

SUPPLEMENT TO “INSTRUMENTAL VARIABLE MODELS
FOR DISCRETE OUTCOMES

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BY ANDREW CHESHER

This document gives additional graphical displays of identified sets for the binary outcome, binary endogenous variable case considered in Section 3.1.3 of the paper.

S1. THE STRUCTURES CONSIDERED

IDENTIFIED SETS ARE ILLUSTRATED using probability distributions generated by structures in which binary Y and X are generated by a triangular linear equation system

$$Y = 1[a_0 + a_1X + \varepsilon > 0], \quad X = 1[b_0 + b_1Z + \eta > 0].$$

Latent variates ε and η are jointly normally distributed and distributed independently of an instrumental variable Z :

$$\begin{bmatrix} \varepsilon \\ \eta \end{bmatrix} | Z \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix} \right).$$

In the displays that follow, the values of b_1 and r are varied, and $a_0 = 0$, $a_1 = 0.5$, $b_0 = 0$.

Let Φ denote the standard normal distribution function. In all the cases considered, the structural equation for binary Y in the structures that generate probability distributions is

$$Y = \begin{cases} 0, & 0 < U \leq \Phi(-0.5X), \\ 1, & \Phi(-0.5X) < U \leq 1, \end{cases}$$

with $U \equiv \Phi(\varepsilon) \sim \text{Unif}(0, 1)$ and $U \perp\!\!\!\perp Z$.

The identifying power of the following model is considered:

$$Y = \begin{cases} 0, & 0 < U \leq p(X), \\ 1, & p(X) < U \leq 1, \end{cases} \quad U \perp\!\!\!\perp Z, \quad U \sim \text{Unif}(0, 1).$$

Since $X \in \{0, 1\}$, the analysis concerns the identifying power of the model for the values of $p(0)$ and $p(1)$, which in all the cases considered are

$$p(0) = \Phi(0) = 0.5, \quad p(1) = \Phi(-0.5) = 0.308.$$

The instrument takes values in the set

$$\Omega = \{-0.75, -0.5625, -0.375, -0.1875, 0, \\ +0.1875, +0.375, +0.5625, +0.75\}.$$

S2. GRAPHS

In the graphs that follow, horizontal and vertical coordinates are values of, respectively, $p(0)$ and $p(1)$. These lie in the unit square. The line of equality $p(1) = p(0)$ is drawn in red. The value of $(p(0), p(1))$ in the structures that generate the probability distributions is at the intersection of the green horizontal and vertical lines.

At each value of the instrumental variable, there is an identified set comprising two rectangular regions in one of which (drawn red) $p(1) \geq p(0)$, while in the other (drawn blue) $p(1) \leq p(0)$. The rectangles have one vertex in common, where $p(1) = p(0)$. The value at which $p(1) = p(0)$ varies with the value of the instrumental variable.

S2.1. *Increasing Instrument Support*

As the value of the instrument varies across Ω , the identified set of values of $p(0)$ and $p(1)$ is the intersection of the sets obtained at each value of the instrumental variable. The process of constructing the identified set when $r = -0.25$ and $b_1 = 1$ is shown in the sequence of Figures S1–S9. From Figure S3 onward, the intersection (that is, the identified set) is marked in yellow.

In Figures S2–S7, the identified set is disconnected when it contains values such that $p(1) > p(0)$ and $p(1) < p(0)$, but no values such that $p(1) = p(0)$. In Figures S8 and S9, only values with $p(1) < p(0)$ are in the identified set.

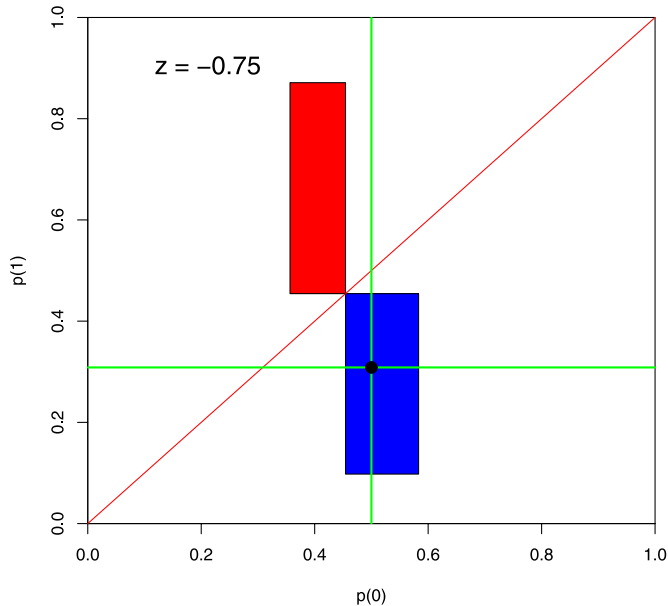


FIGURE S1.—Identified set when $r = -0.25$, $z = -0.75$.

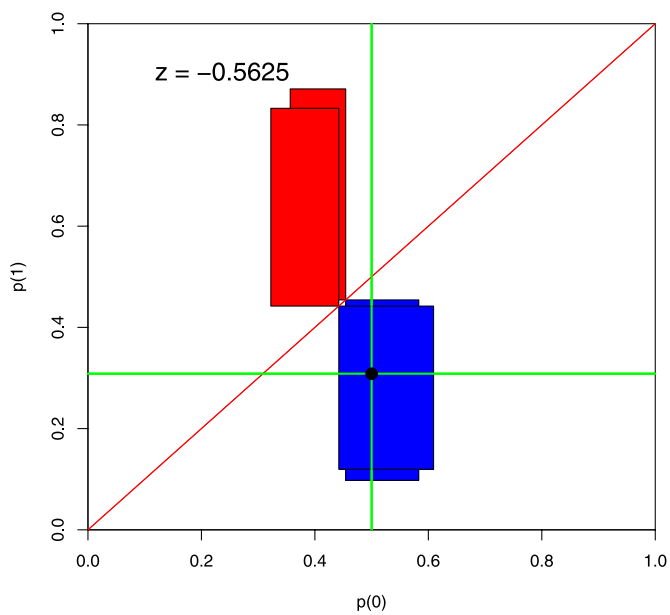


FIGURE S2.—Identified set when $r = -0.25$, $z \in \{-0.75, -0.5765\}$.

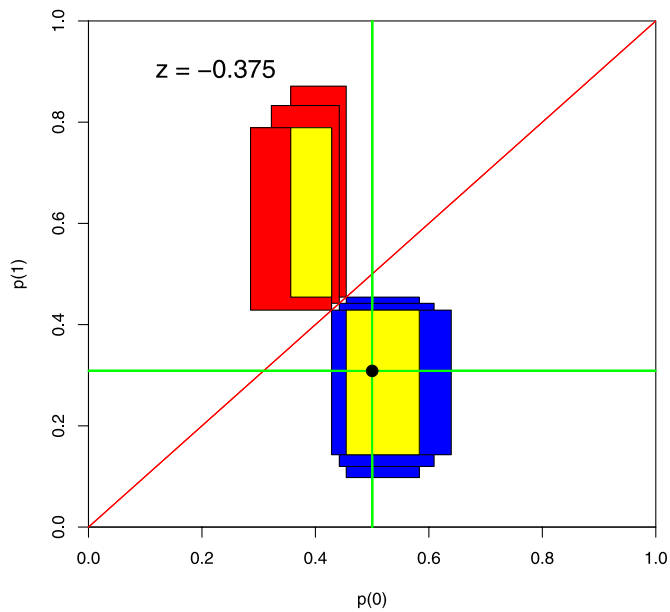
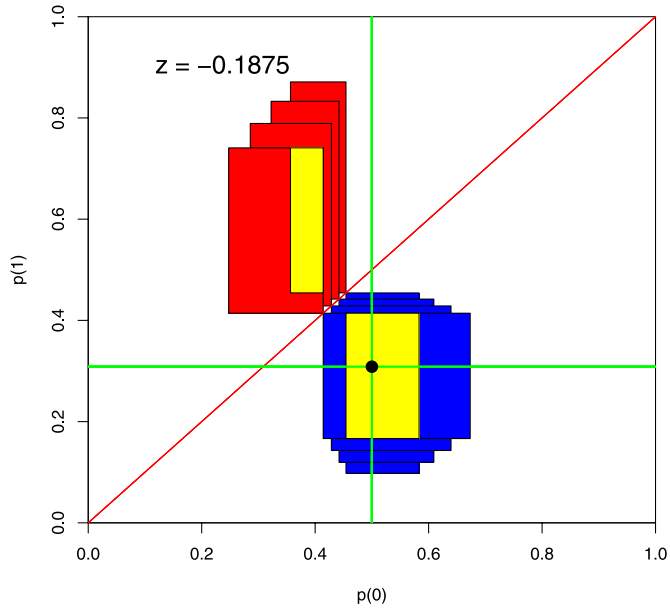
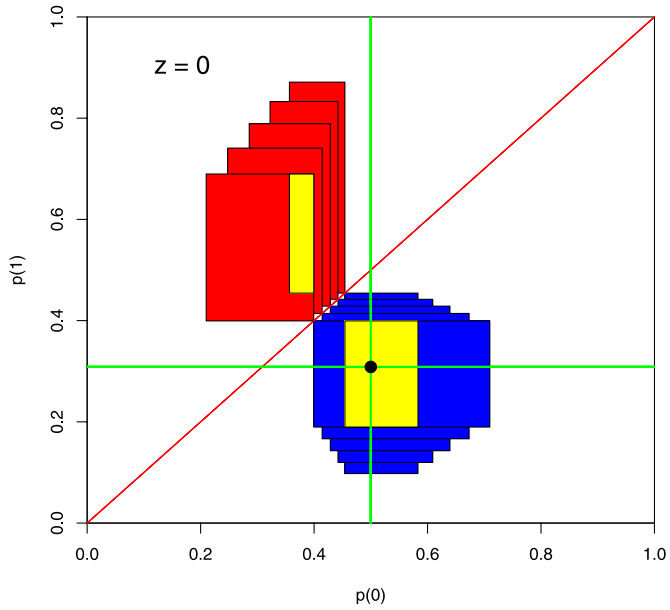
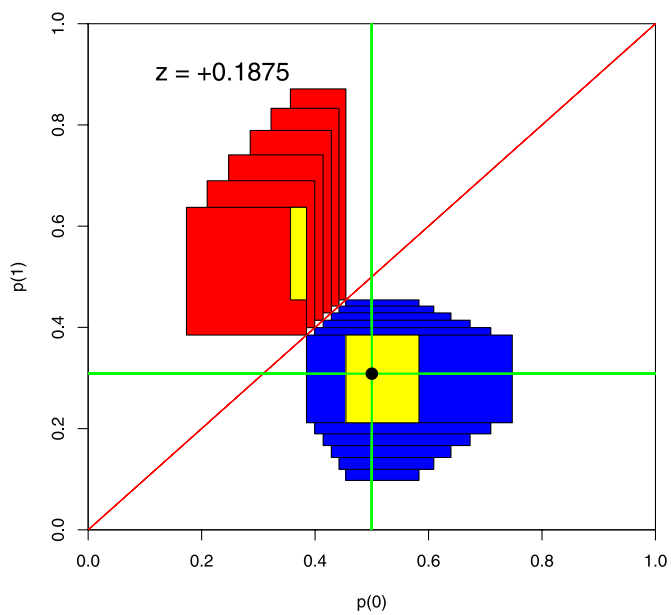
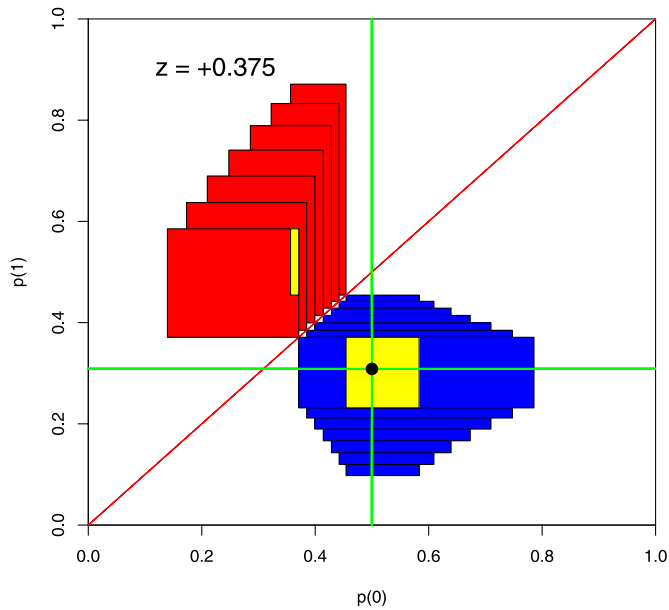
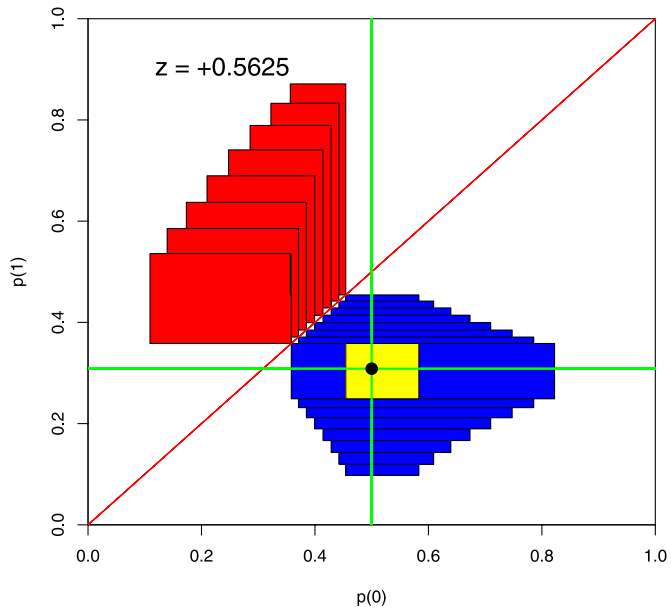
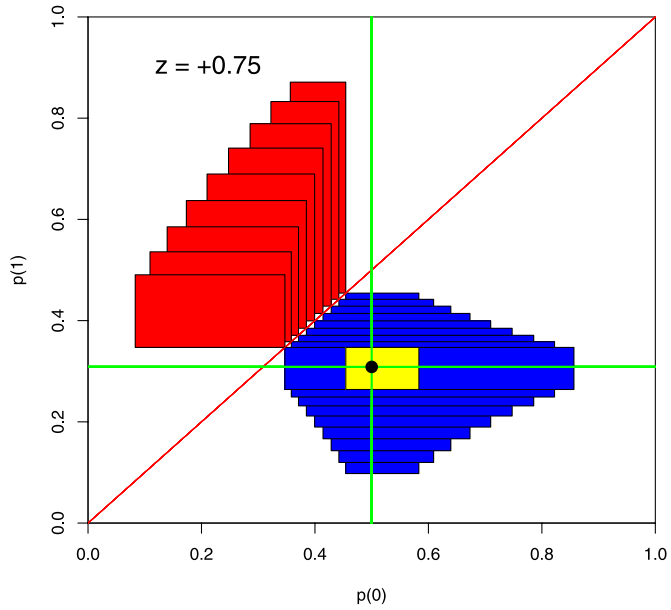


FIGURE S3.—Identified set when $r = -0.25$, $z \in \{-0.75, \dots, -0.375\}$.

FIGURE S4.—Identified set when $r = -0.25$, $z \in \{-0.75, \dots, -0.1875\}$.FIGURE S5.—Identified set when $r = -0.25$, $z \in \{-0.75, \dots, 0\}$.

FIGURE S6.—Identified set when $r = -0.25$, $z \in \{-0.75, \dots, +0.1875\}$.FIGURE S7.—Identified set when $r = -0.25$, $z \in \{-0.75, \dots, +0.375\}$.

FIGURE S8.—Identified set when $r = -0.25$, $z \in \{-0.75, \dots, +0.5625\}$.FIGURE S9.—Identified set when $r = -0.25$, $z \in \{-0.75, \dots, +0.75\}$.

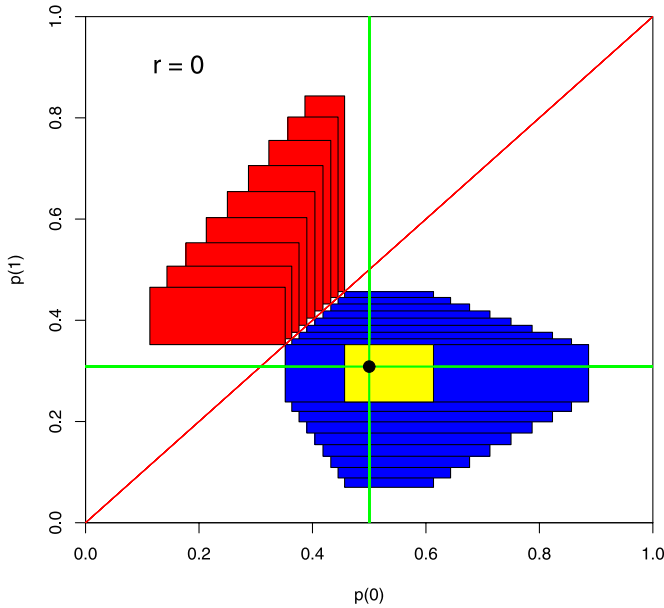


FIGURE S10.—Identified set when $r = 0$, $z \in \{-0.75, \dots, +0.75\}$.

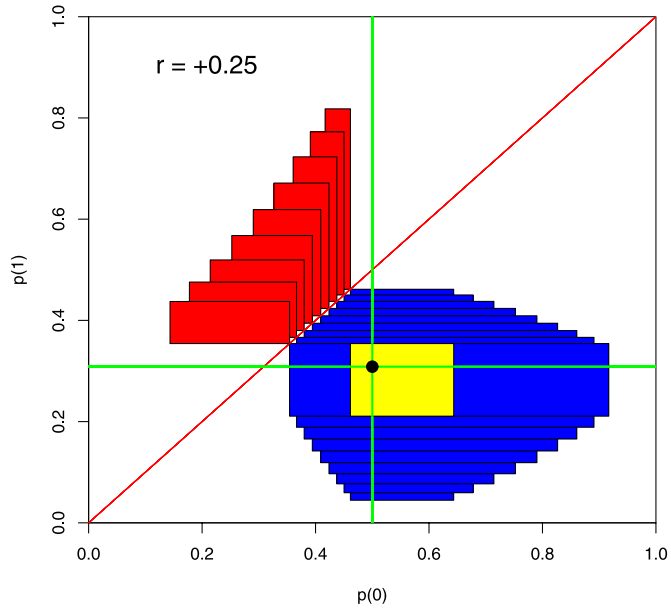
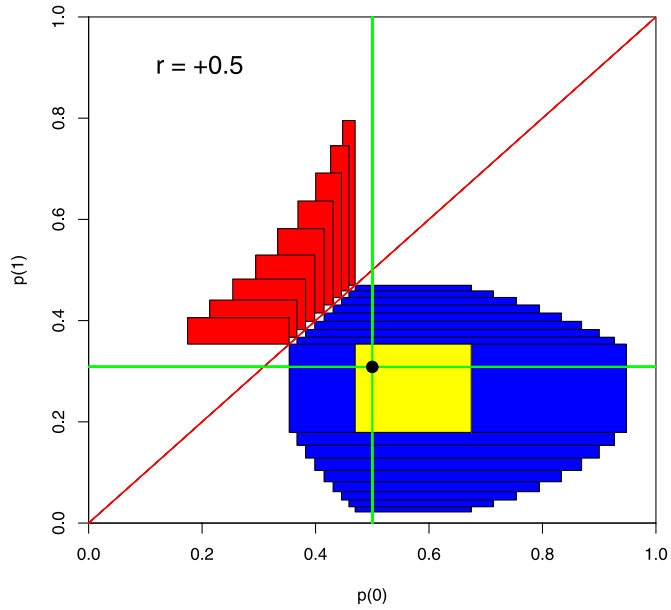
S2.2. Changing Endogeneity

In Figure S9, the value of r is -0.25 as in all the preceding figures. In Figures S10, S11, and S12, this value is increased through the sequence $0.0, +0.25, +0.50$. As this happens, the extent of the identified sets at individual values of Z that have $p(1) \geq p(0)$ is reduced, while the sets that have $p(1) \leq p(0)$ become larger.

S2.3. Widening the Support of the Instrument

Finally, the value of the coefficient b_1 on the instrumental variable in the equation for the endogenous variable is increased.

In Figure S12, the value of b_1 is 1.0 and the value of the correlation r is $+0.5$. Holding the value of the correlation fixed at $+0.5$, Figure S13 shows the identified set when $b_1 = 0.3$ and Figure S14 shows the identified set when $b_1 = 2$. Changing the value of b_1 is equivalent to changing the units of measurement of the instrumental variable and, with Ω held fixed, equivalent to widening its support. At the small value of b_1 (narrow support), the identified set is large in extent and disconnected. At the large value of b_1 , the identified set is small.

FIGURE S11.—Identified set when $r = +0.25$, $z \in \{-0.75, \dots, +0.75\}$.FIGURE S12.—Identified set when $r = +0.5$, $z \in \{-0.75, \dots, +0.75\}$.

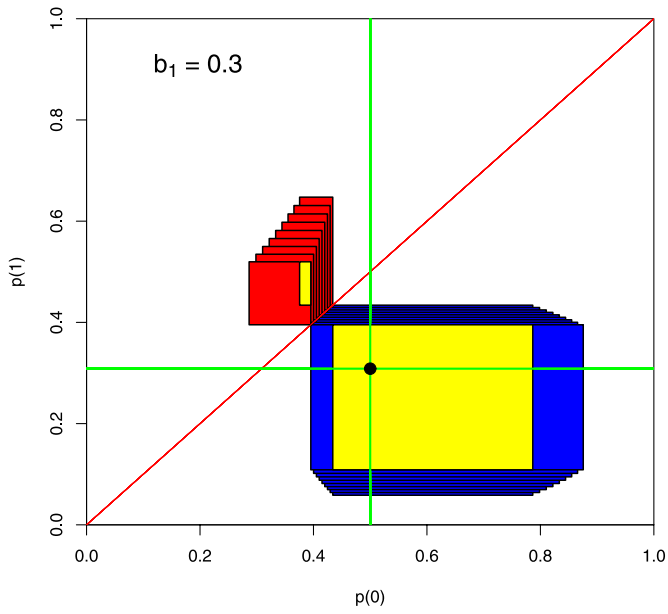


FIGURE S13.—Identified set when $r = +0.5$, $z \in \{-0.75, \dots, +0.75\}$, $b_1 = 0.3$.

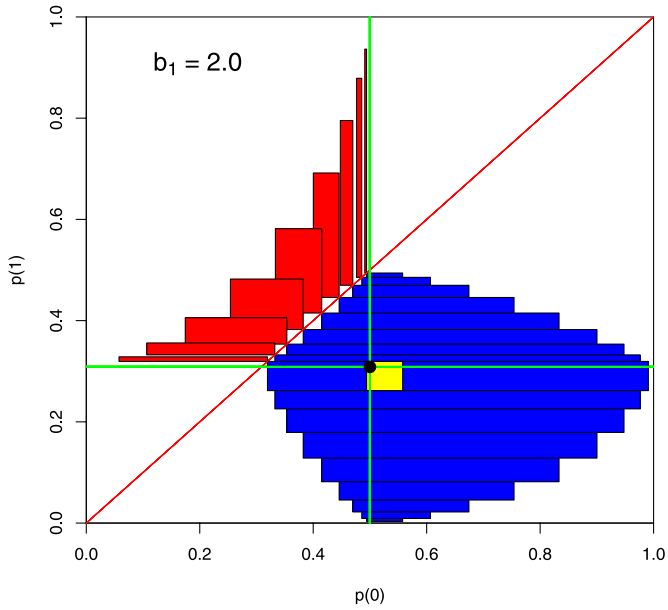


FIGURE S14.—Identified set when $r = +0.5$, $z \in \{-0.75, \dots, +0.75\}$, $b_1 = 2.0$.

*Dept. of Economics, University College London, Gower Street, London WC1E
6BT, U.K. and CeMMAP; andrew.chesher@ucl.ac.uk.*

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