SUPPLEMENT TO "UNCERTAINTY SHOCKS IN A MODEL OF EFFECTIVE DEMAND: COMMENT"

(*Econometrica*, Vol. 86, No. 4, July 2018, 1513–1526)

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This supplement extends our analysis along three dimensions. One, it provides additional analytical results that show how the specification of the preference shock in recursive, Epstein and Zin (1991), preferences affects equilibrium outcomes. Two, it explores the implications of using a risk premium shock instead of a preference shock and additively separable preferences in consumption and leisure. Three, it conducts further sensitivity analysis on the parameters.

S.1. TIME-INVARIANT WEIGHTS

IN THE ARTICLE, we show that the distributional weights in recursive, Epstein and Zin (1991), preferences must sum to 1 when there is an intertemporal preference shock; otherwise, it creates an asymptote in the value function. However, when the weights are constant, they do not need to sum to 1 because it is possible to find a positive monotonic transformation of the value function that eliminates the asymptote and leaves the stochastic discount factor (SDF) unchanged (e.g., van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012)). To demonstrate this point, consider the following recursive preferences:

$$U_t = \left[c_t^{(1-\sigma)/\theta} + \beta\left(E_t\left[U_{t+1}^{1-\sigma}\right]\right)^{1/\theta}\right]^{\theta/(1-\sigma)}, \tag{S.1}$$

where $\theta \equiv (1-\sigma)/(1-1/\psi)$, $\sigma \geq 0$ determines the coefficient of relative risk aversion, $\psi \geq 0$ is the intertemporal elasticity of substitution (IES), $\beta \in (0,1)$ is the subjective discount factor, and E_t is the mathematical expectation operator conditional on information in period t. The distributional weights, 1 and β, do not sum to 1, so there is an asymptote with unit IES. The SDF is given by

$$m_{t,t+1} = \beta\left(\frac{c_t}{c_{t+1}}\right)^{1/\psi}\left(\frac{V_{t+1}^{1-\sigma}}{E_t\left[V_{t+1}^{1-\sigma}\right]}\right)^{1-\frac{1}{\theta}}.$$

To find the positive monotonic transformation of the utility function, apply the following steps:

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 https://doi.org/10.3982/ECTA15405

Step 1: Multiply and divide by $(1-\beta)^{\theta-1}$ to obtain

$$m_{t,t+1} = \beta\left(\frac{c_t}{c_{t+1}}\right)^{1/\psi}\frac{(1-\beta)^{\theta-1}}{(1-\beta)^{\theta-1}}\left(\frac{V_{t+1}^{1-\sigma}}{E_t\left[V_{t+1}^{1-\sigma}\right]}\right)^{1-\frac{1}{\theta}}$$
$$= \beta\left(\frac{c_t}{c_{t+1}}\right)^{1/\psi}\left(\frac{\left((1-\beta)^{\theta/(1-\sigma)}V_{t+1}\right)^{1-\sigma}}{E_t\left[\left((1-\beta)^{\theta/(1-\sigma)}V_{t+1}\right)^{1-\sigma}\right]}\right)^{1-\frac{1}{\theta}}.$$

Step 2: Let $W_t \equiv (1-\beta)^{\theta/(1-\sigma)}V_t$, a positive monotonic transformation of V_t. The SDF becomes

$$m_{t,t+1} = \beta\left(\frac{c_t}{c_{t+1}}\right)^{1/\psi}\left(\frac{W_{t+1}^{1-\sigma}}{E_t\left[W_{t+1}^{1-\sigma}\right]}\right)^{1-\frac{1}{\theta}}.$$

Step 3: Check the properties of W_t. We can rewrite (S.1) by substituting for V_t and V_{t+1} to obtain

$$\left((1-\beta)^{-\theta/(1-\sigma)}W_t\right)^{(1-\sigma)/\theta} = c_t^{(1-\sigma)/\theta} + \beta\left(E_t\left[\left((1-\beta)^{-\theta/(1-\sigma)}W_{t+1}\right)^{1-\sigma}\right]\right)^{1/\theta}$$
$$\rightarrow \quad (1-\beta)^{-1}W_t^{(1-\sigma)/\theta} = c_t^{(1-\sigma)/\theta} + (1-\beta)^{-1}\beta\left(E_t\left[W_{t+1}^{1-\sigma}\right]\right)^{1/\theta} \tag{S.2}$$
$$\rightarrow \quad W_t = \left[(1-\beta)c_t^{(1-\sigma)/\theta} + \beta\left(E_t\left[W_{t+1}^{1-\sigma}\right]\right)^{1/\theta}\right]^{\theta/(1-\sigma)}.$$

The distributional weights in (S.2) sum to 1, while the SDF is the same as when the weights did not sum to 1. However, if a preference shock is included and the distributional weights do not sum to 1, as in the BB model, then a similar transformation of V_t will introduce the preference shock at both t and $t+1$ in the new utility function so the weights will not sum to 1 even after the transformation.

S.2. AUGMENTED DISCOUNT FACTOR DECOMPOSITION

For simplicity, the decomposition of the augmented discount factor given in equation (18) and presented in Figure 2 of the comment is based on an approximation where $c_1^{\text{BB}} = \beta r/(1+\beta)$. We could instead solve for equilibrium c_1^{BB}. Figure S.1 shows that the approximate decomposition presented in the paper is nearly identical to the decomposition based on the exact solution for c_1^{BB}.

S.3. TOY MODEL I: TWO-PERIOD MODEL

The section solves a simple two-period endowment economy with BB preferences that analytically shows the relationship between demand uncertainty and household impatience. We set $\eta = 1$ so $u(c_t, n_t) = c_t$ and $\{a_{t+\tau}\}_{\tau=0}^{\infty} = \{1, a_{t+1}, 0, 0, \dots\}$. We assume $\log(a_{t+1}) \sim \mathbb{N}(-\sigma_a^2/2, \sigma_a^2)$ so $a_{t+1} > 0$ and $E_t a_{t+1} = 1$, but $a_{t+\tau}$ for $\tau \neq 1$ are known with certainty. Then preferences become

$$U_t^{\text{BB}} = \left[(1-\beta)c_t^{(1-\sigma)/\theta} + \beta\left(E_t\left[a_{t+1}^{\theta}c_{t+1}^{1-\sigma}\right]\right)^{1/\theta}\right]^{\theta/(1-\sigma)}.$$

The household receives a unit endowment each period and can save, x_t, at an exogenous net real interest rate $\tilde{r} = 0$. For simplicity, we set $\beta = 1$ so the household's optimality

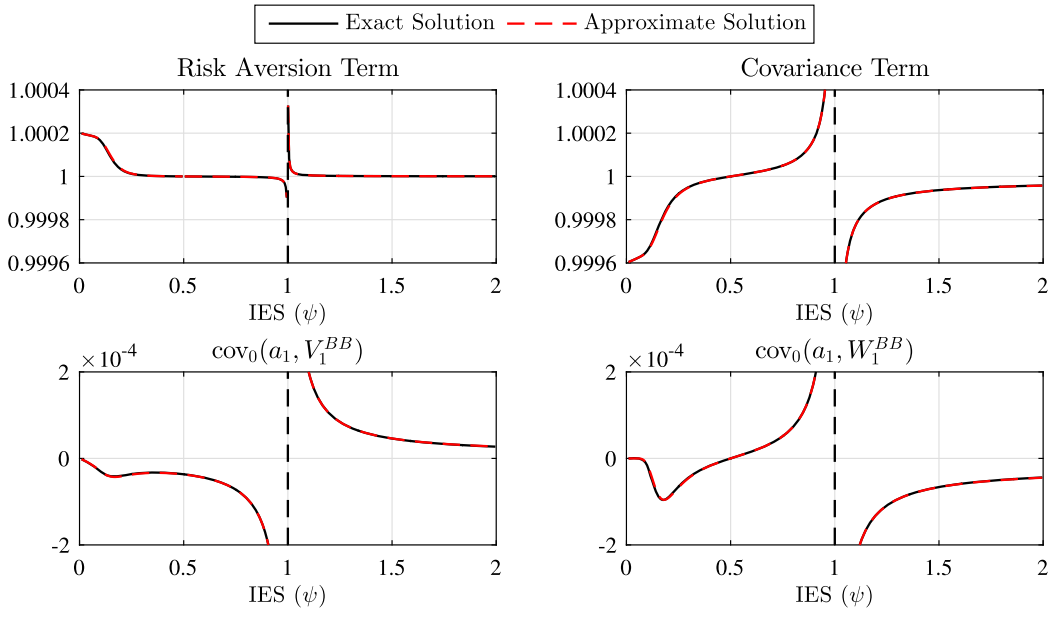


FIGURE S.1.—Key terms in the decomposition of the augmented discount factor in equation (18) of the comment.

condition is given by

$$(1 - x_t)^{-1/\psi} = \left(E_t\left[a_{t+1}^{\theta}\right]\right)^{1/\theta}(1 + x_t)^{-1/\psi}.$$

The household's intertemporal choice between consuming today or tomorrow depends on the value of $\mathbb{B} \equiv (E_t[a_{t+1}^{\theta}])^{1/\theta} = E_t[\exp(\theta \log a_{t+1})]^{1/\theta} = \exp((\theta - 1)\sigma_a^2/2)$, where $\mathbb{B}$ alters the household's impatience relative to the certainty equivalent case. In the special case when $\sigma_a = 0$, $\mathbb{B} = 1$ so $x_t^* = 0$ and $c_t^* = c_{t+1}^* = 1$. When $\sigma_a > 0$, we obtain the following conditions (based on $\sigma > 1$):

1. When $\theta < 1$ (i.e., $\psi > 1$ or $\psi < 1/\sigma$), then $\mathbb{B} < 1$ and $c_t^* > c_{t+1}^*$ (impatient households).
2. When $\theta = 1$, then $\mathbb{B} = 1$ and $c_t^* = c_{t+1}^*$ (certainty equivalent households).
3. When $\theta > 1$, just like in BB's calibration, then $\mathbb{B} > 1$ and $c_t^* < c_{t+1}^*$ (patient households).
4. As $\theta \to +\infty$ ($\psi \to 1$ from below), $\mathbb{B} \to +\infty$ and $c_t^* \to 0$.
5. As $\theta \to -\infty$ ($\psi \to 1$ from above), then $\mathbb{B} \to 0$ and $c_t^* \to c^{\max}$, where $c^{\max}$ is determined by the natural borrowing constraint.

S.4. TOY MODEL II: INFINITE HORIZON MODEL

The section solves a small-open endowment economy-type model using a Campbell–Shiller log-linear approximation that exploits the assumption of log-normal shocks. The benefit of this model is that it is easy to see the asymptote in the solution and the results are based on the shock in BB.

S.4.1. *Model*

A representative household chooses sequences of consumption, c_t, to maximize

$$U_t = \left[a_t(1-\beta)(c_t/c)^{1-\chi} + \beta(Z_t/Z)^{1-\chi}\right]^{1/(1-\chi)},$$

where $\chi = 1/\psi$ is the inverse IES and the risk aggregator, Z_t, is defined as $Z_t \equiv (E_t[U_{t+1}^{1-\sigma}])^{1/(1-\sigma)}$. The preferences are normalized so $U = 1$ in steady state. For simplicity, we assume a_t is given by

$$\hat{a}_t \equiv \log a_t - \log a = \sigma_{a,t-1}\varepsilon_t, \qquad \hat{\sigma}^2_{a,t} \equiv \sigma^2_{a,t} - \sigma^2_a = \sigma_{\sigma^a}\varepsilon_{\sigma,t}, \quad \varepsilon_t, \varepsilon_{\sigma,t} \sim \mathbb{N}(0,1),$$

where a hat denotes log-deviations from the steady state. The household's choices are constrained by $c_t + w_{t+1}/r = w_t$, where w_t is wealth and r is the gross return. The Euler equation is given by

$$1 = E_t\left[\beta r(a_{t+1}/a_t)(c_{t+1}/c_t)^{-\chi}(V_{t+1}/Z_t)^{\chi-\sigma}(1/Z)^{1-\chi}\right],$$

where V_t is the value function that solves the household's constrained optimization problem.

S.4.2. *Log-Linear Solution*

We posit the following minimum state variable solution:

$$\hat{c}_t = A_w\hat{w}_t + A_a\hat{a}_t + A_\sigma\hat{\sigma}^2_{a,t},$$
$$\hat{V}_t = B_w\hat{w}_t + B_a\hat{a}_t + B_\sigma\hat{\sigma}^2_{a,t},$$
$$\hat{w}_{t+1} = C_w\hat{w}_t + C_a\hat{a}_t + C_\sigma\hat{\sigma}^2_{a,t}.$$

A_σ is the main object of interest, since we are concerned with the response of consumption to a demand uncertainty shock. To solve the model, we first log-linearize the value function to obtain

$$\hat{V}_t = (1-\beta)\left[\hat{a}_t/(1-\chi) + \hat{c}_t\right] + \beta\hat{Z}_t,$$
$$\hat{Z}_t = -\log Z + \log\left(E_t\left[\exp\left((1-\sigma)\hat{V}_{t+1}\right)\right]\right)/(1-\sigma).$$

Notice that in log-linearized form, $\hat{a}_t$ enters the value function equation with coefficient $1/(1-\chi)$. It is the presence of this term that will generate the asymptote in A_σ when the IES is equal to 1.

After substituting the guess into the value function and then equating coefficients, we find

$$B_w = (1-\beta)A_w + \beta B_w C_w, \qquad B_a = (1-\beta)/(1-\chi) + (1-\beta)A_a + \beta B_w C_a,$$
$$B_\sigma = (1-\beta)A_\sigma + \beta\left(B_w C_\sigma + (1-\sigma)B_a^2/2\right).$$

Next, we log-linearize the Euler equation to obtain

$$0 = \log(\beta R) - (1-\sigma)\log Z$$
$$+ \log\left(E_t\left[\exp\left(\hat{a}_{t+1} - \hat{a}_t - \chi(\hat{c}_{t+1} - \hat{c}_t) + (\chi-\sigma)(\hat{V}_{t+1} - \hat{Z}_t)\right)\right]\right).$$

As before, we substitute in the unknown decision rules, collect terms, and take expectations. Since the Euler equation must hold at all points in the state space, we obtain the following restrictions:

$$0 = A_w(1 - C_w)\chi, \qquad 0 = (1 - A_a\chi) + A_w C_a \chi,$$
$$0 = \chi(A_\sigma - A_w C_\sigma) + \big(1 - A_a\chi + (\chi - \sigma)B_a\big)^2/2 - (1 - \sigma)(\chi - \sigma)B_a^2/2.$$

In steady state, $c/w = \tilde{r}/r$ where $\tilde{r} = r - 1$, so the log-linear budget constraint is given by $\hat{w}_{t+1} = r\hat{w}_t - \tilde{r}\hat{c}_t$. Substituting in the guess for the final time and equating coefficients yields

$$C_w = r - \tilde{r}A_w, \qquad C_a = -\tilde{r}A_a, \qquad C_\sigma = -\tilde{r}A_\sigma.$$

Thus, we have nine equations and nine unknown coefficients. The system implies $A_w = B_w = C_w = 1$,

$$A_a = 1/(\chi r), \qquad C_a = -\tilde{r}/(\chi r), \qquad B_a = (1 - \beta)/(1 - \chi) + (1 - \beta r)/(\chi r),$$
$$A_\sigma = -\big(\big(1 - A_a\chi + (\chi - \sigma)B_a\big)^2 - (1 - \sigma)(\chi - \sigma)B_a^2\big)/(2\chi r).$$

The gross return, r, is endogenous and must satisfy the steady-state Euler equation, given by

$$\log(\beta r) = \big[(1 - \sigma)^2 B_a^2 - \big(1 - \chi A_a + (\chi - \sigma)B_a\big)^2\big]\sigma_a^2/2$$
$$+ \big[(1 - \sigma)^2 B_\sigma^2 - \big((\chi - \sigma)B_\sigma - \chi A_\sigma\big)^2\big]\sigma_{\sigma^a}^2/2.$$

Notice A_σ depends on B_a. Since B_a has an asymptote when $\chi = 1$ (IES equals 1), so does A_σ. Therefore, it is possible to obtain an arbitrary large consumption response by setting the IES closer to 1. As χ tends to 0 or ∞ (IES moves away from 1), A_σ approaches 0. When the degree of risk aversion, σ, increases, the asymptote has a bigger effect on the consumption response. In the case when $\chi = \sigma$ (expected utility), B_a drops out of the equation for A_σ, so the asymptote disappears.

S.4.3. *Alternative Preferences*

We repeat the same exercise with the alternative preferences,

$$U_t = \big[(1 - a_t\beta)(c_t/c)^{1-\chi} + a_t\beta(Z_t/Z)^{1-\chi}\big]^{1/(1-\chi)},$$

so the weights on current and future utility sum to 1. The log-linear value function is given by

$$\hat{V}_t = (1 - \beta)\hat{c}_t + \beta\hat{Z}_t.$$

Notice the $\hat{a}_t$ term that appeared with the BB preferences drops out. The Euler equation becomes

$$1 = E_t\left[a_t\beta r\left(\frac{1 - a_{t+1}\beta}{1 - a_t\beta}\right)\left(\frac{c_{t+1}}{c_t}\right)^{-\chi}\left(\frac{V_{t+1}}{Z_t}\right)^{\chi-\sigma}\left(\frac{1}{Z}\right)^{1-\chi}\right].$$

Once again, we log-linearize the value function and the Euler equation, plug in the decision rules, and equate coefficients. After solving the system of equations, the new coefficients are given by

$$\begin{aligned} A_a &= -1/\big(\chi r(1-\beta)\big), \qquad B_a = -(1-\beta r)/\big(\chi r(1-\beta)\big), \\ A_\sigma &= -\Big(\big(-\beta/(1-\beta) - A_a\chi + (\chi-\sigma)B_a\big)^2 - (1-\sigma)(\chi-\sigma)B_a^2\Big)/(2\chi r). \end{aligned} \tag{S.3}$$

The asymptote in A_σ disappears, since there is no longer an asymptote in B_a. Also, r is given by

$$\begin{aligned} \log(\beta r) = &\big[(1-\sigma)^2 B_a^2 - \big(-\beta/(1-\beta) - \chi A_a + (\chi-\sigma)B_a\big)^2\big]\sigma_a^2/2 \\ &+ \big[(1-\sigma)^2 B_\sigma^2 - \big((\chi-\sigma)B_\sigma - \chi A_\sigma\big)^2\big]\sigma_{\sigma^a}^2/2. \end{aligned}$$

After substituting r into (S.3), we find $A_\sigma = 0$. To see that result, we guess and verify that $r = 1/\beta$ by noting $\chi A_a = -\beta/(1-\beta)$ and $B_a = 0$. Thus, households are certainty equivalent with respect to intertemporal preference shocks with our alternative preferences that eliminate the asymptote.

S.4.4. *Asymptote*

Figure S.2 plots the response of consumption to a preference volatility shock (A_σ) with the BB preferences and our alternative specification across different IES values. We set the coefficient of relative risk aversion, σ, to 80 and the shock standard deviations, σ_a and σ_{σ^a}, to 0.003—the values in BB. As our analytical solution demonstrates, there is no response of consumption to an increase in volatility with our alternative preferences. In contrast, the BB preferences break certainty equivalence because there is an asymptote in the response of consumption when the IES equals 1. Therefore, values of the IES around 1 magnify the effect of changes in $\hat{\sigma}_a^2$.[1]

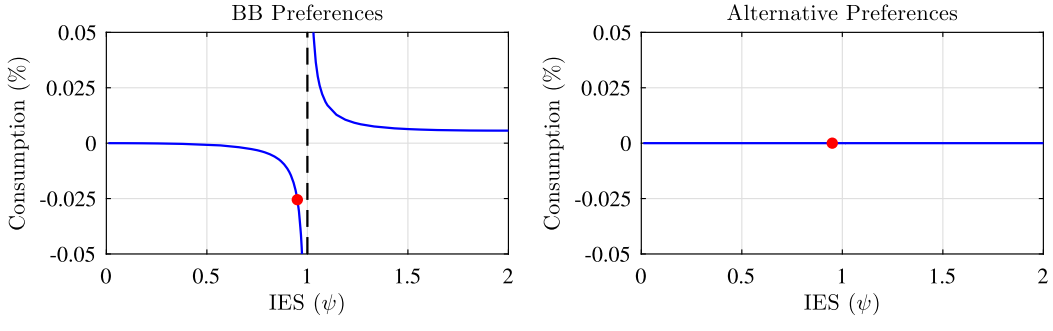


FIGURE S.2.—Impact response of consumption to a change in the standard deviation of the preference shock (A_σ).

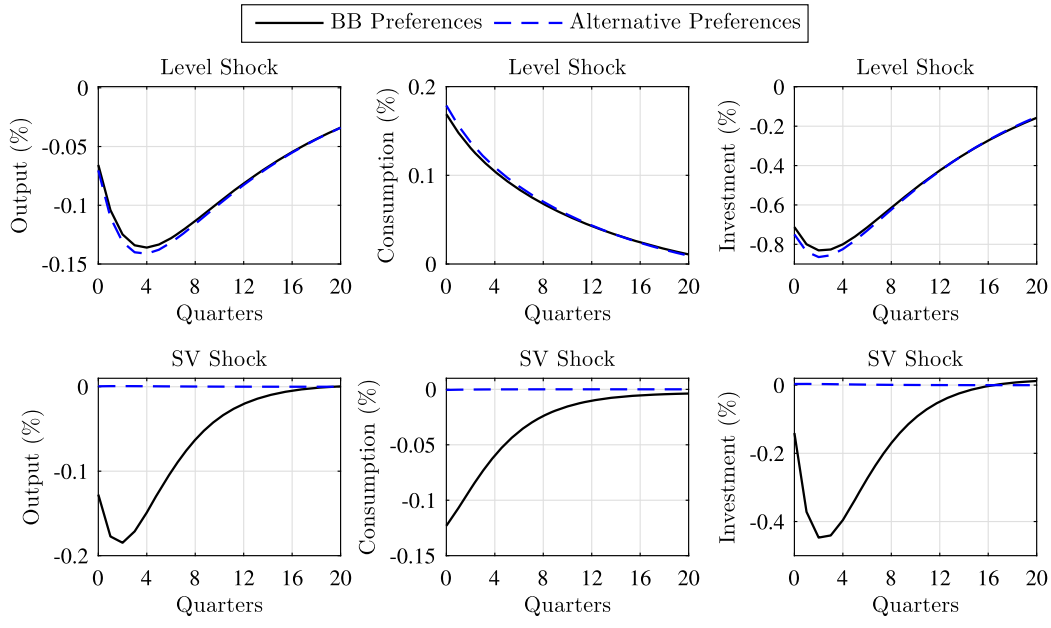


FIGURE S.3.—Responses of output, consumption, and investment to a one standard deviation preference shock.

S.5. IMPULSE RESPONSES: BB VERSUS ALTERNATIVE PREFERENCES

Figure S.3 compares impulse responses to a one standard deviation level and volatility shock to household preferences under BB preferences and our alternative specification. All of the parameters, including the IES, are set to the baseline values in BB. The top row shows the responses to the level shock are nearly identical for the two sets of preferences, which validates our transformation of the shock process. The impulse responses to the other shocks in the model—technology level and volatility shocks—are also mostly unaffected by changing the preference specification. The only time the model behaves differently is in response to higher volatility. The bottom row shows the BB preferences produce economically meaningful declines in output, consumption, and investment. In contrast, the responses to demand uncertainty shocks under our alternative preference specification are so small that it is difficult to see their shape and size when plotted on the same axes.

S.6. COMPARISON WITH RISK PREMIUM UNCERTAINTY SHOCKS

Risk premium shocks are a common alternative to preference shocks because they are a proxy for changes in demand. They also help explain the comovement between consumption and investment because risk premium shocks affect the return on risk-free bonds relative to the return on capital. If we remove the preference shock by setting $a_t = \bar{a}$ and add a risk premium shock to the return on the nominal bond in the BB model, then the

[1]The qualitative results are identical when we solve the model with persistent shocks to household preferences.

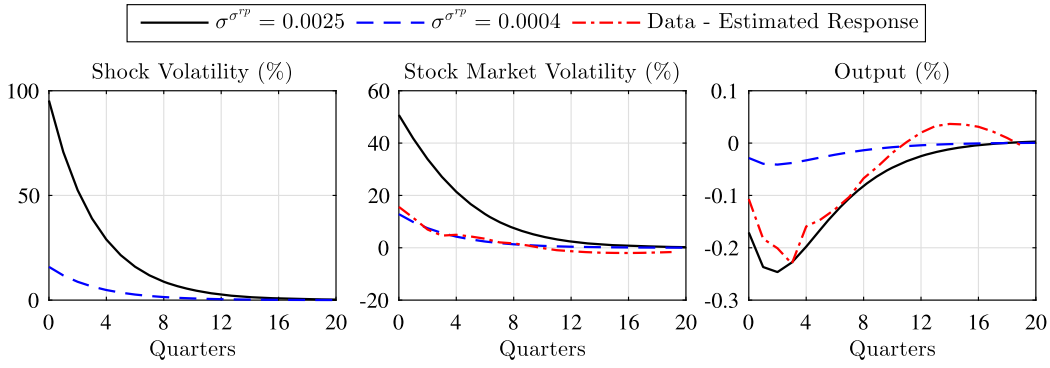


FIGURE S.4.—Responses of output and stock market volatility to a one standard deviation increase in risk premium volatility.

first-order condition for the bond becomes

$$1 = E_t\left[m_{t,t+1} a_t^{rp} r_t/\pi_{t+1}\right],$$

where r_t is the gross nominal interest rate and π_t is the gross inflation rate. Following Smets and Wouters (2007), a_t^{rp} is a risk premium shock that follows the same process as the preference shock.

To match the responses from the VAR, the model requires a very large standard deviation of the risk premium uncertainty shock (Figure S.4). As a result, the model significantly overstates the unconditional and stochastic volatility in the data, as shown in Table S.I. Moreover, the large standard deviation causes the model to overstate the increase in stock market volatility from the VAR. When we decrease the standard deviation of the volatility shock to match stock market volatility, the output response is much smaller than it is in the data even though the unconditional volatilities from the model are still larger than in the data. To test the robustness of our result, we reran BB's impulse response matching exercise, replacing the preference shock with a risk premium shock. However, the algorithm was unable to find parameters that allowed the model to match the VAR.

S.7. SENSITIVITY ANALYSIS I: EXPECTED UTILITY AND ADDITIVE SEPARABILITY

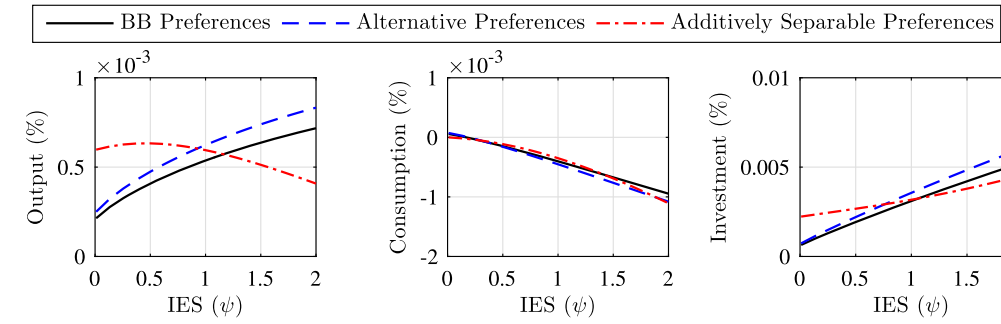
Expected utility is common in the literature. Epstein–Zin preferences collapse to expected utility when $\psi = 1/\sigma$ because the value function drops out of the SDF. Figure S.5

TABLE S.I

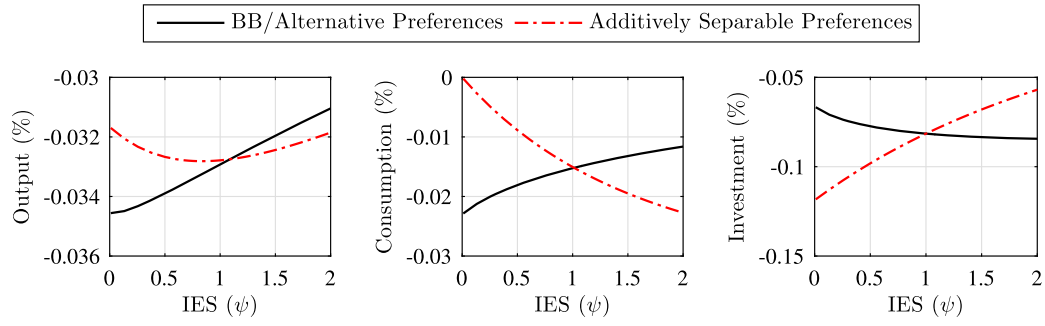
STANDARD DEVIATIONS (%)[a]

	Unconditional Volatility			Stochastic Volatility		
Moment	Data	$\sigma^{\sigma^{rp}} = 0.0025$	$\sigma^{\sigma^{rp}} = 0.0004$	Data	$\sigma^{\sigma^{rp}} = 0.0025$	$\sigma^{\sigma^{rp}} = 0.0004$
Output	1.1	5.1	3.0	0.4	1.5	0.8
Consumption	0.7	1.5	0.9	0.2	0.4	0.2
Investment	3.8	15.8	8.8	1.6	4.5	2.2

[a]The data are based on a sample from 1986 to 2014. The model-based statistics reflect the average from repeated simulations with the same length as the data. Stochastic volatility is measured by the standard deviation of the time-series of 5-year rolling standard deviations. These procedures follow Table 2 from BB.



(a) Impact responses to a preference volatility shock.



(b) Impact responses to a risk premium volatility shock.

FIGURE S.5.—Impact effect on output, consumption, and investment from a one standard deviation volatility shock.

compares the impact responses of real activity to a preference volatility shock (top panel) and a risk premium volatility shock (bottom panel) under expected utility. In addition to showing the effects of the shocks under BB's and our alternative preferences, we also consider additively separable preferences, given by $E_0 \sum_{t=0}^{\infty} \beta^t a_t[(c_t^{1-\sigma} - 1)/(1-\sigma) - \chi n_t^{1+\eta}/(1+\eta)]$, where $1/\eta$ is the Frisch elasticity of labor supply and $1/\sigma$ is the IES. With additively separable preferences, household optimality implies

$$w_t = \chi n_t^{\eta} c_t^{\sigma} \quad \text{and} \quad m_{t,t+1}^{\text{AS}} = \beta(a_{t+1}/a_t)(c_t/c_{t+1})^{\sigma}.$$

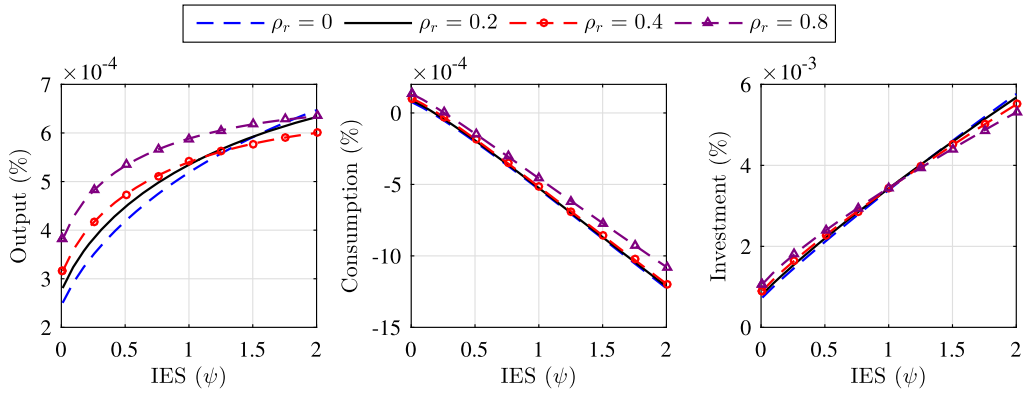
As is common practice, the preference parameter, χ, is set so steady-state labor hours equal 1/3 of available time. The other parameters and equilibrium conditions are the same as the BB model.

A preference shock has a similar effect with the BB preferences and our alternative preferences. The magnitudes are also similar with additively separable preferences. Interestingly, in all three cases, both output and investment increase, while consumption decreases. The comovement problem, however, is resolved by replacing the preference shock with a risk premium shock, regardless of whether the model has multiplicative or additively separable preferences. Once again, the impact responses are similar under additively separable preferences, although they have a different relationship with the IES parameter than in the multiplicative case. By correcting the comovement problem, the responses are slightly larger but still considerably smaller than BB's VAR estimates.

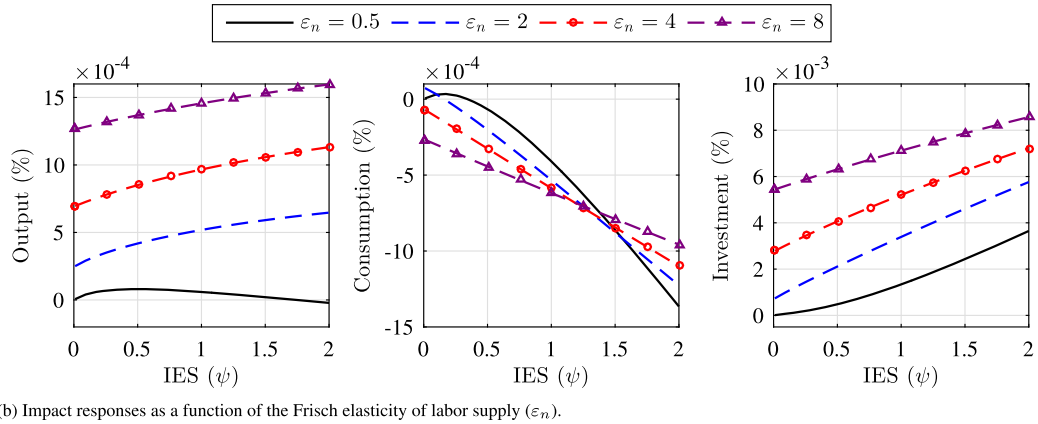
S.8. SENSITIVITY ANALYSIS II: INTEREST RATE INERTIA AND FRISCH ELASTICITY

Figure S.6 provides additional sensitivity analysis on the persistence of the nominal interest rate in the policy rule and the Frisch elasticity of labor supply by reproducing Figure 3 in the Comment. In the BB model, there is no persistence in the Taylor rule, but VAR evidence shows the federal funds rate responds to shocks in a hump-shaped pattern over time. It is also a feature commonly included in DSGE models. BB set the Frisch elasticity of labor supply to 2. We decided to examine other values given its importance for the precautionary labor supply response to uncertainty shocks.

Adding interest rate smoothing has very little effect on the size of the responses. Furthermore, given BB's baseline calibration, it does not fix the comovement problem. In the special case where the capital adjustment cost parameter is near 0, output and investment both decline but the magnitudes are so small it is impossible to find parameters where the model matches the responses from the VAR. The Frisch elasticity of labor supply has a slightly larger effect on the responses, but they are still two orders of magnitude smaller than with the BB preferences, and output and investment both increase. With elasticities near zero, output declines but investment still increases.



(a) Impact responses as a function of the interest rate persistence in the policy rule (ρ_r).



(b) Impact responses as a function of the Frisch elasticity of labor supply (ε_n).

FIGURE S.6.—Impact effect on output, consumption, and investment from a one standard deviation preference volatility shock with our alternative preferences. In each panel, the dashed line shows the response with the parameter value from BB.

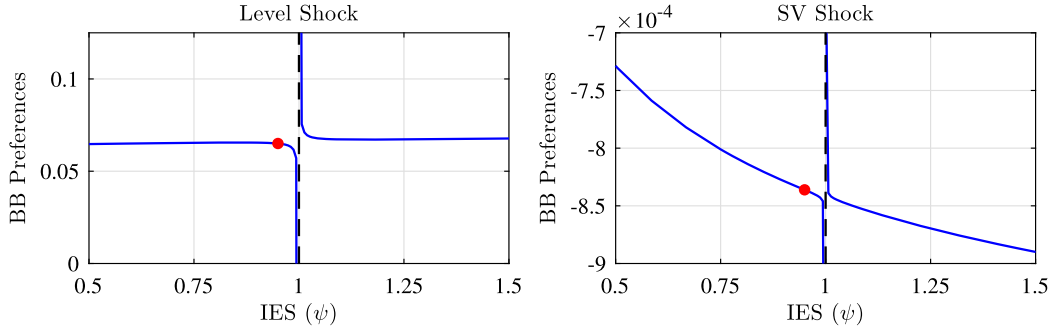


FIGURE S.7.—Responses of output to a one standard deviation increase in the level and volatility of technology.

S.9. ASYMPTOTE WITH TECHNOLOGY SHOCKS

For values of the IES near 1, this section shows that the BB preferences can affect the responses of other shocks in the model besides a preference shock. In their appendix, BB introduced a technology volatility shock that evolves in the same way as the preference volatility shock. We set the standard deviation of the volatility shock, σ^{σ^Z}, so a one standard deviation positive shock generates a 95% increase in volatility, just like the preference volatility shock. The other parameters are set to the values BB estimated, so the responses are directly comparable. Figure S.7 reports the impact effect on output from a one standard deviation increase in the level and volatility of technology as a function of the IES. Once again, with the BB preferences, an asymptote appears with unit IES, and it goes away when we adjust the distributional weights in the utility function so they always sum to 1. Those results show that the effects of the preference shock on the time aggregator spill over to the predictions of other shocks. Given the BB calibration, however, the asymptote only has a large effect on the responses when the IES is close to unity similar to first-moment preference shocks.

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Co-editor Giovanni L. Violante handled this manuscript.

Manuscript received 6 June, 2017; final version accepted 30 December, 2017; available online 24 April, 2018.