

SUPPLEMENT TO “SHADOW INSURANCE”
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APPENDIX A: STYLIZED EXAMPLES OF CAPTIVE REINSURANCE

WE ILLUSTRATE THE BALANCE SHEET MECHANICS of how an operating company can increase statutory capital by ceding reinsurance to an unauthorized captive. We offer three examples to illustrate the three main types of reinsurance: coinsurance, coinsurance with funds withheld, and modified coinsurance.¹ The latter two types differ from coinsurance in that the ceding company retains control of the assets, so the captive does not need to establish a trust fund. The examples show that the three types of reinsurance can achieve the same economic outcomes. We refer the reader to [Loring and Higgins \(1997\)](#) and [Tiller and Tiller \(2009, Chapters 4–5\)](#) for further details.

A.1. *Coinsurance*

In [Figure A.1](#), the operating company starts with \$10 in bonds and no liabilities, so its equity is \$10. For simplicity, the captive is initially a shell company with no assets. In the first step, the operating company sells term life insurance for \$100. The operating company must record a statutory reserve of \$110, which is higher than the GAAP reserve of \$90 because of Regulation XXX. Consequently, its equity is reduced to \$0.

In the second step, the operating company cedes all liabilities to the captive, paying a reinsurance premium of \$100. Reserve credit on reinsurance ceded to an unauthorized reinsurer requires collateral through a trust fund established in or an unconditional letter of credit from a qualified U.S. financial institution ([National Association of Insurance Commissioners \(2011, Appendix A-785\)](#)). Therefore, the captive establishes a trust fund with \$90 in bonds and secures a letter of credit up to \$20 to fund the difference between statutory and GAAP reserves. For simplicity, our example ignores a small fee that the captive would pay to secure the letter of credit. On the liability side, the captive records a GAAP reserve of only \$90 because it is not subject to Regulation XXX.²

¹The types of life reinsurance in the data are coinsurance, modified coinsurance, combination coinsurance, yearly renewable term, and accidental death benefit. The types of annuity reinsurance are coinsurance, modified coinsurance, combination coinsurance, and guaranteed minimum death benefit.

²Our example assumes that the operating company’s domicile does not require mirror reserving, and the captive’s domicile does not count a letter of credit as an admitted asset. If we flip both of these assumptions, the economics of this example remains the same. The captive records the letter of credit as a \$20 asset and holds a statutory reserve of \$110, so its equity remains \$10.

Operating company

(in domicile with tighter capital regulation)

1. Sells insurance for \$100
(with statutory reserve of \$110
and GAAP reserve of \$90).

2. Cedes reinsurance.

A		L		⇒	A		L		⇒	A		L										
Bonds	\$10				Bonds	\$10	Premium	\$100		Reserve	\$110	Equity	\$0		Bonds	\$10				Equity	\$10	
		Equity	\$10				Equity	\$0														

Captive

(in domicile with looser capital regulation)

2. Assumes reinsurance.
Establishes trust with \$90 in bonds.
Secures letter of credit up to \$20.

A		L			⇒	A		L	
		Equity	\$0			Trust: Bonds	\$90	Letter of credit	
						Cash	\$10	Reserve	\$90
								Equity	\$10

FIGURE A.1.—An example of captive reinsurance: Coinsurance. This example illustrates how coinsurance affects the balance sheets of an operating company and an unauthorized captive, both of which are part of the same insurance group. The operating company must hold a statutory reserve of \$110, while the captive can hold a GAAP reserve of \$90.

As a consequence of captive reinsurance, the operating company's balance sheet is restored to its original position with \$10 in equity. The captive ends up with an additional \$10 in cash that it can use for various purposes, including a commission to the operating company or a dividend to the parent company.

A.2. *Coinsurance With Funds Withheld*

The first step in Figure A.2 is the same as in Figure A.1. In the second step, the operating company cedes all liabilities to the captive, paying a reinsurance premium of \$10. The operating company withholds \$90 in the transaction, investing it in bonds. The withheld assets are recorded as a "funds held" liability for the operating company and as a "funds deposited" asset for the captive. The captive secures a letter of credit up to \$20 to fund the difference between statutory and GAAP reserves. On the liability side, the captive records a GAAP reserve of only \$90 because it is not subject to Regulation XXX.

A.3. *Modified Coinsurance*

The first step in Figure A.3 is the same as in Figure A.1. In the second step, the operating company cedes all liabilities to the captive, paying a reinsurance premium of \$10. The operating company withholds \$90 in the transaction, investing it in bonds. The withheld assets are recorded as a "modco reserve" liability for the operating company and as a "modco deposit" asset for the captive. The captive secures a letter of credit up to \$20 to fund the difference between statutory and GAAP reserves. On the liability side, the captive records a GAAP reserve of only \$90 because it is not subject to Regulation XXX.

APPENDIX B: DATA ON COMPANY CHARACTERISTICS

We construct the following company characteristics based on the NAIC annual financial statements (*A.M. Best Company (1999–2013)*). The relevant parts for our construction are Liabilities, Surplus, and Other Funds; Exhibit 5 (Aggregate Reserve for Life Contracts); Exhibit of Life Insurance; and Schedule S Part 6 (Restatement of Balance Sheet to Identify Net Credit for Ceded Reinsurance).

- Log liabilities: The logarithm of as reported total liabilities.
- Leverage: The ratio of as reported total liabilities to as reported total assets.

A.M. Best Company (2011) constructs the following company characteristics as part of the rating process.

- A.M. Best rating: We convert the A.M. Best financial strength rating (coded from A++ to D) to a cardinal measure (coded from 175 to 0%) based on risk-based capital guidelines (*A.M. Best Company (2011, p. 24)*).
- Risk-based capital: A.M. Best capital adequacy ratio, which is the ratio of adjusted capital and surplus to required capital.

Operating company
(in domicile with tighter capital regulation)

1. Sells insurance for \$100
(with statutory reserve of \$110
and GAAP reserve of \$90).

2. Cedes reinsurance, paying \$10 premium.
Invests \$90 in bonds.

<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">A</td> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">L</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Bonds \$10</td> <td style="padding: 5px;">Equity \$10</td> </tr> </table>	A	L	Bonds \$10	Equity \$10	⇒	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">A</td> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">L</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Bonds \$10 Premium \$100</td> <td style="padding: 5px;">Reserve \$110 Equity \$0</td> </tr> </table>	A	L	Bonds \$10 Premium \$100	Reserve \$110 Equity \$0	⇒	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">A</td> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">L</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Bonds \$100</td> <td style="padding: 5px;">Funds withheld \$90 Equity \$10</td> </tr> </table>	A	L	Bonds \$100	Funds withheld \$90 Equity \$10
A	L															
Bonds \$10	Equity \$10															
A	L															
Bonds \$10 Premium \$100	Reserve \$110 Equity \$0															
A	L															
Bonds \$100	Funds withheld \$90 Equity \$10															

Captive
(in domicile with looser capital regulation)

2. Assumes reinsurance.
Secures letter of credit up to \$20.

<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">A</td> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">L</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Equity \$0</td> <td style="padding: 5px;"></td> </tr> </table>	A	L	Equity \$0		⇒	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">A</td> <td style="width: 50%; border-bottom: 1px solid black; text-align: center;">L</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Funds deposited \$90 Letter of credit Cash \$10</td> <td style="padding: 5px;">Reserve \$90 Equity \$10</td> </tr> </table>	A	L	Funds deposited \$90 Letter of credit Cash \$10	Reserve \$90 Equity \$10
A	L									
Equity \$0										
A	L									
Funds deposited \$90 Letter of credit Cash \$10	Reserve \$90 Equity \$10									

FIGURE A.2.—An example of captive reinsurance: Coinsurance with funds withheld. This example illustrates how coinsurance with funds withheld affects the balance sheets of an operating company and an unauthorized captive, both of which are part of the same insurance group. The operating company must hold a statutory reserve of \$110, while the captive can hold a GAAP reserve of \$90.

Operating company
(in domicile with tighter capital regulation)

1. Sells insurance for \$100
(with statutory reserve of \$110
and GAAP reserve of \$90).

2. Cedes reinsurance, paying \$10 premium.
Invests \$90 in bonds.

A		L		⇒	A		L		⇒	A		L											
Bonds	\$10				Bonds	\$10	Premium	\$100		Reserve	\$110	Equity	\$0		Bonds	\$100	Modco reserve	\$90		Equity	\$10		
		Equity	\$10				Equity	\$0				Equity	\$10				Equity	\$10					

Captive
(in domicile with looser capital regulation)

2. Assumes reinsurance.
Secures letter of credit up to \$20.

A		L		⇒	A		L	
		Equity	\$0		Modco deposit	\$90	Letter of credit	\$20
					Cash	\$10	Reserve	\$90
							Equity	\$10

FIGURE A.3.—An example of captive reinsurance: Modified coinsurance. This example illustrates how modified coinsurance affects the balance sheets of an operating company and an unauthorized captive, both of which are part of the same insurance group. The operating company must hold a statutory reserve of \$110, while the captive can hold a GAAP reserve of \$90.

- Current liquidity: A measure of balance sheet liquidity, defined as the ratio of current assets (i.e., unencumbered cash and unaffiliated investments) to total liabilities.
- Return on equity: A measure of profitability, defined as the ratio of net operating gain after taxes to the average capital and surplus over the current and prior year.
- A.M. Best financial size category: A measure of company size (coded from 1 to 15) based on the adjusted policyholders' surplus for the insurance group.

APPENDIX C: PROOFS

PROOF OF PROPOSITION 1: We first rewrite total profit and statutory capital in period t as functions of the state variables: Y_{t-1} , L_{t-1} , \widehat{L}_{t-1} , K_{t-1} , and \widehat{K}_{t-1} . Using equations (2) and (5), we rewrite equation (1) recursively as

$$Y_t = \delta Y_{t-1} + (P_t - V)Q_t.$$

Substituting equations (2) and (3) in equation (4), we have

$$\begin{aligned} K_t &= R_{A,t}K_{t-1} + (1 + \rho)(R_{A,t} - \delta)L_{t-1} + \delta Y_{t-1} \\ &\quad + (P_t - (1 + \rho)V)Q_t + \rho V B_t. \end{aligned}$$

Substituting equations (5) and (6) in equation (7), we have

$$\widehat{K}_t = R_{A,t}\widehat{K}_{t-1} + (1 + \widehat{\rho})(R_{A,t} - \delta)\widehat{L}_{t-1} - \widehat{\rho}V B_t.$$

The first-order condition for the insurance price is

$$\begin{aligned} \frac{\partial J_t}{\partial P_t} &= \frac{\partial Y_t}{\partial P_t} + c_t \frac{\partial K_t}{\partial P_t} \\ &= Q_t + (P_t - V) \frac{\partial Q_t}{\partial P_t} + c_t \left(Q_t + (P_t - (1 + \rho)V) \frac{\partial Q_t}{\partial P_t} \right) = 0, \end{aligned}$$

which implies equation (10). The first-order condition for affiliated reinsurance is

$$\begin{aligned} \frac{\partial J_t}{\partial B_t} &= c_t \frac{\partial K_t}{\partial B_t} + \widehat{c}_t \frac{\partial \widehat{K}_t}{\partial B_t} \\ &= (c_t \rho - \widehat{c}_t \widehat{\rho})V = 0, \end{aligned}$$

which implies equation (11).

Q.E.D.

PROOF OF COROLLARY 1: The partial derivative of marginal cost with respect to affiliated reinsurance is

$$\frac{\partial \Phi_t}{\partial B_t} = \left(\frac{\rho V}{1 + c_t} \right)^2 \frac{\partial c_t}{\partial K_t}.$$

The sign of this partial derivative is determined by

$$\frac{\partial c_t}{\partial K_t} = -\frac{\partial^2 C_t}{\partial K_t^2} + \mathbb{E}_t \left[M_{t+1} \frac{\partial^2 J_{t+1}}{\partial K_t^2} \right] < 0,$$

which follows from the assumption $\partial^2 C_t / \partial K_t^2 > 0$ and $\partial^2 J_{t+1} / \partial K_t^2 < 0$ by Stokey, Lucas, and Prescott (1989, Theorem 4.8).

We rewrite excess capital in period t as

$$W_t = K_t + \widehat{K}_t - (\rho - \widehat{\rho}) \widehat{L}_t.$$

The partial derivative of excess capital with respect to affiliated reinsurance is

$$\begin{aligned} \frac{\partial W_t}{\partial B_t} &= \frac{\partial K_t}{\partial B_t} + \frac{\partial \widehat{K}_t}{\partial B_t} - (\rho - \widehat{\rho}) \frac{\partial \widehat{L}_t}{\partial B_t} \\ &= \frac{\partial P_t}{\partial B_t} \left(Q_t + (P_t - (1 + \rho)V) \frac{\partial Q_t}{\partial P_t} \right) \\ &= \frac{\partial P_t}{\partial B_t} Q_t \varepsilon_t \left(\frac{1}{\varepsilon_t} - 1 + \frac{(1 + \rho)V}{P_t} \right). \end{aligned}$$

The expression inside the parentheses is positive since

$$\begin{aligned} \frac{P_t}{V} \left(1 - \frac{1}{\varepsilon_t} \right) < 1 + \rho &\Leftrightarrow \frac{1 + (1 + \rho)c_t}{1 + c_t} < 1 + \rho \\ &\Leftrightarrow \rho > 0. \end{aligned} \quad Q.E.D.$$

APPENDIX D: A STRUCTURAL MODEL OF THE LIFE INSURANCE MARKET

We develop a structural model to test the prediction that shadow insurance reduces the marginal cost of issuing policies and increases the equilibrium supply in the retail market. We estimate the structural model on the life insurance market, rather than the annuity market, for two reasons. First, as we discussed in Section 4, life insurance accounts for a larger share of affiliated reinsurance than annuities because of Regulation (A)XXX. Second, variable annuities account for most of the annuity market, and data on their rider fees are not readily available.

D.1. *Data on Life Insurance Prices*

Our sample of life insurance premiums is from [Compulife Software \(2002–2012\)](#), which is a computer-based quotation system for insurance agents. We focus on 10-year guaranteed level term life insurance for males aged 30 as representative of the life insurance market. However, we have also examined 20-year policies and older age groups for robustness. We pull quotes for all states at the end of June in each year from 2002 to 2012, for the regular health category and a face amount of \$1 million. We merge the life insurance premiums with the company characteristics in [Appendix B](#) by company name. Whenever the premium is not available for an operating company, we assign the average premium for its insurance group.

Our measure of price is the premium divided by actuarial value. Let π_n be the one-year survival probability at age n , and let $R(m)$ be the zero-coupon Treasury (gross) yield at maturity m . We define the actuarial value of 10-year term life insurance at age n per dollar of death benefit as

$$V(n) = \left(1 + \sum_{m=1}^9 \frac{\prod_{l=0}^{m-1} \pi_{n+l}}{R(m)^m} \right)^{-1} \left(\sum_{m=1}^{10} \frac{\prod_{l=0}^{m-2} \pi_{n+l} (1 - \pi_{n+m-1})}{R(m)^m} \right).$$

We use the appropriate mortality table from the American Society of Actuaries: the 2001 Valuation Basic Table before January 2008 and the 2008 Valuation Basic Table after January 2008. These mortality tables are derived from the actual mortality experience of insured pools, so they account for potential adverse selection. We smooth the transition between the two vintages of the mortality tables by geometric averaging.

D.2. *Empirical Specification*

Operating companies optimally price insurance in an oligopolistic market. Demand is determined by the random coefficients logit model, which can be derived from a discrete choice problem. Since all companies sell the same type of policy, product differentiation is along company characteristics that capture reputation in the retail market. Life insurance is a type of intermediated savings, so the natural alternative is all saving vehicles that are intermediated by financial institutions other than insurance companies. Therefore, we specify the “outside good” as total annual saving by U.S. households in savings deposits, money market funds, and mutual funds ([Board of Governors of the Federal Reserve System \(2013, Table F.100\)](#)).

Let $P_{n,t}$ be the price of insurance sold by company n in year t . Let $\mathbf{x}_{n,t}$ be a vector of observable characteristics of company n in year t , which are determinants of demand. The probability that retail customers with preference

parameters (α, β) buy insurance from company n in year t is

$$q_{n,t}(\alpha, \beta) = \frac{\exp\{\alpha P_{n,t} + \beta' \mathbf{x}_{n,t} + \xi_{n,t}\}}{1 + \sum_{m=1}^N \exp\{\alpha P_{m,t} + \beta' \mathbf{x}_{m,t} + \xi_{m,t}\}},$$

where N is the total number of operating companies. The structural error $\xi_{n,t}$ captures company characteristics that are unobservable to the econometrician.

Let S_t be the demand for the outside good in year t , and let $Q_{n,t}$ be the demand for insurance sold by company n in year t . Let $F(\alpha, \beta)$ denote the distribution of preference parameters, which is multivariate normal with a diagonal covariance matrix. The market share for company n in year t is

$$(D.1) \quad \bar{q}_{n,t} = \frac{Q_{n,t}}{S_t + \sum_{m=1}^N Q_{m,t}} = \int q_{n,t}(\alpha, \beta) dF(\alpha, \beta).$$

The demand elasticity for insurance sold by company n in year t is

$$\varepsilon_{n,t} = -\frac{\partial \log(\bar{q}_{n,t})}{\partial \log(P_{n,t})} = -\frac{P_{n,t}}{\bar{q}_{n,t}} \int \alpha q_{n,t}(\alpha, \beta) (1 - q_{n,t}(\alpha, \beta)) dF(\alpha, \beta).$$

Equation (10) is the optimal pricing equation for each company in Nash equilibrium. Marginal cost varies across operating companies because of differences in the shadow cost of capital. Let $SI_{n,t}$ be a dummy that is 1 if company n uses shadow insurance in year t .³ Let $\mathbf{y}_{n,t}$ be a vector of observable characteristics of company n in year t , which are determinants of marginal cost. We parameterize marginal cost for company n in year t as

$$(D.2) \quad \Phi_{n,t} = \left(1 - \frac{1}{\varepsilon_{n,t}}\right) P_{n,t} = \exp\{\phi SI_{n,t} + \psi' \mathbf{y}_{n,t} + \nu_{n,t}\},$$

where the structural error $\nu_{n,t}$ represents an unobservable cost shock. Shadow insurance reduces marginal cost according to Proposition 1, so we expect that $\phi < 0$.

³The dummy for shadow insurance is 1 if gross life and annuity reserves ceded to shadow reinsurers is positive. We have also considered the share of gross life and annuity reserves ceded to shadow reinsurers, which is a continuous measure between zero and 1. Because there are relatively few companies that use shadow insurance (see Table II), there is little cross-sectional variation in the intensive margin that is useful for identification. Therefore, we report the results based on the dummy for shadow insurance to make clear that our identification is coming from the extensive margin of whether the life insurer uses shadow insurance.

D.3. Identifying Assumption

Because insurance prices are endogenous to demand, we make the following identifying assumption.

ASSUMPTION 1: *The structural error in demand (D.1) satisfies*

$$(D.3) \quad \mathbb{E}[\xi_{n,t} | \mathbf{SI}_{n,t}, \mathbf{x}_{n,t}] = 0.$$

The structural error in marginal cost (D.2) satisfies

$$(D.4) \quad \mathbb{E}[\nu_{n,t} | \mathbf{SI}_{n,t}, \mathbf{y}_{n,t}] = 0.$$

We estimate demand (D.1) and marginal cost (D.2) jointly under Assumption 1. Equation (D.3) says that shadow insurance is uncorrelated with demand, conditional on observable characteristics. A motivation for this identifying assumption is that retail customers only care about shadow insurance insofar as it reduces prices under the hypothesis that it does not increase risk. Another motivation is that retail customers do not bother gathering information about shadow insurance beyond what is already reflected in the A.M. Best rating. This exclusion restriction is plausible because the negative attention from regulators and rating agencies came after 2012 (e.g., [A.M. Best Company \(2013b\)](#), [Lawsky \(2013\)](#), [Kojien and Yogo \(2013\)](#), [Robinson and Son \(2013\)](#), and related media coverage).

The company characteristics in our specification of $\mathbf{x}_{n,t}$ are the A.M. Best rating and the conventional determinants of ratings described in Appendix B: log liabilities, risk-based capital, leverage, current liquidity, return on equity, and a dummy for stock company. Thus, the marginal effect of the A.M. Best rating can be interpreted as soft information used in the rating process that is not captured by these other variables. Given the mean and standard deviation of (α, β) , we invert equation (D.1) to recover the structural errors $\xi_{n,t}$, approximating the integral through simulation. We then construct the moments for demand by interacting the structural error with a vector of instruments, which consists of shadow insurance, company characteristics, and squared characteristics.

Equation (D.4) says that shadow insurance is uncorrelated with the cost shock, conditional on observable characteristics. The implicit assumption is that $\mathbf{y}_{n,t}$ contains all determinants of marginal cost that are also related to shadow insurance. The company characteristics in our specification of $\mathbf{y}_{n,t}$ are the same as those in $\mathbf{x}_{n,t}$, plus year dummies. Given (ϕ, ψ) , we invert equation (D.2) to recover the structural errors $\nu_{n,t}$. We then construct the moments for marginal cost by interacting the structural error with a vector of instruments, which consists of shadow insurance, company characteristics, and year dummies.

We stack the moments for demand and marginal cost and estimate the system by two-step generalized method of moments. The weighting matrix in the

first step is block diagonal in demand and marginal cost, where each block is the inverse of the quadratic matrix of the instruments. The optimal weighting matrix in the second step is robust to heteroscedasticity and correlation between the structural errors for demand and marginal cost.

D.4. *Structural Estimates of the Life Insurance Market*

Columns (1) and (2) of Table D.I report the estimated mean and standard deviation of the random coefficients in demand (D.1). Our preferred specification limits the random coefficients to log liabilities, the A.M. Best rating, and leverage. The mean coefficient on price is -1.33 with a standard error of 0.50 . This implies a demand elasticity of 2.18 for the average company in 2012. The

TABLE D.I
STRUCTURAL ESTIMATES OF THE LIFE INSURANCE MARKET^a

Variable	Demand		Marginal Cost (3)
	Mean (1)	Standard Deviation (2)	
Price	-1.33 (0.50)		
Shadow insurance			-0.13 (0.03)
Log liabilities	2.71 (0.05)	0.24 (0.11)	0.02 (0.01)
A.M. Best rating	0.13 (0.08)	0.12 (0.58)	-0.07 (0.03)
Risk-based capital	-0.07 (0.07)		0.01 (0.02)
Leverage	0.11 (0.09)	0.33 (0.15)	-0.04 (0.02)
Current liquidity	0.09 (0.06)		0.00 (0.01)
Return on equity	-0.21 (0.03)		0.04 (0.02)
Stock company	0.07 (0.10)		0.01 (0.03)
Observations	1711		

^aThe random coefficients logit model of demand (D.1) and marginal cost (D.2) are estimated jointly by generalized method of moments. The specification for marginal cost includes year dummies, whose coefficients are not reported for brevity. The instruments for demand are shadow insurance, company characteristics, and squared characteristics. The instruments for marginal cost are shadow insurance, company characteristics, and year dummies. The coefficients are standardized, and heteroscedasticity-robust standard errors are reported in parentheses. The sample consists of U.S. life insurers from 2002 to 2012, which are matched to term life insurance prices from Compulife Software.

mean coefficient on log liabilities is 2.71, and the mean coefficient on the A.M. Best rating is 0.13. That is, demand is positively related to both company size and the A.M. Best rating. The standard deviation of the random coefficient on log liabilities is 0.24 and statistically significant. Similarly, the standard deviation of the random coefficient on leverage is 0.33 and statistically significant.

Column (3) of Table D.I reports the estimated coefficients for marginal cost (D.2). Shadow insurance reduces marginal cost by 13% with a standard error of 3%. Other important determinants of marginal cost are the A.M. Best rating and leverage. Marginal cost decreases by 7% per one standard deviation increase in the A.M. Best rating. Similarly, marginal cost decreases by 4% per one standard deviation increase in leverage.

We have attempted to estimate a richer model in which price and risk-based capital also have random coefficients. However, the standard deviations of the random coefficients on price and risk-based capital converge to zero, and large standard errors reveal that the richer model is poorly identified. Similarly, we were not able to identify a richer model in which the covariance matrix for the random coefficients is not diagonal. The identification problem arises from the fact that the variation in aggregate market shares can only identify a limited covariance structure for the random coefficients.

D.5. Retail Market in the Absence of Shadow Insurance

The structural estimates in Table D.I allow us to estimate counterfactual insurance prices and market size in the absence of shadow insurance. We first set $SI_{n,t} = 0$ in equation (D.2) to estimate the counterfactual marginal cost for each company in the absence of shadow insurance. We then solve for the new price vector that satisfies the equilibrium conditions for demand (D.1) and supply (D.2). We summarize our findings in Section 5.

APPENDIX E: POTENTIAL IMPACT OF SHADOW INSURANCE ON RISK AND EXPECTED LOSS

We first show that ratings are unrelated to shadow insurance. This finding is consistent with the hypothesis that ratings correctly reflect the absence of risk in shadow insurance. However, this finding is also consistent with an alternative hypothesis that ratings do not adequately reflect the presence of risk, which is a potential concern given the evidence in Section 6. Therefore, we quantify the potential risk of shadow insurance under the alternative hypothesis based on publicly available data and plausible assumptions.

E.1. Relation Between Ratings and Shadow Insurance

According to A.M. Best Company (2013b), ratings and risk-based capital fully reflect the risk of shadow insurance. In Table E.I, we empirically investi-

gate the relation between ratings and shadow insurance, which reveals the perceived magnitude of risk. Appendix B describes how we convert the A.M. Best rating to a cardinal measure and also describes the conventional determinants of ratings that we use as regressors. We standardize ratings and all regressors that are not dummy variables, so that the coefficients have a straightforward interpretation.

In column (1) of Table E.I, we estimate the relation between ratings and a dummy for shadow insurance by ordinary least squares.⁴ Our simplest specification controls for only year and A.M. Best financial size category, whose coefficients are not reported for brevity. A coefficient of 0.03 on shadow insurance has the wrong sign if we expect shadow insurance to increase risk. However,

TABLE E.I
RELATION BETWEEN RATINGS AND SHADOW INSURANCE^a

Variable	A.M. Best Rating			Risk-Based Capital
	OLS		IV	
	(1)	(2)	(3)	
Shadow insurance	0.03 (0.06)	0.00 (0.06)	0.25 (0.34)	-0.02 (0.05)
Log liabilities		0.17 (0.04)	0.13 (0.06)	0.15 (0.05)
Risk-based capital		0.13 (0.02)	0.15 (0.02)	
Leverage		-0.01 (0.03)	0.01 (0.03)	-0.46 (0.06)
Current liquidity		0.08 (0.02)	0.06 (0.02)	0.18 (0.04)
Return on equity		0.03 (0.02)	0.03 (0.02)	0.01 (0.02)
Stock company		0.05 (0.06)	0.02 (0.07)	0.12 (0.06)
R^2	0.60	0.62	0.63	0.33
Observations	6641	6641	6351	6641

^aColumns (1) and (2) estimate the relation between A.M. Best ratings and company characteristics by ordinary least squares (OLS). Column (3) estimates the same relation by instrumental variables (IV), where the instrument for shadow insurance is the market share for term life insurance in 1999, interacted with a dummy for stock company in 1999 (see Appendix B). Column (4) estimates the relation between risk-based capital and company characteristics by OLS. All specifications include dummies for year and A.M. Best financial size category, whose coefficients are not reported for brevity. The coefficients are standardized, and the standard errors in parentheses are robust to heteroscedasticity and correlation within insurance group. The sample consists of U.S. life insurers from 2002 to 2012.

⁴See footnote 3 for why we use a dummy instead of a continuous measure for shadow insurance.

the coefficient is economically small and statistically insignificant, as ratings are only 0.03 standard deviations higher for life insurers that use shadow insurance. In column (2), we show that the coefficient on shadow insurance is robust to controlling for the conventional determinants of ratings. In [Kojen and Yogo \(2013\)](#), we also showed that the results are robust to controlling for nonlinearities through squared characteristics.

Because we do not know the proprietary model used by A.M. Best Company, omitted variables could explain the absence of a negative relation between ratings and shadow insurance. For example, A.M. Best Company could have soft information that is positively related to both ratings and the use of shadow insurance. We could address this concern through instrumental variables, but the challenge is that many known characteristics that correlate with shadow insurance (see Section 3) are also direct determinants of ratings.

Our instrument is the market share for term life insurance in 1999, interacted with a dummy for stock company in 1999. For each company, we calculate its market share as the face amount of term life insurance in force divided by the sum across all companies. The motivation for the instrument is that Regulation XXX had a stronger impact on life insurers with more presence in the term life insurance market. The interaction accounts for the fact that among those companies affected by Regulation XXX, the stock companies have a stronger incentive to take advantage of the captives laws after 2002 ([Mayers and Smith \(1981\)](#)). The market share in 1999 is plausibly exogenous to ratings after 2002, conditional on the conventional determinants of ratings, because Regulation XXX applies only to new policies issued after 2000 and does not apply retroactively to existing liabilities.

We cannot test whether the instrument is exogenous to ratings. However, we can verify that the instrument is not an obvious direct determinant of ratings in 1999, prior to changes in regulation that preceded shadow insurance. In Table E.II, we estimate the relation between ratings and company characteristics in 1999 by ordinary least squares. The coefficient on the instrument is economically small and statistically insignificant. Ratings increase by only 0.02 standard deviations per one standard deviation increase in the instrument. If the instrument were a direct determinant of ratings, we would have expected the coefficient to be economically large and statistically significant.

In column (3) of Table E.I, we estimate the relation between ratings and shadow insurance by instrumental variables.⁵ The coefficient on shadow insurance again has the wrong sign, as ratings are 0.25 standard deviations higher for life insurers that use shadow insurance. However, the coefficient is statistically insignificant with a standard error of 0.34. Interestingly, the coefficients on the conventional determinants have the expected signs with higher ratings awarded to life insurers that are larger and have higher risk-based capital, more

⁵In a first-stage regression that is not reported, the instrument is a highly relevant predictor of shadow insurance with an F -statistic of 21 ([Stock and Yogo \(2005\)](#)).

TABLE E.II
RELATION BETWEEN RATINGS AND COMPANY
CHARACTERISTICS IN 1999^a

Variable	Coefficient
Instrument for SI	0.02 (0.02)
Log liabilities	0.20 (0.06)
Risk-based capital	0.09 (0.03)
Leverage	0.00 (0.05)
Current liquidity	0.06 (0.03)
Return on equity	-0.04 (0.03)
Stock company	0.18 (0.06)
R^2	0.68
Observations	826

^aThe relation between A.M. Best ratings and company characteristics is estimated by ordinary least squares. The specification includes dummies for A.M. Best financial size, whose coefficients are not reported for brevity. The coefficients are standardized, and the standard errors in parentheses are robust to heteroscedasticity and correlation within insurance group. The sample consists of U.S. life insurers in 1999.

liquid assets, and higher profitability. Overall, the evidence in Table E.I does not suggest an economically meaningful negative relation between ratings and shadow insurance.

In addition to ratings, A.M. Best Company (2013b) claims to adjust risk-based capital for shadow insurance. In column (4) of Table E.I, we investigate the relation between risk-based capital and shadow insurance by ordinary least squares. Risk-based capital is negatively related to shadow insurance, but the coefficient is economically small and statistically insignificant. Risk-based capital is only 0.02 standard deviations lower for life insurers that use shadow insurance.

E.2. Potential Impact of Shadow Insurance on Risk

The evidence in Table E.I is consistent with the hypothesis that ratings and risk-based capital correctly reflect the absence of risk in shadow insurance. However, this evidence is also consistent with an alternative hypothesis that

ratings and risk-based capital do not adequately reflect the presence of risk. We now quantify the potential risk of shadow insurance under the alternative hypothesis based on publicly available data and plausible assumptions. The fact that accurate risk assessments are difficult highlights the importance of more transparency.

We start with accounting identities and a simple rating framework for an operating company that cedes reinsurance to a shadow reinsurer. Let A and L be the operating company's assets and liabilities, so its equity is $E = A - L$. We define leverage as L/A and risk-based capital as $RBC = E/(\kappa L)$, where the risk charge $\kappa > 0$ summarizes the risk profile of assets and liabilities. Let \widehat{A} and \widehat{L} be the shadow reinsurer's assets and liabilities, so its equity is $\widehat{E} = \widehat{A} - \widehat{L}$. Liabilities \widehat{L} are observable based on reinsurance ceded by the operating company to the shadow reinsurer. However, we do not observe \widehat{E} (equivalently \widehat{A}) or the risk profile of \widehat{A} or \widehat{L} . Therefore, we make the following assumption based on the evidence in Section 6.

ASSUMPTION 2: Shadow reinsurers do not have equity (i.e., $\widehat{E} = 0$). The risk profile of reinsurance ceded is identical to assets and liabilities that remain on the balance sheet, so the risk charge on \widehat{L} is κ .

We now ask how the operating company's balance sheet would change if shadow insurance were moved back on balance sheet. Assumption 2 yields simple adjustments to risk-based capital and leverage based on publicly available data.

PROPOSITION 2: Under Assumption 2, the adjusted risk-based capital is

$$(E.1) \quad \frac{E + \widehat{E}}{\kappa(L + \widehat{L})} = \frac{RBC \times L}{L + \widehat{L}}.$$

The adjusted leverage is

$$(E.2) \quad \frac{L + \widehat{L}}{A + \widehat{A}} = \frac{L + \widehat{L}}{A + L}.$$

Our adjustment reduces risk-based capital from 208% to 155%, or by 53 percentage points, for the average company using shadow insurance in 2012. According to equation (E.1), risk-based capital falls because equity does not change, but the capital required to support the additional liabilities (i.e., the denominator of the ratio) rises. The difference between reported and adjusted risk-based capital has increased from 10 percentage points in 2002 to 53 percentage points in 2012, as shadow insurance \widehat{L} has grown relative to liabilities L that remain on balance sheet.

We ultimately do not know how ratings would be adjusted for shadow insurance because they are based on a proprietary model and soft information that are not publicly available. However, we could get a sense of the potential magnitude by assuming that ratings are a direct function of risk-based capital. Under this assumption, we first convert the A.M. Best rating to the equivalent risk-based capital based on the guideline table in *A.M. Best Company* (2011, p. 24). For example, a rating of A is equivalent to risk-based capital of 145%. We then apply equation (E.1) to obtain the adjusted risk-based capital, which implies an adjusted rating by the same guideline table. We find that the rating drops by 3 notches from A to B+ for the average company using shadow insurance in 2012.

In Appendix F, we estimate the term structure of default probabilities by A.M. Best rating. These estimates imply default probabilities for each company, corresponding to the reported rating versus the adjusted rating. The adjusted ratings imply a 10-year cumulative default probability of 3.0% for the average company using shadow insurance in 2012, which is 3.5 times higher than that implied by the reported ratings.

E.3. *Potential Impact of Shadow Insurance on Expected Loss*

We can use the A.M. Best rating to estimate expected loss because it reflects a life insurer's claims-paying ability without support from the state guaranty associations. Let $\Pr(m|\text{Rating})$ be the marginal default probability between years $m - 1$ and m , conditional on the rating. Let θ be the loss ratio conditional on default, which we estimate to be 0.25 (see Appendix F). Let $R(m)$ be the zero-coupon Treasury (gross) yield at maturity m (Gürkaynak, Sack, and Wright (2007)). For each company, we estimate the present value of expected loss as

$$\sum_{m=1}^{15} \frac{\Pr(m|\text{Rating})\theta L}{R(m)^m}.$$

To estimate the expected loss adjusted for shadow insurance, we modify this formula by using the adjusted rating instead and replacing L with $L + \hat{L}$.

The expected loss based on reported ratings and liabilities is \$4.9 billion for the industry in 2012. The expected loss increases to \$14.4 billion when ratings and liabilities are adjusted for shadow insurance. The difference between adjusted and reported expected loss grew from \$0.1 billion in 2002 to \$9.5 billion in 2012. Since state guaranty associations ultimately pay off all liabilities by assessing the surviving companies, this expected loss represents an externality to the life insurers not using shadow insurance. State taxpayers also bear a share of the cost because guaranty association assessments are tax deductible.

To put these estimates of expected loss into perspective, we estimate the total capacity of state guaranty funds. All states cap annual guaranty association as-

assessments, typically at 2% of recent life insurance and annuity premiums. Following Gallanis (2009), we estimate the total capacity of state guaranty funds as the maximum annual assessment aggregated across all states, projected to remain constant over the next 10 years. As a share of the total capacity of state guaranty funds, the expected loss for the industry grew from 7% in 2002 to 26% in 2012.

APPENDIX F: DEFAULT PROBABILITIES AND LOSS CONDITIONAL ON DEFAULT

We describe the term structure of default probabilities and the loss ratio conditional on default, which we use to estimate expected loss in Appendix E.

F.1. *Term Structure of Default Probabilities*

We use the term structure of impairment rates from A.M. Best Company (2013a). A.M. Best Company designates an insurer as financially impaired upon the first regulatory action that restricts its activity (i.e., liquidation, supervision, rehabilitation, receivership, conservatorship, a cease-and-desist order, suspension, license revocation, or administrative order). They estimate the impairment rates by pooled method of moments, using the universe of A.M. Best rated companies from 1977 to 2012. Their sample covers 5097 companies that account for 98% of the U.S. insurance industry by premium volume. A.M. Best Company (2013a, Exhibit 2) reports the cumulative impairment rates from one to fifteen years by rating category. We calculate the marginal impairment rate between years $m - 1$ and m as the first difference of the cumulative impairment rates, which we denote as $\omega(m|\text{Rating})$.

A.M. Best Company's impairment rates have three drawbacks for our application. First, their sample includes property and casualty insurers, and they do not have separate estimates just for life insurers. Second, their estimates are subject to survivorship bias because insurers are dropped from the sample when their ratings are withdrawn.⁶ Third, we do not know the precision of their estimates because standard errors are not reported. Unfortunately, we could not obtain the data necessary to replicate their study. Although we have a complete list of impairments (A.M. Best Company (2013c, pp. 20–34)), we do not have the universe of A.M. Best rated companies from 1977 to 2012.

An impaired insurer could subsequently default on policyholder claims. A default occurs when a state regulator liquidates an insolvent insurer, and guaranty associations provide coverage to the policyholders in their state. To estimate the probability of default conditional on impairment, we merge the

⁶Ratings can be withdrawn for various reasons including voluntary liquidations, mergers and acquisitions, company request, lack of proper financial information for evaluation, and substantial changes that make the rating process inapplicable.

list of life insurer insolvencies from 1991 to 2012 (Peterson (2013)) with the list of life insurer impairments (A.M. Best Company (2013c, pp. 20–34)). Since there are 325 impairments of which 71 led to insolvency, we estimate the probability of default conditional on impairment to be 0.22.

We estimate the marginal default probability as the marginal impairment rate times the probability of default conditional on impairment:

$$\Pr(m|\text{Rating}) = \omega(m|\text{Rating}) \times 0.22.$$

We use an analogous formula for the cumulative default probability. Our estimates are consistent but potentially biased because of sampling correlation between the impairment rate and the probability of default conditional on impairment. We cannot quantify the magnitude of the bias because we do not know the precision of the impairment rates.

F.2. Loss Ratio Conditional on Default

For each life insurer insolvency from 1991 to 2012, we have the associated costs and total liabilities from Peterson (2013). The associated costs are the sum of funds necessary for reinsurance assumed, claims paid by the guaranty associations, and expenses incurred by the guaranty associations, less assets recovered. We estimate the loss ratio as the sum of associated costs divided by the sum of total liabilities aggregated across all insolvencies, which is 0.25.

REFERENCES

- A.M. BEST COMPANY (1999–2013): *Best's Statement File: Life/Health, United States*. Oldwick, NJ: A.M. Best Company. [3]
- (2011): *Best's Credit Rating Methodology: Global Life and Non-Life Insurance Edition*. A.M. Best Methodology. Oldwick, NJ: A.M. Best Company. [3,17]
- (2013a): *Best's Impairment Rate and Rating Transition Study—1977 to 2012*. A.M. Best Special Report. Oldwick, NJ: A.M. Best Company. [18]
- (2013b): *Rating Factors for Organizations Using Life Captive Reinsurers*. A.M. Best Special Report. Oldwick, NJ: A.M. Best Company. [10,12,15]
- (2013c): *U.S. Life/Health Impairment Review: L/H Impairments Hold at Half-Century Low; Accident and Health Remains Trouble Spot*. A.M. Best Special Report. [18,19]
- BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM (2013): *Financial Accounts of the United States: Historical Annual Tables 2005–2012*. Federal Reserve Statistical Release, No. Z.1. Washington, DC: Board of Governors of the Federal Reserve System. [8]
- COMPULIFE SOFTWARE (2002–2012): *Compulife Historical Data*. Harrodsburg, KY: Compulife Software. [8]
- GALLANIS, P. G. (2009): “NOLHGA, the Life and Health Insurance Guaranty System, and the Financial Crisis of 2008–2009,” Unpublished Manuscript, National Organization of Life and Health Insurance Guaranty Associations. [18]
- GÜRKAYNAK, R. S., B. SACK, AND J. H. WRIGHT (2007): “The U.S. Treasury Yield Curve: 1961 to the Present,” *Journal of Monetary Economics*, 54, 2291–2304. [17]
- KOIJEN, R. S. J., AND M. YOGO (2013): “Shadow Insurance,” Working Paper 19568, NBER. [10,14]

- LAWSKY, B. M. (2013): “Shining a Light on Shadow Insurance: A Little-Known Loophole That Puts Insurance Policyholders and Taxpayers at Greater Risk,” Unpublished Manuscript, New York State Department of Financial Services. [10]
- LORING, D. W., AND G. M. HIGGINS (1997): “Introduction to Life Reinsurance 101,” *Record of the Society of Actuaries*, 23, 1–25. [1]
- MAYERS, D., AND C. W. SMITH JR. (1981): “Contractual Provisions, Organizational Structure, and Conflict Control in Insurance Markets,” *Journal of Business*, 54, 407–434. [14]
- NATIONAL ASSOCIATION OF INSURANCE COMMISSIONERS (2011): *Accounting Practices and Procedures Manual*. Kansas City, MO: National Association of Insurance Commissioners. [1]
- PETERSON, P. A. (2013): *Insolvency Cost Information Files*. Herndon, VA: National Organization of Life and Health Insurance Guaranty Associations. [19]
- ROBINSON, S., AND M. SON (2013): “The Captive Triangle: Where Life Insurers’ Reserve and Capital Requirements Disappear,” Report 156495, Moody’s Investors Service. [10]
- STOCK, J. H., AND M. YOGO (2005): “Testing for Weak Instruments in Linear IV Regression,” in *Identification and Inference for Econometric Models: Essays in Honor of Thomas Rothenberg*, ed. by D. W. Andrews and J. H. Stock. Cambridge: Cambridge University Press, Chapter 5, 80–108. [14]
- STOKEY, N. L., R. E. LUCAS JR., AND E. C. PRESCOTT (1989): *Recursive Methods in Economic Dynamics*. Cambridge, MA: Harvard University Press. [7]
- TILLER, J. E., JR. AND D. F. TILLER (2009): *Life, Health and Annuity Reinsurance* (Third Ed.). Winsted, CT: ACTEX Publications. [1]

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