y}_0|\bar{\mathbf{p}}_0\big) \\
&\overset{(3)}{=} \Pr\big(\mu \le h^{-1}(\bar{\mathbf{p}}_0, \bar{y}_0)|\bar{\mathbf{p}}_0\big) \\
&\overset{(4)}{=} \Pr\big(\mu \le h^{-1}(\bar{\mathbf{p}}_0, \bar{y}_0)\big) \\
&\overset{(5)}{=} F_\mu\big(h^{-1}(\bar{\mathbf{p}}_0, \bar{y}_0)\big).
\end{aligned}$$

The first equality follows by definition; the second follows by the representation of peer achievement in (4.3); the third follows by A4; and the fourth by A3. Therefore, $\mu = h^{-1}(\bar{\mathbf{p}}_0, \bar{y}_0)$ is identified by the joint distribution of $(\bar{Y}_{-i}^*, \bar{\mathbf{P}}_{-i})$. □

Proof of identification of quantile structural function Following the proof in Imbens and Newey (2009),

$$\begin{aligned}
&F_{Y_i^*|\bar{Y}_{-i}^*,\mu}\big(Y_i^*|\bar{Y}_{-i}^*, \mu\big) \\
&\quad = \Pr\big(Y_i^* \le y_0|\bar{y}_0, \mu_0\big) \\
&\quad = \Pr\big(q\big(\bar{Y}_{-i}^*, \mu, \theta_i\big) \le y_0|\bar{y}_0, \mu_0\big) \\
&\quad = \Pr\big(\theta_i \le q^{-1}(\bar{y}_0, \mu_0, y_0)|\bar{y}_0, \mu_0\big) \\
&\quad = \Pr\big(\theta_i \le q^{-1}(\bar{y}_0, \mu_0, y_0)\big) \\
&\quad = F_{\theta_i}\big(q^{-1}(\bar{y}_0, \mu_0, y_0)\big) \\
&\quad = q^{-1}(\bar{y}_0, \mu_0, y_0).
\end{aligned}$$

Since the inverse of the structural function is identified, the function itself is also identified on the joint support of $(\bar{Y}_{-i}^*, \mu, \theta_i)$. □

A.2 *Interpretation of contextual effects*

Fruehwirth (2012) provided a detailed explanation of the interpretation of contextual effects using a simple linear-in-means context, but a condensed intuition is provided here to aid in interpreting the above results. First, consider contextual effects. If there were no contemporaneous peer spillovers and students did not choose effort, then $\bar{\mathbf{X}}_{-ict}$ would only affect i's achievement through the characteristics that enter his achievement directly. This is the way that contextual effects are generally thought about in the literature. In this case, we would expect that increasing, say, the percentage of peers with high parental education would have a positive effect on i's achievement. However, when student i is able to choose effort, it is unclear whether increasing $\bar{\mathbf{X}}_{-ict}$ will have a positive or a negative effect. For instance, higher peer parental education may substitute for a student's own effort. On the other hand, any amount of effort may also be more productive as a result of the "better" peer group, suggesting a higher level of optimal effort. Finally, when there are spillovers from peer effort, conditional on a given level of peer achievement, a higher level of peer parental education suggests a lower level of peer effort. Given these three countervailing effects, the sign of $\hat{\boldsymbol{\beta}}_4$ is indeterminate. A similar conclusion holds for classroom productivity, μ, suggesting that the assumption of an upward bias from unobserved correlated effects may not hold.

A.3 *Supplemental tables*

TABLE A.1. Summary statistics by race.[a]

	White		Nonwhite	
	Mean	Std. Dev.	Mean	Std. Dev.
Reading score (standardized)	0.4723	0.8975	−0.2195	0.8827
Male	0.5056	0.5000	0.4891	0.4999
Parent HS/some post-sec.	0.6191	0.4856	0.7858	0.4103
Parent 4-year degree+	0.3241	0.4680	0.1030	0.3040
Characteristics of Classroom				
Avg. peer reading	0.3127	0.3963	0.0607	0.4356
Avg. white peer reading	0.4633	0.4126	0.3453	0.5031
Avg. nonwhite peer reading	−0.1230	0.5399	−0.2338	0.4377
% White ach. level 1 or 2	0.1676	0.1367	0.1793	0.1976
% Nonwhite ach. level 1 or 2	0.3033	0.2917	0.3896	0.2167
% Nonwhite	0.2395	0.2104	0.5311	0.2671
% Parent with HS degree	0.6471	0.2130	0.7156	0.1948
% Parent with 4-year+	0.2745	0.2366	0.2061	0.2070
Class size	23.03	3.507	22.13	3.719
No peers of other race	0.1455	0.3526	0.0674	0.2507
Teacher with adv. degree	0.2827	0.4503	0.2536	0.4351
Teacher experience	12.74	9.688	12.07	9.878
N	623,986		321,997	

[a]Author's calculations using North Carolina Education Research Data Center, end of grade exams. The sample is restricted to grades 4 and 5 and academic years 1997–1998 to 2001–2002.

TABLE A.2. Summary statistics by apparent random assignment.[a]

	Estimation Sample		Random Assignment[b]	
	Mean	Std. Dev.	Mean	Std. Dev.
Reading score (standardized)	0.2238	0.9527	0.2105	0.9525
Male	0.4992	0.5000	0.5004	0.5000
Parent HS/some post-sec.	0.6696	0.4704	0.6830	0.4653
Parent 4-year degree+	0.2583	0.4377	0.2415	0.4280
Characteristics of Classroom				
Avg. peer reading	0.2095	0.4127	0.1961	0.4078
Avg. white peer reading	0.4393	0.4482	0.4157	0.4400
Avg. nonwhite peer reading	−0.1824	0.4772	−0.1925	0.4821
% White ach. level 1 or 2	0.1733	0.1567	0.1795	0.1562
% Nonwhite ach. level 1 or 2	0.3700	0.2320	0.3722	0.2343
% Nonwhite	0.3797	0.2184	0.3717	0.2177
% Parent with HS degree	0.6648	0.2090	0.6779	0.1998
% Parent with 4-year+	0.2594	0.2283	0.2428	0.2168
Class size	22.87	3.444	22.82	3.409
Teacher with adv. degree	0.2680	0.4429	0.2723	0.4451
Teacher experience	12.29	9.745	12.30	9.740
N	552,208		396,553	

[a]Author's calculations using North Carolina Education Research Data Center, end of grade exams. The sample is restricted to grades 4 and 5 and academic years 1997–1998 to 2001–2002. Only classrooms with at least two students of each race are included.

[b]Apparent random assignment schools are those that, for a given school year, had a p-value of 0.1 for the joint test that the difference between classroom and school characteristics is significantly different from 0.

TABLE A.3. Quantile regression with lagged peer achievement.[a]

	Quantiles				
	0.1	0.3	0.5	0.7	0.9
White ($N = 344{,}885$)					
Avg. white reading $_{t-1}$	0.0138**	0.0226***	0.0267***	0.0258***	0.0302***
	[0.0055]	[0.0040]	[0.0034]	[0.0040]	[0.0051]
Avg. nonwhite reading $_{t-1}$	0.0120***	0.0111***	0.0101***	0.0089***	0.0156***
	[0.0035]	[0.0026]	[0.0022]	[0.0026]	[0.0033]
% Nonwhite	−0.1373***	−0.1228***	−0.1218***	−0.1120***	−0.1091***
	[0.0094]	[0.0069]	[0.0059]	[0.0069]	[0.0088]
% Male	−0.0577***	−0.0368**	−0.0363***	−0.0339**	−0.0501***
	[0.0199]	[0.0145]	[0.0124]	[0.0145]	[0.0186]
% Parents HS degree	0.026	0.0266*	0.0025	−0.0246	−0.0136
	[0.0216]	[0.0157]	[0.0134]	[0.0156]	[0.0199]
% Parents 4-year degree	0.1349***	0.1289***	0.1008***	0.0849***	0.0837***
	[0.0213]	[0.0155]	[0.0133]	[0.0154]	[0.0196]

Table A.3. (Continued.)

	Quantiles				
	0.1	0.3	0.5	0.7	0.9
Nonwhite ($N = 207{,}323$)					
Avg. white reading $_{t-1}$	0.0469***	0.0416***	0.0346***	0.0326***	0.0292***
	[0.0059]	[0.0045]	[0.0040]	[0.0040]	[0.0054]
Avg. nonwhite reading $_{t-1}$	−0.0015	−0.003	0.0003	−0.0067	−0.0007
	[0.0060]	[0.0046]	[0.0041]	[0.0042]	[0.0057]
% Nonwhite	−0.1789***	−0.1667***	−0.1572***	−0.1372***	−0.1125***
	[0.0114]	[0.0088]	[0.0078]	[0.0078]	[0.0105]
% Male	−0.0563**	−0.0559***	−0.0661***	−0.0665***	−0.0749***
	[0.0259]	[0.0199]	[0.0176]	[0.0176]	[0.0236]
% Parents HS degree	−0.0481*	−0.0252	−0.0318*	−0.0166	−0.0158
	[0.0283]	[0.0219]	[0.0193]	[0.0193]	[0.0258]
% Parents 4-year degree	0.0208	0.0274	0.0462**	0.0725***	0.0904***
	[0.0287]	[0.0221]	[0.0195]	[0.0194]	[0.0260]

[a]Quantile regressions estimated separately by race. Also included are dummy variables for male, parent with high school degree, parent with 4-year degree, lagged achievement, teacher experience, experience^2, and teacher advanced degree. School-by-year fixed effects, grade fixed effects, and the constant also are included. School-by-year fixed effects are calculated by mean regression and included as controls in quantile regressions. Standard errors are given in brackets and are not corrected for clustering or generated regressors. *, significant at 10%; **, significant at 5%; ***, significant at 1%.

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