*Econometrica Supplementary Material*

# SUPPLEMENT TO "STRUCTURAL CHANGE AND THE KALDOR FACTS IN A GROWTH MODEL WITH RELATIVE PRICE EFFECTS AND NON-GORMAN PREFERENCES": ONLINE APPENDIX

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## APPENDIX B

### B.1. *Proofs of Lemmas 1–3*

#### B.1.1. *Proof of Lemma 1*

PROOF: Equation (2) corresponds to the expenditure function

$$e\big(P_G(t), P_S(t), V_i(t)\big) = \left[\varepsilon\left[V_i(t) + \frac{\nu}{\gamma}\left[\frac{P_G(t)}{P_S(t)}\right]^{\gamma} + \frac{1}{\varepsilon} - \frac{\nu}{\gamma}\right]\right]^{1/\varepsilon} P_S(t). \tag{B.1}$$

First, note that non-negativity of consumption bundles is fulfilled since $\frac{\partial e(\cdot)}{\partial P_G(t)} = \nu[\frac{e(\cdot)}{P_S(t)}]^{1-\varepsilon}[\frac{P_S(t)}{P_G(t)}]^{1-\gamma} > 0$ and $\frac{\partial e(\cdot)}{\partial P_S(t)} = [\frac{e(\cdot)}{P_S(t)}]^{1-\varepsilon}[[\frac{e(\cdot)}{P_S(t)}]^{\varepsilon} - \nu[\frac{P_G(t)}{P_S(t)}]^{\gamma}] \geq 0$ is ensured by (3) (remember that $\gamma \geq \varepsilon$). Then, according to the integrability theorem, the utility function represents a locally non-satiated preference relation if and only if the Slutsky matrix $\mathbf{H}$ is symmetric and negative semidefinite and satisfies $\mathbf{H}\cdot\mathbf{P} = \mathbf{0}$, where $\mathbf{P}$ is the vector of prices. The Hessian of (B.1) can be written as

$$\mathbf{H} = \Xi \begin{pmatrix} \frac{P_S(t)}{P_G(t)} & -1 \\ -1 & \frac{P_G(t)}{P_S(t)} \end{pmatrix},$$

where $\Xi = \nu[\frac{e(\cdot)}{P_S(t)}]^{1-2\varepsilon} P_G(t)^{\gamma-1} P_S(t)^{-\gamma}[\nu(1-\varepsilon)[\frac{P_G(t)}{P_S(t)}]^{\gamma} - (1-\gamma)[\frac{e(\cdot)}{P_S(t)}]^{\varepsilon}]$. Symmetry and the regularity condition are then straightforward. The eigenvalues of $\mathbf{H}$ are 0 and $\Xi[\frac{P_S(t)}{P_G(t)} + \frac{P_G(t)}{P_S(t)}]$. So both eigenvalues are less than or equal to zero (and the matrix is negative semidefinite) if and only if condition (3) holds. *Q.E.D.*

#### B.1.2. *Proof of Lemma 2*

PROOF: Optimization implies that the marginal rate of technical substitution is equal to the relative factor price, that is, $\frac{w(t)}{R(t)} = \frac{\alpha}{1-\alpha}\frac{K_j(t)}{L_j(t)}$, $j = G, S$. With

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$R(t)=A$ and (9), this gives (12) and (16). Next, $R(t)$ has to equalize the valued marginal product across all sectors. This yields

$$R(t)=A=(1-\alpha)\left[\frac{L(t)}{K_G(t)+K_S(t)}\right]^{\alpha}P_j(t)\exp[g_j t], \quad j=G,S,$$

where (16) has been used. Solving for $P_j(t)$ gives (14). Finally, with (16), the production functions can be rewritten as (15). *Q.E.D.*

B.1.3. *Proof of Lemma 3*

PROOF: (i) This proof is an application of Roy's identity that states $x_j^i(t)=-\frac{\partial V(\cdot)/\partial P_j(t)}{\partial V(\cdot)/e_i(t)}$ for $j=G,S$. We have $\partial V(\cdot)/\partial P_G(t)=-\nu P_G(t)^{\gamma-1}P_S(t)^{-\gamma}$, $\partial V(\cdot)/\partial P_S(t)=-e_i(t)^{\varepsilon}P_S(t)^{-\varepsilon-1}+\nu P_G(t)^{\gamma}P_S(t)^{-\gamma-1}$, and $\partial V(\cdot)/e_i(t)=e_i(t)^{\varepsilon-1}P_S(t)^{-\varepsilon}$.

Part (ii) follows immediately from (17) and (18).

(iii) The Allen–Uzawa formula for the elasticity of substitution reads $\sigma_i(t)=\frac{\partial^2 e(\cdot)}{\partial P_S(t)\,\partial P_G(t)}\frac{\partial e(\cdot)}{\partial P_G(t)}\frac{\partial e(\cdot)}{\partial P_S(t)}e(\cdot)$, where $e(\cdot)$ is the expenditure function defined in (B.1). Note that $\frac{\partial e(\cdot)}{\partial P_G(t)}$ and $\frac{\partial e(\cdot)}{\partial P_S(t)}$ are derived in the proof of Lemma 1 and $\frac{\partial^2 e(\cdot)}{\partial P_S(t)\,\partial P_G(t)}$ is given by the off-diagonal element of the Slutsky matrix $\mathbf{H}$. Plugging in the expressions, simplifying, and substituting $e(\cdot)$ by $e_i(t)$, we obtain (19). We have $\sigma_i(t)\leq 1$ because of (3) and $\gamma\geq\varepsilon$. This completes part (iii). *Q.E.D.*

## B.2. *The Specified Subclass of PIGL Preferences*

The most general (indirect) form of PIGL preferences can be written as (see Muellbauer (1975))

$$V(\mathbf{P},e_i)=\frac{1}{\vartheta}\left[\frac{e_i}{a(\mathbf{P})}\right]^{\vartheta}-b(\mathbf{P}), \tag{B.2}$$

with $\vartheta\neq 0$.[1] $e_i$ is the expenditure level of household $i$, $\mathbf{P}$ is the price vector, $a(\mathbf{P})$ is a linearly homogeneous function, and $b(\mathbf{P})$ is homogeneous of degree zero. For $\vartheta=1$ we obtain the Gorman form and for $\vartheta=-1$ the quadratic

[1]For $\vartheta=0$, we have the "PIGLOG" case with

$$V(\mathbf{P},e_i)=\frac{\log[e_i]}{\log[\tilde{a}(\mathbf{P})]}-\tilde{b}(\mathbf{P}),$$

where again $\tilde{a}(\mathbf{P})$ is a linearly homogeneous function and $\tilde{b}(\mathbf{P})$ is homogeneous of degree zero (see Muellbauer (1975)).

demand system. Finally, with $\frac{\partial b(\mathbf{P})}{\partial \mathbf{P}} = \mathbf{0}$, we obtain the class of homothetic preferences.

In the following, I show that the class of preferences specified in this paper is the most general class of intratemporal preferences defined over two sectors implying a behavior which is jointly consistent with a constant (negative) growth rate of the expenditure share devoted to one sector (see Figure 1) and a constant (positive) growth rate of per-capita expenditures (one of the Kaldor facts) in an environment where the relative price changes at a constant rate also (see Figure 2). The PIGL class of preferences is a natural starting point because it is the most general class of utility functions which avoids an aggregation problem. PIGL preferences guarantee that there exists a representative expenditure level in the sense that a household with this specific expenditure level exhibits the same expenditure shares as the aggregate economy. With PIGL preferences, this representative expenditure level is just a function of the per-capita expenditure level as well as of a summary statistic of the expenditure level distribution. Moreover, with additive separable intertemporal utility, as Crossley and Low (2011) showed, it is a necessary condition that the intratemporal preferences are part of the PIGL class, in order for the intertemporal elasticity of substitution to be constant. Since a constant intertemporal elasticity of substitution is indispensable to be consistent with the Kaldor facts, the PIGL class of preferences is the only natural starting point.

With (B.2), the marginal utility of consumption expenditures is $a(\mathbf{P})^{-\vartheta} e_i^{\vartheta-1}$. Now suppose that all relative prices change at constant growth rates (as in Ngai and Pissarides (2007)). Then, in order to fulfill the Kaldor facts, the marginal utility of consumption expenditures (and $e_i$) has to grow at a constant rate, too. With changing relative prices, this is only possible if $a(\mathbf{P}) = \bar{a} \prod_j P_j^{\alpha_j}$, where the $\alpha_j$'s sum up to 1 and $\bar{a}$ is a (positive) constant. Since utility functions are invariant with respect to positive monotonic transformations, we can normalize $\bar{a} = 1$ without loss of generality. Then, utility can be rewritten as

$$V(\mathbf{P}, e_i) = \frac{1}{\vartheta} \left[ \frac{e_i}{\prod_j P_j^{\alpha_j}} \right]^{\vartheta} - b(\mathbf{P}).$$

Applying Roy's identity, we then obtain the following expenditure share devoted to sector $j$:

$$\eta_j^i = \alpha_j + \frac{\partial b(\mathbf{P})}{\partial P_j} \left[ \frac{e_i}{a(\mathbf{P})} \right]^{-\vartheta} P_j.$$

Now suppose the expenditure share of sector $j = G$ changes at a constant rate. This is only possible if we have $\alpha_G = 0$ and $b(\mathbf{P}) = P_G^{\gamma}\bar{b}(\mathbf{P})$, where $\bar{b}(\mathbf{P})$ is homogeneous of degree $-\gamma$ and independent of $P_G$.

To sum up, the following conditions must hold in the two-sector case: $a(\mathbf{P}) = P_S$ and $b(\mathbf{P}) = \frac{\nu}{\gamma}[\frac{P_G}{P_S}]^{\gamma}$. Hence, we obtain the functional form of (2), where $\vartheta = \varepsilon$. In the paper, the restrictions on $\gamma$ and $\varepsilon$ are chosen such that the expenditure elasticity of demand of sector $G$ and the elasticity of substitution between the two sectors are smaller than or equal to unity (which is the empirically relevant case).

## B.3. *Further Extensions*

### B.3.1. *The Symmetric Counterpart of (2)*

Clearly, (2) does exclude the case where both sectors enter the preferences symmetrically. The symmetric counterpart of (2) can be written as

$$V(P_G, P_S, e_i) = \frac{1}{\varepsilon}\left[\frac{e_i}{P_G^{\varrho}P_S^{1-\varrho}}\right]^{\varepsilon} - \frac{\nu}{\gamma}\left[\frac{P_G}{P_S}\right]^{\gamma} - \frac{1}{\varepsilon} + \frac{\nu}{\gamma}.$$

Then, the regularity conditions are given by

$$\varrho\left[\frac{e_i}{P_G^{\varrho}P_S^{1-\varrho}}\right]^{\varepsilon} \geq -\nu\left[\frac{P_G}{P_S}\right]^{\gamma},$$

$$(1-\varrho)\left[\frac{e_i}{P_G^{\varrho}P_S^{1-\varrho}}\right]^{\varepsilon} \geq \nu\left[\frac{P_G}{P_S}\right]^{\gamma},$$

$$\left[\frac{e_i}{P_G^{\varrho}P_S^{1-\varrho}}\right]^{\varepsilon}(1-2\varrho-\gamma) - (1-\varepsilon)\nu\left[\frac{P_G}{P_S}\right]^{\gamma} \geq \frac{\varrho(\varrho-1)}{\nu}\left[\frac{e_i}{P_G^{\varrho}P_S^{1-\varrho}}\right]^{2\varepsilon}\left[\frac{P_G}{P_S}\right]^{-\gamma}.$$

The first two ensure non-negative consumption, whereas the third condition makes sure that the Slutsky matrix is negative semidefinite. But—as demonstrated in Appendix B.2 of this supplement—the expenditure share devoted to the goods sector changes only at a constant rate if $\varrho = 0$. For the symmetric case, the elasticity of substitution between the two goods is

$$\sigma_i = 1 - \varepsilon - (1-\varrho)\frac{[\gamma - (1-\varrho)\varepsilon]}{1-\eta_G^i} + \varrho\frac{\varrho\varepsilon + \gamma}{\eta_G^i},$$

where $\eta_G^i$ is the expenditure share of individual $i$ devoted to the goods sector.

B.3.2. *Adding a Third Sector With a Constant Expenditure Share*

By changing the preferences to

$$V(P_G, P_H, P_S, e_i) = \frac{1}{\varepsilon}\left[\frac{e_i}{P_H^{\kappa} P_S^{1-\kappa}}\right]^{\varepsilon} - \frac{\nu}{\gamma}\left[\frac{P_G}{P_S}\right]^{\gamma} - \frac{1}{\varepsilon} + \frac{\nu}{\gamma}, \tag{B.3}$$

it is possible to allow for a third sector $H$ with an expenditure elasticity of demand equal to unity. This implies that the expenditure share of sector $H$ is constant and equal to $\kappa$. We now have all three cases: a luxury $S$, a necessity $G$, and a "neutral" sector $H$. The intratemporal preferences (B.3) are consistent with balanced growth on the aggregate as long as $P_H$ and $P_S$ change at constant rates (which may differ).

B.3.3. *Hump-Shape Evolution of the Manufacturing Expenditure Share*

Suppose we have

$$V(P_M, P_F, P_S, e_i) = \frac{1}{\varepsilon}\left[\frac{e_i}{P_S}\right]^{\varepsilon} - \frac{\nu}{\gamma}\frac{[P_F^{1-\varphi} + P_M^{1-\varphi}]^{\gamma/(1-\varphi)}}{P_S^{\gamma}} - \frac{1}{\varepsilon} + \frac{\nu}{\gamma}.$$

What is called "goods" in the main text is now represented by a CES aggregate defined over manufacturing $M$ and food $F$. Hence, depending on the elasticity of substitution $\varphi$ and relative price evolution between manufacturing and food, there will be an additional structural change *within* the goods sector. Applying Roy's identity and aggregating gives the following demand system:

$$X_j^i = \nu\phi\left[\frac{E}{N}\right]^{1-\varepsilon} P_S^{\varepsilon}\frac{[P_F^{1-\varphi} + P_M^{1-\varphi}]^{\gamma/(1-\varphi)}}{P_S^{\gamma}}\frac{P_j^{-\varphi}}{P_F^{1-\varphi} + P_M^{1-\varphi}}, \quad j = M, F, \tag{B.4}$$

and

$$X_S^i = \frac{E}{NP_S}\left[1 - \nu\phi\left[\frac{P_S}{e_i}\right]^{\varepsilon}\frac{[P_F^{1-\varphi} + P_M^{1-\varphi}]^{\gamma/(1-\varphi)}}{P_S^{\gamma}}\right]. \tag{B.5}$$

The expenditure share devoted to manufacturing can then be expressed as $\eta_M = \eta_G\tilde{\eta}_M$, where $\eta_G = \nu\phi[\frac{E}{N}]^{-\varepsilon}P_S^{\varepsilon}\frac{[P_F^{1-\varphi}+P_M^{1-\varphi}]^{\gamma/(1-\varphi)}}{P_S^{\gamma}}$ is the expenditure share of the entire goods sector (i.e., manufacturing plus food) and $\tilde{\eta}_M = \frac{P_M^{1-\varphi}}{P_F^{1-\varphi}+P_M^{1-\varphi}}$ represents manufacturing expenditures relative to total goods expenditures. As in the main text, this framework is consistent with balanced growth as long as the price of the service sector changes at a constant rate (however, the expenditure share of the goods sector will no longer decline at a constant rate). Depending on the parameter $\varphi$ and the evolution of the relative price $\frac{P_F}{P_M}$, any dynamic of food's and manufacturing's expenditure share can be generated.

In the following, I illustrate that even in the simple case, where the prices of all sectors change at constant (but potentially different) rates, we can observe a non-monotonic dynamic in the manufacturing expenditure share. With constant growth rates of all relative prices, we have

$$\frac{\dot{\tilde{\eta}}_M(t)}{\tilde{\eta}_M(t)} = (1-\varphi)\big[1-\tilde{\eta}_M(t)\big](g_{P_M} - g_{P_F}). \tag{B.6}$$

Hence, $\tilde{\eta}_M$ is either monotonically increasing or decreasing, depending on whether $\varphi \gtreqless 1$ and $g_{P_M} \gtreqless g_{P_F}$. Then, the expenditure share devoted to goods is

$$\frac{\dot{\eta}_G(t)}{\eta_G(t)} = -\varepsilon[g_E - n - g_{P_S}] - \gamma(g_{P_S} - g_{P_F}) + \gamma[g_{P_M} - g_{P_F}]\tilde{\eta}_M(t). \tag{B.7}$$

Since $\frac{\dot{\eta}_M(t)}{\eta_M(t)} = \frac{\dot{\eta}_G(t)}{\eta_G(t)} + \frac{\dot{\tilde{\eta}}_M(t)}{\tilde{\eta}_M(t)}$, the growth rate of the manufacturing sector is a linear function of $\tilde{\eta}_M(t)$.

In the following, let us assume that $\tilde{\eta}_M$ is monotonically increasing over time (since, for instance, $\varphi < 1$ and $g_{P_M} > g_{P_F}$). Then, $\eta_M(t)$ is hump-shaped over time if

$$\big[\gamma - (1-\varphi)\big](g_{P_M} - g_{P_F}) < 0,$$

and

$$-\varepsilon[g_E - n - g_{P_S}] - \gamma(g_{P_S} - g_{P_F}) + (1-\varphi)(g_{P_M} - g_{P_F}) > 0.$$

The intuition is that although the expenditure share of the entire goods sector is monotonically declining due to an income and substitution effect, there is also a shifting within the good sector away from food towards manufacturing. This shifting within the good sector is strong in the beginning, which leads to an increasing expenditure share of manufacturing. Eventually, however, the declining share of goods becomes dominating and $\eta_M$ declines.

### B.4. *Steady State With Neoclassical Production Functions With Identical Labor Intensities*

Suppose the production functions are

$$Y_G(t) = A_G(t)F\big[K_G(t), \exp(\tilde{g}t)L_G(t)\big], \tag{B.8}$$

$$Y_S(t) = \exp(\tilde{g}_S t)F\big[K_S(t), \exp(\tilde{g}t)L_S(t)\big], \tag{B.9}$$

and

$$\text{(B.10)} \quad Y_I(t) = F\big[K_I(t), \exp(\tilde{g}t)L_I(t)\big],$$

instead of (7) and (8). $\tilde{g}$ is the exogenous rate of Harrod-neutral technical progress and $F[\cdot]$ is an identical neoclassical production function across sectors (with constant returns to scale and positive but diminishing marginal products). $A_G(t)$ and $\exp(\tilde{g}_S t)$ are consumption sector specific Hicks-neutral productivity terms which are time-varying. Because the neoclassical production function $F[\cdot]$ is identical, changes in the Hicks-neutral productivity terms will one-to-one determine changes in relative prices. If the Hicks-neutral productivity terms are not time-varying, we have the same specifications as in Kongsamut, Rebelo, and Xie (2001) and Foellmi and Zweimueller (2008), which abstract from relative price effects. If $F[\cdot]$ is a Cobb–Douglas, there is no difference between Hicks-neutral and Harrod-neutral technical change and the specified technologies coincide with the ones in Ngai and Pissarides (2007).

For the growth rates of relative prices, we have

$$g_{P_G}(t) - g_{P_S}(t) = \tilde{g}_S - \frac{\dot{A}_G(t)}{A_G(t)} \quad \text{and} \quad g_{P_I}(t) - g_{P_S}(t) = \tilde{g}_S.$$

Since the production functions are identical apart from the Hicks-neutral productivity terms, capital intensities must equalize across sectors, that is,

$$k_j(t) \equiv \frac{K_j(t)}{\exp(\tilde{g}t)L_j(t)} = k(t), \quad j = G, S, I,$$

where $k(t) \equiv \frac{K(t)}{\exp(\tilde{g}t)L(t)}$ is capital per efficiency units of labor. As in the main text, we chose the price of the investment good as a numéraire. Since $F[\cdot]$ has constant returns to scale, we can write the firms' first order conditions as

$$r(t) = f'\big(k(t)\big) - \delta \quad \text{and} \quad w(t) = \exp(\tilde{g}t)\big[f\big(k(t)\big) - k(t)f'\big(k(t)\big)\big],$$

where $f(k(t)) \equiv F[k(t), 1]$. Asset market clearing implies $\int_0^1 a_i(t)\,di = \frac{K(t)}{N(t)} = k(t)\exp(\tilde{g}t)L(0)$. Summing up both sides of (26) over all households and plugging in the expressions for the interest and wage rate and using the asset market clearing condition, we get

$$\text{(B.11)} \quad \dot{k}(t) = f\big(k(t)\big) - k(t)[\delta + n + \tilde{g}] - \tilde{e}(t),$$

where $\tilde{e}(t) \equiv \frac{E(t)}{\exp(\tilde{g}t)L(t)}$ is detrended per-capita consumption. In terms of detrended variables, the Euler equation and the transversality conditions are

$$\text{(B.12)} \quad \frac{\dot{\tilde{e}}(t)}{\tilde{e}(t)} = \frac{f'(k(t)) - \delta - \rho + \varepsilon\tilde{g}_S}{1 - \varepsilon} - \tilde{g},$$

and

$$\text{(B.13)} \qquad \lim_{t\to\infty} k(t)\exp\left[-\int_0^t f'\big(k(\varsigma)\big)-\delta-n-\tilde{g}\,d\varsigma\right]=0.$$

For a given $k(0)$, equations (B.11)–(B.13) define the aggregate equilibrium dynamics. If $\tilde{g}_S=0$ (and the relative price between service and investment good does not change), these equations are *identical* to the ones in a one-sector neoclassical growth model with exogenous Harrod-neutral technical progress. If $\tilde{g}_S\neq 0$, an additional constant shows up in the Euler equation but the system of differential equations remains qualitatively identical. (For this, we see that it is crucial that the Harrod-neutral technical progress in the service sector occurs at a constant rate. In contrast, Harrod-neutral progress in the good sector and consequently the *growth rate* of the relative price between good and services can change over time.)

There exists a globally stable steady state where the detrended $k(t)$ and $\tilde{e}(t)$ are constant. The steady state values are implicitly given by $f'(k^*)=\delta+\rho-\varepsilon\tilde{g}_S+(1-\varepsilon)\tilde{g}$ and $\tilde{e}^*=f(k^*)-k^*[\delta+n+\tilde{g}]$. In the steady state, the aggregate capital stock, expenditure, and output grow at the constant rate $\tilde{g}+n$. To ensure finite utility and the transversality condition, we need $\rho-n>\varepsilon\tilde{g}$ and $f'(k^*)>\delta-n-\tilde{g}$.

For $\tilde{g}=\frac{A-\delta-\rho+\varepsilon g_S}{1-(1-\alpha)\varepsilon}$, $\tilde{g}_S=g_S-\alpha\tilde{g}$, and $\frac{\dot{A}_G(t)}{A_G(t)}=g_G-\alpha\tilde{g}$, $\forall t$, the growth rates of $E(t)$, $K(t)$, $\eta_G(t)$, $P_G(t)$, $P_S(t)$, and $w(t)$ in the steady state are quantitatively identical to the ones in Proposition 2 in the main text.

### B.5. *Modeling the Input–Output Structure*

Suppose that the final investment good, consumption good, and service are produced according to the technology

$$Y_j(t)=A_j(t)\Phi_j\big(X^{va}_{G,j},X^{va}_{S,j},X^{va}_{I,j}\big), \quad j=G,S,I,$$

where $X^{va}_{G,j}$, $X^{va}_{S,j}$, and $X^{va}_{I,j}$ are value-added of the consumption good, service, and investment good sector used to produce final output of sector $j$. $\Phi_j(\cdot)$ is a neoclassical production function and $A_j(t)$ is a sector specific Hicks-neutral productivity term. Given the production function, firm's optimization will tell you how much U.S. $ value-added of industry $G$, $S$, and $I$ is generated if one U.S. $ is spent on final uses of sector $j$. For CES production functions $\Phi_j(\cdot)$, this link between value-added and final output is illustrated in equations (6) and (7) in Herrendorf, Rogerson, and Valentinyi (2013). Suppose the value-added production function of sector $j$ can be written as

$$Y^{va}_j=F\big[K_j(t),\exp[\tilde{g}t]L_j(t)\big], \quad j=G,S,I,$$

where $K_j(t)$ and $L_j(t)$ are capital and labor used in value-added industry $j = G, S, I$. $X_G^j(t)$, $X_S^j(t)$, and $X_I^j(t)$ are consumption goods, services, and investment goods used as intermediate inputs by sector $j$. Then, it is fully specified what spending a U.S. \$ on final use $j$ implies in terms of factor employment on the sectoral value-added level.

If $A_j(t) = \exp[\breve{g}_j t]$ and $\Phi_j(\cdot)$ and $F[\cdot]$ are Cobb–Douglas, the derived production functions in terms of final output are Cobb–Douglas also and the model is consistent with balanced growth on the aggregate. This is identical to the result in Ngai and Pissarides (2007), who showed in Sections IV and V how intermediate inputs or several capital goods can be introduced in their framework. (See also Ngai and Samaniego (2009) for an empirical application with the Cobb–Douglas intermediate input structure.)

Alternatively, as long as $A_S(t) = \exp(\tilde{g}_S t)$ and $A_I(t) = 1$, the reduced form of the final output production function is

$$Y_G(t) = A_G(t) F\big[\tilde{K}_G(t), \exp[\tilde{g}t]\tilde{L}_G(t)\big], \tag{B.14}$$

$$Y_S(t) = \exp(\tilde{g}_S t) F\big[\tilde{K}_S(t), \exp(\tilde{g}t)\tilde{L}_S(t)\big], \tag{B.15}$$

and

$$Y_I(t) = F\big[\tilde{K}_I(t), \exp(\tilde{g}t)\tilde{L}_I(t)\big], \tag{B.16}$$

where $\tilde{K}_j(t)$ and $\tilde{L}_j(t)$ are factors used in all value-added industries to produce final output $j$. This coincides with the reduced form analyzed in Appendix B.4 of this supplement. So the model would still be consistent with balanced growth on the aggregate.

### B.6. *Equilibrium Dynamics With Factor Intensity Differences*

Suppose that, instead of (7), the technologies are

$$Y_j(t) = \frac{\exp[g_j t]}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}} L_j(t)^{\alpha_j} K_j(t)^{1-\alpha_j}, \quad j = G, S,$$

with $\alpha_j \in (0, 1)$, $j = G, S$, and $\alpha_G \neq \alpha_S$. We assume that the relative factor endowments are the same for all households, that is, $\frac{a_i(0)}{l_i} = \frac{K(0)}{L(0)}$, $\forall i$. Under this assumption, changes in the expenditure shares—which will now affect the relative factor reward $\frac{R(t)}{w(t)}$—do not affect the inequality measurement $\phi$.[2] Finally, we assume that condition (3) holds with strict inequality.[3]

[2]Without this assumption, the joint $l_i$ and $a_i(0)$ distribution would have to be specified and multiple equilibria could potentially arise.

[3]This assumption shortens the subsequent proofs, since a separate discussion of the case in which—by coincidence—$\eta_G(0) = 1$ can be avoided.

With factor intensity differences, Lemma 2 will not hold anymore. Together with zero profits and market clearing, firms' cost minimization yields $w(t)L_G(t) = \alpha_G \eta_G(t)E(t)$ and $w(t)L_S(t) = \alpha_S[1 - \eta_G(t)]E(t)$. Combining these expressions with the labor market clearing condition gives

$$w(t) = \frac{E(t)}{L(t)}\big[\alpha_S + \eta_G(t)[\alpha_G - \alpha_S]\big]. \tag{B.17}$$

The AK technology of the investment goods sector is unchanged. Consequently, we still have $r = R - \delta = A - \delta$. Equilibrium prices are given by

$$P_j(t) = \exp[-g_j t]w(t)^{\alpha_j} A^{1-\alpha_j}, \quad j = G, S. \tag{B.18}$$

Combining (23) with (B.17), (B.18), and the definition of $L(t)$, we obtain

$$\eta_G(t) = \tilde{\nu}\left[\frac{E(t)}{L(t)}\right]^{\alpha_G\gamma-(\gamma-\varepsilon)\alpha_S-\varepsilon} L(0)^{-\varepsilon}\big[\alpha_S + \eta_G(t)(\alpha_G - \alpha_S)\big]^{\alpha_G\gamma-(\gamma-\varepsilon)\alpha_S}, \tag{B.19}$$

where $\tilde{\nu} \equiv \nu\phi A^{\varepsilon+\alpha_S(\gamma-\varepsilon)-\alpha_G\gamma} \exp[((\gamma - \varepsilon)g_S - \gamma g_G)t]$. Differentiating (B.19) with respect to time gives

$$\begin{aligned} \frac{\dot{\eta}_G(t)}{\eta_G(t)} &= \hat{\gamma}\left[\frac{\dot{E}(t)}{E(t)} - n\right] + (\gamma - \varepsilon)g_S - \gamma g_G \\ &\quad + [\hat{\gamma} + \varepsilon]\frac{\dot{\eta}_G(t)[\alpha_G - \alpha_S]}{\alpha_S + \eta_G(t)[\alpha_G - \alpha_S]}, \end{aligned} \tag{B.20}$$

where $\hat{\gamma} \equiv \alpha_G\gamma - (\gamma - \varepsilon)\alpha_S - \varepsilon$. With (B.17) and (B.18), the Euler equation (25) can be written as

$$\big[1 - \varepsilon(1 - \alpha_S)\big]\left[\frac{\dot{E}(t)}{E(t)} - n\right] = A - \delta - \rho + \varepsilon g_S - \frac{\varepsilon\alpha_S\dot{\eta}_G(t)[\alpha_G - \alpha_S]}{\alpha_S + \eta_G(t)[\alpha_G - \alpha_S]}. \tag{B.21}$$

Finally, the law of motion of the capital stock is given by $\frac{\dot{K}(t)}{K(t)} = A - \delta + \frac{w(t)L(t)}{K(t)} - \frac{E(t)}{K(t)}$. With (B.17), this can be written as

$$\frac{\dot{K}(t)}{K(t)} = A - \delta - \frac{E(t)}{K(t)}\big[1 - \alpha_S - \eta_G(t)(\alpha_G - \alpha_S)\big]. \tag{B.22}$$

Equations (B.20), (B.21), (B.22) and the transversality condition define the evolution of $\eta_G(t)$, $E(t)$, and $K(t)$. $K(0)$ is exogenously given. The non-predetermined $E(0)$ implicitly pins down $\eta_G(0)$ according to (B.19).[4]

[4]For any given $E(t)$, a unique $\eta_G(t) \in (0, 1)$ fulfills (B.19). To see this, note that at $\eta_G(t) = 0$, the left-hand side (LHS) of (B.19) is zero, whereas the right-hand side (RHS) is $\in (0, 1)$ (the

A constant growth path (CGP) is defined according to Acemoglu and Guerrieri (2008) as an equilibrium growth path along which expenditures grow at a constant rate. We have the following proposition:

PROPOSITION 1: *Suppose we have*

$$(\gamma-\varepsilon)g_S-\gamma g_G+\left[\alpha_G\gamma-(\gamma-\varepsilon)\alpha_S-\varepsilon\right]\frac{A-\delta-\rho+\varepsilon g_S}{1-(1-\alpha_S)\varepsilon}<0, \tag{B.23}$$

*and let us denote asymptotic values by a superscript* $A$ (*i.e.*, $z^A=\lim_{t\to\infty}z(t)$, *for* $z=\eta_G, g_E, g_K, g_w, g_{\eta_G}$). *Then, there exists a globally saddle-path stable CGP with*

$$\eta_G^A=0,$$
$$g_E^A-n=g_K^A-n=g_w^A=\frac{A-\delta-\rho+\varepsilon g_S}{1-(1-\alpha_S)\varepsilon},$$
$$g_{\eta_G}^A=(\gamma-\varepsilon)g_S-\gamma g_G+\left[\alpha_G\gamma-(\gamma-\varepsilon)\alpha_S-\varepsilon\right]\left[g_E^A-n\right]<0.$$

PROOF: With the expressions for $g_E^A$ and $g_{\eta_G}^A$, (B.20) and (B.21) can be rewritten as

$$\frac{\dot\eta_G(t)}{\eta_G(t)}=\left[\frac{\dot E(t)}{E(t)}-g_E^A\right]\hat\gamma+(\hat\gamma+\varepsilon)\frac{\dot\eta_G(t)[\alpha_G-\alpha_S]}{\alpha_S+\eta_G(t)[\alpha_G-\alpha_S]}+g_{\eta_G}^A,$$

and

$$\frac{\dot E(t)}{E(t)}-g_E^A=-\frac{\varepsilon\alpha_S\dot\eta_G(t)[\alpha_G-\alpha_S]}{[1-\varepsilon(1-\alpha_S)][\alpha_S+\eta_G(t)[\alpha_G-\alpha_S]]}. \tag{B.24}$$

Solving these two equations for $\dot\eta_G(t)$ gives

$$\dot\eta_G(t)=\frac{g_{\eta_G}^A\eta_G(t)\left[\eta_G(t)+\dfrac{\alpha_S}{\alpha_G-\alpha_S}\right]}{\dfrac{\alpha_S}{\alpha_G-\alpha_S}+\eta_G(t)\left[1-(\hat\gamma+\varepsilon)+\alpha_S\varepsilon\dfrac{\hat\gamma}{q}\right]},$$

upper bound is ensured by the strict inequality of condition (3)). On the contrary, with $\eta_G(t)=1$ we have LHS $=1>$ RHS. Hence, since both are continuous functions, there is at least one intersection between 0 and 1. Finally, since the LHS is linear and the RHS is either concave or convex on the entire domain, we have at most two intersections. Hence, LHS and RHS cross exactly once between 0 and 1.

where $q \equiv 1 - \varepsilon(1 - \alpha_S)$. Hence, $\dot{\eta}_G(t)$ is zero if and only if $\eta_G(t) = 0$ (note that $\eta_G(t) \in [0, 1]$). The equilibrium with $\dot{\eta}_G(t) = \eta_G(t) = 0$ is stable since

$$\left.\frac{\partial \dot{\eta}_G(t)}{\partial \eta_G(t)}\right|_{\eta_G(t)=0} = g^A_{\eta_G} < 0.$$

Hence, no matter where we start, $\eta_G(t)$ will always converge to $\eta^A_G = 0$ and consequently $\frac{\dot{E}(t)}{E(t)}$ will converge to $g^A_E$ (see (B.24)). Hence, asymptotically we have $\frac{\dot{K}(t)}{K(t)} = A - \delta - \frac{E(t)}{K(t)}[1 - \alpha_S]$ and $E(t)$ grows at a constant rate. This is exactly the same structure as in the equilibrium of the main text. Then, by the same argument as in the proof of Proposition 2, the transversality condition is violated unless $\frac{\dot{K}(t)}{K(t)} = \frac{\dot{E}(t)}{E(t)}$. *Q.E.D.*

The CGP is very similar to the one in Acemoglu and Guerrieri (2008). But, the important difference to Acemoglu and Guerrieri (2008) is that this model features an income effect (as long as $\varepsilon > 0$). In contrast to the asymptotic equilibrium in the main text (see Proposition 3), the structural change is now also governed by the sectoral differences in the output elasticities of labor. The intuition is the same as in Acemoglu and Guerrieri (2008): We have capital deepening, whereby the relative factor price of labor, $\frac{w(t)}{R(t)}$, increases over time. This increases the relative price of the sector, which is more labor intensive. Finally, according to the substitution effect, this relative price drift affects the structural change. Condition (B.23) ensures $g^A_{\eta_G} < 0$ and guarantees global stability.

### B.7. *Relating $\phi$ to an Atkinson Index*

The Atkinson index (Atkinson (1970)) is defined as

$$\mathcal{I}_A\left(\zeta, \{e_i(t)\}^1_{i=0}\right) = 1 - \frac{N(t)}{E(t)}\left[\int_0^1 e_i(t)^{1-\zeta}\, di\right]^{1/(1-\zeta)},$$

with the parameter $\zeta \geq 0$ being the relative inequality aversion. Then, we can write

$$\phi(t) = \left[1 - \mathcal{I}_A\left(\varepsilon, \{e_i(t)\}^1_{i=0}\right)\right]^{1-\varepsilon},$$

that is, $\phi$ is a negative, monotonic transformation of the Atkinson inequality index with $\zeta = \varepsilon$. Hence, $\phi$ is ordinally equivalent to the inverse of an Atkinson index. This justifies our interpretation of $\phi$ as an inverse measurement of expenditure inequality fulfilling the principle of transfers, scale invariance, and decomposability (see Cowell (2000)).

### B.8. *Data Sources, Description, and Sample Selection*

This section documents the data sources and the construction of the variables as well as the selection of the samples. The paper uses the following data sources:

- *Bureau of Economic Analysis (BEA)*: The BEA divides total personal consumption expenditure of the United States into "goods" and "services" (see BEA's NIPA Table 1.1.5). At the most detailed level of consumption expenditure categories, this classification can be seen in the underlying detail Table 2.4.5U. The classification follows the recommendations of the System of National Accounts (SNA), which ensures international comparability. Moreover, the BEA provides price indices for the different expenditure categories (see NIPA Table 1.1.4) and the corresponding real quantities (see NIPA Table 1.1.3). Finally, the population data are taken from NIPA Table 7.1. The paper uses the yearly data of the years 1946–2012.

In NIPA Tables 1.1.3–1.1.5, the BEA differentiates between durable and non-durable goods. Figures B.8–B.10 in this supplement show the trend in the expenditure share, relative price, and relative real quantity if durable goods are excluded.

- *Consumer Expenditure Survey (CEX)*: The paper uses the CEX survey data after the introduction of the "NEW ID" in 1986. The data of the years 1986–2010 are obtained from ICPSR. The data of the year 2011 are obtained directly from the Bureau of Labor Statistics (BLS). The expenditure of a specific household over a 3 month period is considered as an observation. The BEA's classification of total personal consumption expenditures into goods and services is redone with the CEX data. This replication is subject to smaller consistency issues due to differences in the level of aggregation. Moreover, one has to keep in mind that the BEA data include purchases by nonprofit institutions serving households and by U.S. government civilian and military personnel stationed abroad, whereas the CEX data only cover household expenditure. Also, some fringe benefit payments made by an employer are accounted in the BEA data, but are not reported in the CEX data.

The following CEX categories are considered as services: food away from home; shelter; utilities, fuels and public services; other vehicle expenses; public transportation; health care; personal care; education; cash contributions; personal insurance; and pensions. For homeowners, the imputed renting value is taken as shelter expenditures. The remaining categories are considered as goods. I drop all observations with missing income/transfer reports (177,000 observations), non-positive food expenditures (additional 1,006 observations), or with an expenditure share of goods $\notin [0, 1]$ (additional 99 observations). This leaves me with 477,730 observations (i.e., 3 month expenditure spells and 177,419 households).

In each year, I group the observations into five income quintiles according to the total household labor earnings after taxes plus transfers per OECD modified equivalence scale. For each quintile $q = 1, \dots, 5$, the (average) expenditure share of goods is calculated as

$$\eta_G^q(t) = \frac{1}{|\mathcal{N}_q(t)|} \sum_{i \in \mathcal{N}_q(t)} \eta_G^i(t), \tag{B.25}$$

and the average expenditure per-equivalence scale is calculated as

$$e_q(t) = \frac{1}{|\mathcal{N}_q(t)|} \sum_{i \in \mathcal{N}_q(t)} e_i(t), \tag{B.26}$$

where $\mathcal{N}_q(t)$ is the set of households that belong to quintile $q$ in year $t$. Figure 4 plots $\eta_G^q(t)$ for $q = 1, \dots, 5$ and $t = 1986, \dots, 2011$. Figure 7 shows the scatter plot between $\log \eta_G^q(t)$ and $\log e_q(t)$ for all quintiles and years (after allowing for a distinct intercept for each year).

The regressions in columns (1) and (2) in Table I are on the disaggregate level. The additional controls in column (2) are: "Children share" and "Elderly share," which measure the share of household members with age $< 18$ and $\geq 65$, respectively. "Residence indicators," which consist of four regional indicators (northeast; midwest; south; west), a rural/urban dummy as well as indicators of different population sizes of the city of residence (more than 4M; 1.2–4M; 0.33–1.19M; 125–329.9k; less than 125k). "Family size indicators" consists of 11 groups (family size of one, two, ..., ten and above ten). And finally "Ref. person controls," which consist of the age, the sex, seven skill level indicators (never attended school; elementary school; high school, less than h.s. graduate; h.s. graduate; college, less than college graduate; college graduate (associate's D. and bachelor's D.); graduate school (master's D. and Professional/Doctorate's D.)) and four race indicators (white; black; native american; asian) of the reference person.

In columns (4) and (5) of Table I, durable goods are excluded. The following categories are considered as durable goods: old and new vehicles; house furnishing, and equipment.

This supplement provides the regressions corresponding to columns (1) and (2) in Table I with CEX diary data of the year 2010. In the diary data, the logarithmized labor and capital earnings after taxes and deductions plus transfers per equivalent scale is used as an instrument for the expenditure level. The following categories are considered as service: shelter (mortgage payments, rents and buying of land and houses); food and drinks away from home; fuels and electricity, public services; clothing repair services; vehicle leasing; vehicle repair and maintenance services; house maintenance services; healthcare services; entertainment services; sports equipment rental; personal care; school tuition. The remaining categories are considered as "goods." Observations are monthly expenditures of a particular household. The regressions include month fixed effects. The sample selection and the additional control variables in column (2) of Table B.II are identical to the ones in Table I.

- *Panel Study of Income Dynamics (PSID)*: The PSID wave of 2009 includes relatively detailed expenditure data. The following expenditure categories are considered as "goods": Food at Home (F12, F18, F22), Car Purchase Cash-down Payments (F64), Car Loan Payments (F67), Car Lease Payments (F71, F72), Additional Car/Lease Payments (F79), Gasoline (F80B), Household Furnishings (F88), and Clothing (F89). The following expenditure categories are considered as "services": Property Tax Expenses (A21), Owner Insurance (A22), Mortgage Payments (A25), Rent (A31), Home Repairs (F87), Utilities (A41–A45B), Health Insurance (H63), Hospital/Nursing Home Expenses (H64), Doctor/Outpatient Surgery/Dental Expenses (H70), Prescriptions/Other Medical Services (H76), Donations (M2A–M12B), Child Care (F6D), Food Delivered to Home (F20, F24), Food Eaten Out (F21, F25), Car Insurance (F77), Car Repair (F80A), Parking (F80C), Bus/Train Fares (F81A), Cab Fares (F81B), Other Transportation Expenses (F81C), School Expenses (F83), Other School Expenses (F86), Trips/Vacations (F90), Other Recreation (F91), Support of Others (G106), and Voluntary Pension Contributions of Head and Wife (P15, P85). Since expenditures of different categories are reported for different time intervals, all expenditures are annualized. The expenditure share of goods is "good expenditure" relative to "good expenditure" plus "service expenditure." These PSID data are used in column (3) of Table I. The logarithmized expenditure level per equivalent scale is instrumented by the logarithmized "Total Labor Income" plus "Total Transfer Income" plus "Social Security Income" per OECD equivalent scale. "Children share" and "Elderly share" are measured as in the CEX data. "Residence indicators" consist of five regional indicators (northeast; northcentral; south; west; alaska&hawaii) as well as six indicators of different population sizes of the city of residence (Central counties of at least 1M population; Fringe counties of at least 1M population; Counties in metropolitan areas; Urban population of 20k or more; Urban population of less than 20k; Completely rural areas). "Family size indicators" consist of the same 11 groups as in the CEX data. "Family head controls" consist of the age, the sex, seven skill level indicators (constructed as in the CEX data), and seven race indicators (white; black; hispanic; indian; asian; pacific-islander; other-race) of the family "head."
- *Eurostat*: As the BEA data, the expenditure data are based on the Classification of Individual Consumption by Purpose (COICOP) recommended by the System of National Accounts (SNA). From Eurostat I, use the 3 digit COICOP expenditure and price data (see tables "nama_co3_c" and "nama_co3_p"). The following categories are considered as "goods": Food and non-alcoholic beverages; Alcoholic beverages, tobacco, and narcotics; Clothing and footwear;

Electricity, gas, and other fuels; Furnishings, household equipment, and routine maintenance of the house; Medical products, appliances, and equipment; Purchase of vehicles; Telephone and telefax equipment; Audio-visual, photographic, and information processing equipment; Other major durables for recreation and culture; Other recreational items and equipment, gardens, and pets; Newspapers, books, and stationery; Personal care products; and Personal effects n.e.c. The following categories are considered as "service": Actual rentals for housing; Imputed rentals for housing; Maintenance and repair of the dwelling; Water supply and miscellaneous services relating to the dwelling; Out-patient services; Hospital services; Operation of personal transport equipment; Transport services; Postal services; Telephone and telefax services; Recreational and cultural services; Package holidays; Education; Restaurants and hotels; Social protection; Insurance; Financial services n.e.c.; and Other services n.e.c. The category "Prostitution" is excluded since there are too many missing values. At this disaggregate level, Eurostat provides data for the following 23 countries: Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, and United Kingdom. These 23 countries and the BEA data of the United States are included in Figure B.12. Eurostat provides price indices for the different expenditure subcategories (although for a shorter time period for some countries). The price index of "goods" and "services" is calculated as a Fisher chained price index. The real quantities are calculated as the nominal expenditure of goods/services divided by the corresponding price index. The ratio of the price indices is plotted in Figure B.13. The relative real quantity of services is illustrated in Figure B.14. Due to the lack of reliable price data on the disaggrate level, Portugal is not included in these two figures.

- *Living Costs and Food Survey (LCF)*: In Figures B.3 and B.4, I use U.K. survey data provided in the "Family Spending" files by Office for National Statistics (ONS). The expenditure categories of recent years are not comparable with the years before 2001/2002. So I use data covering the years 2001–2011. In each year, the households are grouped into the five quintiles according to gross family income. For each income quintile, I calculate the average weekly expenditure per capita by aggregating "all expenditure groups 1–12" and dividing by "number of persons per household." Expenditure on "goods" is defined as "all expenditure groups 1–12" minus "housing (4)," "health (6)," "transport (7)," "communication (8)," "recreation & culture (9)," "education (10)," and "restaurant & hotel (11)." The expenditure share on goods of quintile $i$ is calculated as the sum of expenditures on "goods" of all households belonging to quintile $i$ divided by the sum of "all expenditure groups 1–12" of households belonging to quintile $i$.

B.9. *Additional Figures and Tables*

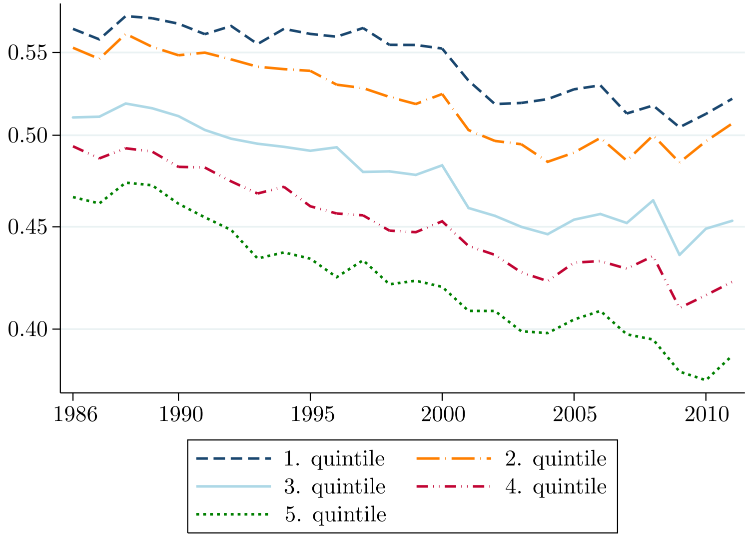


FIGURE B.1.—Cross-sectional variation excluding shelter. The figure plots the expenditure share devoted to goods for each income quintile of the U.S. on a logarithmic scale when shelter expenditures are excluded. As in Figure 4, the quintiles refer to total household labor earnings after tax plus transfers per OECD modified equivalence scale. The sample consists of expenditure data of 477,730 quarters (and 177,419 households). Source: Consumer Expenditure Survey interview data obtained from the BLS for the year 2011 and from the ICPSR for all other years.

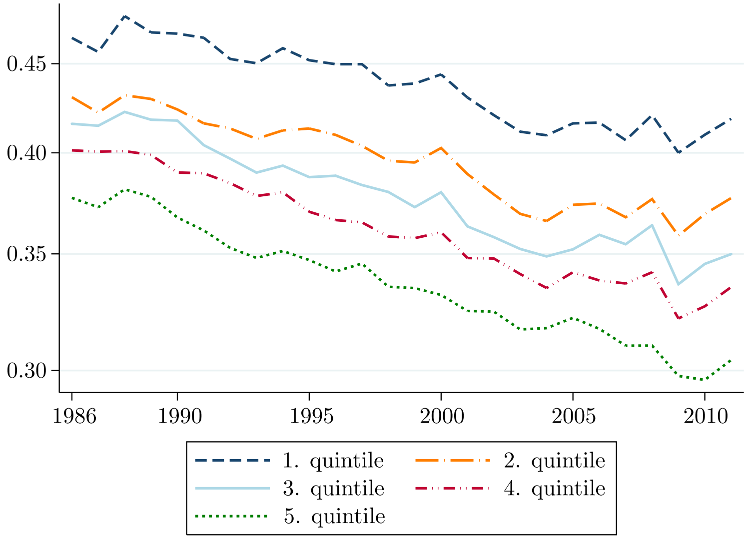


FIGURE B.2.—Expenditure structure with total income instead of labor earnings to form the quintiles. This figure plots the expenditure share devoted to goods for each income quintile of the U.S. on a logarithmic scale. The expenditure share devoted to goods is calculated as in Figure 4. However, in contrast to Figure 4, the income quintiles refer to total household income after taxes plus transfers per OECD modified equivalence scale. The sample consists of expenditure data of 477,627 quarters. Source: Consumer Expenditure Survey interview data obtained from the BLS for the year 2011 and from the ICPSR for all other years.

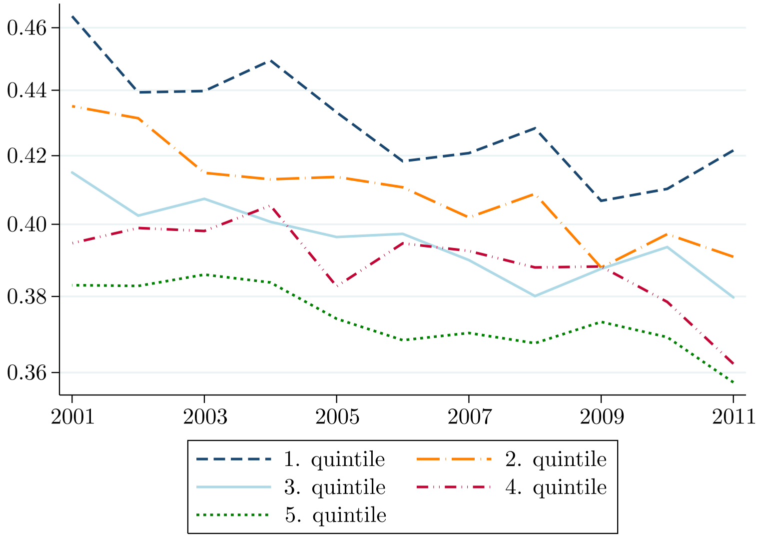


FIGURE B.3.—Cross-sectional variation in U.K. data. The figure plots the expenditure share devoted to goods for each income quintile of the U.K. on a logarithmic scale. The following expenditure categories are considered as services: housing, health, transport, communication, recreation & culture, education, restaurant & hotel. All other categories are considered as goods. The income quintiles refer to gross household income. Source: ONS, Living Costs and Food Survey (LCF).

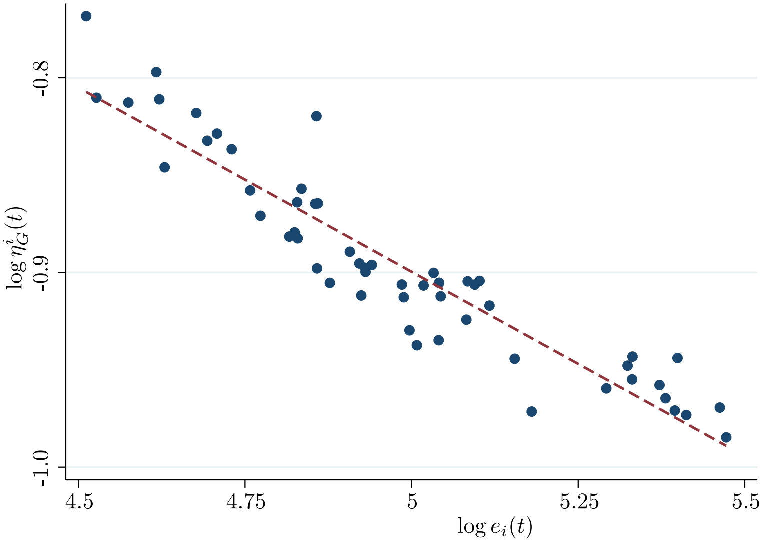


FIGURE B.4.—Scatter plot of cross-sectional variation in U.K. data. The figure depicts the partial correlation between the logarithmized average weekly expenditure level per household member and the logarithmized average expenditure share of goods of a given income quintile, where we allowed in each year for a separate (distinct) intercept. The slope of the fitted line is $-0.1892$. This slope is the same as if we regressed the logarithmized expenditure share on the logarithmized expenditure level per household member and time dummies. The $R^2$ of this underlying regression is 0.8731 and the standard error of the slope coefficient is 0.0114. Source: ONS, Living Costs and Food Survey (LCF).

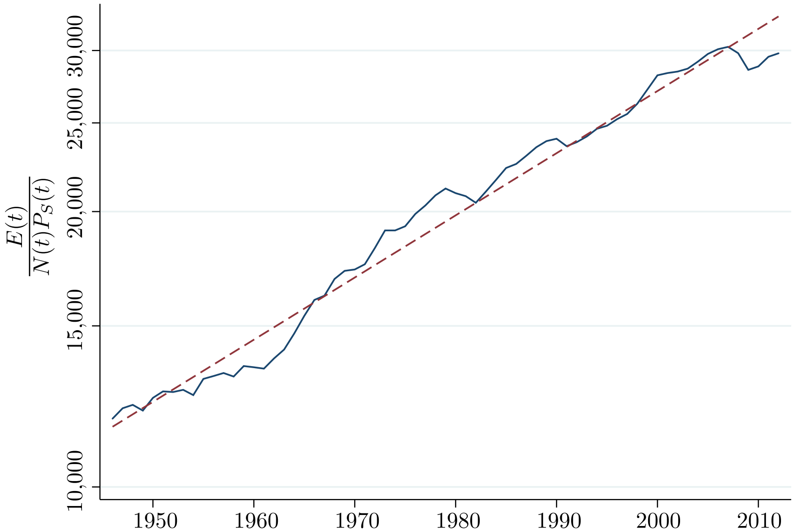


FIGURE B.5.—Per-capita expenditures in terms of services. The figure plots per-capita personal consumption expenditures in terms of services in the U.S. on a logarithmic scale. The price of services is normalized to 1 in the year 2005. The dashed line represents the predicted values obtained by regressing the logarithmized expenditures on time and a constant. The estimated slope coefficient and its standard error are 0.0156 and 0.00028. respectively. The regression attains an $R^2$ of 0.9791. Source: BEA, NIPA Tables 1.1.4 and 1.1.5 as well as Table 7.1 for the population data.

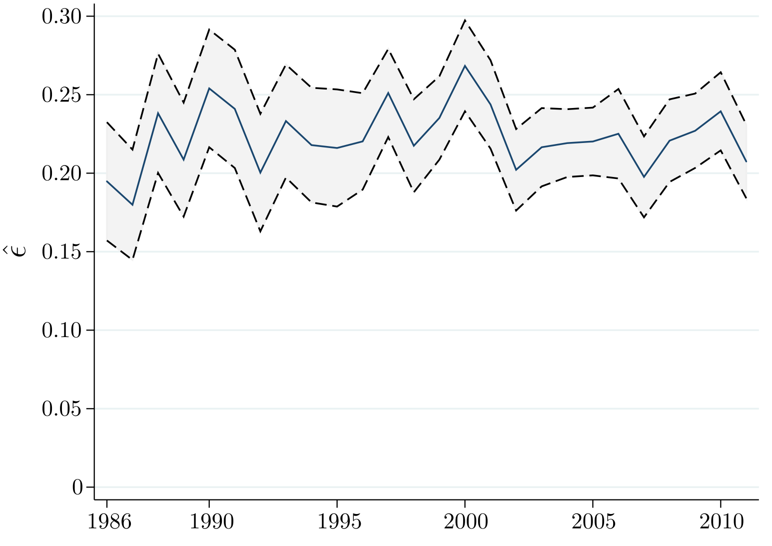


FIGURE B.6.—Estimates of $\varepsilon$ over time. The figure plots the estimates for $\varepsilon$ and its 95 percent confidence band obtained by running the specification of column (2) of Table I for each year separately. The regressions include quarter fixed effects.

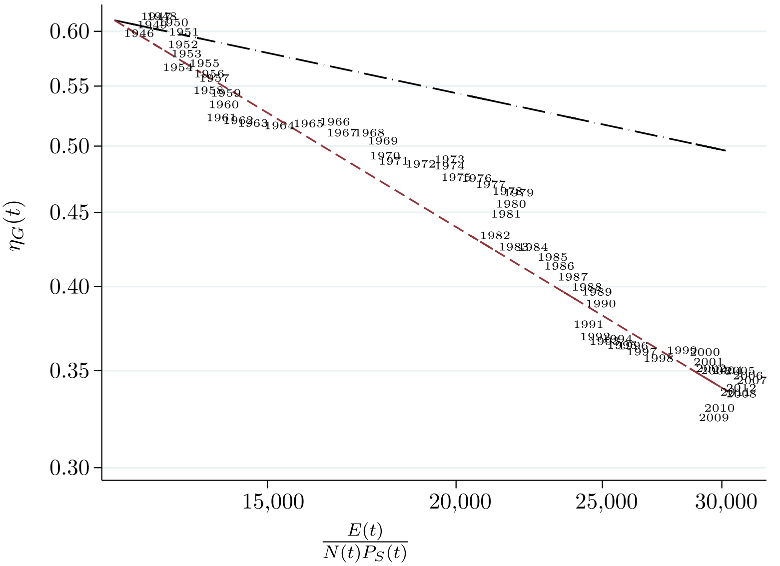


FIGURE B.7.—Share of goods versus aggregate per-capita expenditures. The figure depicts the scatter plot between per-capita personal consumption expenditures in terms of services and the expenditure share devoted to goods in the United States. Both axes are measured on a logarithmic scale. The price of services is normalized to 1 in the year 2005. The red dashed line represents the predicted values obtained by regressing the logarithmized expenditure share on the logarithmized per-capita expenditures and a constant. The estimated slope coefficient and its standard error are $-0.6301$ and 0.0166, respectively. The absolute value of this slope can be interpreted as the required $\varepsilon$ if we want to explain the entire structural change solely by an income effect. (See (23) and note that the slope in this figure is in line with (40) and $\gamma = 0$.) The simple regression attains an $R^2$ of 0.9570. Source: BEA, NIPA Tables 1.1.4 and 1.1.5 as well as Table 7.1 for the population data. The black dashed line is the predicted share of goods with $\varepsilon = 0.2222$, which we estimated in the cross-section (see Figure 7). We see that the model can—even with $\gamma = 0$—fit the aggregate data pretty well. However, this requires an $\varepsilon$ which is, compared to cross-sectional data, inconsistently high. I would like to thank Richard Rogerson for suggesting to illustrate this comparison.

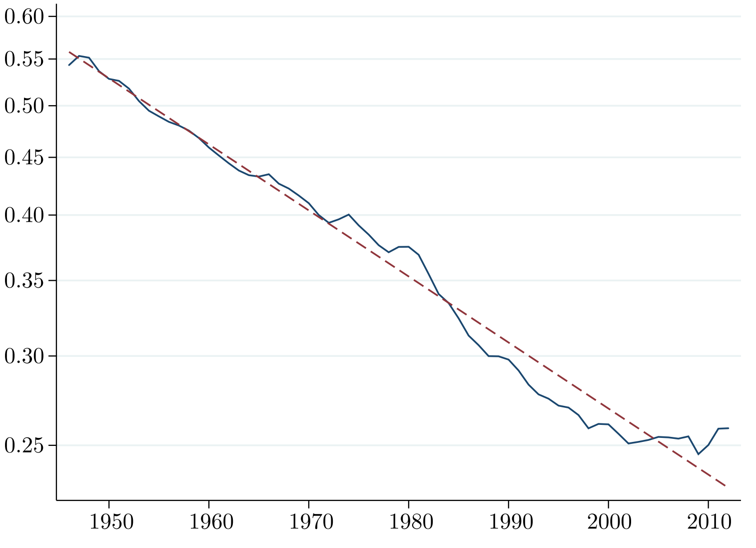


FIGURE B.8.—Expenditure share of non-durable goods. The figure plots the share of personal consumption expenditures devoted to non-durable goods in the United States on a logarithmic scale. The dashed line represents the predicted values obtained by regressing the logarithmized expenditure share on time and a constant. The estimated slope coefficient and its standard error are −0.0135 and 0.00025, respectively. The regression attains an $R^2$ of 0.9787. Source: BEA, NIPA Table 1.1.5.

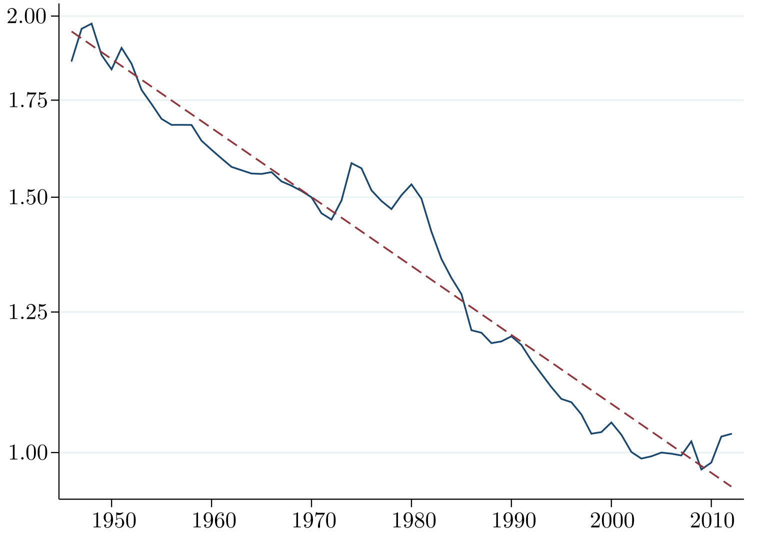


FIGURE B.9.—Relative price between non-durable goods and services. The figure plots the relative consumer price between non-durable goods and services on a logarithmic scale. The dashed line represents the predicted values obtained by regressing the logarithmized relative price on a constant and time. The estimated slope coefficient and its standard error are −0.0110 and 0.00030, respectively. The regression attains an $R^2$ of 0.9532. Source: BEA, NIPA Table 1.1.4.

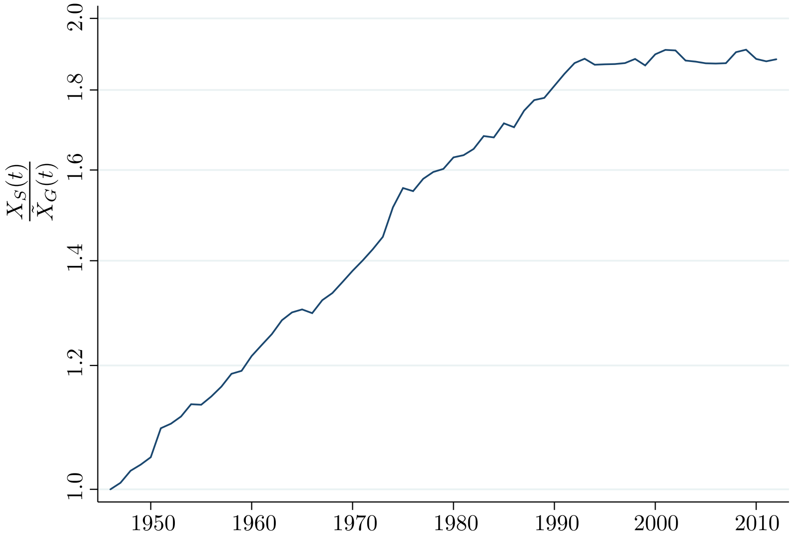


FIGURE B.10.—Real quantity of services relative to non-durable goods. The figure plots the real quantity of services relative to non-durable goods, $\tilde{X}_G(t)$, on a logarithmic scale. The relative quantity is normalized to 1 in the year 1946. Source: BEA, NIPA Table 1.1.3.

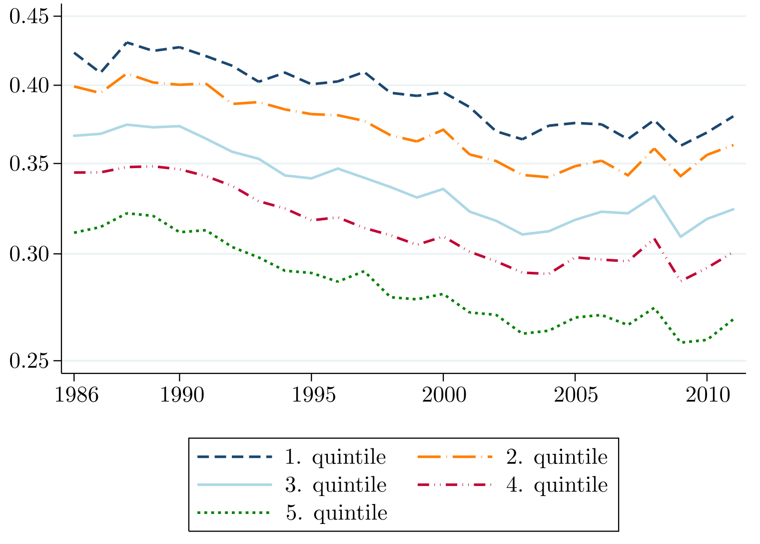


FIGURE B.11.—Cross-sectional variation excluding durables. The figure plots the expenditure share devoted to goods for each income quintile of the United States on a logarithmic scale when durable goods are excluded. As in Figure 4, the quintiles refer to total household labor earnings after tax plus transfers per OECD modified equivalence scale. The sample consists of expenditure data of 425,402 quarters. Source: Consumer Expenditure Survey interview data obtained from the BLS for the year 2011 and from the ICPSR for all other years.

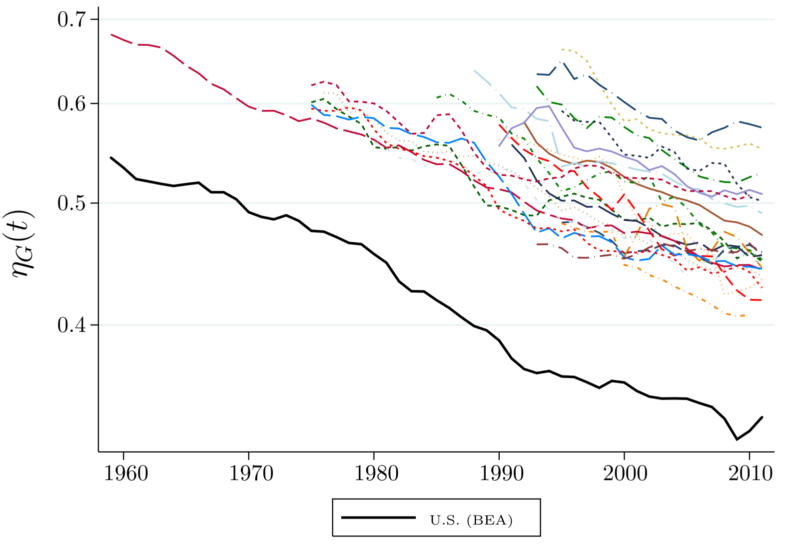


FIGURE B.12.—Expenditure share of goods: OECD countries. The figure plots the expenditure share of goods on a logarithmic scale in 23 European OECD countries and the United States. If we regress in the OECD sample (excluding the U.S.) the logarithm of the share of goods on a country fixed effect and the year, the slope coefficient is −0.0087 with a standard error of 0.0001. The $R^2$ of the regression is 0.9295. Source: Eurostat and BEA.

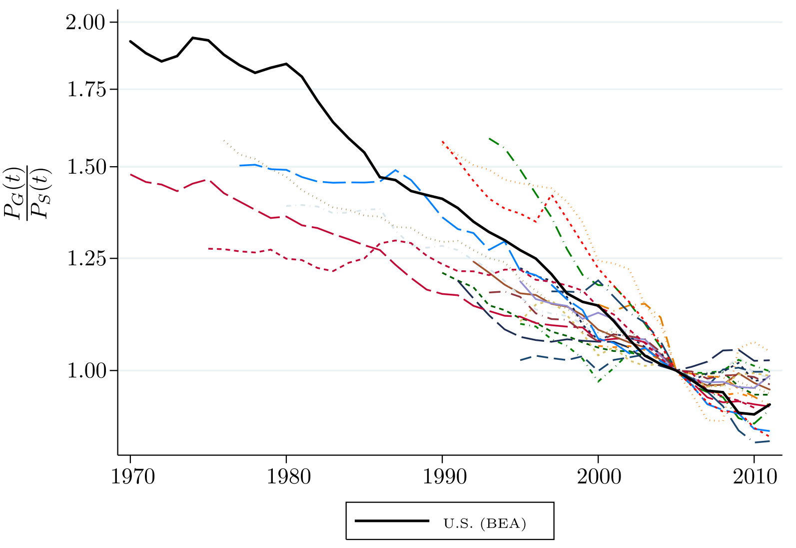


FIGURE B.13.—Relative price of goods: OECD countries. The figure plots the relative price of goods on a logarithmic scale in 22 European OECD countries and the United States. The relative price is normalized to 1 in the year 2005. If we regress in the OECD sample (excluding the U.S.) the logarithm of the relative price on a country fixed effect and the year, the slope coefficient is −0.0138 with a standard error of 0.0003. The $R^2$ of the regression is 0.8472. Source: Eurostat and BEA.

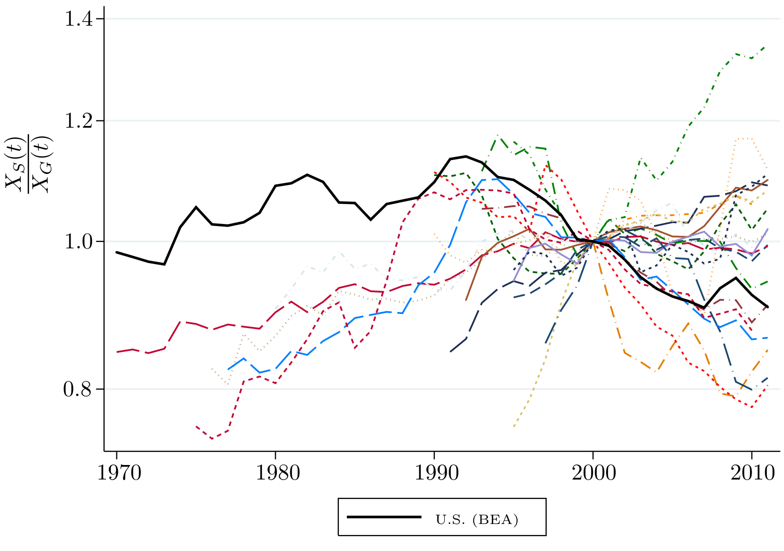


FIGURE B.14.—Relative real quantity of services: OECD countries. The figure plots the real quantity of services relative to goods on a logarithmic scale of 22 European OECD countries and the United States. The relative quantity is normalized to 1 in the year 2000. Source: Eurostat and BEA.

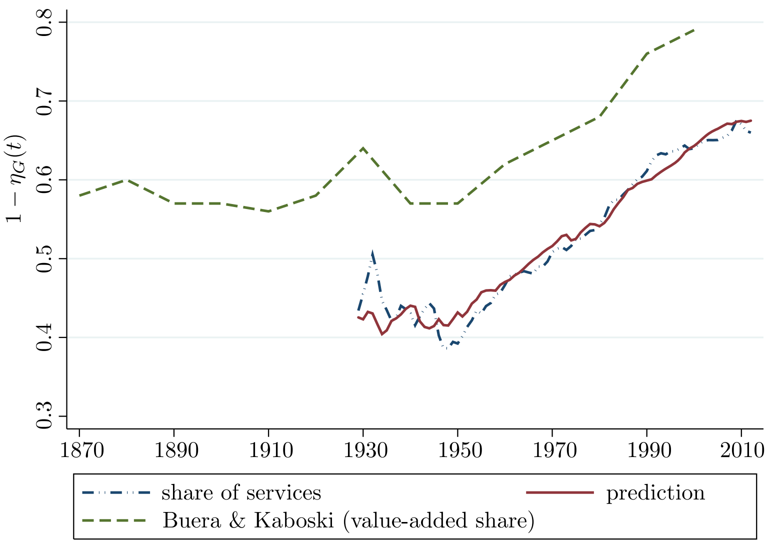


FIGURE B.15.—Predicting the late rise of the service economy. The blue dashed line represents the expenditure share devoted to services, $1-\eta_G(t)$, in the BEA data back to 1929. The red line is the model's implied expenditure share using $\hat{\varepsilon}=0.182$ and $\hat{\gamma}=0.410$ (as suggested by the cross-sectional estimation) as well as the observed relative prices $\frac{P_G(t)}{P_S(t)}$ and per-capita expenditure in terms of services $\frac{E(t)}{N(t)P_S(t)}$. The green dashed line represents the share of services in value-added as in Buera and Kaboski (2012b). The level difference comes from the fact that the BEA categorizes final consumption expenditures into goods and services, whereas for the value-added data, industries are categorized as services or manufacturing. Source: BEA, NIPA Tables 1.1.4, 1.1.5, and 7.1 and Buera and Kaboski (2012b) for the value-added data.

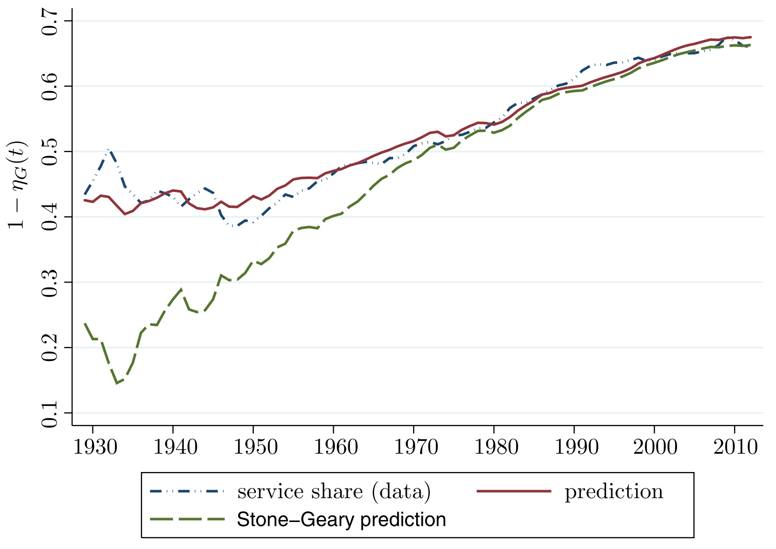


FIGURE B.16.—Stone–Geary's prediction. The blue dashed line represents the expenditure share devoted to services, $1 - \eta_G(t)$, in the BEA data back to 1929. The red line is—as in Figure B.15—the cross-sectional back in time prediction using this paper's functional form. The green dashed line represents the back in time prediction if we use the function form implied by a "generalized Stone–Geary" utility function $U(X_G(t), X_S(t)) = [\omega_G^{1/\sigma}(X_G(t) - \bar{X}_G)^{(\sigma-1)/\sigma} + (1-\omega_G)^{1/\sigma} X_S(t)^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$. This functional form implies for the expenditure share of goods of the representative agent: $\eta_G(t) = \frac{\omega_G P_G(t)^{1-\sigma}}{\omega_G P_G(t)^{1-\sigma} + (1-\omega_G) P_S(t)^{1-\sigma}} + \frac{(1-\omega_G) P_S(t)^{1-\sigma}}{\omega_G P_G(t)^{1-\sigma} + (1-\omega_G) P_S(t)^{1-\sigma}} [\frac{\bar{X}_G P_G(t) N(t)}{E(t)}]$. Estimating this functional form by nonlinear least squares using cross-sectional data of the different income quintiles 1986–2011 gives: $\hat{\omega}_G = 0.285$, $\hat{\sigma} = 0.719$, and $\hat{\bar{X}}_G = 649.19$. $\hat{\bar{X}}_G$ can be interpreted as quarterly subsistence spending per-equivalent scale in 2005 good prices. The subsistence spending are (on average) 16.37 percent in 1986. (All the estimates are comparable to the ones in Herrendorf, Rogerson, and Valentinyi (2013). Adding a (potentially negative) subsistence level in services leaves the fit unchanged.) Calibrating the per-capita subsistence spending, $\frac{\bar{X}_G P_G(t) N(t)}{E(t)}$, to 16.37 percent in 1986 and $\omega$ and $\sigma$ to the values of 0.285 and 0.719, the green dashed line shows the predicted expenditure share spent on services using aggregate BEA data. The generalized Stone–Geary specification implies that an asymptotic expenditure elasticity of demand is unity for all sectors. Consequently, relatively large subsistence levels are needed in order to get reasonable income effects in later years. But these required subsistence levels imply an even larger income effect in earlier years and the model generates rather the opposite of a "late rise of the service sector."

TABLE B.I

CROSS-SECTIONAL ESTIMATION OF $\varepsilon$ WITH TOTAL INCOME AS THE INSTRUMENT[a]

| | (1) | (2) |
|---|---|---|
| $-\log e_i(t)$ | 0.205*** | 0.195*** |
| | (0.002) | (0.003) |
| Children share | | 0.128*** |
| | | (0.004) |
| Elderly share | | −0.042*** |
| | | (0.003) |
| Residence indicators | No | Yes |
| Family size indicators | No | Yes |
| Ref. person controls | No | Yes |
| Observations | 477,627 | 425,315 |
| $R^2$ | 0.013 | 0.047 |
| Method | IV | IV |

[a]Dependent variable: $\log \eta_G^i(t)$. Standard errors in parentheses. *** significant at 1 percent, ** significant at 5 percent, * significant at 10 percent. All regressions include quarter fixed effects (104 groups). The logarithmized expenditure level per equivalent scale is instrumented by the logarithmized total income after taxes plus transfers per equivalent scale. "Children share" and "Elderly share" measure the share of household members with age $< 18$ and $\geq 65$, respectively. "Residence indicators" consists of regional indicators (four groups), a rural/urban dummy, as well as indicators of different population densities of the city of residence (five groups). "Family size indicators" consists of 11 groups. "Ref. person controls" consists of the age, the sex, skill-level indicators (seven groups) and race indicators (four groups) of the reference person.

TABLE B.II

CROSS-SECTIONAL ESTIMATION OF $\varepsilon$ USING DIARY DATA OF THE YEAR 2010[a]

| | (1) | (2) |
|---|---|---|
| $-\log e_i(t)$ | 0.244*** | 0.300*** |
| | (0.025) | (0.039) |
| Children share | | 0.025 |
| | | (0.058) |
| Elderly share | | −0.023 |
| | | (0.041) |
| Residence indicators | No | Yes |
| Family size indicators | No | Yes |
| Ref. person controls | No | Yes |
| Observations | 9,050 | 9,017 |
| $R^2$ | 0.206 | 0.225 |
| Method | IV | IV |

[a]Dependent variable: $\log \eta_G^i(t)$. Standard errors in parentheses. *** significant at 1 percent, ** significant at 5 percent, * significant at 10 percent. All regressions include month fixed effects (12 groups). The logarithmized expenditure level per equivalent scale is instrumented by the logarithmized labor and capital earnings after taxes and deductions plus transfers per equivalent scale. The following broad expenditure categories are considered as services: shelter (mortgage payments, rents and buying of land and houses); food and drinks away from home; fuels and electricity, public services; clothing repair services; vehicle leasing; vehicle repair and maintenance services; house maintenance services; healthcare services; entertainment services; sports equipment rental; personal care; school tuition. "Children share" and "Elderly share" measure the share of household members with age $< 18$ and $\geq 65$, respectively. "Residence indicators" consists of regional indicators (four groups), a rural/urban dummy, as well as indicators of different population densities of the city of residence (five groups). "Family size indicators" consist of 11 groups. "Ref. person controls" consists of the age, the sex, skill-level indicators (seven groups), and race indicators (four groups) of the reference person. Source: Diary data of the Consumer Expenditure Survey from the Bureau of Labor Statistics.

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