Supplement to "How do tax progressivity and household heterogeneity affect Laffer curves?"

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Appendix

A.1 *Definition of a recursive competitive equilibrium*

We call an equilibrium of the growth adjusted economy a stationary equilibrium.[1] Let $\Phi^M(k^z, e^w, u^m, u^w, a^m, a^w, F^{Mw}, j)$ be the measure of married households with the corresponding characteristics and $\Phi^S(k^z, e, u, a, \iota, F_S^\iota, j)$ be the measure of single households. We now define such a stationary recursive competitive equilibrium as follows:

Definition. 1. The value functions $V^M(\Phi^M)$ and $V^S(\Phi^S)$ and policy functions, $c^z(\Phi^M)$, $k^z(\Phi^M)$, $n^m(\Phi^M)$, $n^w(\Phi^M)$, $c(\Phi^S)$, $k(\Phi^S)$, and $n(\Phi^S)$ solve the consumers' optimization problem given the factor prices and initial conditions.

2. Markets clear:

$$K^z + B^z = \int k^z \, d\Phi^M + \int k^z \, d\Phi^S,$$

$$L^z = \int \left(n^m w^{zm} + n^w w^{zf}\right) d\Phi^M + \int \left(nw^z\right) d\Phi^S,$$

$$\int c^z \, d\Phi^M + \int c^z \, d\Phi^S + (\mu + \delta)K^z + G^z = \left(K^z\right)^\alpha \left(L^z\right)^{1-\alpha}.$$

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[1] The associated BGP can of course trivially be constructed by scaling all appropriate variables by the growth factor Z_t.

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3. The factor prices satisfy:

$$w^z = (1-\alpha)\left(\frac{K^z}{L^z}\right)^{\alpha},$$
$$r = \alpha\left(\frac{K^z}{L^z}\right)^{\alpha-1} - \delta.$$

4. The government budget balances:

$$g^z\left(2\int d\Phi^M + \int d\Phi^S\right) + \int_{j<65,n=0} T^z\, d\Phi^M + \int_{j<65,n=0} T^z\, d\Phi^S + G^z + (r-\mu)B^z$$
$$= \int\left(\tau_k r\left(k^z + \Gamma^z\right) + \tau_c c^z + \tau_l^M\left(\frac{n^m w^{mz} + n^w w^{wz}}{1+\tilde{\tau}_{ss}}\right)\right) d\Phi^M$$
$$+ \int\left(\tau_k r\left(k^z + \Gamma^z\right) + \tau_c c^z + \tau_l^S\left(\frac{n w^z}{1+\tilde{\tau}_{ss}}\right)\right) d\Phi^S.$$

5. The social security system balances:

$$\Psi^z\left(\int_{j\geq 65} d\Phi^M + \int_{j\geq 65} d\Phi^S\right) = \frac{\tilde{\tau}_{ss} + \tau_{ss}}{1+\tilde{\tau}_{ss}}\left(\int_{j<65}\left(n^m w^{mz} + n^w w^{wz}\right) d\Phi^M + \int_{j<65} n w^z\, d\Phi^S\right).$$

6. The assets of the dead are uniformly distributed among the living:

$$\Gamma^z\left(\int \omega(j)\, d\Phi^M + \int \omega(j)\, d\Phi^S\right) = \int\left(1-\omega(j)\right)k^z\, d\Phi^M + \int\left(1-\omega(j)\right)k^z\, d\Phi^S.$$

A.2 *The labor income tax Laffer curve in a simple static economy with a representative household*

Here we argue that, given our choice of the utility function, there is always a Laffer curve, in the sense that tax revenue initially rises, but eventually falls as average tax rates increase as long as households have some non-labor income, either through government transfers or capital income. We demonstrate this in a static, representative household economy, but the argument extends directly to our dynamic economy with heterogeneous households.

Consider a simple static consumer optimization problem with preferences of the form used in this paper:

$$\max_{c,h} \log(c) - \chi\frac{h^{1+\eta}}{1+\eta}$$
$$\text{s.t. } c = g + wh(1-\tau),$$

where g is the government transfer, c is consumption and h is labor supply. We assume that the government transfers back to the consumer a share s of its tax revenues, and thus $g = swh\tau$.

Combining the first order condition with respect to h and c we obtain:[2]

$$\chi h^{\eta} = \frac{w(1-\tau)}{g + w(1-\tau)h} = \frac{w(1-\tau)}{sw\tau h + w(1-\tau)h}$$

so that:

$$h = \left(\frac{1-\tau}{\chi(s\tau + 1 - \tau)}\right)^{1/(1+\eta)}.$$

How labor supply depends on the tax rate τ depends crucially on s, and thus on the extent to which households receive non-labor income (here in the form of government transfer income). In the extreme case where $s = 0$, then the government wastes all tax revenue, the only source of household income is labor income, and labor supply is given by:

$$h = \left(\frac{1}{\chi}\right)^{1/(1+\eta)}$$

and is independent of τ. In this case, total tax revenue is given by:

$$\mathrm{TR}(\tau) = wh(\tau)\tau = w\left(\frac{1}{\chi}\right)^{1/(1+\eta)}\tau$$

which is an increasing linear function of τ, and thus there is no Laffer curve. In contrast, for any $s > 0$, we have $h(\tau = 1) = 0$, and labor supply is strictly decreasing in τ:

$$\frac{\partial h(\tau)}{\partial \tau} = -\frac{1}{1+\eta}\left(\frac{1-\tau}{\chi(s\tau+1-\tau)}\right)^{-\eta/(1+\eta)}\left(\frac{s\tau}{\chi(s\tau+1-\tau)^2}\right) < 0$$

for all $\tau > 0$, and $\frac{\partial h(\tau=0)}{\partial \tau} = 0$. Thus, for all $s > 0$ total tax revenue $\mathrm{TR}(\tau) = wh(\tau)\tau$ satisfies $\mathrm{TR}(\tau = 0) = \mathrm{TR}(\tau = 1) = 0$, as well as $\frac{\partial \mathrm{TR}(\tau=0)}{\partial \tau} > 0$. Therefore, there is a well-behaved Laffer curve with revenue-maximizing tax rate $\tau \in (0, 1)$.

Figure S1 shows how tax revenues change with τ for 4 different values of s: $s = 0$, $s = 0.3$, $s = 0.6$ and $s = 0.9$, assuming that $w = 1$, $\eta = 1/0.6$ and $\chi = 1$. Note that we obtain $\frac{\partial h}{\partial \tau} < 0$ even if $s = 0$ as long as the household has other, non-labor income sources, such as capital income (as in our full dynamic model). This is shown using the implicit function theorem since with capital income the optimal hours choice h has no closed-form solution. This result in turn again leads to a Laffer curve with interior revenue maximizing tax rate $\tau \in (0, 1)$.

A.3 *The impact of tax progressivity in a complete markets model with a representative agent: Proof of Proposition 4.1*

Proof. FOCs:

$$\chi h^{\eta} = \lambda\theta_0(1-\theta_1)H^{\theta_1 - 1}h^{-\theta_1},$$

$$\lambda = \frac{1}{c}.$$

[2] In the dynamic model we obtain an identical condition, but where g would also include asset income.

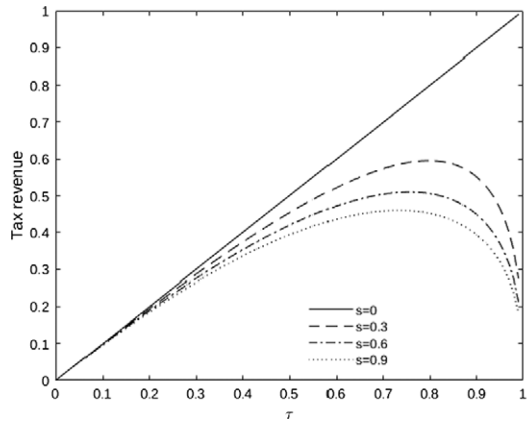

Figure S1. The Laffer curves in a static model with a representative household for different values of s.

In equilibrium, $H = h$, so:

$$h^{1+\eta} = \frac{\theta_0(1-\theta_1)}{\chi}\lambda.$$

Taking logs and solving for $\log(h)$:

$$\log(h) = \frac{1}{1+\eta}\big(\log(\lambda) + \log(\theta_0) + \log(1-\theta_1) - \log(\chi)\big)$$

so that:

$$\frac{\partial \log(h)}{\partial \theta_1} = -\frac{1}{(1+\eta)(1-\theta_1)} < 0.$$

This is the "partial equilibrium effect" of the change in progressivity, which we have described in the main text for a general utility function.

From the firms' FOCs:

$$w = (1-\alpha)K^{\alpha}H^{-\alpha},$$
$$r = \alpha K^{\alpha-1}H^{1-\alpha} - \delta.$$

Equilibrium (steady-state) conditions:

$$c + k = \theta_0\left(\frac{wh}{wH}\right)^{1-\theta_1} + k(1+r) + s(wH - \theta_0),$$
$$1 = \beta\left(1 + \alpha\left(\frac{k}{h}\right)^{\alpha-1} - \delta\right),$$
$$h^{1+\eta} = \frac{\theta_0(1-\theta_1)}{c\chi}.$$

From the second equation:

$$\frac{k}{h} = \left(\frac{\frac{1}{\beta} + \delta - 1}{\alpha} \right)^{\frac{1}{\alpha-1}}.$$

This implies that in equilibrium:

$$w = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha} = (1-\alpha)\left(\frac{\frac{1}{\beta} + \delta - 1}{\alpha} \right)^{\frac{\alpha}{\alpha-1}},$$

$$r + \delta = \alpha\left(\frac{k}{h}\right)^{\alpha-1} = \alpha\left(\frac{\frac{1}{\beta} + \delta - 1}{\alpha} \right)$$

which do not depend on either θ_0 or θ_1.

Then from the third equation:

$$c = \frac{\theta_0(1-\theta_1)}{\chi h^{1+\eta}}.$$

Plugging this all into the first equation:

$$\frac{\theta_0(1-\theta_1)}{\chi h^{1+\eta}} + \left(\frac{k}{h}\right)h = \theta_0 + \left(\frac{k}{h}\right)h(1+r) + s(wh - \theta_0)$$

or:

$$\frac{\theta_0(1-\theta_1)}{\chi} = \theta_0(1-s)h^{1+\eta} + \left(r\left(\frac{1+\beta(\delta-1)}{\alpha\beta}\right)^{\frac{1}{\alpha-1}} + sw \right) h^{2+\eta} \tag{S1}$$

which pins down h.

Differentiating both sides with respect to θ_1, we get:

$$-\frac{\theta_0}{\chi} = \left((1+\eta)\theta_0(1-s)h^{\eta} + \left(\left(\frac{1+\beta(\delta-1)}{\alpha\beta}\right)^{\frac{1}{\alpha-1}} r + sw \right)(2+\eta)h^{1+\eta} \right) \frac{\partial h}{\partial \theta_1}$$

so that:

$$\frac{\partial h}{\partial \theta_1} = -\frac{\theta_0}{\chi\left((1+\eta)\theta_0(1-s) + \left(\left(\frac{k}{h}\right) r + sw \right)(2+\eta)h \right) h^{\eta}} < 0.$$

We are also interested in $\frac{\partial \mathrm{TR}}{\partial \theta_1}$. We have:

$$\frac{\partial \mathrm{TR}}{\partial \theta_1} = w\frac{\partial h}{\partial \theta_1} < 0.$$

In a similar way, differentiating both sides of (S1) with respect to θ_0, we get:

$$\frac{1-\theta_1}{\chi} = (1-s)h^{1+\eta} + \left((1+\eta)\theta_0(1-s)h^{\eta} + \left(\left(\frac{1+\beta(\delta-1)}{\alpha\beta}\right)^{\frac{1}{\alpha-1}} r + sw \right)(2+\eta)h^{1+\eta} \right) \frac{\partial h}{\partial \theta_0}$$

so that:

$$\frac{\partial h}{\partial \theta_0} = \frac{\dfrac{1-\theta_1}{\chi} - (1-s)h^{1+\eta}}{\chi\left((1+\eta)\theta_0(1-s) + \left(\left(\dfrac{k}{h}\right)r + sw\right)(2+\eta)h\right)h^{\eta}}.$$

Since (S1) implies that $\frac{1-\theta_1}{\chi} > (1-s)h^{1+\eta}$, we get:

$$\frac{\partial h}{\partial \theta_0} > 0. \qquad \square$$

A.4 *Proof of Proposition 4.2*

Proof. FOCs with respect to c_H and c_L:

$$\frac{1}{c_{L,t}} = \lambda, \qquad \frac{1}{c_{H,t}} = \lambda \quad \Rightarrow \quad c_{L,t} = c_{H,t} = c.$$

FOCs with respect to h_H and h_L:

$$\chi h_{L,t}^{\eta} = \lambda\theta_0(1-\theta_1)\left(\frac{we^{-a}}{AE}\right)^{1-\theta_1} h_{L,t}^{-\theta_1},$$

$$\chi h_{H,t}^{\eta} = \lambda\theta_0(1-\theta_1)\left(\frac{we^{a}}{AE}\right)^{1-\theta_1} h_{H,t}^{-\theta_1}$$

which implies:

$$\left(\frac{h_H}{h_L}\right)^{\theta_1+\eta} = \left(e^{2a}\right)^{1-\theta_1} \quad \Rightarrow \quad \frac{h_H}{h_L} = e^{\frac{2a(1-\theta_1)}{\theta_1+\eta}}$$

so

$$\frac{\partial(h_H/h_L)}{\partial\theta_1} = -\frac{2a(1+\eta)}{(\theta_1+\eta)^2} e^{\frac{2a(1-\theta_1)}{\theta_1+\eta}} < 0$$

and

$$\frac{\partial^2(h_H/h_L)}{\partial\theta_1\partial a} = -\frac{2(1+\eta)}{(\theta_1+\eta)^2} e^{\frac{2a(1-\theta_1)}{\theta_1+\eta}} < 0. \qquad \square$$

A.5 *Proof of Proposition 4.3*

Proof. Let $U_{\bar{h}}(w_i)$ be the utility from working $h = \bar{h}$ hours for the individual with productivity w_i, and let U_0 be the utility from not working. We have:

$$U_{\bar{h}}(w_i) = \log\left(\theta_0\left(\frac{w_i\bar{h}}{AE}\right)^{1-\theta_1} + T\right) - F$$

and

$$U_0 = \log(T).$$

The individual with productivity w_i decides to work if and only if $U_{\bar{h}}(w_i) \geq U_0$.

Suppose there is $\underline{w}$ such that $U_{\bar{h}}(\underline{w}) = U_0$ or:

$$\log\left(\theta_0\left(\frac{\underline{w}\bar{h}}{AE}\right)^{1-\theta_1} + T\right) - F = \log(T). \tag{S2}$$

Equation (S2) implicitly defines $\underline{w}$ as a function of θ_1.

We have:

$$\frac{\partial U_{\bar{h}}(\underline{w})}{\partial \theta_1} = -\frac{\theta_0\left(\frac{\underline{w}\bar{h}}{AE}\right)^{1-\theta_1} \times \log\left(\frac{\underline{w}\bar{h}}{AE}\right)}{\theta_0\left(\frac{\underline{w}\bar{h}}{AE}\right)^{1-\theta_1} + T} > 0$$

since $\underline{w}\bar{h} < AE$ (AE is the average earnings of employed individuals), and thus $\log(\frac{\underline{w}\bar{h}}{AE}) < 0$.

We also have:

$$\frac{\partial U_{\bar{h}}(\underline{w})}{\partial w} = \frac{\theta_0(1-\theta_1)\bar{h}^{1-\theta_1}\underline{w}^{-\theta_1}}{\theta_0\left(\frac{w_i\bar{h}}{AE}\right)^{1-\theta_1} + T} > 0$$

and thus:

$$\frac{\partial \underline{w}}{\partial \theta_1} = \frac{\frac{\partial U_{\bar{h}}(\underline{w})}{\partial \theta_1}}{\frac{\partial U_{\bar{h}}(\underline{w})}{\partial w}} < 0.$$

This means that higher tax progressivity parameter (θ_1) leads to higher labor market participation.

Now, let $\mathrm{TR}(w_i)$ be the tax revenues collected from the productivity type $w_i > \underline{w}$:

$$\mathrm{TR}(w_i) = w_i\bar{h} - \theta_0\left(\frac{w_i\bar{h}}{AE}\right)^{1-\theta_1}$$

so that:

$$\frac{\partial \mathrm{TR}(w_i)}{\partial \theta_1} = \theta_0\left(\frac{w_i\bar{h}}{AE}\right)^{1-\theta_1} \times \log\left(\frac{w_i\bar{h}}{AE}\right).$$

We get that $\frac{\partial \mathrm{TR}(w_i)}{\partial \theta_1} > 0$ for $w_i\bar{h} > AE$, and $\frac{\partial \mathrm{TR}(w_i)}{\partial \theta_1} < 0$ for $w_i\bar{h} < AE$. □

A.6 *Balanced growth with labor participation margin*

As is well-known,[3] for balanced growth we need to assume labor-augmenting technological progress. In this case, consumption, investment, output and capital all grow at the rate of labor-augmenting technical progress, while hours worked remain constant.

[3] See King, Plosser, and Rebelo (2002) for details.

King, Plosser, and Rebelo (2002) show that the momentary preferences that deliver first-order optimality conditions consistent with these requirements can take one of the following two forms:

$$U(c,n) = \frac{1}{1-\nu} c^{1-\nu} v(n) \quad \text{if } 0 < \nu < 1 \text{ or } \nu > 1,$$
$$U(c,n) = \log(c) + v(n) \quad \text{if } \nu = 1.$$

To reformulate the household problem recursively, one replaces consumption with its growth-adjusted version in both the household's budget constraint and the household's objective function (see the next subsection for the details). With the second version of the momentary utility function, such "adjustment terms" drop out into a separate additive term which can be ignored:

$$\begin{aligned} & E_t \sum_{j=j_0}^{J} \beta^j \big[\log(c_{t,j}) + v(n_j) - F\mathbb{1}_{[n_j>0]}\big] \\ &= E_t \sum_{j=j_0}^{J} \beta^j \big[\log(c_{t,j}/Z_t) + v(n_j) - F\mathbb{1}_{[n_j>0]} + \log(Z_t)\big] \\ &= E_t \sum_{j=j_0}^{J} \beta^j \big[\log\big(c_j^z\big) + v(n_j) - F\mathbb{1}_{[n_j>0]}\big] + E_t \sum_{j=j_0}^{J} \beta^j \log(Z_t), \end{aligned}$$

where $c_j^z = c_{t,j}/Z_t$.

This procedure would not work with the first version of the momentary utility function. Proceeding the same way, we would obtain:

$$\begin{aligned} & E_t \sum_{j=j_0}^{J} \beta^j \left[\frac{1}{1-\nu} c_{t,j}^{1-\nu} v(n_j) - F\mathbb{1}_{[n_j>0]}\right] \\ &= E_t \sum_{j=j_0}^{J} \tilde{\beta}^j \left[\frac{1}{1-\nu} \big(c_j^z\big)^{1-\nu} v(n_j)\right] - E_t \sum_{j=j_0}^{J} \beta^j F\mathbb{1}_{[n_j>0]}, \end{aligned}$$

where $\tilde{\beta} = \beta Z^{1-\nu}$. This means that as time passes by, fixed participation costs become "more important" for the houshold (since it uses the original discount factor, β).

A.7 *Recursive formulation of the household problem*

Married households of age j_0 in period t maximize

$$U = E_t \sum_{j=j_0}^{J} \omega(j) \left(\log(c_{t,j}) - \chi^m \frac{\big(n_{t,j}^m\big)^{1+\eta^m}}{1+\eta^m} - \chi^w \frac{\big(n_{t,j}^w\big)^{1+\eta^w}}{1+\eta^w} - F \cdot \mathbb{1}_{[n_{t,j}^w>0]}\right)$$

subject to the sequence of budget constraints:

$$c_{t,j}(1+\tau_c)+k_{t+1,j+1}=\begin{cases}(k_{t,j}+\Gamma_t)\big(1+r_t(1-\tau_k)\big)+g_t+W^L_{t,j}, & \text{if } j<65,\\ (k_{t,j}+\Gamma_t)\big(1+r_t(1-\tau_k)\big)+g_t+\Psi_t, & \text{if } j\geq 65,\end{cases}$$

where W^L is the household labor income (and unemployment benefits in case wife doesn't work):

$$W^L_{t,j}=\big(W^{L,m}_{t,j}+W^{L,w}_{t,j}\big)\big(1-\tau_{ss}-\tau_l\big(W^{L,m}_{t,j}+W^{L,w}_{t,j}\big)\big)+\big(1-\mathbb{1}_{[n^w_{t,j}>0]}\big)T_t,$$

$W^{L,m}_{t,j}$ and $W^{L,w}_{t,j}$ are the labor incomes of the two household members:

$$W^{L,\iota}_{t,j}=\frac{n^\iota_{t,j}w_t e^{a^\iota+\gamma^\iota_0+\gamma^\iota_1 e^\iota_{t,j}+\gamma^\iota_2(e^\iota_{t,j})^2+\gamma^\iota_3(e^\iota_{t,j})^3+u^\iota_{t,j}}}{1+\tilde{\tau}_{ss}},\quad \iota=m,w$$

which depend on the individual's fixed type a^ι, experience $e^\iota_{t,j}$ (which we assume equals age for men) and productivity shock $u^i_{t,j}$.

To reformulate this household problem recursively, we divide the budget constraints by the technology level Z_t. Recall that with our normalization of Z_0 and K_0, we have $Z_t=Y_t$. Also, recall that on the balanced growth path, $\Gamma^z=\Gamma_t/Z_t$, $g^z=g_t/Z_t$, $\Psi^z=\Psi_t/Z_t$, $T^z=T_t/Z_t$, $w^z=w_t/Z_t$ and r_t must remain constant. We define $c^z_j=c_{t,j}/Z_t$ and $k^z_j=k_{t,j}/Z_t$ and conjecture that they do not depend on the calendar time t either. This allows us to rewrite the budget constraints as:

$$c^z_j(1+\tau_c)+k^z_{j+1}(1+\mu)=\begin{cases}\big(k^z_j+\Gamma^z\big)\big(1+r(1-\tau_k)\big)+g^z+W^L_j, & \text{if } j<65,\\ \big(k^z_j+\Gamma^z\big)\big(1+r(1-\tau_k)\big)+g^z+\Psi^z, & \text{if } j\geq 65.\end{cases}$$

Substituting $c_{t,j}=c^z_j Z_t$ into the objective function, we get an additive term that depends only on the sequence of Z_t and drops out of the maximization problem, and finally get the recursive formulation stated in the main text.

Similar trasformation can be applied for the single households.

A.8 *Tax function*

Given the tax function

$$ya=\theta_0 y^{1-\theta_1}$$

we employ, the average tax rate is defined as

$$ya=\big(1-\tau(y)\big)y$$

and thus

$$\theta_0 y^{1-\theta_1}=\big(1-\tau(y)\big)y$$

and thus

$$\begin{aligned} 1-\tau(y) &= \theta_0 y^{-\theta_1}, \\ \tau(y) &= 1-\theta_0 y^{-\theta_1}, \\ T(y) &= \tau(y)y = y-\theta_0 y^{1-\theta_1}, \\ T'(y) &= 1-(1-\theta_1)\theta_0 y^{-\theta_1}. \end{aligned}$$

Thus the tax wedge for any two incomes (y_1, y_2) is given by

$$1-\frac{1-T'(y_2)}{1-T'(y_1)} = 1-\left(\frac{y_2}{y_1}\right)^{-\theta_1} = 1-\frac{1-\tau(y_2)}{1-\tau(y_1)} \tag{S3}$$

and therefore independent of the scaling parameter θ_0.[4] Thus by construction one can raise average taxes by lowering θ_0 and not change the progressivity of the tax code, since (as long as tax progressivity is defined by the tax wedges) the progressivity of the tax code[5] is uniquely determined by the parameter θ_1. Heathcote, Storesletten, and Violante (2017) estimate the parameter $\theta_1 = 0.18$ for all households. Above we let θ_1 vary by family type.

A.9 *Estimation of returns to experience and shock processes from the PSID*

We take the log of equation (15) and estimate a log(wage) equation using data from the non-poverty sample of the PSID 1968–1997. Equation (16) is estimated using the residuals from (15).

To control for selection into the labor market, we use Heckman's 2-step selection model. For people who are working and for which we observe wages, the wage depends on a 3rd order polynomial in age (men), t, or years of labor market experience (women), e, as well as dummies for the year of observation, D:

$$\log(w_{it}) = \phi_i\big(constant + D_t'\zeta + \gamma_1 e_{it} + \gamma_2 e_{it}^2 + \gamma_3 e_{it}^3 + u_{it}\big). \tag{S4}$$

Age and labor market experience are the only observable determinants of wages in the model apart from gender. The probability of participation (or selection equation)

[4] It should be noted that the last inequality only holds in the absence of additional lumpsum transfers.

[5] Note that

$$1-\tau(y) = \frac{1-T'(y)}{1-\theta_1} > 1-T'(y)$$

and thus as long as $\theta_1 \in (0,1)$ we have that

$$T'(y) > \tau(y)$$

and thus marginal tax rates are higher than average tax rates for all income levels.

depends on various demographic characteristics, Z:

$$\Phi(participation) = \Phi(Z_{it}'\xi + \upsilon_{it}). \tag{S5}$$

The variables included in Z are marital status, age, the number of children, years of schooling, time dummies, and an interaction term between years of schooling and age. To obtain the parameters, σ^{ι}, ρ^{ι} and $\sigma_{\alpha^{\iota}}$ we obtain the residuals $u_i t$ and use them to estimate the below equation by fixed effects estimation:

$$u_{it} = \alpha_i + \rho u_{it-1} + \epsilon_{it}. \tag{S6}$$

The parameters can be found in Table 2.

A.10 *Matching of individuals in marriage*

Single households face an age-dependent probability, $M(j)$, of becoming married, whereas married households face an age-dependent probability, $D(j)$, of divorce. There is assortative matching in the marriage market, in the sense that there is a greater chance of marrying someone with similar ability, a fact that singles rationally foresee.

To implement assortative matching numerically, we introduce the match index, M_n, in the simulation stage of our computational algorithm. M_n is a convex combination of a random shock, $\varsigma \sim U[0, 1]$ and permanent ability, a:

$$M_n = (1 - \varphi)\varsigma + \varphi a, \tag{S7}$$

where $\varphi \in [0, 1]$. Single men and women matched to get married in this period are sorted, within their gender, based on M_n, and assigned the partner of the opposite gender with the same rank. The parameter, φ, thus determines the degree of assortative matching, based on ability. If $\varphi = 0$, then matching is random and if $\varphi = 1$ spouses will have identical ability.

Singles have rational expectations with respect to potential partners. The matching function in Equation (S7) implies conditional probabilities for marrying someone of ability, a', given an individual's own ability, a. Conditional on gender, age and permanent ability, we also keep track of the distribution of singles with respect to assets, labor market experience, female participation costs and idiosyncratic productivity shocks. A single individual can thus have a rational expectation about a potential partner with respect to these characteristics and the expectation will be conditional on the individual's own gender, age and permanent ability.

In Section 6 we calibrate the parameter φ to match the correlation of the wages of married couples in the data. We model the normal distributions of abilities, $a \sim N(0, \sigma_a^{\iota 2})$, using Tauchen's (1986) method and 5 discrete values of a, placed at $\{-1.5\sigma_a^{\iota}$, $-0.75\sigma_a^{\iota}, 0, 0.75\sigma_a^{\iota}, 1.5\sigma_a^{\iota}\}$. Given our calibrated value of φ we obtain the below matrix of marriage probabilities across ability levels:

$$\phi^{-\iota}\left(a|a^{\iota};\varphi\right)=\begin{bmatrix}0.509 & 0.442 & 0.049 & 0.000 & 0.000\\ 0.189 & 0.325 & 0.404 & 0.081 & 0.000\\ 0.071 & 0.258 & 0.343 & 0.256 & 0.072\\ 0.000 & 0.076 & 0.401 & 0.330 & 0.193\\ 0.000 & 0.000 & 0.046 & 0.445 & 0.509\end{bmatrix}.$$

The reason that this matrix is not exactly symmetric is that it comes out of our simulation with 160,000 households.

A.11 *Details on the sensitivity analysis with respect to the size of the labor supply elasticity*

In this appendix we provide the details of our sensitivity analysis with respect to the elasticity of labor supply along the *intensive* margin. In Figure S2 we plot Laffer curves for different levels of tax progressivity when we double the Frisch labor supply elasticity of both males and females (left panel, $\eta^m = 1/0.8$ and $\eta^w = 1/1.6$) and when we cut it in half (right panel, $\eta^m = 1/0.2$ and $\eta^w = 1/0.4$), and recalibrate the model to match the same data moments as the original benchmark model.

We observe that the intensive margin labor supply elasticity has significant impact, both on the level of the Laffer curve, but also on the location of its peak. Qualitatively, a more elastic intensive margin labor supply reduces the ability of the government to raise higher revenue through higher averages taxes (the level of the Laffer curve), and makes that ability more sensitive to the progressivity of the tax code, in that the impact on the peak of a more progressive tax system is stronger with more elastic labor supply. We discuss the main quantitative implications in the main text.

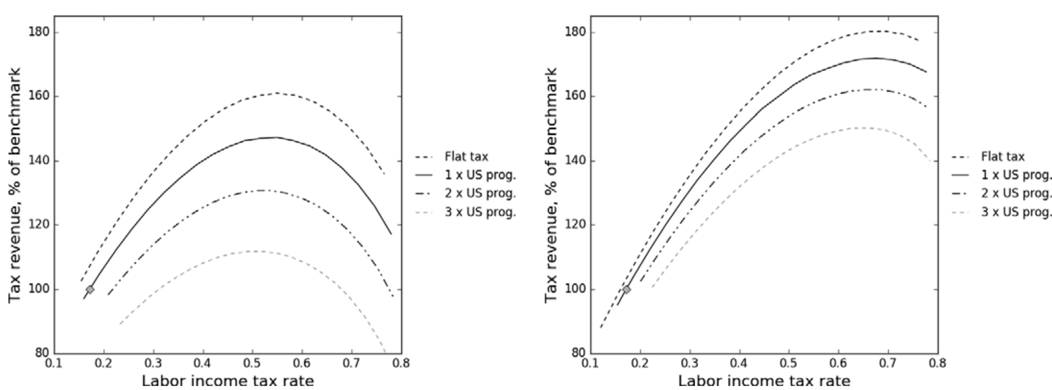

FIGURE S2. Laffer curves by tax progressivity, high (left panel) and low (right panel) intensive margin labor supply elasticity.

A.12 *Additional tables and figures*

TABLE S1. Tax functions by country and family type, OECD 2000–2007.

	Married 0C		Married 1C		Married 2C		Single 0C	
Country	θ_0	θ_1	θ_0	θ_1	θ_0	θ_1	θ_0	θ_1
Austria	0.926427	0.150146	1.003047	0.198779	1.076124	0.23796	0.854448	0.175967
Canada	0.901481	0.155047	0.981109	0.228148	1.066354	0.296329	0.789222	0.147083
Denmark	0.787587	0.229954	0.874734	0.305302	0.920347	0.331685	0.690296	0.220311
Finland	0.868634	0.223116	0.92298	0.261043	0.976928	0.293236	0.763024	0.207634
France	0.917449	0.119957	0.944289	0.133912	1.019455	0.174277	0.85033	0.137575
Germany	0.892851	0.203455	0.956596	0.238398	1.022274	0.272051	0.77908	0.198354
Greece	1.060959	0.161687	1.088914	0.178131	1.127027	0.19963	1.019879	0.228461
Iceland	0.872072	0.194488	0.932844	0.243148	0.990471	0.287094	0.784118	0.153982
Ireland	0.946339	0.162836	1.101397	0.282089	1.187044	0.326003	0.85533	0.188647
Italy	0.900157	0.15939	0.949843	0.198573	1.00814	0.241968	0.822067	0.153275
Japan	0.948966	0.073769	0.971621	0.086518	0.992375	0.097036	0.916685	0.121497
Luxembourg	0.947723	0.15099	1.024163	0.190363	1.113409	0.231438	0.849657	0.163415
Netherlands	0.958121	0.219349	1.004174	0.245393	1.025102	0.256418	0.863586	0.272312
Norway	0.838322	0.148316	0.894721	0.194368	0.932718	0.218213	0.76396	0.146082
Portugal	0.948209	0.119169	0.97794	0.138682	1.009808	0.157309	0.882183	0.132277
Spain	0.923449	0.130171	0.93517	0.134039	0.949941	0.14052	0.862569	0.164186
Sweden	0.782747	0.166797	0.865716	0.240567	0.919471	0.276415	0.717018	0.217619
Switzerland	0.925567	0.116475	0.968531	0.136431	1.008289	0.15569	0.878904	0.128988
UK	0.908935	0.165287	0.994826	0.233248	1.049323	0.273376	0.836123	0.168479
US	0.873964	0.108002	0.940772	0.158466	1.006167	0.203638	0.817733	0.1106

TABLE S2. Distribution of households (with a head between 20 and 64 years of age) by the number of children and marital status, IPUMS USA, 2000–2007.

	Marital status		
# of children	Single	Married	Total
0	29.28	20.86	50.15
1	7.49	13.27	20.76
2	4.41	14.26	18.67
3	1.65	5.81	7.46
4	0.50	1.61	2.11
5	0.14	0.42	0.56
6	0.04	0.14	0.18
7	0.01	0.05	0.07
8	0.00	0.02	0.03
9+	0.00	0.02	0.02
Total	43.54	56.46	100.00

TABLE S3. Labor income taxes paid by income deciles (benchmark calibration).

Income Decile	Share of Total	Cumulative Share
1	0.000	0.000
2	0.011	0.011
3	0.022	0.033
4	0.036	0.069
5	0.050	0.119
6	0.067	0.187
7	0.093	0.279
8	0.133	0.412
9	0.200	0.612
10	0.388	1.000

TABLE S4. Relative tax progressivity in the OECD 2000–2007 (sensitivity analysis).

Country	$y_w = y_m$	$y_w = 0.41y_m$	$y_w = 0.1y_m$
Japan	0.82	0.74	0.77
Switzerland	1.06	0.97	0.93
Portugal	1.04	0.99	0.91
U.S.	1.00	1.00	1.00
France	1.08	1.03	1.07
Spain	1.16	1.08	1.13
Norway	1.19	1.23	1.37
Luxembourg	1.37	1.31	1.25
Italy	1.28	1.31	1.40
Austria	1.42	1.37	1.41
Canada	1.34	1.41	1.59
U.K.	1.31	1.46	1.74
Greece	1.48	1.47	1.67
Iceland	1.35	1.49	1.61
Germany	1.63	1.61	1.60
Sweden	1.72	1.63	1.75
Ireland	1.61	1.65	1.78
Finland	1.66	1.73	1.88
Netherlands	1.98	1.85	1.99
Denmark	1.82	1.88	2.05

Note: The table displays tax progressivity across countries relative to the U.S. under varying assumptions about the ratio between female and male incomes for married couples. The middle column is the benchmark assumption of $y_w = 0.41y_m$ in the CPS (2001–2007) that is used earlier in the paper.

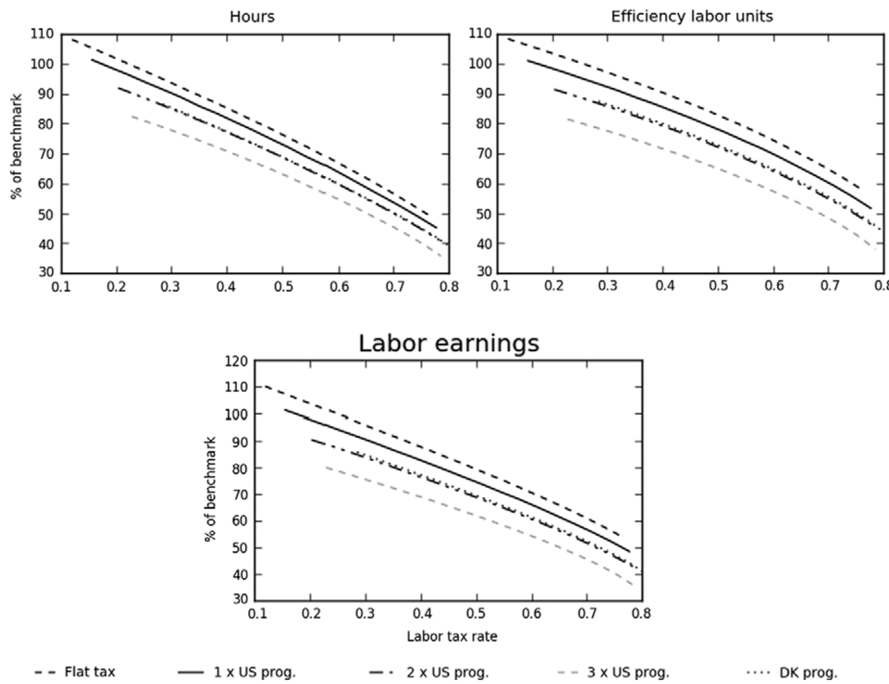

FIGURE S3. Labor supply and earnings statistics by tax progressivity and level for g-Laffer curves.

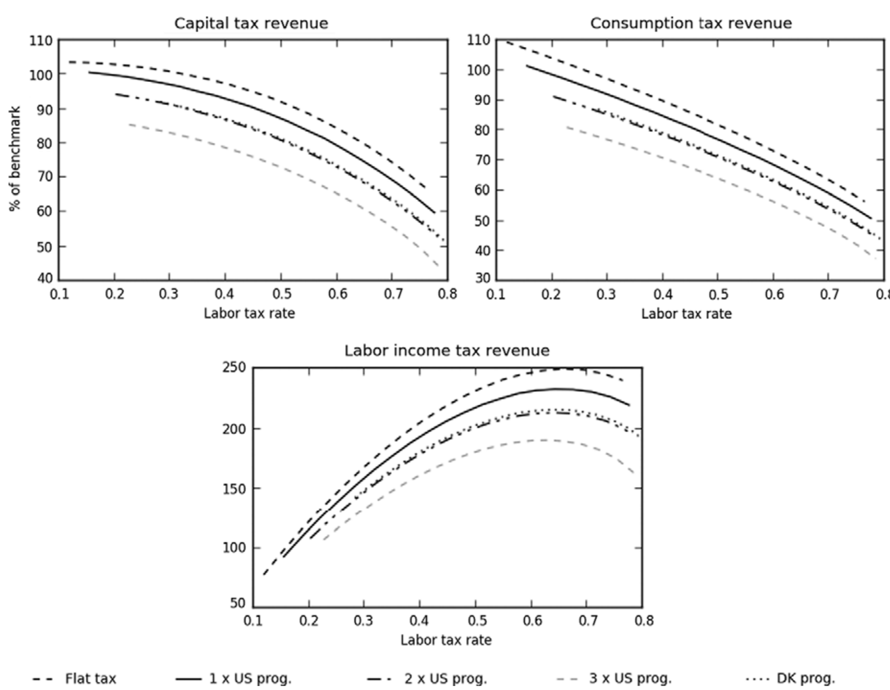

FIGURE S4. The impact of tax progressivity on revenue from labor income taxes, consumption taxes and capital income taxes.

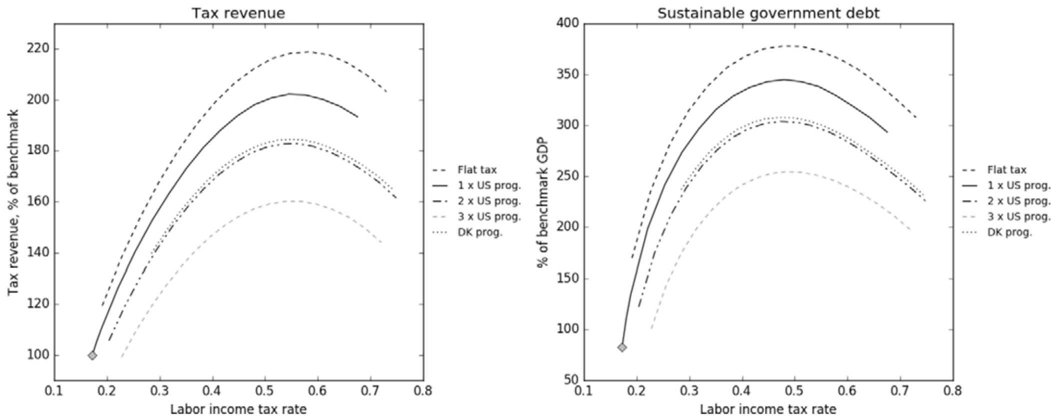

Figure S5. Tax revenue and maximum sustainable debt level by tax level and progressivity.

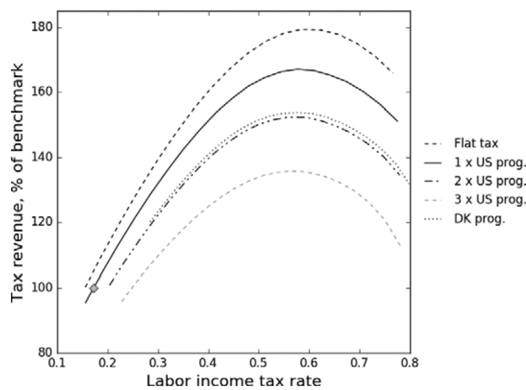

Figure S6. The impact of tax progressivity on the s-Laffer curve (wasting additional revenue).

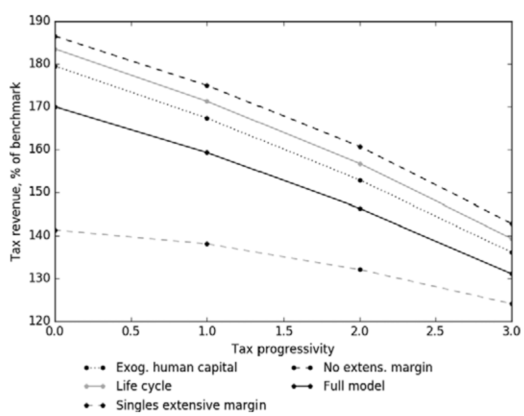

Figure S7. The impact of tax progressivity on maximum revenue in different models.

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