Econometrica Supplementary Material

SUPPLEMENT TO "THE 'NEW' ECONOMICS OF TRADE AGREEMENTS: FROM TRADE LIBERALIZATION TO REGULATORY CONVERGENCE?"

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APPENDIX: DERIVATIONS AND PROOFS

IN THIS APPENDIX, we provide derivations and proofs not included in the main body of the paper.

A.1. *Proof That $\breve{A}(|a_i^J - \gamma^J|)$ Satisfies Assumption 1*

Define $A^J = \breve{A}(|a_i^J - \gamma^J|)$, $\breve{A}' < 0$, $\breve{A}'' < 0$, $\gamma^H > \gamma^F$. Let us consider three regions:

1. $a > \gamma^H > \gamma^F$, then $\frac{\partial A^H}{\partial a} = A'(a - \gamma^H) < 0$ and $\frac{\partial A^F}{\partial a} = A'(a - \gamma^F) < 0$ and $A'(a - \gamma^H) > A'(a - \gamma^F)$. Meanwhile, $A^H > A^F$. So we have $\frac{d \log A^H(a)}{da} = \frac{A'(a-\gamma^H)}{A^H} > \frac{A'(a-\gamma^F)}{A^F} = \frac{d \log A^F(a)}{da}$.
2. $\gamma^H > a > \gamma^F$, then $\frac{\partial A^H}{\partial a} = -A'(a - \gamma^H) > 0$ and $\frac{\partial A^F}{\partial a} = A'(a - \gamma^F) < 0$. Meanwhile, A^H and A^F are both positive. So we have $\frac{d \log A^H(a)}{da} = \frac{A'(a-\gamma^H)}{A^H} > 0 > \frac{A'(a-\gamma^F)}{A^F} = \frac{d \log A^F(a)}{da}$.
3. $\gamma^H > \gamma^F > a$, then $\frac{\partial A^H}{\partial a} = -A'(a - \gamma^H) > 0$ and $\frac{\partial A^F}{\partial a} = -A'(a - \gamma^F) > 0$ and $-A'(a - \gamma^H) > -A'(a - \gamma^F)$. Meanwhile, $A^F > A^H$. So we have $\frac{d \log A^H(a)}{da} = \frac{-A'(a-\gamma^H)}{A^H} > \frac{-A'(a-\gamma^F)}{A^F} = \frac{d \log A^F(a)}{da}$.

Hence $\breve{A}(|a_i^J - \gamma^J|)$ is log-supermodular in a_i^J and γ^J. In addition, $A_{aa} = \frac{\breve{A}''\breve{A} - (\breve{A}')^2}{\breve{A}^2}$ is always negative when $\breve{A}'' < 0$.

A.2. *Proof of Lemma 2*

To prove Lemma 2, we make use of the zero-profit conditions

$$\frac{1}{\sigma - 1}\big(N^J \tilde{c}_J^J\big(a_J^J, P^J\big(\boldsymbol{n}, a_H^J, a_F^J\big)\big) + (1+\nu)N^K \tilde{c}_J^K\big(a_J^K, P^K\big(\boldsymbol{n}, a_H^K, a_F^K\big)\big)\big)$$
$$= \Phi\big(\big|a_J^J - a_J^K\big|\big), \quad J = H, F,$$

where $\tilde{c}_J^J = \lambda_J^J c_J^J(a_J^J, P^J(\boldsymbol{n}, a_H^J, a_F^J))$ and $\tilde{c}_J^K = \lambda_J^K c_J^K(a_J^K, P^K(\boldsymbol{n}, a_H^K, a_F^K))$.

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We prove the claims of Lemma 2 for standards in the home country market, with the proof for standards in the foreign country market proceeding in an analogous fashion. To establish $\frac{dn_H}{da_H^H} < 0$ and $\frac{dn_F}{da_H^H} > 0$ totally differentiate the zero profit conditions with respect to n_H, n_F, and a_H^H;

$$\begin{aligned} &\frac{N^H}{\sigma-1}\left[\frac{\partial \tilde{c}_H^H}{\partial a_H^H} da_H^H + \frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial a_H^H} da_H^H + \frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial n_H} dn_H + \frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial n_F} dn_F\right] \\ &\quad + (1+\nu)\frac{N^F}{\sigma-1}\left[\frac{\partial \tilde{c}_H^F}{\partial P^F}\frac{\partial P^F}{\partial n_H} dn_H + \frac{\partial \tilde{c}_H^F}{\partial P^F}\frac{\partial P^F}{\partial n_F} dn_F\right] = \left[\Phi'\left(\left|a_H^H - a_H^F\right|\right)\right] da_H^H, \end{aligned} \quad \text{(A1)}$$

$$\begin{aligned} &\frac{N^F}{\sigma-1}\left[\frac{\partial \tilde{c}_F^F}{\partial P^F}\frac{\partial P^F}{\partial n_H} dn_H + \frac{\partial \tilde{c}_F^F}{\partial P^F}\frac{\partial P^F}{\partial n_F} dn_F\right] \\ &\quad + (1+\nu)\frac{N^H}{\sigma-1}\left[\frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial P^H}{\partial a_H^H} da_H^H + \frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial P^H}{\partial n_H} dn_H + \frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial P^H}{\partial n_F} dn_F\right] = 0. \end{aligned} \quad \text{(A2)}$$

But the home firm chooses a_H^H to satisfy the first-order condition for profit maximization

$$\frac{\partial \pi_H^H}{\partial a_H^H} = \frac{N^H}{\sigma-1}\frac{\partial \tilde{c}_H^H}{\partial a_H^H} - \Phi'\left(\left|a_H^H - a_H^F\right|\right) = 0,$$

which we may substitute into (A1) to arrive at the home and foreign totally differentiated zero-profit conditions evaluated at the profit-maximizing choices:

$$\begin{aligned} &\frac{N^H}{\sigma-1}\left[\frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial a_H^H} da_H^H + \frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial n_H} dn_H + \frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial n_F} dn_F\right] \\ &\quad + (1+\nu)\frac{N^F}{\sigma-1}\left[\frac{\partial \tilde{c}_H^F}{\partial P^F}\frac{\partial P^F}{\partial n_H} dn_H + \frac{\partial \tilde{c}_H^F}{\partial P^F}\frac{\partial P^F}{\partial n_F} dn_F\right] = 0, \end{aligned} \quad \text{(A3)}$$

$$\begin{aligned} &\frac{N^F}{\sigma-1}\left[\frac{\partial \tilde{c}_F^F}{\partial P^F}\frac{\partial P^F}{\partial n_H} dn_H + \frac{\partial \tilde{c}_F^F}{\partial P^F}\frac{\partial P^F}{\partial n_F} dn_F\right] \\ &\quad + (1+\nu)\frac{N^H}{\sigma-1}\left[\frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial P^H}{\partial a_H^H} da_H^H + \frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial P^H}{\partial n_H} dn_H + \frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial P^H}{\partial n_F} dn_F\right] = 0. \end{aligned} \quad \text{(A4)}$$

Solving (A4) for dn_F, substituting into (A3) and simplifying yields

$$\frac{dn_H}{da_H^H} = \frac{-\dfrac{\partial P^H}{\partial a_H^H}\dfrac{\partial P^F}{\partial n_F}}{\left[\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}\right]}. \quad \text{(A5)}$$

The denominator of the expression in (A5) is strictly positive provided for $\iota^H > 1$ and $\iota^F > 1$, while the term in the numerator is composed of the product of two negative terms, and hence is positive as well. Hence, $\frac{dn_H}{da_H^H} < 0$ as claimed in Lemma 2.

To establish that $\frac{dn_F}{da_H^H} > 0$, we solve (A4) for dn_H and substitute the resulting expression into (A3) and simplify to arrive at

$$\frac{dn_F}{da_H^H} = \frac{\dfrac{\partial P^H}{\partial a_H^H}\dfrac{\partial P^F}{\partial n_H}}{\left[\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}\right]} > 0.$$

A.3. *Proof of Lemma 3*

The eight derivatives involve two calculations that need to be performed for all $J \in \{H, F\}$ and $J' \in \{H, F\}$, where D^J is an indicator variable that is equal to 1 for $J = H$ and equal to -1 for $J = F$:

$$\begin{aligned}
\frac{dP^J}{da_{J'}^J} &= \frac{\partial P^J}{\partial a_{J'}^J} + \frac{\partial P^J}{\partial n_H}\frac{dn_H}{da_{J'}^J} + \frac{\partial P^J}{\partial n_F}\frac{dn_F}{da_{J'}^J} \\
&= \frac{\partial P^J}{\partial a_{J'}^J} + \frac{\partial P^J}{\partial n_H}\left(\frac{-D^J\dfrac{\partial P^J}{\partial a_{J'}^J}\dfrac{\partial P^K}{\partial n_F}}{\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}}\right) + \frac{\partial P^J}{\partial n_F}\left(\frac{D^J\dfrac{\partial P^J}{\partial a_{J'}^J}\dfrac{\partial P^K}{\partial n_H}}{\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}}\right) \\
&= \frac{\partial P^J}{\partial a_{J'}^J}\left[1 - \left(\frac{\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}}{\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}}\right)\right] = 0, \\
\frac{dP^K}{da_{J'}^J} &= \frac{\partial P^K}{\partial n_H}\frac{dn_H}{da_{J'}^J} + \frac{\partial P^K}{\partial n_F}\frac{dn_F}{da_{J'}^J} \\
&= \frac{\partial P^K}{\partial n_H}\left(\frac{-D^J\dfrac{\partial P^J}{\partial a_{J'}^J}\dfrac{\partial P^{J'}}{\partial n_F}}{\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}}\right) + \frac{\partial P^K}{\partial n_F}\left(\frac{D^J\dfrac{\partial P^J}{\partial a_{J'}^J}\dfrac{\partial P^K}{\partial n_H}}{\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}}\right) \\
&= \frac{\partial P^J}{\partial a_{J'}^J}\left(\frac{\dfrac{\partial P^K}{\partial n_F}\dfrac{\partial P^K}{\partial n_H} - \dfrac{\partial P^K}{\partial n_F}\dfrac{\partial P^K}{\partial n_H}}{\dfrac{\partial P^H}{\partial n_H}\dfrac{\partial P^F}{\partial n_F} - \dfrac{\partial P^H}{\partial n_F}\dfrac{\partial P^F}{\partial n_H}}\right) = 0.
\end{aligned}$$

A.4. *Proof of Proposition 1*

We begin with the expression for world welfare:

$$\begin{aligned}
\Omega = \sum_J L^J - N^H \log(P^H) - N^F \log(P^F) + \frac{\sigma}{\sigma - 1} z^H n_F N^H \tilde{c}_F^H \\
+ \frac{\sigma}{\sigma - 1} z^F n_H N^F \tilde{c}_H^F - N^H \frac{s^H}{1 - s^H} - N^F \frac{s^F}{1 - s^F}.
\end{aligned}$$

We first prove that global efficiency requires $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$. We then turn to the efficiency of $\bar{a} = \widetilde{\boldsymbol{a}}$.

Evaluating the derivatives of Ω with respect to net trade taxes and consumption subsidies at the levels $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$ yield

$$\begin{aligned}
\frac{d\Omega}{dz^H}\bigg|_{z^H=z^F=0,s^H=s^F=1/\sigma} &= -\frac{N^H}{P^H}\frac{dP^H}{dz^H} - \frac{N^F}{P^F}\frac{dP^F}{dz^H} + qn_F N^H \tilde{c}_F^H, \\
\frac{d\Omega}{dz^F}\bigg|_{z^H=z^F=0,s^H=s^F=1/\sigma} &= -\frac{N^H}{P^H}\frac{dP^H}{dz^F} - \frac{N^F}{P^F}\frac{dP^F}{dz^F} + qn_H N^F \tilde{c}_H^F, \\
\frac{d\Omega}{ds^H}\bigg|_{z^H=z^F=0,s^H=s^F=1/\sigma} &= -\frac{N^H}{P^H}\frac{dP^H}{ds^H} - \frac{N^F}{P^F}\frac{dP^F}{ds^H} - N^H\left(\frac{\sigma}{\sigma-1}\right)^2, \\
\frac{d\Omega}{ds^F}\bigg|_{z^H=z^F=0,s^H=s^F=1/\sigma} &= -\frac{N^H}{P^H}\frac{dP^H}{ds^F} - \frac{N^F}{P^F}\frac{dP^F}{ds^F} - N^F\left(\frac{\sigma}{\sigma-1}\right)^2.
\end{aligned}$$

To establish that $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$ are efficient, we show that $\frac{d\Omega}{dz^H}|_{z^H=z^F=0,s^H=s^F=1/\sigma} = 0$ and $\frac{d\Omega}{ds^F}|_{z^H=z^F=0,s^H=s^F=1/\sigma} = 0$, with $\frac{d\Omega}{dz^F}|_{z^H=z^F=0,s^H=s^F=1/\sigma} = 0$ and $\frac{d\Omega}{ds^F}|_{z^H=z^F=0,s^H=s^F=1/\sigma} = 0$ then following under analogous arguments.

Efficient Net Trade Taxes $z^H = z^F = 0$. We first show that $\frac{d\Omega}{dz^H}|_{z^H=z^F=0,s^H=s^F=1/\sigma} = 0$, noting that p_H^H, p_H^F, and p_F^F are independent of z^H with z^H impacting directly only the price of the foreign brand in the home market p_F^H. Total per-capita spending on differentiated goods equals one, so we have

$$n_H p_H^H c_H^H + n_F p_F^H c_F^H = 1; \qquad n_H p_H^F c_H^F + n_F p_F^F c_F^F = 1. \tag{A6}$$

Using (A6), we can then write

$$\begin{aligned}
-\frac{N^H}{P^H}\frac{dP^H}{dz^H} &= \left(\frac{1}{\sigma-1}\right)N^H\left[p_H^H c_H^H \frac{dn_H}{dz^H} + p_F^H c_F^H \frac{dn_F}{dz^H}\right] - n_F N^H \tilde{c}_F^H, \\
-\frac{N^F}{P^F}\frac{dP^F}{dz^H} &= \left(\frac{1}{\sigma-1}\right)N^F\left[p_H^F c_H^F \frac{dn_H}{dz^H} + p_F^F c_F^F \frac{dn_F}{dz^H}\right],
\end{aligned}$$

and, therefore,

$$\begin{aligned}
&\frac{d\Omega}{dz^H}\bigg|_{z^H=z^F=0,s^H=s^F=1/\sigma} \\
&\quad = \frac{1}{\sigma-1}\left[\left(p_H^H N^H c_H^H + p_H^F N^F c_H^F\right)\frac{dn_H}{dz^H} + \left(p_F^F N^F c_F^F + p_F^H N^H c_F^H\right)\frac{dn_F}{dz^H}\right] + \frac{1}{\sigma-1} n_F N^H \tilde{c}_F^H.
\end{aligned}$$

When $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$, we also have

$$p_J^J = \lambda_J^J, \qquad p_K^J = (1+\nu)\lambda_K^J, \tag{A7}$$

and, therefore,

$$\begin{aligned}&\left.\frac{d\Omega}{dz^H}\right|_{z^H=z^F=0,s^H=s^F=1/\sigma}\\&\quad=\frac{1}{\sigma-1}\left\{\left[N^H\tilde{c}_H^H+(1+\nu)N^F\tilde{c}_H^F\right]\frac{dn_H}{dz^H}+\left[N^F\tilde{c}_F^F+(1+\nu)N^H\tilde{c}_F^H\right]\frac{dn_F}{dz^H}\right\}\\&\quad\quad+n_FN^H\tilde{c}_F^H.\end{aligned}\tag{A8}$$

For (A8) to equal zero requires, beginning from $z^H=z^F=0$ and $s^H=s^F=1/\sigma$, a small increase in the net tariff on home imports generates additional tariff revenue (in the amount $n_FN^H\tilde{c}_F^H$) that is just offset by the loss in differentiated goods production associated with the induced entry and exit (in the amount $[N^H\tilde{c}_H^H+(1+\nu)N^F\tilde{c}_H^F]\frac{dn_H}{dz^H}+[N^F\tilde{c}_F^F+(1+\nu)N^H\tilde{c}_F^H]\frac{dn_F}{dz^H}$).

To derive expressions for $\frac{dn_H}{dz^H}$ and $\frac{dn_F}{dz^H}$, we use the home and foreign zero-profit conditions

$$\begin{aligned}&\frac{1}{\sigma-1}\left[N^H\tilde{c}_H^H\left(P^H\left(z^H,n_H,n_F\right)\right)+(1+\nu)N^F\tilde{c}_H^F\left(P^F(n_H,n_F)\right)\right]\\&\quad=\Phi\left(\left|a_H^H-a_H^F\right|\right),\end{aligned}\tag{A9}$$

$$\begin{aligned}&\frac{1}{\sigma-1}\left[N^F\tilde{c}_F^F\left(P^F(n_H,n_F)\right)+(1+\nu)N^H\tilde{c}_F^H\left(P_F^H\left(z^H\right),P^H\left(z^H,n_H,n_F\right)\right)\right]\\&\quad=\Phi\left(\left|a_F^H-a_F^F\right|\right),\end{aligned}\tag{A10}$$

where we have suppressed the dependency of consumption and price indices on product characteristics and have made explicit the direct dependency of consumption, prices and price indices on z^H. Totally differentiating (A9) and (A10) yield

$$\frac{dn_H}{dz^H}=\frac{\begin{pmatrix}(1+\nu)\dfrac{d\tilde{c}_F^H}{dp_F^H}\dfrac{dp_F^H}{dz^H}\left[\left(\dfrac{N^H}{N^F}\right)\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{dP^H}{dn_F}+(1+\nu)\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{dP^F}{dn_F}\right]\\-\dfrac{\partial P^H}{\partial z^H}\dfrac{dP^F}{dn_F}\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]\end{pmatrix}}{\left[\dfrac{dP^H}{dn_H}\dfrac{dP^F}{dn_F}-\dfrac{dP^H}{dn_F}\dfrac{dP^F}{dn_H}\right]\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]},\tag{A11}$$

and

$$\frac{dn_F}{dz^H}=\frac{\begin{pmatrix}-(1+\nu)\dfrac{d\tilde{c}_F^H}{dp_F^H}\dfrac{dp_F^H}{dz^H}\left[\left(\dfrac{N^H}{N^F}\right)\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{dP^H}{dn_H}+(1+\nu)\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{dP^F}{dn_H}\right]\\+\dfrac{\partial P^H}{\partial z^H}\dfrac{dP^F}{dn_H}\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]\end{pmatrix}}{\left[\dfrac{dP^H}{dn_H}\dfrac{dP^F}{dn_F}-\dfrac{dP^H}{dn_F}\dfrac{dP^F}{dn_H}\right]\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]}.\tag{A12}$$

Substituting (A11) and (A12) back into (A8) and rearranging then yield

$$\begin{aligned}
&\left.\frac{d\Omega}{dz^H}\right|_{z^H=z^F=0,s^H=s^F=1/\sigma}=0\\
&\Leftrightarrow\quad \left[N^H\tilde{c}_H^H+(1+\nu)N^F\tilde{c}_H^F\right]\\
&\quad\times\left\{(1+\nu)\frac{d\tilde{c}_F^H}{dp_F^H}\frac{dp_F^H}{dz^H}\left[\left(\frac{N^H}{N^F}\right)\frac{d\tilde{c}_H^H}{dP^H}\frac{dP^H}{dn_F}+(1+\nu)\frac{d\tilde{c}_H^F}{dP^F}\frac{dP^F}{dn_F}\right]\right.\\
&\quad\left.-\frac{\partial P^H}{\partial z^H}\frac{dP^F}{dn_F}\left[\frac{d\tilde{c}_H^H}{dP^H}\frac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\frac{d\tilde{c}_H^F}{dP^F}\frac{d\tilde{c}_F^H}{dP^H}\right]\right\}\\
&\quad-\left[N^F\tilde{c}_F^F+(1+\nu)N^H\tilde{c}_F^H\right]\\
&\quad\times\left\{(1+\nu)\frac{d\tilde{c}_F^H}{dp_F^H}\frac{dp_F^H}{dz^H}\left[\left(\frac{N^H}{N^F}\right)\frac{d\tilde{c}_H^H}{dP^H}\frac{dP^H}{dn_H}+(1+\nu)\frac{d\tilde{c}_H^F}{dP^F}\frac{dP^F}{dn_H}\right]\right.\\
&\quad\left.-\frac{\partial P^H}{\partial z^H}\frac{dP^F}{dn_H}\left[\frac{d\tilde{c}_H^H}{dP^H}\frac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\frac{d\tilde{c}_H^F}{dP^F}\frac{d\tilde{c}_F^H}{dP^H}\right]\right\}\\
&\quad+n_FN^H\tilde{c}_F^H\left[\frac{dP^H}{dn_H}\frac{dP^F}{dn_F}-\frac{dP^H}{dn_F}\frac{dP^F}{dn_H}\right]\left[\frac{d\tilde{c}_H^H}{dP^H}\frac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\frac{d\tilde{c}_H^F}{dP^F}\frac{d\tilde{c}_F^H}{dP^H}\right]=0.
\end{aligned}$$

We now make use of the following:

$$\begin{aligned}
&\frac{d\tilde{c}_H^H}{dP^H}=(\sigma-1)\frac{\tilde{c}_H^H}{P^H};\qquad \frac{d\tilde{c}_H^F}{dP^F}=(\sigma-1)\frac{\tilde{c}_H^F}{P^F};\qquad \frac{d\tilde{c}_F^F}{dP^F}=(\sigma-1)\frac{\tilde{c}_F^F}{P^F};\\
&\frac{d\tilde{c}_F^H}{dP^H}=(\sigma-1)\frac{\tilde{c}_F^H}{P^H};\qquad \frac{d\tilde{c}_F^H}{dp_F^H}=-\sigma\frac{\tilde{c}_F^H}{p_F^H},
\end{aligned}$$

and also

$$\begin{aligned}
&\frac{dP^H}{dn_F}=\frac{1}{1-\sigma}P^Hp_F^Hc_F^H;\qquad \frac{dP^H}{dn_H}=\frac{1}{1-\sigma}P^Hp_H^Hc_H^H;\\
&\frac{dP^F}{dn_F}=\frac{1}{1-\sigma}P^Fp_F^Fc_F^F;\qquad \frac{dP^F}{dn_H}=\frac{1}{1-\sigma}P^Fp_H^Fc_H^F;\\
&\frac{\partial P^H}{\partial z^H}=P^Hn_F\tilde{c}_F^H;\qquad \frac{dp_F^H}{dz^H}=\lambda_F^H.
\end{aligned}$$

With this, the above can be simplified to

$$\left.\frac{d\Omega}{dz^H}\right|_{z^H=z^F=0,s^H=s^F=1/\sigma}=0\quad\Leftrightarrow\quad \tilde{c}_H^H\left[n_F\tilde{c}_F^F-1\right]-(1+\nu)\tilde{c}_H^F\left[n_F(1+\nu)\tilde{c}_F^H-1\right]=0.$$

But using (A6) and (A7), we then have

$$\tilde{c}_H^H\left[n_F\tilde{c}_F^F-1\right]-(1+\nu)\tilde{c}_H^F\left[n_F(1+\nu)\tilde{c}_F^H-1\right]=-\tilde{c}_H^Hn_H(1+\nu)\tilde{c}_H^F+(1+\nu)\tilde{c}_H^Fn_H\tilde{c}_H^H=0.$$

This establishes that global efficiency requires $z^H=z^F=0$.

Efficient Consumption Subsidies $s^H = s^F = 1/\sigma$. We next show that $\frac{d\Omega}{ds^F}|_{z^H=z^F=0,s^H=s^F=1/\sigma} = 0$, noting that p_H^F and p_F^F are independent of s^H with s^H impacting directly only the prices of the home and the foreign brand in the home market, p_H^H and p_F^H. Again using (A6), we can then write

$$-\frac{N^H}{P^H}\frac{dP^H}{ds^H} = \left(\frac{1}{\sigma-1}\right)N^H\left[p_H^H c_H^H \frac{dn_H}{ds^H} + p_F^H c_F^H \frac{dn_F}{ds^H}\right] + \left(\frac{\sigma}{\sigma-1}\right)n_H N^H \tilde{c}_H^H + \left(\frac{\sigma}{\sigma-1}\right)n_F(1+\nu)N^H \tilde{c}_F^H,$$

$$-\frac{N^F}{P^F}\frac{dP^F}{ds^H} = \left(\frac{1}{\sigma-1}\right)N^F\left[p_H^F c_H^F \frac{dn_H}{ds^H} + p_F^F c_F^F \frac{dn_F}{ds^H}\right],$$

and, therefore,

$$\left.\frac{d\Omega}{ds^H}\right|_{z^H=0=z^F,s^H=\frac{1}{\sigma}=s^F} = \left(\frac{1}{\sigma-1}\right)\left[\left[p_H^H N^H c_H^H + p_H^F N^F c_H^F\right]\frac{dn_H}{ds^H} + \left[p_F^F N^F c_F^F + p_F^H N^H c_F^H\right]\frac{dn_F}{ds^H}\right] + \left(\frac{\sigma}{\sigma-1}\right)\left[n_H N^H \tilde{c}_H^H + n_F(1+\nu)N^H \tilde{c}_F^H\right] - N^H\left(\frac{\sigma}{\sigma-1}\right)^2.$$

Using (A7) and (A6) then delivers

$$\left.\frac{d\Omega}{ds^H}\right|_{z^H=0=z^F,s^H=\frac{1}{\sigma}=s^F} = \left(\frac{1}{\sigma-1}\right)\left[\left[N^H \tilde{c}_H^H + (1+\nu)N^F \tilde{c}_H^F\right]\frac{dn_H}{ds^H} + \left[N^F \tilde{c}_F^F + (1+\nu)N^H \tilde{c}_F^H\right]\frac{dn_F}{ds^H} - N^H\left(\frac{\sigma}{\sigma-1}\right)\right]. \tag{A13}$$

Our goal is to show that the right-hand side of (A13) is equal to zero.

To derive expressions for $\frac{dn_H}{ds^H}$ and $\frac{dn_F}{ds^H}$, we again use the home and foreign zero-profit conditions, which we now write as

$$\left(\frac{1}{\sigma-1}\right)\left[N^H \tilde{c}_H^H\left(p_H^H(s^H), P^H(s^H, n_H, n_F)\right) + (1+\nu)N^F \tilde{c}_H^F\left(P^F(n_H, n_F)\right)\right] = \Phi\left(\left|a_H^H - a_H^F\right|\right), \tag{A14}$$

$$\left(\frac{1}{\sigma-1}\right)\left[N^F \tilde{c}_F^F\left(P^F(n_H, n_F)\right) + (1+\nu)N^H \tilde{c}_F^H\left(P_F^H(s^H), P^H(s^H, n_H, n_F)\right)\right] = \Phi\left(\left|a_F^H - a_F^F\right|\right). \tag{A15}$$

Totally differentiating (A14) and (A15) yield

$$\frac{dn_H}{ds^H}=\frac{\left(\begin{array}{l}(1+\nu)\dfrac{d\tilde{c}_F^H}{dp_F^H}\dfrac{dp_F^H}{ds^H}\left[\left(\dfrac{N^H}{N^F}\right)\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{dP^H}{dn_F}+(1+\nu)\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{dP^F}{dn_F}\right]\\ \quad-\dfrac{\partial P^H}{\partial s^H}\dfrac{dP^F}{dn_F}\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]\end{array}\right)}{\left[\dfrac{dP^H}{dn_H}\dfrac{dP^F}{dn_F}-\dfrac{dP^H}{dn_F}\dfrac{dP^F}{dn_H}\right]\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]}$$
$$-\frac{\dfrac{d\tilde{c}_H^H}{dp_H^H}\dfrac{dp_H^H}{ds^H}\left[\dfrac{d\tilde{c}_F^F}{dP^F}\dfrac{dP^F}{dn_F}+(1+\nu)\left(\dfrac{N^H}{N^F}\right)\dfrac{d\tilde{c}_F^H}{dP^H}\dfrac{dP^H}{dn_F}\right]}{\left[\dfrac{dP^H}{dn_H}\dfrac{dP^F}{dn_F}-\dfrac{dP^H}{dn_F}\dfrac{dP^F}{dn_H}\right]\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]},$$

and

$$\frac{dn_F}{ds^H}=\frac{\left(\begin{array}{l}-(1+\nu)\dfrac{d\tilde{c}_F^H}{dp_F^H}\dfrac{dp_F^H}{ds^H}\left[\left(\dfrac{N^H}{N^F}\right)\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{dP^H}{dn_H}+(1+\nu)\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{dP^F}{dn_H}\right]\\ \quad+\dfrac{\partial P^H}{\partial s^H}\dfrac{dP^F}{dn_H}\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]\end{array}\right)}{\left[\dfrac{dP^H}{dn_H}\dfrac{dP^F}{dn_F}-\dfrac{dP^H}{dn_F}\dfrac{dP^F}{dn_H}\right]\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]}$$
$$+\frac{\dfrac{d\tilde{c}_H^H}{dp_H^H}\dfrac{dp_H^H}{ds^H}\left[\dfrac{d\tilde{c}_F^F}{dP^F}\dfrac{dP^F}{dn_H}+(1+\nu)\left(\dfrac{N^H}{N^F}\right)\dfrac{d\tilde{c}_F^H}{dP^H}\dfrac{dP^H}{dn_H}\right]}{\left[\dfrac{dP^H}{dn_H}\dfrac{dP^F}{dn_F}-\dfrac{dP^H}{dn_F}\dfrac{dP^F}{dn_H}\right]\left[\dfrac{d\tilde{c}_H^H}{dP^H}\dfrac{d\tilde{c}_F^F}{dP^F}-(1+\nu)^2\dfrac{d\tilde{c}_H^F}{dP^F}\dfrac{d\tilde{c}_F^H}{dP^H}\right]}.$$

Substituting these expressions back into (A13), using the price derivatives recorded above and in addition noting that

$$\frac{\partial P^H}{\partial s^H}=-P^H\frac{\sigma}{\sigma-1};\qquad \frac{dp_F^H}{ds^H}=-q_F^H(1+\nu);\qquad \frac{dp_H^H}{ds^H}=-q_H^H,$$

and using as well the expressions for efficient prices in (A7), we then have

$$\left.\frac{d\Omega}{ds^H}\right|_{z^H=z^F=0,s^H=s^F=1/\sigma}=0.$$

This establishes that global efficiency requires $s^H=s^F=1/\sigma$. Notice that this argument does not require product characteristics to be set at the efficient level, only that $z^H=z^F=0$.

Efficient Employment Subsidies. While we do not introduce employment subsidies into our formal analysis, we have noted in the text that in our setting the global social planner has a degree of freedom when choosing between a consumption subsidy and an employment subsidy for addressing the monopoly markup distortion. Specifically, we claimed that, with s denoting the (common) subsidy for consumption of differentiated products and ω denoting the (common) rate of employment subsidy, efficiency is achieved by any combination of s and ω that satisfies $(1-s)(1-\omega)=1-1/\sigma$. With ω set to 0 we have just established that efficiency implies $s=1/\sigma$. We now argue that with s set to 0 efficiency can be equally well attained by setting $\omega=1/\sigma$.

To see this, note that the joint global revenue needed for a home-country employment subsidy at rate ω_H and foreign-country employment subsidy at rate ω_F is given by

$$N^H\left[\left(\frac{\omega_H}{(1-\omega_H)(1-s^H)}\right)n_H p_H^H c_H^H+\left(\frac{\omega_F(1+\nu)}{(1-\omega_F)(1-s^H)\iota_F}\right)n_F p_F^H c_F^H\right]$$
$$+N^F\left[\left(\frac{\omega_F}{(1-\omega_F)(1-s^F)}\right)n_F p_F^F c_F^F+\left(\frac{\omega_H(1+\nu)}{(1-\omega_H)(1-s^F)\iota_H}\right)n_H p_H^F c_H^F\right]$$

which, with $s^H=s^F\equiv s=0$ and $z^H=0=z^F$ and when $\omega_H=\omega_F\equiv\omega$, collapses to $[N^H+N^F](\frac{\omega}{(1-\omega)})$. A comparison with the expression for world welfare in (14) in the body of the paper then confirms that the first best can be achieved with $s=1/\sigma$ or with $\omega=1/\sigma$, or more generally with any combination of s and ω that satisfies $(1-s)(1-\omega)=1-1/\sigma$.

Efficient Standards $\bar{a}=\tilde{\boldsymbol{a}}$. We next prove that global efficiency is achieved when we also have $\bar{a}=\tilde{\boldsymbol{a}}$. With net trade taxes and consumption subsidies set at their efficient levels $z^H=z^F=0$ and $s^H=s^F=\frac{1}{\sigma}$, the expression for world welfare becomes

$$\Omega=\sum_J L^J-N^H\log(P^H)-N^F\log(P^F)-\frac{N^H+N^F}{\sigma-1}.$$

The first-order conditions are

$$\frac{d\Omega}{da_K^J}=-\frac{N^H}{P^H}\frac{dP^H}{da_K^J}-\frac{N^F}{P^F}\frac{dP^F}{da_K^J}=0\quad\text{for all } J\in\{H,F\}\text{ and } K\in\{H,F\},$$

and by Lemma 3 these conditions are satisfied at the profit-maximizing characteristics choices. This establishes that the first-order conditions for global efficiency are satisfied at the profit maximizing characteristics choices $\tilde{\boldsymbol{a}}$.

Second-Order Conditions. We now consider in detail the second-order conditions for efficiency, focusing on the planner's choice of standards. To illustrate why this choice raises particular questions about the second-order conditions, we first derive the slope of the world welfare contours in Figure 1. With net tariffs and consumption subsidies fixed at the efficient levels, world welfare is given by

$$\Omega=\sum_J L^J-N^H\log(P^H)-N^F\log(P^F)-N^H\frac{1}{\sigma-1}-N^F\frac{1}{\sigma-1}.$$

Using $P^H \equiv (\frac{[N^H(P^H)^{\sigma-1}]}{N^H})^{\frac{1}{\sigma-1}}$, $P^F \equiv (\frac{[N^F(P^F)^{\sigma-1}]}{N^F})^{\frac{1}{\sigma-1}}$, we now transform the expression for world welfare to the equivalent expression

$$\Omega = \sum_J L^J - N^H \log\left(\left(\frac{[N^H(P^H)^{\sigma-1}]}{N^H}\right)^{\frac{1}{\sigma-1}}\right) - N^F \log\left(\left(\frac{[N^F(P^F)^{\sigma-1}]}{N^F}\right)^{\frac{1}{\sigma-1}}\right) - N^H\frac{1}{\sigma-1} - N^F\frac{1}{\sigma-1},$$

or

$$\Omega = \sum_J L^J - \frac{1}{\sigma-1}\{N^H\log([N^H(P^H)^{\sigma-1}]) + N^F\log([N^F(P^F)^{\sigma-1}]) - N^H[\log(N^H)-1] - N^F[\log(N^F)-1]\}.$$

Totally differentiating yields

$$\left.\frac{d[N^F(P^F)^{\sigma-1}]}{d[N^H(P^H)^{\sigma-1}]}\right|_{d\Omega=0} = -\left(\frac{P^H}{P^F}\right)^{1-\sigma}. \tag{A16}$$

According to (A16), for $\sigma > 1$, the slope is flatter than -1 to the right of the N^F/N^H ray (where $P^H > P^F$) and it is steeper than -1 to the left of the N^F/N^H ray (where $P^H < P^F$). Figure 1 depicts the world welfare indifference curve passing through the point labeled Q, which corresponds to the equilibrium under profit-maximizing choices of product characteristics when net tariffs and consumption subsidies are set at the efficient levels.

This raises the question whether the second-order conditions for the planner's choice of standards are globally met. Specifically, we seek conditions under which the point labeled Q in Figure 1 is preferred to the extremes where either the planner sets product attributes to maximize global welfare when $n_F = 0$ or $n_H = 0$.

To explore this question, we first define the following variables:

$$Y \equiv [N^F(P^F)^{\sigma-1}]; \qquad X \equiv [N^H(P^H)^{\sigma-1}],$$
$$Z_H \equiv (\sigma-1)\Phi(|a_H^H - a_H^F|); \qquad Z_F \equiv (\sigma-1)\Phi(|a_F^H - a_F^F|),$$
$$\mu_H \equiv (1+\nu)^{\sigma-1}\left(\frac{A_H^H}{A_H^F}\right)^{\sigma}\left(\frac{\lambda_H^H}{\lambda_H^F}\right)^{1-\sigma} > 1; \qquad \mu_F \equiv (1+\nu)^{\sigma-1}\left(\frac{A_F^F}{A_F^H}\right)^{\sigma}\left(\frac{\lambda_F^F}{\lambda_F^H}\right)^{1-\sigma} > 1,$$
$$B_H \equiv \frac{Z_H}{(1+\nu)^{1-\sigma}(A_H^F)^{\sigma}(\lambda_H^F)^{1-\sigma}}; \qquad B_F \equiv \frac{Z_F}{(A_F^F)^{\sigma}(\lambda_F^F)^{1-\sigma}}.$$

Then we have

$$\pi_H = 0: \quad Y = B_H - \mu_H X \quad \text{and} \quad \pi_F = 0: \quad Y = B_F - \frac{1}{\mu_F}X.$$

The point Q in Figure 1 is defined by $\pi_H = 0$ and $\pi_F = 0$ yielding

$$X = \frac{B_H - B_F}{\mu_H - \dfrac{1}{\mu_F}}; \qquad Y = \frac{\mu_H B_F - \dfrac{1}{\mu_F} B_H}{\mu_H - \dfrac{1}{\mu_F}},$$

where these expressions are evaluated at the profit-maximizing product characteristic choices for both home and foreign firms. Notice that we have $\mu_H > \frac{1}{\mu_F}$, so we must have $B_H > B_F$ for $X > 0$ at the point Q.

Now let μ_H' be the slope of the home zero profit line and and B_H' be its intercept when the planner sets the attributes $\bar{a}_H^H$ and $\bar{a}_H^F$ for home produced goods *at the levels that maximize global welfare when* $n_F = 0$. Note that $Y = \mu_H'(\frac{N^F}{N^H})X$ is the equation that satisfies $n_F = 0$ in these circumstances. We solve for the corresponding $Q_F' = (X', Y')$, where

$$X' = \frac{B_H'}{\mu_H'\left(1 + \dfrac{N^F}{N^H}\right)}; \qquad Y' = \frac{B_H'}{1 + \dfrac{N^H}{N^F}}.$$

Global welfare at this Q_F' is

$$\Omega_{Q_F'} = -\left(N^H + N^F\right)\log B_H' + N^H \log \mu_H' + N^H \log\left(1 + \frac{N^F}{N^H}\right) + \log N^F \log\left(1 + \frac{N^H}{N^F}\right).$$

Suppose that when the planner sets $z^H = 0$, it is possible for her to find a a_F^F and a_F^H with $a_F^F < a_F^H$, *while leaving the standards for home firms as above*, such that when $n_F > 0$ firms in both countries earn zero profits. Take an arbitrary pair of such standards, $\check{a}_F^F$ and $\check{a}_F^H$ and call the resulting point $\check{Q} = (\check{X}, \check{Y})$. Notice, of course, that these standards are not optimal for the planner when firms are active in both countries. At the point of intersection of the zero profit lines,

$$\check{X} = \frac{B_H' - \check{B}_F}{\mu_H' - \dfrac{1}{\check{\mu}_F}}, \qquad \check{Y} = \frac{\mu_H' \check{B}_F - \dfrac{1}{\check{\mu}_F} B_H}{\mu_H' - \dfrac{1}{\check{\mu}_F}}.$$

Note that the B_H' and μ_H' are the same as above (since we have not changed the standards facing home firms), while we use a check above the B_F and μ_F to remind ourselves that these are associated with the arbitrary standards, $\check{a}_F^F$ and $\check{a}_F^H$. The resulting global welfare is

$$\Omega_{\check{Q}} = -N^H \log\left(B_H' - \check{B}_F\right) - N^F \log\left(\mu_H' \check{B}_F - \frac{1}{\check{\mu}_F} B_H'\right) + \left(N^H + N^F\right)\log\left(\mu_H' - \frac{1}{\check{\mu}_F}\right).$$

The difference is

$$\Omega_{\check{Q}} - \Omega_{Q_F'} = N^H \log \frac{\mu_H' B_H' - B_H'/\check{\mu}_F}{\mu_H' B_H' - \mu_H' \check{B}_F} + N^F \log \frac{\mu_H' B_H' - B_H'/\check{\mu}_F}{\mu_H' \check{B}_F - B_H'/\check{\mu}_F}$$
$$- N^H \log\left(1 + \frac{N^F}{N^H}\right) - N^F \log\left(1 + \frac{N^H}{N^F}\right)$$

$$= N^H \log \frac{D_1 + D_2}{D_1} + N^F \log \frac{D_1 + D_2}{D_2}$$
$$- N^H \log\left(1 + \frac{N^F}{N^H}\right) - N^F \log\left(1 + \frac{N^H}{N^F}\right),$$

where $D_1 \equiv \mu'_H B'_H - \mu'_H \check{B}_F > 0$ and $D_2 \equiv \mu'_H \check{B}_F - B'_H / \check{\mu}_F > 0$.

To show $\Omega_{\check{Q}} - \Omega_{Q'_F} \geq 0$, requires

$$(N^H)^{N^H} (N^F)^{N^F} (D_1 + D_2)^{N^H + N^F} - (N^H + N^F)^{N^H + N^F} (D_1)^{N^H} (D_2)^{N^F} \geq 0.$$

Now normalize so that $N^H + N^F = 2$ and rearrange to get

$$(N^H)^{N^H} (2 - N^H)^{2 - N^H} - 4\left(\frac{D_1}{D_1 + D_2}\right)^{N^H} \left(\frac{D_2}{D_1 + D_2}\right)^{2 - N^H} \geq 0.$$

Note that $(D_1)^{N^H} (1 - D_1)^{2 - N^H}$ is maximized at $D_1/(1 - D_1) = N^H/(2 - N^H) \Rightarrow \frac{D_1}{D_1 + D_2} = N^H/2$ and $\frac{D_2}{D_1 + D_2} = (2 - N^H)/2$. So the expression above is greater than or equal to

$$(N^H)^{N^H} (2 - N^H)^{2 - N^H} - 4\left(\frac{N^H}{2}\right)^{N^H} \left(\frac{2 - N^H}{2}\right)^{2 - N^H} = 0.$$

So we have proven that $\Omega_{\check{Q}} - \Omega_{Q'_F} \geq 0$, that is, the planner prefers $\check{Q}$ to Q'_F for arbitrary $\check{a}^F_F$ and $\check{a}^H_F$ such that $n_F > 0$ and all firms break even. But Q is the social optimum when all firms are active. Clearly, $\Omega_Q \geq \Omega_{\check{Q}}$. So

$$\Omega_Q - \Omega_{Q'_F} \geq 0.$$

An analogous argument shows that Q also welfare-dominates an extreme where the planner sets attributes to maximize global welfare when $n_H = 0$.

Unilateral Incentives to Deviate From Efficient Consumption Subsidies. We next show that there is no need for an NTA that stipulates zero net trade taxes on all goods and covers product standards to also cover consumption subsidies provided that National Treatment (NT) is imposed, as we observed in the text. To this end, we position the home and foreign consumption subsidies initially at the efficient level $1/\sigma$, and ask whether a country has a unilateral incentive to deviate (with trade taxes and standards all held to efficient levels). A first observation is that the world prices are functions of trade taxes but independent of consumption subsidies in this model, so there is no need to negotiate over consumption subsidies for purposes of eliminating terms-of-trade manipulation. Hence we need only consider the incentive to use consumption subsidies for purposes of delocation.

With net trade taxes set to zero, the home country's choice of consumption subsidy s^H will impact p^H_H and p^H_F according to $p^H_H = (1 - s^H) q^H_H$; $p^H_F = (1 - s^H)(1 + \nu) q^H_F$, and similarly the foreign country's choice of consumption subsidy s^F will impact p^F_F and p^F_H according to $p^F_F = (1 - s^F) q^F_F$; $p^F_H = (1 - s^F)(1 + \nu) q^F_H$.

Focusing on the home country choice of s^H and beginning from the efficient point, in the context of Figure 1 a slight increase in s^H will shift both the home zero profit line and the foreign zero profit in (toward the y-axis). Totally differentiating the home zero profit line with respect to s^H and $(P^H)^{\sigma-1}$ yields $\frac{d[N^H(P^H)^{\sigma-1}]}{ds^H}|_{\pi_H=0} = \frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)}$. Hence, the home zero profit line shifts in (toward the y-axis in Figure 1) with a small increase in s^H by the amount $\frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)}$. But totally differentiating the foreign zero profit line with respect to s^H and $(P^H)^{\sigma-1}$ yields $\frac{d[N^H(P^H)^{\sigma-1}]}{ds^H}|_{\pi_F=0} = \frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)}$. Hence, the foreign zero profit line shifts in with a small increase in s^H by the exact same amount $\frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)}$. This implies that $(P^F)^{\sigma-1}$ is left unchanged by the increase in s^H, and hence implies that foreign welfare (which is given by $\Omega^F = L^F - N^F\log(P^F) - N^F\frac{1}{\sigma-1}$) is unaffected by the small increase in s^H. But given that s^H was initially positioned at the efficient level, it is impossible for home welfare to rise if foreign welfare does not fall. We may thus conclude that the home country cannot improve its welfare with a small unilateral deviation from $s^H = \frac{1}{\sigma}$. And with

$$\left.\frac{d[N^H(P^H)^{\sigma-1}]}{ds^H}\right|_{\pi_H=0} = \frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)} = \left.\frac{d[N^H(P^H)^{\sigma-1}]}{ds^H}\right|_{\pi_F=0}$$

starting from any level of s^H, it is easy to see that the same argument applies globally for unilateral deviations from $s^H = \frac{1}{\sigma}$ of any size. Therefore, we may conclude that in the presence of NT, an NTA does not need to cover the consumption subsidies for each country.

Unilateral Incentives to Deviate From Efficient Employment Subsidies. Finally, we noted in the text that, unlike with consumption subsidies, there *is* a unilateral incentive to deviate from efficient policies with a small employment subsidy, implying that employment subsidies must be constrained in an efficient NTA. To see this, let us begin from free trade and efficient consumption subsidies and no employment subsidy, plus efficient standards, and consider the home country welfare, which is given by $\Omega^H(\boldsymbol{a}^E, \boldsymbol{p}^E) = L^H - N^H\log P^H(\boldsymbol{a}^E, \boldsymbol{p}^E) - N^H\frac{1}{\sigma-1}$. Suppose, beginning from these efficient policies, the home country were to introduce a small employment subsidy. The revenue consequences of a sufficiently small employment subsidy would be inconsequential (second order); but a small employment subsidy would increase the profits of home firms and shift the home zero profit line in (toward the y-axis in Figure 1) while leaving the profits of foreign firms unchanged and thereby leaving the foreign zero profit line unaffected. This implies that P^H would fall (while P^F would rise), yielding a first-order increase in home welfare Ω^H. Hence, and distinct from consumption subsidies, countries have a unilateral incentive to deviate from efficient policies with employment subsidies.[1]

An Efficient Agreement Does not Need to Specify a Consumption Subsidy. As long as the choice of consumption subsidy satisfies national treatment (and net trade barriers are zero), countries will unilaterally chose the efficient consumption subsidy for any fixed set of product characteristics. For any fixed set of characteristics, welfare is given by

$$\Omega = -\log P^H - \log P^F + z^H n_F \tilde{c}_F^H + z^F n_H \tilde{c}_H^F - s^H G^H - s^F G^F,$$

[1]We have illustrated the incentive to defect from efficient policies with employment subsidies by focusing on the delocation incentives that exist with such policies, but there are also terms-of-trade incentives that arise with employment subsidies and that are absent with consumption subsidies in this model.

where

$$G^J = n_J \frac{\sigma}{\sigma - 1} \tilde{c}_J^J + n_K \iota_J \frac{\sigma}{\sigma - 1} \tilde{c}_K^J = \frac{C_D^J P^J}{1 - s^J} = \frac{1}{1 - s^J} = Q^J C_D^J,$$

$$P^J = \left[n_J \left(A_J^J\right)^\sigma \left(p_J^J\right)^{1-\sigma} + n_K \left(A_K^J\right)^\sigma \left(p_K^J\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = Q^J \left(1 - s^J\right),$$

where

$$Q^J = \left[n_J \left(A_J^J\right)^\sigma \left(q_J^J\right)^{1-\sigma} + n_K \left(A_K^J\right)^\sigma \left(\iota_J q_K^J\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

Consider an initial situation where $z^H = z^F = 0$ and note, using equation (7) in the main text that the *ad valorem* cost of serving either foreign market is $1 + \nu \equiv \iota$. Using the above conditions, the welfare function can be written as

$$\Omega = -\log Q^H - \log Q^F - \log\left(1 - s^H\right) - \log\left(1 - s^F\right) - s^H Q^H C_D^H - s^F Q^F C_D^F,$$

$$\left. \frac{d\Omega}{ds^H} \right|_{z^H = z^F = 0} = -\frac{1}{Q^H} \frac{dQ^H}{ds^H} - \frac{1}{Q^F} \frac{dQ^F}{ds^H} + \frac{1}{1 - s^H} - Q^H C_D^H - s^H \frac{dQ^H C_D^H}{ds^H} - s^F \frac{dQ^F C_D^F}{ds^H}.$$

Using (9) in the body of the paper we can simplify this expression since $\frac{1}{1-s^H} = Q^H C_D^H$.

$$\left. \frac{d\Omega}{ds^H} \right|_{z^H = z^F = 0} = -\frac{1}{Q^H} \frac{dQ^H}{ds^H} - \frac{1}{Q^F} \frac{dQ^F}{ds^H} - s^H \frac{dQ^H C_D^H}{ds^H} - s^F \frac{dQ^F C_D^F}{ds^H}.$$

To evaluate this expression, consider

$$\frac{dQ^H}{ds^H} = \frac{1}{1 - \sigma} \frac{Q^H}{\left(Q^H\right)^{1-\sigma}} \left[\left(A_H^H\right)^\sigma \left(q_H^H\right)^{1-\sigma} \frac{dn_H}{ds^H} + \left(A_F^H\right)^\sigma \left(\iota q_F^H\right)^{1-\sigma} \frac{dn_H}{ds^H} \right]$$

$$= \frac{Q^H}{1 - \sigma} \left[\frac{\left(A_H^H\right)^\sigma \left(q_H^H\right)^{1-\sigma}}{\left(Q^H\right)^{1-\sigma}} \frac{dn_H}{ds^H} + \frac{\left(A_F^H\right)^\sigma \left(\iota q_F^H\right)^{1-\sigma}}{\left(Q^H\right)^{1-\sigma}} \frac{dn_H}{ds^H} \right].$$

Note that $n_H (A_H^H)^\sigma (q_H^H)^{1-\sigma} / (Q^H)^{1-\sigma}$ is the expenditure share of Home for Home's production. So $P^H C_D^H n_H (A_H^H)^\sigma (q_H^H)^{1-\sigma} / (Q^H)^{1-\sigma} = n_H p_H^H c_H^H$. Since $P^H C_D^H = 1$, $n_H (A_H^H)^\sigma (q_H^H)^{1-\sigma} / (Q^H)^{1-\sigma} = n_H p_H^H c_H^H$, so $(A_H^H)^\sigma (q_H^H)^{1-\sigma} / (Q^H)^{1-\sigma} = p_H^H c_H^H$. Using this property, $\frac{dQ^H}{ds^H} = \frac{Q^H}{1-\sigma} (p_H^H c_H^H \frac{dn_H}{ds^H} + p_F^H c_F^H \frac{dn_F}{ds^H})$. Similar steps give $\frac{dQ^F}{ds^H} = \frac{Q^F}{1-\sigma} (p_H^F c_H^F \frac{dn_H}{ds^H} + p_F^F c_F^F \frac{dn_F}{ds^H})$.

From (9) in the body of the paper, it follows that $\frac{dQ^F C_D^F}{ds^H} = \frac{d(\frac{1}{1-s^F})}{ds^H} = 0$. However, this implies $\frac{dQ^F C_D^F}{ds^H} = q_F^F c_F^F \frac{dn_F}{ds^H} + \iota q_H^F c_H^F \frac{dn_H}{ds^H} = 0$. Since $(1 - s^F)(q_F^F c_F^F \frac{dn_F}{ds^H} + \iota q_H^F c_H^F \frac{dn_H}{ds^H}) = \frac{dQ^F}{ds^H} (\sigma - 1)$, it follows that this term is also zero.

Using these results to cancel terms, we are left with an expression that is a function of home country factors alone (i.e., only unilateral considerations matter):

$$\left. \frac{d\Omega}{ds^H} \right|_{z^H = z^F = 0} = -\frac{1}{Q^H} \frac{dQ^H}{ds^H} - s^H \frac{dQ^H C_D^H}{ds^H}$$

$$= -\frac{1}{Q^H} \frac{Q^H}{1 - \sigma} \left(p_H^H c_H^H \frac{dn_H}{ds^H} + p_F^H c_F^H \frac{dn_F}{ds^H} \right) - s^H \left(q_H^H c_H^H \frac{dn_H}{ds^H} + \iota q_F^H c_F^H \frac{dn_F}{ds^H} \right)$$

$$= \frac{1}{\sigma - 1}\left(p_H^H c_H^H \frac{dn_H}{ds^H} + p_F^H c_F^H \frac{dn_F}{ds^H}\right) - \frac{s^H}{1-s^H}\left(p_H^H c_H^H \frac{dn_H}{ds^H} + p_F^H c_F^H \frac{dn_F}{ds^H}\right).$$

Hence, when $s^H = 1/\sigma$, welfare is maximized.

A.5. *Proof of Proposition 2*

We look for the Nash equilibrium choices of product standards in an FTA without NT. By an FTA, we mean that the two governments are constrained to set $\tau^J = 0$, $e_J = 0$, and we also have $s^J = 1/\sigma$. Consider the outcome from free entry when $\bar{a}_H^F = a_{\min}$, $\bar{a}_F^H = a_{\max}$ and a_H^H and a_F^F are at their profit-maximizing levels in response to these extreme standards for imports. There are three possible outcomes: (i) $n_H > 0$ and $n_F > 0$; (ii) $n_H > 0$ and $n_F = 0$; (iii) $n_F > 0$ and $n_H = 0$.

Case (i): If $n_H > 0$ and $n_F > 0$ when $\bar{a}_H^F = a_{\min}$, $\bar{a}_F^H = a_{\max}$ and a_H^H and a_F^F are at their profit-maximizing levels in response to these extreme standards for imports, neither government can induce “complete delocation”; that is, exit by all firms in the other country. As long as there are active firms in both countries, each government has an incentive to push its standard for import goods to the extreme, since doing so (given the other government’s policy) always reduces the local price index by the arguments in Figure 1. Given the pair of extreme standards for import goods, the Nash response for each government is to set the standard for local products equal to the profit maximizing level.

Case (ii): Now the home government can induce complete delocation and it has an incentive to do so. It will set its standard for import products high enough to ensure $n_F = 0$. There will be a range of standards that achieve this, including $\bar{a}_F^H = a_{\max}$; all of them are best responses so any can be part of a Nash equilibrium (with the same consequences for other variables). But given that a_F^H is chosen such that $n_F = 0$, the incentives facing the foreign government are different. It does not use a_H^F to induce delocation, since such a strategy is bound to fail. Instead it “accepts”that all differentiated products will be imported and it trades off the desirability of the import products given local tastes and variety. By setting $a_H^F = \hat{a}^F$, the foreign government selects the optimal variant in the eyes of consumers in country F, considering both the direct effect on utility and the indirect effect on prices. By setting a_H^F at the profit maximizing level for home firms, it maximizes variety. It will choose a standard somewhere between these two. Arguing in this way, it is straightforward to establish that the best response for a_H^F is strictly between $\hat{a}^F$ and a_H^H. Similarly, the best response for a_H^H will be strictly between a_H^F and $\hat{a}^H$.

Case (iii) is similar.

On the Interplay Between Better Suitability and Delocation. In the text leading up to the statement of Proposition 2, we described how the local incentive to deviate from efficient standards reflects a combination of product suitability and delocation motives. Here we show that the product suitability motive may or may not be operative on the margin in the Nash equilibrium, but the delocation motive *always* is operative.

To this end, it is first helpful to express $\frac{dn_H}{da_F^H}$ and $\frac{dn_F}{da_F^H}$ evaluated at an arbitrary a_F^H. Following the same steps as in appendix Section A.2 but not requiring a_F^H to satisfy the first-order condition for profit maximization yields the following expressions for $\frac{dn_H}{da_F^H}$ and $\frac{dn_F}{da_F^H}$ evaluated at an arbitrary a_F^H:

$$\frac{dn_H}{da_F^H} = \frac{\left[\frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial n_F} + (1+\nu)\frac{\partial \tilde{c}_H^F}{\partial P^F}\frac{\partial P^F}{\partial n_F}\right]\left[\frac{N^H}{\sigma-1}(1+\nu)\frac{\partial \tilde{c}_F^H}{\partial a_F^H} - \Phi'(|a_F^H - a_F^F|)\right]}{\left(\frac{\partial P^H}{\partial n_H}\frac{\partial P^F}{\partial n_F} - \frac{\partial P^H}{\partial n_F}\frac{\partial P^F}{\partial n_H}\right)\left(\frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial \tilde{c}_F^F}{\partial P^F} - (1+\nu)^2\frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial \tilde{c}_H^F}{\partial P^F}\right)} - \frac{\frac{\partial P^H}{\partial a_F^H}\frac{\partial P^F}{\partial n_F}}{\frac{\partial P^H}{\partial n_H}\frac{\partial P^F}{\partial n_F} - \frac{\partial P^H}{\partial n_F}\frac{\partial P^F}{\partial n_H}}, \tag{A17}$$

$$\frac{dn_F}{da_F^H} = \frac{-\left(\frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial n_F} + (1+\nu)\frac{\partial \tilde{c}_H^F}{\partial P^F}\frac{\partial P^F}{\partial n_F}\right)\left[\frac{N^H}{\sigma-1}(1+\nu)\frac{\partial \tilde{c}_F^H}{\partial a_F^H} - \Phi'(|a_F^H - a_F^F|)\right]}{\left(\frac{\partial P^H}{\partial n_H}\frac{\partial P^F}{\partial n_F} - \frac{\partial P^H}{\partial n_F}\frac{\partial P^F}{\partial n_H}\right)\left(\frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial \tilde{c}_F^F}{\partial P^F} - (1+\nu)^2\frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial \tilde{c}_H^F}{\partial P^F}\right)} + \frac{\frac{\partial P^H}{\partial a_F^H}\frac{\partial P^F}{\partial n_H}}{\frac{\partial P^H}{\partial n_H}\frac{\partial P^F}{\partial n_F} - \frac{\partial P^H}{\partial n_F}\frac{\partial P^F}{\partial n_H}}. \tag{A18}$$

It is clear that the term $[\frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial P^H}{\partial n_F} + (1+\nu)\frac{\partial \tilde{c}_H^F}{\partial P^F}\frac{\partial P^F}{\partial n_F}]$ is negative while the terms $[\frac{\partial P^H}{\partial n_H}\frac{\partial P^F}{\partial n_F} - \frac{\partial P^H}{\partial n_F}\frac{\partial P^F}{\partial n_H}]$ and $[\frac{\partial \tilde{c}_H^H}{\partial P^H}\frac{\partial \tilde{c}_F^F}{\partial P^F} - (1+\nu)^2\frac{\partial \tilde{c}_F^H}{\partial P^H}\frac{\partial \tilde{c}_H^F}{\partial P^F}]$ are positive, so the sign of the first term in (A17) will be opposite the sign of $(\frac{N^H}{\sigma-1}(1+\nu)\frac{\partial \tilde{c}_F^H}{\partial a_F^H} - \Phi'(|a_F^H - a_F^F|))$ while the sign of the first term in (A18) will be the same as the sign of $(\frac{N^H}{\sigma-1}(1+\nu)\frac{\partial \tilde{c}_F^H}{\partial a_F^H} - \Phi'(|a_F^H - a_F^F|))$. And as Lemma 3 confirms, the sign of the second term in (A17) is negative while the sign of the second term in (A18) is positive.

Evaluated at the profit-maximizing choice of a_F^H, the associated first-order condition assures that $\frac{N^H}{\sigma-1}(1+\nu)\frac{\partial \tilde{c}_F^H}{\partial a_F^H} - \Phi'(|a_F^H - a_F^F|) = 0$ and so the first term in each of the expressions (A17) and (A18) is zero. But when these expressions are evaluated at a level of a_F^H above the profit-maximizing choice, we have $\frac{N^H}{\sigma-1}(1+\nu)\frac{\partial \tilde{c}_F^H}{\partial a_F^H} - \Phi'(|a_F^H - a_F^F|) < 0$ making the first term in (A17) positive and, therefore, working to overturn the second term in (A17), and making the first term in (A18) negative and therefore working to overturn the second term in (A18). And when these expressions are evaluated at a level of a_F^H below the profit-maximizing choice, we have $\frac{N^H}{\sigma-1}(1+\nu)\frac{\partial \tilde{c}_F^H}{\partial a_F^H} - \Phi'(|a_F^H - a_F^F|) > 0$ making the first term in (A17) negative and therefore working to reinforce the second term in (A17), and making the first term in (A18) positive and therefore working to reinforce the second term in (A18).

Now consider Figure A1, which depicts n_H and n_F as a function of a_F^H. To draw the n_H and n_F curves, we use expressions (A17) and (A18). The point in the figure labeled a_F^{H1} is where n_F takes its maximum value, and the point in the figure labeled a_F^{H2} is where n_H takes its minimum value. According to (A17) and (A18) evaluated at the profit maximizing levels of a_F^F and a_H^F, $a_F^{H1} < a_F^{H2}$ as depicted. Also depicted in the figure is the local

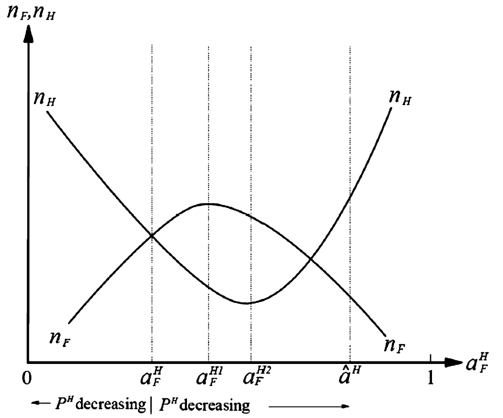

FIGURE A1.—Number of firms as function of a_F^H.

ideal $\hat{a}^H$. And finally, as noted in the figure, P^H falls as we move away from the profit-maximizing level a_F^H in either direction.

Several observations follow from Figure A1. Moving left from the profit maximizing level a_F^H, P^H falls due to the delocation associated with the fall in a_F^H, with n_F falling and n_H rising as foreign firms are delocated to the home-country market. So the incentive for the home country to defect toward the left from the efficient profit maximizing a_F^H is due to delocation. But moving right from the profit maximizing level a_F^H, P^H falls despite the fact that initially n_F is rising and n_H is falling. So the incentive to defect toward the right from the efficient profit maximizing a_F^H is initially—in the interval $((a_F^H, a_F^{H1})$—*not* due to delocation; it is due instead to the direct impact on P^H of having imports adopt a characteristic that is a little closer to the Home ideal $\hat{a}^H$, and this direct impact dominates the (anti-)delocation effects here. Once we move into the interval (a_F^{H1}, a_F^{H2}), *both* n_H and n_F are falling with further increases in a_F^H, so again the incentive for the home country to keep raising a_F^H in this interval to lower P^H is not due to delocation, but must still be due to the domination of the direct impact on P^H of having imports adopt a characteristic that is a little closer to the Home ideal $\hat{a}^H$. In the interval $(a_F^{H2}, \hat{a}^H)$, we now have delocation *and* the direct impact described above both helping to push P^H lower. But for the interval $(\hat{a}^H, a_{\max})$, the direct effect is now going the wrong way so it is the delocation effect that dominates at this point and keeps P^H falling.

Finally, notice that Figure A1 shows the number of foreign firms as being still positive at $\hat{a}^H$, which, if a general property, would mean that *only* the delocation motive operates in the neighborhood of the case (ii) Nash equilibrium. On the other hand, if n_F hits zero at a standard smaller than $\hat{a}^H$, then the “last little bit of standard”could provide benefits both via delocation and via product suitability. It can be shown that both possibilities can arise. Hence the product suitability motive may or may not be operative on the margin in the Nash equilibrium, but the delocation motive *always* is operative.

A.6. *Regulatory Convergence: FTA versus NTA*

As argued in the text, when Nash standards are set at their extreme limits, a transition from an FTA to an NTA will involve regulatory convergence. To see this is also true for an FTA that involves complete delocation, suppose that $n_F = 0$. This implies: $(P^H)^{\sigma-1} =$

$\frac{(p_H^H)^{\sigma-1}}{n_H(A_H^H)^\sigma}$, $(P^F)^{\sigma-1} = \frac{(p_H^F)^{\sigma-1}}{n_H(A_H^F)^\sigma}$. Since firms are only active in the home country, the zero profit condition gives: $n_H = \frac{N^H+N^F}{(\sigma-1)\Phi(a_H^H-a_H^F)}$.

The choice of standard in each location will attempt to minimize the relevant local price index. Substituting n_H into the relevant price index and maximizing with respect to the local standard generates the following best response functions:

$$\frac{\Phi'(\bar{a}_H^H-\bar{a}_H^F)}{\Phi(\bar{a}_H^H-\bar{a}_H^F)}\frac{d(\bar{a}_H^H-\bar{a}_H^F)}{d\bar{a}_H^J} = \sigma\frac{A_a(\bar{a}_H^J,\gamma_H^J)}{A(\bar{a}_H^J,\gamma_H^J)} + (1-\sigma)\eta(\bar{a}_H^J), \quad J\in\{H,F\}. \tag{A19}$$

The Nash equilibrium is the pair of standards that satisfy these two equations.

Regulatory convergence is confirmed by comparing the first-order conditions in (A19) that characterize the Nash standards to those generated by profit-maximization and, therefore, the globally efficient standards. For the representative home firm, the optimal choices of $\tilde{a}_H^H$ and $\tilde{a}_H^F$ when $\mathbf{z}=0$ and $\mathbf{s}=1/\sigma$ satisfy

$$\begin{aligned}&\frac{\Phi'(\tilde{a}_H^H-\tilde{a}_H^F)}{\Phi(\tilde{a}_H^H-\tilde{a}_H^F)}\frac{d(\tilde{a}_H^H-\tilde{a}_H^F)}{d\tilde{a}_H^J}\\&\quad=\Lambda_H^J(\tilde{a}_H^H,\tilde{a}_H^F)\left[\sigma\frac{A_a(\tilde{a}_H^J,\gamma^J)}{A(\tilde{a}_H^J,\gamma^J)}+(1-\sigma)\eta(\tilde{a}_H^J)\right], \quad J\in\{H,F\},\end{aligned} \tag{A20}$$

where $\Lambda_H^J(\tilde{a}_H^H,\tilde{a}_H^F)$ is the fraction of its global operating profits that the representative home firm earns in market J. But with $\Lambda_H^J(\tilde{a}_H^H,\tilde{a}_H^F)<1$ for $J\in\{H,F\}$, it follows from (15) in the body of the paper and (A20) that $|\tilde{a}_H^H-\tilde{a}_H^F|<|\bar{a}_H^H-\bar{a}_H^F|$; and thus an efficient NTA delivers regulatory convergence.

A.7. *A Smarter OTA Without National Treatment*

To illustrate the possibility of a smarter OTA, let us take an initial equilibrium under the FTA with active firms in both countries and with $\bar{a}_F^H=a_{\max}$ and $\bar{a}_H^F=a_{\min}$. Suppose we were to depict the zero-profit lines for home and foreign firms when all firms are free to choose their profit-maximizing characteristics for sales in their local market but are subject to these extreme regulations in their export markets. In such circumstances, each zero-profit would be downward sloping, just as in Figure 1. Moreover, it will often be the case that the $\pi_H=0$ line would have a (negative) slope greater than one in absolute value, and the $\pi_F=0$ line would have a (negative) slope less than one in absolute value, just as in the earlier figure.

Now suppose that we contemplate a trade agreement with zero *net* tariffs, just as with an FTA, but now with $\tau^H=\tau^F=-e_H=-e_F\equiv\tau>0$. As we know, equilibrium prices and quantities depend only on net trade taxes and so are independent of τ. Home welfare in these circumstances would be given by $\Omega^H=L_H+\tau(\frac{\sigma}{\sigma-1})(\tilde{M}^H-\tilde{E}_H)-N^H\log(P^H)-N^H\frac{1}{\sigma-1}$, where aggregate home imports and home exports are $\tilde{M}^H=N^Hn_F\lambda_F^H(A_F^H)^\sigma(p_F^H)^{-\sigma}(P^H)^{\sigma-1}$, $\tilde{E}_H=N^Fn_H\lambda_H^F(A_H^F)^\sigma(p_H^F)^{-\sigma}(P^F)^{\sigma-1}$.

Would the home government still wish to apply the extreme standard of $\bar{a}_F^H=a_{\max}$ in such circumstances, as it would with free trade? Recall that under the FTA, the delocation motive operates on the margin. Were the home country to slightly ease its regulation of imports to something a bit less than $\bar{a}_F^H=a_{\max}$, it would induce entry by foreign firms and exit by home firms; that is, it would reverse the last bit of delocation. The increase in n_F would contribute to greater imports. Also, since $\bar{a}_F^H$ now is closer to $\hat{a}^H$, import products would be more attractive which also contributes to greater imports. Finally, the shift of $\bar{a}_F^H$ away from the level that minimizes the local price index P^H eases competition in the home market, which further contributes to a rise in imports. Overall, the easing of standards causes imports to rise. Meanwhile, the fall in the number of home firms and the fall in the foreign price index spell a reduction in home exports. The expansion in home imports and the contraction of home exports generate an increase in home tax revenues, as tariff collections rise and export subsidy outlays fall.

The net effect on home welfare combines the adverse effect of the cut in $\bar{a}_F^H$ on the home price index and the favorable effect on total tax revenues. Note, however, that the marginal welfare loss from an increase in P^H is independent of τ, whereas the marginal gain from the increased tax revenues rises linearly with τ. It follows that there must exist a τ large enough that the positive effect dominates.[2] In short, when τ is sufficiently large, the home government's best response to any set of foreign standards will be to choose a standard for imports strictly less than $a_{\max}$. By analogous arguments, the foreign government will choose an import standard $\bar{a}_H^F$ that is strictly greater than $a_{\min}$. In other words, the positive tariffs and offsetting export subsidies induce both governments to moderate their regulation of imports. Finally, if the home and foreign zero profit lines under an FTA are, respectively, steeper and flatter than a line with slope minus one, global welfare will be higher under a smart trade agreement with $\boldsymbol{\tau} > 0$ than under an FTA with $\boldsymbol{\tau} = 0$.

Although countries may be able to design a smarter OTA that improves upon an FTA, there are no values of $\tau^H = -e_F$ and $\tau^F = -e_H$ that would permit an OTA without national treatment to deliver the first-best level of global welfare. To see this, begin at the profit-maximizing standards illustrated in Figure 1. Suppose first that τ^H and τ^F are set to be positive and consider the welfare effects of a small increase in $\bar{a}_F^H$. By Lemma 2, foreign firms would enter and home firms would exit. By Lemma 3, there would be no first-order change in either price index. Meanwhile, the increase in $\bar{a}_F^H$ from the level that is profit-maximizing for foreign firms makes the import product more attractive to home consumers. Together, the increases in n_F and A_H^F imply that imports $\tilde{M}^H$ would rise, which would generate a gain in tariff revenues. Meanwhile, the exit by home firms reduces home exports $\tilde{E}_H$, so home outlays for export subsidies would fall. In combination, the home country's tax revenues grow, with no first-order effect on its price index. This combination represents a gain in welfare for the home country, and hence we have that no positive τ^H and τ^F exist to discourage deviation from the first-best standards. Suppose instead that the countries set τ^H and τ^F to be negative. In that case, the home government could deviate by reducing its standard $\bar{a}_F^H$ slightly below the efficient level and raise domestic welfare with an increase in trade tax revenues and no first-order effect on the home price index.[3] So, negative tariffs (with positive export taxes) also do not discourage deviations in standard setting. Evidently, a smarter OTA, no matter how smart, cannot deliver the first best.

[2]Since $\tilde{M}^H$ and $\tilde{E}_H$ depend only on net trade taxes, and thus are independent of τ, the gain in tax revenues generated by a reduction in $\bar{a}_F^H$ grows linearly with τ, without bound.

[3]When $\bar{a}_F^H$ is reduced below the profit-maximizing level for foreign firms, n_F falls, A_F^H falls, and n_H rises. So imports fall, exports rise, and the sum of outlays for import subsidies and proceeds from export taxes will rise. Meanwhile, the home price index is unaffected to first order, so the deviation must be beneficial to the home country.

A.8. *Nash Standards With National Treatment*

Suppose that the countries have concluded an FTA that mandates free trade and subsidies that counteract markup pricing; that is, we take $\tau^J = e_J = 0$ and $s^J = 1/\sigma$ for $J \in \{H, F\}$. The agreement also includes a mandate for national treatment in regulatory policy. We ask, What characteristics will the two governments choose in a Nash equilibrium of standard setting, if there are no further constraints on their choices?

When all brands sold in country J bear the same characteristic, $\bar{a}^J$, the demand shifters take the common value $\bar{A}^J \equiv A(\bar{a}^J, \gamma^J)$. Then we can write the price index for country J simply as $P^J = (\bar{A}^J)^{\frac{\sigma}{1-\sigma}}[n_J(p_J^J)^{1-\sigma} + n_K(p_K^J)^{1-\sigma}]^{\frac{1}{1-\sigma}}$ for $J \in \{H, F\}$. Solving this pair of equations for the number of firms in each country gives $n_J = \frac{(\bar{A}^K)^\sigma(p_K^K)^{1-\sigma}(P^J)^{1-\sigma} - (\bar{A}^J)^\sigma(p_K^J)^{1-\sigma}(P^K)^{1-\sigma}}{(\bar{A}^H)^\sigma(\bar{A}^F)^\sigma[(p_H^H)^{1-\sigma}(p_F^F)^{1-\sigma} - (p_F^H)^{1-\sigma}(p_H^F)^{1-\sigma}]}$, provided that the solution yields nonnegative values for both n_H and n_F. The denominator of this expression is always positive. It follows that firms are active in both countries if and only if the numerators are positive for both $J = H$ and $J = F$. Using the pricing equations (8)–(10) in the body of the paper, this is equivalent to $(\bar{A}^H/\bar{A}^F)^\sigma(\bar{\lambda}^H/\bar{\lambda}^F)^{1-\sigma}(1+\nu)^{\sigma-1} > (P^H/P^F)^{1-\sigma} > (\bar{A}^H/\bar{A}^F)^\sigma(\bar{\lambda}^H/\bar{\lambda}^F)^{1-\sigma}(1+\nu)^{1-\sigma}$, where $\bar{\lambda}^J \equiv \lambda(\bar{a}^J)$.

Assuming for the moment that firms are active in both countries, we can use the two zero-profit conditions to solve for the equilibrium price indices. We find $(P^J)^{\sigma-1} = \frac{\sigma[\Phi(|\bar{a}^H - \bar{a}^F|)]}{N^J(\bar{A}^J)^\sigma(\bar{\lambda}^J)^{1-\sigma}[1+(1+\nu)^{1-\sigma}]}$, $J \in \{H, F\}$. Then, in a Nash equilibrium, each government chooses its standard to minimize its price index, given the standard of the other. The best-response functions that follow from the first-order conditions imply

$$\frac{\Phi'(\bar{a}^H - \bar{a}^F)}{\Phi(\bar{a}^H - \bar{a}^F)} \frac{d(\bar{a}^H - \bar{a}^F)}{d\bar{a}^J} = \sigma \frac{A_a(\bar{a}^J, \gamma^J)}{A(\bar{a}^J, \gamma^J)} + (1 - \sigma)\eta(\bar{a}^J), \quad J \in \{H, F\}.$$

A.9. *Nash Standards Under MR When Adaptation Costs Are Large*

If adaptation costs are large, each firm will choose only one characteristic and invoke mutual recognition to serve the export market. A firm then chooses a to maximize

$$A^J(a_J)^\sigma \lambda(a_J)^{1-\sigma} N^J (P^J)^{\sigma-1} + A^K(a_J)^\sigma \lambda(a_J)^{1-\sigma}(1+\nu)^{1-\sigma} N^K (P^K)^{\sigma-1}.$$

The first-order condition implies:

$$\begin{aligned}
&A^J(a_J)^\sigma \lambda(a_J)^{1-\sigma} N^J (P^J)^{\sigma-1}\left[\sigma \frac{\partial \log(A^J(a_J))}{\partial a_J} + (1-\sigma)\frac{\partial \log(\lambda(a_J))}{\partial a_J}\right] \\
&\quad + A^K(a_J)^\sigma \lambda(a_J)^{1-\sigma}(1+\nu)^{1-\sigma} N^K (P^K)^{\sigma-1} \\
&\quad \times \left[\sigma \frac{\partial \log(A^K(a_J))}{\partial a_J} + (1-\sigma)\frac{\partial \log(\lambda(a_J))}{\partial a_J}\right] \\
&= 0.
\end{aligned}$$

Thus,

$$\begin{aligned}&\left(A_H^H\right)^\sigma N^H\left(P^H\right)^{\sigma-1}\left[\sigma\frac{\partial\log\left(A^H(a_H)\right)}{\partial a_H}+(1-\sigma)\frac{\partial\log\left(\lambda(a_H)\right)}{\partial a_H}\right]\\&\quad+\left(A_H^F\right)^\sigma(1+\nu)^{1-\sigma}N^F\left(P^F\right)^{\sigma-1}\left[\sigma\frac{\partial\log\left(A^F(a_H)\right)}{\partial a_H}+(1-\sigma)\frac{\partial\log\left(\lambda(a_H)\right)}{\partial a_H}\right]\\&=0,\end{aligned}\tag{A21}$$

and

$$\begin{aligned}&\left(A_F^F\right)^\sigma N^F\left(P^F\right)^{\sigma-1}\left[\sigma\frac{\partial\log\left(A^F(a_F)\right)}{\partial a_F}+(1-\sigma)\frac{\partial\log\left(\lambda(a_F)\right)}{\partial a_F}\right]\\&\quad+A\left(a_F^H\right)^\sigma(1+\nu)^{1-\sigma}N^H\left(P^H\right)^{\sigma-1}\left[\sigma\frac{\partial\log\left(A^H(a_F)\right)}{\partial a_F}+(1-\sigma)\frac{\partial\log\left(\lambda(a_F)\right)}{\partial a_F}\right]\\&=0.\end{aligned}\tag{A22}$$

In addition, the two zero profit conditions imply

$$\begin{aligned}N^H\left(P^H\right)^{\sigma-1}&=\frac{\left[\left(A_F^F\right)^\sigma\lambda_F^{1-\sigma}-(1+\nu)^{1-\sigma}\left(A_H^F\right)^\sigma\lambda_H^{1-\sigma}\right]\sigma\Phi(0)}{\lambda_H^{1-\sigma}\lambda_F^{1-\sigma}\left[\left(A_H^H\right)\left(A_F^F\right)^\sigma-(1+\nu)^{1-\sigma}\left(A_F^H\right)^\sigma(1+\nu)^{1-\sigma}\left(A_F^F\right)^\sigma\right]},\\N^F\left(P^F\right)^{\sigma-1}&=\frac{\left[\left(A_H^H\right)^\sigma\lambda_H^{1-\sigma}-(1+\nu)^{1-\sigma}\left(A_F^H\right)^\sigma\lambda_F^{1-\sigma}\right]\sigma\Phi(0)}{\lambda_H^{1-\sigma}\lambda_F^{1-\sigma}\left[\left(A_H^H\right)\left(A_F^F\right)^\sigma-(1+\nu)^{1-\sigma}\left(A_F^H\right)^\sigma(1+\nu)^{1-\sigma}\left(A_F^F\right)^\sigma\right]}.\end{aligned}$$

Substituting into (A21) and (A22) gives

$$\begin{aligned}&\left(A_H^H\right)^\sigma\left[\left(A_F^F\right)^\sigma\lambda_F^{1-\sigma}-(1+\nu)^{1-\sigma}\left(A_H^F\right)^\sigma\lambda_H^{1-\sigma}\right]\left[\sigma\frac{\partial\log\left(A^H(a_H)\right)}{\partial a_H}+(1-\sigma)\frac{\partial\log\left(\lambda(a_H)\right)}{\partial a_H}\right]\\&\quad+(1+\nu)^{1-\sigma}\left(A_H^F\right)^\sigma\left[\left(A_H^H\right)^\sigma\lambda_H^{1-\sigma}-(1+\nu)^{1-\sigma}\left(A_F^H\right)^\sigma\lambda_F^{1-\sigma}\right]\\&\quad\times\left[\sigma\frac{\partial\log\left(A^F(a_H)\right)}{\partial a_H}+(1-\sigma)\frac{\partial\log\left(\lambda(a_H)\right)}{\partial a_H}\right]=0,\\&\left(A_F^F\right)^\sigma\left[\left(A_H^H\right)^\sigma\lambda_H^{1-\sigma}-(1+\nu)^{1-\sigma}\left(A_F^H\right)^\sigma\lambda_F^{1-\sigma}\right]\left[\sigma\frac{\partial\log\left(A^F(a_F)\right)}{\partial a_F}+(1-\sigma)\frac{\partial\log\left(\lambda(a_F)\right)}{\partial a_F}\right]\\&\quad+(1+\nu)^{1-\sigma}\left(A_F^H\right)^\sigma\left[\left(A_F^F\right)^\sigma\lambda_F^{1-\sigma}-(1+\nu)^{1-\sigma}\left(A_H^F\right)^\sigma\lambda_H^{1-\sigma}\right]\\&\quad\times\left[\sigma\frac{\partial\log\left(A^H(a_F)\right)}{\partial a_F}+(1-\sigma)\frac{\partial\log\left(\lambda(a_F)\right)}{\partial a_F}\right]=0.\end{aligned}$$

The Nash equilibrium with MR is the solution of these two equations for a_H and a_F.

A.10. *Demand in the Presence of a Consumption Externality*

Here, we derive an explicit expression in the presence of a consumption externality ($\xi < 1$) for the industry-level price index $\mathcal{P}^J$ that enters (2) and (3) in the body of the paper. As in the body of the paper, for ease of notation, we define

$$A_i^J \equiv (1-\xi)A^{*J} + \xi A(a_i^J, \gamma^J); \qquad \mathcal{A}_i^J \equiv A(a_i^J, \gamma^J)$$

and hence by (1) and (4) in the body of the paper per-capita utility in country J for $\xi \le 1$ is given by

$$U^J = 1 + C_Y^J + \log\left(\left\{\sum_{i\in\Theta^H} A_i^J (c_i^J)^\beta + (1-\xi)\left[\mathcal{A}_i^J - A^{*J}\right](c_{i\mu}^J)^\beta\right\}^{\frac{1}{\beta}}\right).$$

The first-order conditions for the utility-maximizing choice of c_i^J imply $(C_D^J)^{-\beta} A_i^J (c_i^J)^\beta = p_i^J c_i^J$, summing over i, $(C_D^J)^{-\beta}\sum_i A_i^J (c_i^J)^\beta = \sum_i p_i^J c_i^J$. We define $\mathcal{P}^J$ so that $\mathcal{P}^J C_D^J = \sum_i p_i^J c_i^J$. Then $\mathcal{P}^J = (C_D^J)^{-\beta-1}\sum_i A_i^J (c_i^J)^\beta$.

Also, from the first-order conditions, $c_i^J = (p_i^J)^{\frac{1}{\beta-1}} (A_i^J)^{\frac{-1}{\beta-1}} (C_D^J)^{\frac{\beta}{\beta-1}} \Rightarrow A_i^J (c_i^J)^\beta = (p_i^J)^{\frac{\beta}{\beta-1}} (A_i^J)^{\frac{-1}{\beta-1}} (C_D^J)^{\frac{\beta}{\beta-1}}$. Hence we have $\mathcal{P}^J = (C_D^J)^{-\beta-1} (p_i^J)^{\frac{\beta}{\beta-1}} (A_i^J)^{\frac{-1}{\beta-1}} (C_D^J)^{\frac{\beta}{\beta-1}} = (C_D^J)^{\frac{1}{\beta-1}} (p_i^J)^{\frac{\beta}{\beta-1}} (A_i^J)^{\frac{-1}{\beta-1}}$.

Note that with $c_i^J = c_{i\mu}^J$ we can write $C_D^J = [\sum_i \mathcal{A}_i^J (c_i^J)^\beta]^{\frac{1}{\beta}} = (C_D^J)^{\frac{\beta}{\beta-1}} [\sum_i \mathcal{A}_i^J (p_i^J)^{\frac{\beta}{\beta-1}} \times (A_i^J)^{\frac{-\beta}{\beta-1}}]^{\frac{1}{\beta}}$, and therefore $(C_D^J)^{\frac{-1}{\beta-1}} = [\sum_i \mathcal{A}_i^J (p_i^J)^{\frac{\beta}{\beta-1}} (A_i^J)^{\frac{-\beta}{\beta-1}}]^{\frac{1}{\beta}}$, which implies $C_D^J = [\sum_i^J \mathcal{A}_i^J (p_i^J)^{\frac{\beta}{\beta-1}} (A_i^J)^{\frac{-\beta}{\beta-1}}]^{\frac{-(\beta-1)}{\beta}}$. Substituting yields $\mathcal{P}^J = [\sum_i \mathcal{A}_i^J (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^J)^{\frac{-\beta}{\beta-1}}]^{\frac{-1}{\beta}} \times (p_i^J)^{\frac{\beta}{\beta-1}} (A_i^J)^{\frac{-1}{\beta-1}}$, or finally using $\sigma \equiv \frac{1}{1-\beta}$

$$\mathcal{P}^J = \frac{\sum_i (p_i^J)^{1-\sigma}(A_i^J)^\sigma}{\left[\sum_i \mathcal{A}_i^J (p_i^J)^{1-\sigma}(A_i^J)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}} = \left[\frac{\sum_i (A_i^J)^\sigma (p_i^J)^{1-\sigma}}{\sum_i \left(\frac{\mathcal{A}_i^J}{A_i^J}\right)(A_i^J)^\sigma (p_i^J)^{1-\sigma}}\right]^{\frac{\sigma}{\sigma-1}} P^J,$$

where the second equality follows from the expression for P^J given in (5) in the body of the paper.

A.11. *Proof of Proposition 6*

In the text, we derived the following expressions which implicitly define the efficient prices for $\xi \in [0, 1]$:

$$p_J^{JE}(\xi) = p_J^{JE}(1)\left[\left(\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}}\right)\left(\frac{P^{JE}(\xi)}{\mathcal{P}^{JE}}\right)^{(\frac{\sigma-1}{\sigma})}\right],$$

$$p_K^{JE}(\xi) = p_K^{JE}(1)\left[\left(\frac{A_K^{JE}(\xi)}{\mathcal{A}_K^{JE}}\right)\left(\frac{P^{JE}(\xi)}{\mathcal{P}^{JE}}\right)^{(\frac{\sigma-1}{\sigma})}\right],$$

where $\mathcal{P}^{JE}$ is the efficient industry-level (and brand-level) price index in country J when $\xi = 1$. We claimed that for $\xi < 1$, $p_H^{HE}(\xi) < p_H^{HE}(1)$, $p_F^{HE}(\xi) > p_F^{HE}(1)$, and that the relationship between $p_J^{FE}(\xi)$ and $p_J^{FE}(1)$ depends on the form of product differentiation; if different versions of a brand are horizontally differentiated, $p_F^{FE}(\xi) < p_F^{FE}(1)$

and $p_F^{HE}(\xi) > p_F^{HE}(1)$, whereas if they are vertically differentiated, $p_F^{FE}(\xi) > p_F^{FE}(1)$ and $p_F^{HE}(\xi) < p_F^{HE}(1)$.

Efficient pricing can be implemented with a combination of efficient net trade taxes and efficiency consumption subsidies, namely

$$\tau^{JE}(\xi) + e_{KE}(\xi) = (1+\nu)\left[\frac{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}}{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}} - 1\right], \quad J \in \{H, F\},$$

$$s^{JE}(\xi) = \frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right)\left[1 - \left(\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}}\right)\left(\frac{P^{JE}(\xi)}{\mathcal{P}^{JE}}\right)^{(\frac{\sigma-1}{\sigma})}\right], \quad J \in \{H, F\}.$$

We claimed that $\tau^{HE}(\xi) + e_{FE}(\xi) > 0$ and $s^{HE}(\xi) > \frac{1}{\sigma}$ for all demand shifters that satisfy Assumption 1. In the foreign country, $\tau^{FE}(\xi) + e_{HE}(\xi) > 0$ and $s^{FE}(\xi) > \frac{1}{\sigma}$ if versions of brand i are horizontally differentiated, whereas $\tau^{FE}(\xi) + e_{HE}(\xi) < 0$ and $s^{FE}(\xi) < \frac{1}{\sigma}$ if they are vertically differentiated. To establish these claims, we need to examine $\text{sgn}[(\frac{A_J^{KE}(\xi)}{\mathcal{A}_J^{KE}})(\frac{P^{KE}(\xi)}{\mathcal{P}^{KE}})^{(\frac{\sigma-1}{\sigma})} - 1]$ for $J \in \{H, F\}$.

First, we prove another claim made in the text, namely, that under the efficient consumption subsidies and net trade taxes and the implied vector of efficient prices (which we denoted by $\mathbf{p}^E(\xi)$), and in combination with the vector of efficient product characteristics (which we denoted by $\mathbf{a}^E$), we have

$$\mathcal{P}^{JE}(\xi) = \mathcal{P}^{JE} = P^{JE} \quad \text{for } J \in \{H, F\},$$

where $\mathcal{P}^{JE}(\xi)$ is defined by (18) in the body of the paper using $\mathbf{a}^E$ and $\mathbf{p}^E(\xi)$ and $\mathcal{P}^{JE} = \mathcal{P}^{JE}(1)$. To show that $\mathcal{P}^{HE}(\xi) = \mathcal{P}^{HE}$ (the steps to show $\mathcal{P}^{FE}(\xi) = \mathcal{P}^{FE}$ are analogous), we first write $\mathcal{P}^{HE}$ as

$$\mathcal{P}^{HE} = \left[n_H\left(\mathcal{A}_H^{HE}\right)^\sigma\left(p_H^{HE}(1)\right)^{1-\sigma} + n_F\left(\mathcal{A}_F^{HE}\right)^\sigma\left(p_F^{HE}(1)\right)^{1-\sigma}\right]^{\frac{-1}{\sigma-1}},$$

where we have used $A_H^{HE}(\xi = 1) = \mathcal{A}_H^{HE}$ and $A_F^{HE}(\xi = 1) = \mathcal{A}_F^{HE}$. Then, using the definition of P^H and the relationship between P^H and $\mathcal{P}^H$, we have

$$\mathcal{P}^{HE}(\xi) = \frac{\left[P^{HE}(\xi)\right]^{-(\sigma-1)}}{\left[n_H\frac{\mathcal{A}_H^{HE}}{A_H^{HE}(\xi)}\left(A_H^{HE}(\xi)\right)^\sigma\left(p_H^{HE}(\xi)\right)^{1-\sigma} + n_F\frac{\mathcal{A}_F^{HE}}{A_F^{HE}(\xi)}\left(A_F^{HE}(\xi)\right)^\sigma\left(p_F^{HE}(\xi)\right)^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}}.$$

Plugging the expressions for $p_H^{HE}(\xi)$ and $p_F^{HE}(\xi)$ into the denominator of the above expression and simplifying then yields

$$\frac{\left[P^{HE}(\xi)\right]^{-(\sigma-1)}}{\left[n_H\frac{\mathcal{A}_H^{HE}}{A_H^{HE}(\xi)}\left(A_H^{HE}(\xi)\right)^\sigma\left(p_H^{HE}(\xi)\right)^{1-\sigma} + n_F\frac{\mathcal{A}_F^{HE}}{A_F^{HE}(\xi)}\left(A_F^{HE}(\xi)\right)^\sigma\left(p_F^{HE}(\xi)\right)^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}}$$

$$
\begin{aligned}
&= \frac{\left(\mathcal{P}^{HE}\right)^{-(\sigma-1)}}{\left[\begin{array}{l} n_H \dfrac{\mathcal{A}_H^{HE}}{A_H^{HE}(\xi)}\left(A_H^{HE}(\xi)\right)^{\sigma}\left(p_H^{HE}(1)\right)^{1-\sigma}\left(\dfrac{A_H^{HE}(\xi)}{\mathcal{A}_H^{HE}}\right)^{1-\sigma} \\ \quad + n_F \dfrac{\mathcal{A}_F^{HE}}{A_F^{HE}(\xi)}\left(A_F^{HE}(\xi)\right)^{\sigma}\left(p_F^{HE}(1)\right)^{1-\sigma}\left(\dfrac{A_F^{HE}(\xi)}{\mathcal{A}_F^{HE}}\right)^{1-\sigma}\end{array}\right]^{\frac{\sigma}{\sigma-1}}} \\
&= \left[n_H\left(\mathcal{A}_H^{HE}\right)^{\sigma}\left(p_H^{HE}(1)\right)^{1-\sigma} + n_F\left(\mathcal{A}_F^{HE}\right)^{\sigma}\left(p_F^{HE}(1)\right)^{1-\sigma}\right]^{\frac{-1}{\sigma-1}} \\
&= \mathcal{P}^{HE}.
\end{aligned}
$$

Having established that $\mathcal{P}^{HE}(\xi) = \mathcal{P}^{HE}$, we turn to examine $\operatorname{sgn}[(\frac{A_H^{HE}(\xi)}{\mathcal{A}_H^{HE}})(\frac{P^{HE}(\xi)}{\mathcal{P}^{HE}})^{(\frac{\sigma-1}{\sigma})} - 1]$. Using $\mathcal{P}^{HE}(\xi) = \mathcal{P}^{HE}$ and the relationship between P^H and $\mathcal{P}^H$, we have

$$
\begin{aligned}
&\left(\frac{A_H^{HE}(\xi)}{\mathcal{A}_H^{HE}}\right)\left(\frac{P^{HE}(\xi)}{\mathcal{P}^{HE}}\right)^{(\frac{\sigma-1}{\sigma})} \\
&\quad = \left(\frac{A_H^{HE}(\xi)}{\mathcal{A}_H^{HE}}\right)\left(\frac{P^{HE}(\xi)}{\mathcal{P}^{HE}(\xi)}\right)^{(\frac{\sigma-1}{\sigma})} \\
&\quad = \left(\frac{A_H^{HE}(\xi)}{\mathcal{A}_H^{HE}}\right) \\
&\qquad \times \left[\frac{n_H \dfrac{\mathcal{A}_H^{HE}}{A_H^{HE}(\xi)}\left(A_H^{HE}(\xi)\right)^{\sigma}\left(p_H^{HE}(\xi)\right)^{1-\sigma} + n_F \dfrac{\mathcal{A}_F^{HE}}{A_F^{HE}(\xi)}\left(A_F^{HE}(\xi)\right)^{\sigma}\left(p_F^{HE}(\xi)\right)^{1-\sigma}}{n_H\left(A_H^{HE}(\xi)\right)^{\sigma}\left(p_H^{HE}(\xi)\right)^{1-\sigma} + n_F\left(A_F^{HE}(\xi)\right)^{\sigma}\left(p_F^{HE}(\xi)\right)^{1-\sigma}}\right] \\
&\quad = \frac{n_H\left(A_H^{HE}(\xi)\right)^{\sigma}\left(p_H^{HE}(\xi)\right)^{1-\sigma} + n_F\left[\dfrac{\dfrac{A_H^{HE}(\xi)}{\mathcal{A}_H^{HE}}}{\dfrac{A_F^{HE}(\xi)}{\mathcal{A}_F^{HE}}}\right]\left(A_F^{HE}(\xi)\right)^{\sigma}\left(p_F^{HE}(\xi)\right)^{1-\sigma}}{n_H\left(A_H^{HE}(\xi)\right)^{\sigma}\left(p_H^{HE}(\xi)\right)^{1-\sigma} + n_F\left(A_F^{HE}(\xi)\right)^{\sigma}\left(p_F^{HE}(\xi)\right)^{1-\sigma}} < 1,
\end{aligned}
$$

where the inequality follows for $\xi < 1$ from the ranking of efficient product characteristics, that is, $a_H^{HE} > a_F^{HE} \Rightarrow A_H^{HE}(\xi)/\mathcal{A}_H^{HE} < A_F^{HE}(\xi)/\mathcal{A}_F^{HE}$. An analogous argument establishes that

$$
\left(\frac{A_F^{HE}(\xi)}{\mathcal{A}_F^{HE}}\right)\left(\frac{P^{HE}(\xi)}{\mathcal{P}^{HE}}\right)^{(\frac{\sigma-1}{\sigma})} > 1.
$$

Similarly, in the foreign country, we have

$$
\left(\frac{A_F^{FE}(\xi)}{\mathcal{A}_F^{FE}}\right)\left(\frac{P^{FE}(\xi)}{\mathcal{P}^{FE}}\right)^{(\frac{\sigma-1}{\sigma})} < 1, \qquad \left(\frac{A_H^{FE}(\xi)}{\mathcal{A}_H^{FE}}\right)\left(\frac{P^{FE}(\xi)}{\mathcal{P}^{FE}}\right)^{(\frac{\sigma-1}{\sigma})} > 1,
$$

for the case of horizontal differentiation, because then $a_F^{FE} < a_H^{FE} \Rightarrow A_F^{FE}(\xi)/\mathcal{A}_F^{FE} < A_H^{FE}(\xi)/\mathcal{A}_H^{FE}$. However, with vertical differentiation, $a_F^{FE} < a_H^{FE} \Rightarrow A_F^{FE}(\xi)/\mathcal{A}_F^{FE} > A_H^{FE}(\xi)/\mathcal{A}_H^{FE}$, so the two inequalities are reversed.

Finally, in the text we also claimed that the additional consumption subsidies and net trade taxes implied by efficient intervention in the presence of the consumption externality are revenue neutral, implying that global welfare under the efficient policies when $\xi < 1$ is given by $\Omega(\xi) = \sum_J L^J - \sum_J N^J \log \mathcal{P}^{JE}(\boldsymbol{\xi}) - \sum_J N^J \frac{1}{\sigma-1} = \sum_J L^J - \sum_J N^J \log \mathcal{P}^{JE} - \sum_J N^J \frac{1}{\sigma-1}$, the same level of global welfare that is reached under efficient policies when $\xi = 1$.

Note that the trade tax revenue goes from zero under the efficient policies when $\xi = 1$ to the amount

$$\sum_J N^J \frac{\sigma}{\sigma-1}(1+\nu) \cdot \left[\frac{\left(\frac{A_K^{JE}(\xi)}{\mathcal{A}_K^{JE}} \right)}{\left(\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}} \right)} - 1 \right] \times \left[n_{KE} \tilde{c}_K^{JE} \right], \tag{A23}$$

under the efficient policies when $\xi < 1$: the change in trade tax revenue is therefore given by (A23). The increase in consumption subsidy payments is given by

$$\sum_J \Bigg\{ N^J \frac{\sigma}{\sigma-1} \left(\frac{1}{\sigma} + \frac{\sigma-1}{\sigma} \left[1 - \left(\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}} \right) \left(\frac{P^J\big(p_J^{JE}(\xi), p_K^{JE}(\xi); \mathbf{n}_E, a_J^{JE}, a_K^{JE}\big)}{\mathcal{P}^{JE}} \right)^{\frac{\sigma-1}{\sigma}} \right] \right)$$
$$\times \left[n_{JE} \tilde{c}_J^{JE} + n_{KE}\big(1+\nu+\tau^{JE}(\xi)+e_{KE}(\xi)\big) \tilde{c}_K^{JE} \right] - N^J \frac{1}{\sigma-1} \left[n_{JE} \tilde{c}_J^{JE} + n_{KE}(1+\nu) \tilde{c}_K^{JE} \right] \Bigg\}$$

which can be simplified to

$$\sum_J \left\{ -N^J \frac{1}{\sigma-1} n_{KE}(1+\nu) \tilde{c}_K^{JE} + N^J n_{JE} \tilde{c}_J^{JE} + N^J \frac{\sigma}{\sigma-1} n_{KE}(1+\nu) \frac{\left[\frac{A_K^{JE}(\xi)}{\mathcal{A}_K^{JE}} \right]}{\left[\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}} \right]} \tilde{c}_K^{JE} \right.$$
$$- N^J \left[\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}} \right] \left(\frac{P^J\big(p_J^{JE}(\xi), p_K^{JE}(\xi); \mathbf{n}_E, a_J^{JE}, a_K^{JE}\big)}{\mathcal{P}^{JE}} \right)^{\frac{\sigma-1}{\sigma}}$$
$$\left. \times \left[n_{JE} \tilde{c}_J^{JE} + n_{KE}(1+\nu) \frac{\left[A_K^{JE}(\xi)/\mathcal{A}_K^{JE} \right]}{\left[A_J^{JE}(\xi)/\mathcal{A}_J^{JE} \right]} \tilde{c}_K^{JE} \right] \right\}.$$

Hence, in going from $\xi = 1$ to $\xi < 1$ the change in revenue implied by the efficient trade taxes and consumption subsidies is given by

$$\begin{aligned}\Delta\text{Rev} = \sum_J \{ & N^J\left(\frac{\sigma}{\sigma-1}\right)(1+\nu)\left[\frac{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}}{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}} - 1\right]\left[n_{KE}\tilde{c}_K^{JE}\right] \\ & + N^J\left(\frac{1}{\sigma-1}\right)n_{KE}(1+\nu)\tilde{c}_K^{JE} \\ & - N^J n_{JE}\tilde{c}_J^{JE} - N^J\left(\frac{\sigma}{\sigma-1}\right)n_{KE}(1+\nu)\frac{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}}{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}}\tilde{c}_K^{JE} \\ & + N^J\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}}\left[\frac{P^J\left(p_J^{JE}(\xi), p_K^{JE}(\xi); \mathbf{n}_E, a_J^{JE}, a_K^{JE}\right)}{\mathcal{P}^{JE}}\right]^{\frac{\sigma-1}{\sigma}} \\ & \times\left[n_{JE}\tilde{c}_J^{JE} + n_{KE}(1+\nu)\frac{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}}{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}}\tilde{c}_K^{JE}\right],\end{aligned}$$

which simplifies to

$$\begin{aligned}\Delta\text{Rev} = \sum_J N^J\Bigg\{ & n_{KE}(1+\nu)\tilde{c}_K^{JE}\left\{\frac{A_K^{JE}(\xi)}{\mathcal{A}_K^{JE}}\left[\frac{P^J\left(p_J^{JE}(\xi), p_K^{JE}(\xi); \mathbf{n}_E, a_J^{JE}, a_K^{JE}\right)}{\mathcal{P}^{JE}}\right]^{\frac{\sigma-1}{\sigma}} - 1\right\} \\ & + n_{JE}\tilde{c}_J^{JE}\left\{\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}}\left[\frac{P^J\left(p_J^{JE}(\xi), p_K^{JE}(\xi); \mathbf{n}_E, a_J^{JE}, a_K^{JE}\right)}{\mathcal{P}^{JE}}\right]^{\frac{\sigma-1}{\sigma}} - 1\right\}\Bigg\}.\end{aligned}$$

Using $\mathcal{P}^H(\mathbf{a}^E, \mathbf{p}^E(\xi)) = \mathcal{P}^{HE}$ and the relationship between P^H and $\mathcal{P}^H$, we then have

$$\begin{aligned}\Delta\text{Rev} = \sum_J & \frac{N^J}{n_H\left(A_H^{HE}(\xi)\right)^{\sigma}\left(p_H^{HE}(\xi)\right)^{1-\sigma} + n_F\left(A_F^{HE}(\xi)\right)^{\sigma}\left(p_F^{HE}(\xi)\right)^{1-\sigma}} \\ & \times\left\{n_{KE}(1+\nu)\tilde{c}_K^{JE}\cdot\left[n_{JE}\left[\frac{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}}{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}} - 1\right](\left[A_J^{JE}(\xi)\right]^{\sigma}\left(p_J^{JE}(\xi)\right)^{1-\sigma}\right]\right. \\ & \left. + n_{JE}\tilde{c}_J^{JE}\cdot\left[n_{KE}\left[\frac{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}}{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}} - 1\right]\left[A_K^{JE}(\xi)\right]^{\sigma}\left[p_K^{JE}(\xi)\right]^{1-\sigma}\right]\right\},\end{aligned}$$

which can be rewritten as

$$\begin{aligned}\Delta\text{Rev} = \sum_J & \frac{N^J}{n_H\left(A_H^{HE}(\xi)\right)^{\sigma}\left(p_H^{HE}(\xi)\right)^{1-\sigma} + n_F\left(A_F^{HE}(\xi)\right)^{\sigma}\left(p_F^{HE}(\xi)\right)^{1-\sigma}} n_{KE}n_{JE} \\ & \times\frac{\tilde{c}_J^{JE}\tilde{c}_K^{JE}}{\left(P^J\left(p_J^{JE}(\xi), p_K^{JE}(\xi); \mathbf{n}_E, a_J^{JE}, a_K^{JE}\right)\right)^{\sigma-1}} \\ & \times\left\{(1+\nu)\frac{p_J^{JE}(\xi)}{\lambda_J^{JE}(\xi)}\left[\frac{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}}{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}} - 1\right] + \frac{p_K^{JE}(\xi)}{\lambda_K^{JE}(\xi)}\left[\frac{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}}{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}} - 1\right]\right\}\end{aligned}$$

which implies $\Delta\text{Rev} = 0$ if and only if $(1+\nu)\frac{p_J^{JE}(\xi)}{\lambda_J^{JE}(\xi)}\Big[\frac{\frac{A_K^{JE}(\xi)}{\mathcal{A}_K^{JE}}}{\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}}} - 1\Big] + \frac{p_K^{JE}(\xi)}{\lambda_K^{JE}(\xi)}\Big[\frac{\frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}}}{\frac{A_K^{JE}(\xi)}{\mathcal{A}_K^{JE}}} - 1\Big] = 0$. But substituting in the expressions for $p_J^{JE}(\xi)$ and $p_K^{JE}(\xi)$ yields

$$
\begin{aligned}
&(1+\nu)\frac{p_J^{JE}(\xi)}{\lambda_J^{JE}(\xi)}\left[\frac{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}}{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}} - 1\right] + \frac{p_K^{JE}(\xi)}{\lambda_K^{JE}(\xi)}\left[\frac{A_J^{JE}(\xi)/\mathcal{A}_J^{JE}}{A_K^{JE}(\xi)/\mathcal{A}_K^{JE}} - 1\right] \\
&\quad = (1+\nu)\left[\frac{P^J\big(p_J^{JE}(\xi), p_K^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_K^{JE}\big)}{\mathcal{P}^{JE}}\right]^{\frac{\sigma-1}{\sigma}} \\
&\quad\quad \times \left[\frac{A_K^{JE}(\xi)}{\mathcal{A}_K^{JE}} - \frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}} + \frac{A_J^{JE}(\xi)}{\mathcal{A}_J^{JE}} - \frac{A_K^{JE}(\xi)}{\mathcal{A}_K^{JE}}\right] \\
&\quad = 0.
\end{aligned}
$$

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