Econometrica Supplementary Material

SUPPLEMENT TO "MENU COSTS, MULTIPRODUCT FIRMS, AND AGGREGATE FLUCTUATIONS"

(*Econometrica*, Vol. 79, No. 4, July 2011, 1139–1180)

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APPENDIX 1: REGULAR PRICE ALGORITHM

THIS SECTION closely follows the description in Kehoe and Midrigan (2008). The algorithm is based on the idea that a price is a *regular price* if the store charges it frequently in a window adjacent to that observation. For each period, I compute the mode of prices p_t^M in a window that includes the previous five prices, the current price, and the next five periods.[1] Given the modal price in this window, the regular price is constructed recursively as follows. Set the initial period's regular price equal to the modal price.[2] For each subsequent period, if the store charges the modal price in that period, and at least one-third of prices in the window are equal to the modal price, set the regular price equal to the modal price. Otherwise, set the regular price equal to the last period's regular price. Finally, I would like to eliminate regular price changes that occur in the absence of changes in the store's actual price if the actual and regular price coincide in the period before or after the regular price change. Thus, if the steps above generate a path for regular prices such that a change in the regular price occurs in the absence of a change in the actual price, I replace the last period's regular price with the current period's actual price if the regular and actual prices coincide in the current period. Similarly, I replace the current period's regular price with the last period's actual price if the two coincided in the previous period. These latter steps ensure that the regular price tracks the posted price as closely as possible and that changes in the regular price do not occur several weeks prior to/after a change in the posted price.

So far I have described our algorithm intuitively. Here I provide the precise algorithm I used to compute the regular price. The algorithm is characterized by three parameters. I choose $l = 5$ (the size of the window: number of weeks before/after current period used to compute modal price, $l = 2$ months when applied to the monthly BLS data), $c = 1/3$ (the cutoff used to determine whether a price is temporary), $a = 0.5$ (the number of periods in the window with available price required so as to compute a modal price).

I apply the algorithm below for each good separately. Let p_t be the price in period t and let T be the length of the price series.

1. For each time period $t \in (1 + l, T - l)$.

If the number of periods with available data in $(t - l, \dots, t + l)$ is $\geq 2al$,

[1] I perform this calculation only if at least one-half of prices in this window are available.

[2] If, in the window around this price, more than half of the data are missing, I set the initial regular price equal to the actual price.

 DOI: 10.3982/ECTA6735

let $p_t^M = \text{mode}(p_{t-l}, \dots, p_{t+l})$
let $f_t =$ fraction of periods (with available data) in this window such that $p_t = p_t^M$.
Else, set f_t, $p_t^M = 0$ (missing).
2. Define p_t^R using the following recursive algorithm.
If $p_{1+l}^M \neq 0$, set $p_{1+l}^R = p_{1+l}^M$ (initial value).
Else, set $p_{1+l}^R = p_{1+l}$,
for $t = 2 + l, \dots, T$,
if ($p_t^M \neq 0$ & $f_t > c$ & $p_t = p_t^M$), set $p_t^R = p_t^M$
else, set $p_t^R = p_{t-1}^R$.
3. Repeat the following steps 5 times:
Let $\mathcal{R} = \{t : p_t^R \neq p_{t-1}^R \ \& \ p_{t-1}^R \neq 0 \ \& \ p_t^R \neq 0\}$ be the set of periods with regular price change.
Let $\mathcal{C} = \{t : p_t^R = p_t \ \& \ p_t^R \neq 0 \ \& \ p_t \neq 0\}$ be the set of periods in which the store charges regular price.
Let $\mathcal{P} = \{t : p_{t-1}^R = p_{t-1} \ \& \ p_{t-1}^R \neq 0 \ \& \ p_{t-1} \neq 0\}$ be the set of periods in which the store's last period price was the regular price.
Set $p_{\{\mathcal{R}\cap\mathcal{C}\}-1}^R = p_{\{\mathcal{R}\cap\mathcal{C}\}}$. Set $p_{\{\mathcal{R}\cap\mathcal{P}\}}^R = p_{\{\mathcal{R}\cap\mathcal{P}\}-1}$.

APPENDIX 2: COMPUTATIONAL ALGORITHM

I discuss here the solution method used to compute optimal decision rules given a guess for how aggregate variables evolve with the state. Recall that the firm's problem is characterized by

$$V^R(\boldsymbol{\mu}_{-1}^R, e; g, \boldsymbol{\Lambda}) = \max_{\mu_i^R} \Bigg(\sum_{i=1}^N e_i^{1-\gamma}(\mu_i^R - 1)\mu_i^{R,-\gamma}\hat{P}^{\gamma-1} - \phi^R + \beta EV(\boldsymbol{\mu}_{-1}^{R\prime}, e'; g', \boldsymbol{\Lambda}') \Bigg),$$

$$V^T(\boldsymbol{\mu}_{-1}^R, e; g, \boldsymbol{\Lambda}) = \max_{\mu_i} \Bigg(\sum_{i=1}^N e_i^{1-\gamma}(\mu_i - 1)\mu_i^{-\gamma}\hat{P}^{\gamma-1} - \kappa + \beta EV(\boldsymbol{\mu}_{-1}^{R\prime}, e'; g', \boldsymbol{\Lambda}') \Bigg),$$

$$V^N(\boldsymbol{\mu}_{-1}^R, e; g, \boldsymbol{\Lambda}) = \Bigg(\sum_{i=1}^N e_i^{1-\gamma}(\mu_{i,-1}^R - 1)\mu_{i,-1}^{R,-\gamma}\hat{P}^{\gamma-1} - \kappa + \beta EV(\boldsymbol{\mu}_{-1}^{R\prime}, e'; g', \boldsymbol{\Lambda}') \Bigg).$$

I use a projection-based approximation method.[3] Specifically, I approximate the value functions (associated with each different option: V^R, V^T, V^N), as well as the expected continuation value EV (the latter is not necessary, but greatly speeds up execution since it reduces the number of times I compute the integral on the right hand side of the Bellman equation), using splines (a combination of linear and cubic). This approximation reduces the problem to that of finding a set of coefficients on the basis function (I compute multivariate basis functions from univariate ones using tensor products) that solves the system of functional equations at a finite grid of nodes in the state space. I have increased the number of nodes (and thus basis functions) along each dimension to the point at which further increases produce no significant effect on optimal price rules and also render the distance between the two sides of each functional equation at points other than the collocation nodes (at which this distance is zero by construction) insignificant (the maximum relative errors are on the order of 10^{-4}; average relative errors are on the order of 10^{-5}). Depending on the problem, I use up to 15–20 basis functions in the price space and a small (3–5) number of nodes in the aggregate state space. I approximate integrals using Gaussian quadrature, again using more nodes in the idiosyncratic (productivity shocks) space (9–11 quadrature points). Finally, I solve the optimization problem on μ^R in the V^R equation (the one associated with a temporary price change is a static problem) using a derivative-free method. The assumption that e is common across goods (as well as the unit root in a_i) reduces the problem to one dimensional and I use the very robust golden search method to bracket the optimum (derivative-based methods are somewhat less stable given the kinks and the need for a robust solution method across different sets of parameter values used in calibration). The calibration exercise is conducted using a simplex-based (Nelder–Mead) method.

APPENDIX 3: ECONOMY WITH $\phi > 0$

I describe the problem of a single-product retailer for simplicity. Relative to the economy discussed in text, there are several additional options the retailer can undertake, and I discuss each of these below. Moreover, the state of the retailer now includes both its old regular price (markup) and its old posted price (markup).

A. *Recursive Formulation*

Let $V^{\Delta R}$ denote the value of changing the regular price. Given discounting, it never pays off to pay the cost of changing the regular price without actually using it (as a posted price). Hence, in the period in which the regular price

[3]See Miranda and Fackler (2002) for a detailed description of these methods as well as a toolkit that facilitates their implementation.

changes, $p_t = p_t^R$. Hence, if such a firm changes its regular price, it also changes the posted price and thus the cost of exercising this option is $\phi + \phi^R$.[4]

Let $V^{\Delta p}$ denote the value of a firm to (a) deviate from its old regular price and (b) change its posted price. This option reflects the retailer either initiating a sale or continuing a sale but changing the sale price. The cost of exercising this option is $\phi + \kappa$.

Let V_T^R denote the value (to a firm that posted a price other than the regular price in the previous period) of charging a price equal to the regular price. Such a return entails a physical change in the posted price and hence costs a fixed cost ϕ.

Let V_T^T denote the value (to a firm that posted a price other than the regular price in the previous period) of continuing to charge its old posted price. This option involves a deviation from the regular price and hence costs κ.

Finally, let V_R^R denote the value (to a firm that last posted its regular price) of continuing to post its old regular price. This option costs 0.

Let $V_{\max}^R = \max(V^{\Delta R}, V^{\Delta P}, V_R^R)$ denote the envelope of the options the firm can exercise if its posted price coincided with its regular price in the previous period. Let $V_{\max}^T = \max(V^{\Delta R}, V^{\Delta P}, V_T^R, V_T^T)$ be the envelope of the options the firm can exercise if its posted price deviated from the regular price in the previous period. The five Bellman equations that characterize the retailer's problem are

$$
\begin{aligned}
V^{\Delta R}(\mu_{-1}^R, \mu_{-1}, e) &= \max_{\mu^R} e^{1-\gamma}(\mu^R - 1)\mu^{R,-\gamma} - \phi - \phi^R \\
&\quad + \beta E V_{\max}^R\left(\mu^R \frac{e}{\varepsilon' e'}, \mu^R \frac{e}{\varepsilon' e'}, e'\right), \\
V^{\Delta P}(\mu_{-1}^R, \mu_{-1}, e) &= \max_{\mu} e^{1-\gamma}(\mu - 1)\mu^{-\gamma} - \phi - \kappa \\
&\quad + \beta E V_{\max}^T\left(\mu_{-1}^R \frac{e}{\varepsilon' e'}, \mu \frac{e}{\varepsilon' e'}, e'\right), \\
V_T^R(\mu_{-1}^R, \mu_{-1}, e) &= e^{1-\gamma}(\mu_{-1}^R - 1)(\mu_{-1}^R)^{-\gamma} - \phi \\
&\quad + \beta E V_{\max}^R\left(\mu_{-1}^R \frac{e}{\varepsilon' e'}, \mu_{-1}^R \frac{e}{\varepsilon' e'}, e'\right), \\
V_T^T(\mu_{-1}^R, \mu_{-1}, e) &= e^{1-\gamma}(\mu_{-1} - 1)\mu_{-1}^{-\gamma} - \kappa \\
&\quad + \beta E V_{\max}^T\left(\mu_{-1}^R \frac{e}{\varepsilon' e'}, \mu_{-1} \frac{e}{\varepsilon' e'}, e'\right),
\end{aligned}
$$

[4]Unless $p_{t-1} = p_t^R$. The assumptions I make on the process for the retailer's idiosyncratic states ensure that this is a zero probability event (otherwise the retailer would have found it optimal to change its regular price in the previous period).

$$V_R^R(\mu_{-1}^R, \mu_{-1}, e) = e^{1-\gamma}(\mu_{-1}^R - 1)(\mu_{-1}^R)^{-\gamma} + \beta E V_{\max}^R\left(\mu_{-1}^R \frac{e}{\varepsilon' e'}, \mu_{-1}^R \frac{e}{\varepsilon' e'}, e'\right).$$

Notice here that V_T^R and V_R^R only differ because a retailer that has had a temporary price in the previous period must pay a fixed cost ϕ to return to the old regular price. I keep track of them separately only so as to deal in the discontinuity at $\mu_{-1} = \mu_{-1}^R$: at all other points, the two values are identical. This is what $V_{\max}^T$ and $V_{\max}^R$ capture: both allow the option of charging the old regular price, but doing so entails a fixed cost ϕ at $V_{\max}^T$ and nothing at $V_{\max}^R$. Thus, when the retailer undertakes a temporary price change, it recognizes that it has to pay ϕ again to be able to change its posted price to a regular price.

B. *Decision Rules*

The decision rules are similar to those discussed in text. The additional feature of this model is that sometimes prices will be unchanged during a sale, when the gains from a posted price change do not exceed the fixed cost of changing posted prices, ϕ.

C. *A Calibration*

I now illustrate my claim that to match the data, the model indeed requires a very small cost of changing posted prices. The moments I target are similar to those in the economy studied in text. I add one additional moment so as to pin down ϕ: the frequency of times a price changes during a sale. Recall that this number is equal to 0.67 in the data. Intuitively, this statistic pins down ϕ since this parameter primarily determines the flexibility of posted prices.

Table A.I reports the moments I use, both in the model and in the data. Notice that this model predicts that 67% of the sales prices change during periods of sale, and thus as frequently in the data. As for the other moments, these are very similar to those in the data and in the model studied in text with $\phi = 0$.

The reason an economy with $\phi > 0$ produces a similar fit to the economy I studied in text can be seen in Table A.II in which I report the parameter values that offer the best fit to the data. As earlier, the low cost shocks arrive fairly infrequently ($\alpha = 0.15$), and the menu cost of changing the regular price is relative large, $\phi^R = 0.0639$, relative to the cost of deviating from the regular price, $\kappa = 0.0286$. Importantly, I estimate a menu cost of changing posted prices that is quite low (0.16×10^{-3})—much smaller than the cost of changing regular prices. The only role this second menu cost plays is in generating price stickiness during episodes of sale. But as shown above, periods with sales are, in fact, periods with fairly flexible prices: the menu cost must be therefore very small to account for this feature of the data.

TABLE A.I

ECONOMY WITH $\phi > 0$, SINGLE-PRODUCT RETAILER

Moments	Data	Model
Used in calibration		
1. Frequency of price changes	0.34	0.34
2. Fraction of price changes that are temporary	0.97	0.94
3. Frequency regular price changes	0.029	0.029
4. Probability a temporary price spell ends	0.47	0.47
5. Probability temp. price returns to old regular	0.86	0.91
6. Fraction of periods with temporary prices	0.25	0.24
7. Fraction of periods with price below regular (sale)	0.22	0.24
8. Fraction of goods sold when price below regular	0.37	0.41
9. Mean size of price changes	0.20	0.20
10. Mean size of regular price changes	0.11	0.13
11. Fraction of price changes during a sale	0.67	0.67
Additional moments		
1. Fraction of prices at annual mode	0.58	0.53
2. Fraction of prices below annual mode	0.31	0.35
3. Frequency with which annual mode changes	0.61	0.79
4. Fraction of prices at quarterly mode	0.70	0.70
5. Fraction of prices below quarterly mode	0.22	0.26
6. Frequency with which quarterly mode changes	0.32	0.34
7. Fraction of times sales price changes	0.82	1
8. Std. dev. size of price changes	0.18	0.12
9. Kurtosis price changes	3.15	1.60
10. Fraction changes $< 1/2$ mean	0.36	0.25
11. Fraction changes $< 1/4$ mean	0.19	0.23
12. Std. dev. size of regular price changes	0.08	0.02
13. Kurtosis regular price changes	4.02	1.26
14. Fraction regular price changes $< 1/2$ mean	0.25	0
15. Fraction regular changes $< 1/4$ mean	0.08	0

TABLE A.II

PARAMETER VALUES, ECONOMY WITH $\phi > 0$

Calibrated parameters	
σ_a	0.0227
α	0.1543
ρ	0.5232
$\bar{e}$	0.7350
ϕ^R, relative to SS revenue	0.0639
ϕ, relative to SS revenue	0.1634×10^{-3}
κ, relative to SS revenue	0.0286
Assigned parameters	
γ	3
β (annual)	0.96

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Manuscript received September, 2006; final revision received February, 2010.