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CORRIGENDUM TO CAPITAL INVESTMENT IN "ASSORTATIVE MATCHING WITH LARGE FIRMS"

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THIS DOCUMENT CORRECTS AN ERROR in Eeckhout and Kircher (2018) in the sign of an underived condition for positive assortative matching (PAM thereafter) within those extensions that allows for generic capital investment: It occurs in the applications of the main theory to *The skill premium with generic capital investment* and *The misallocation debate*; see page 104. This note provides the correct condition, proves it, and adjusts the accompanying example.[1] The correct condition is

$$\begin{aligned}&\hat{F}_{xy}\hat{F}_{lr}\hat{F}_{kk} - \hat{F}_{xy}\hat{F}_{lk}\hat{F}_{rk} - \hat{F}_{xk}\hat{F}_{yk}\hat{F}_{lr} \\ &\quad \le \hat{F}_{xr}\hat{F}_{yl}\hat{F}_{kk} - \hat{F}_{xr}\hat{F}_{yk}\hat{F}_{lk} - \hat{F}_{xk}\hat{F}_{yl}\hat{F}_{rk}.\end{aligned} \tag{1}$$

Background: Eeckhout and Kircher (2018) considered a competitive economy with a given distribution of firm types y and worker types x. Firms produce output according to production function $f(x, y, l)$, where x is the worker type hired by firm y and l is the number of such workers. Output is strictly concave in l. Define $F(x, y, l, r) := rf(x, y, l/r)$ as the output of r such firms that employ altogether l such workers. The paper derives a condition when higher firm types employ higher worker types. Such positive assortative matching requires

$$F_{xy}F_{lr} \ge F_{yl}F_{xr}, \tag{2}$$

where subscripts denote cross-partial derivatives and arguments are omitted here.

In Section 4, pages 104–106, of Eeckhout and Kircher (2018), an additional capital investment is introduced. Firms can now produce according to $\hat{f}(x, y, l, k)$, which is con-

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[1]The code that generates the accompanying plot for this example and checks for the correct condition can be found at https://github.com/crisla/Capital-Investment-in-Assortative-Matching-with-large-Firms.

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cave in (l, k) where k is a generic capital good. In a small open economy, the price i per unit of capital is exogeneous. Using $f(x, y, l) := \max_k \hat{f}(x, y, l, k) - ik$, one can use the condition in the previous paragraph to obtain a sorting condition.

An inequality on page 104 and in Appendix B of Eeckhout and Kircher (2018) represents this sorting condition directly in terms of $\hat{f}$, or more precisely, in terms of the total output $\hat{F}(x, y, l, r, k) := r\hat{f}(x, y, l/r, k/r)$ that can be produced by r such firms that employ in total l such workers and k units of capital. This had a sign mistake, and the inequality has the opposite (and therefore incorrect) sign. The correct condition for positive assortative matching in this case is (1).

The remainder of this note proves this result, and revisits the numerical illustration. Using an appropriate functional form for this illustration that gives rise to positive assortative matching with the correct sorting condition reveals the same qualitative features that were discussed in the original paper.

PROOF OF CONDITION (1) FOR PAM WITH GENERIC CAPITAL INVESTMENT

Consider the production function

$$F(x, y, l, r) = \max_k \hat{F}(x, y, l, r, k) - ik. \tag{3}$$

We assume that $\hat{F}$ is twice differentiable; strictly concave in each of l, r, and k; and displays CRS in l, r, and k (so that F has CRS in r and l). Problem (3) can be rewritten by explicitly solving the maximization problem, that is,

$$F(x, y, l, r) = \hat{F}\big(x, y, l, r, k^*\big) - ik^*, \tag{4}$$

where k^* depends on (x, y, l, r) and solves the first-order condition:[2]

$$\hat{F}_k\big(x, y, l, r, k^*(x, y, l, r)\big) = i.$$

Calculating the gradient of k^* will be useful for further simplifications below. Applying the implicit function theorem, we have

$$\left[\frac{\partial k^*}{\partial x}, \frac{\partial k^*}{\partial y}, \frac{\partial k^*}{\partial l}, \frac{\partial k^*}{\partial r}\right] = \left[-\frac{\hat{F}_{xk}}{\hat{F}_{kk}}, -\frac{\hat{F}_{yk}}{\hat{F}_{kk}}, -\frac{\hat{F}_{lk}}{\hat{F}_{kk}}, -\frac{\hat{F}_{rk}}{\hat{F}_{kk}}\right]. \tag{5}$$

We know that PAM arises only if (2) holds along the assignment, and PAM arises if (2) holds strictly everywhere. Using the formulation in (4), the second derivatives in (2) can be rewritten as

$$\begin{aligned}&[F_{xy}, F_{lr}, F_{xr}, F_{yl}] \\ &\quad = \left[\hat{F}_{xy} + \hat{F}_{xk}\frac{\partial k^*}{\partial y}, \hat{F}_{lr} + \hat{F}_{lk}\frac{\partial k^*}{\partial r}, \hat{F}_{xr} + \hat{F}_{rk}\frac{\partial k^*}{\partial x}, \hat{F}_{yl} + \hat{F}_{yk}\frac{\partial k^*}{\partial l}\right].\end{aligned} \tag{6}$$

[2]The second-order condition is automatically fulfilled by concavity of $\hat{F}$ in k.

Substituting (5) into (6) and then replacing the terms in (2), the PAM condition becomes

$$\left(\hat{F}_{xy} - \frac{\hat{F}_{xk}\hat{F}_{yk}}{\hat{F}_{kk}}\right)\left(\hat{F}_{lr} - \frac{\hat{F}_{lk}\hat{F}_{rk}}{\hat{F}_{kk}}\right) \geq \left(\hat{F}_{xr} - \frac{\hat{F}_{xk}\hat{F}_{rk}}{\hat{F}_{kk}}\right)\left(\hat{F}_{yl} - \frac{\hat{F}_{yk}\hat{F}_{lk}}{\hat{F}_{kk}}\right). \tag{7}$$

Multiplying both sides of (7) by $\hat{F}_{kk}$, the sign of the inequality changes as $\hat{F}_{kk} < 0$. Simplifying, we get (1).

APPLICATION TO THE MISALLOCATION DEBATE

Given this new condition, the production function $\tilde{f}(x, y, l, k) = a(\eta(xk)^{\rho} + (1-\eta) \times (yl)^{\rho})^{\frac{\gamma}{\rho}}$ does not satisfy the conditions for PAM. This is the functional form used in the illustration of page 105 and in Appendix B of Eeckhout and Kircher (2018), which constructs the equilibrium using the first-order conditions valid only under PAM. Here, we redo the exercise with a different functional form that does satisfy the PAM condition (1) with the parameter values in Adamopoulos and Restuccia (2014):

$$\tilde{f}(x, y, l, k) = a\left(\eta k^{\rho} + (1-\eta)\left[y\left(\frac{l}{r}\right)^{x}\right]^{\rho}\right)^{\frac{\gamma}{\rho}}. \tag{8}$$

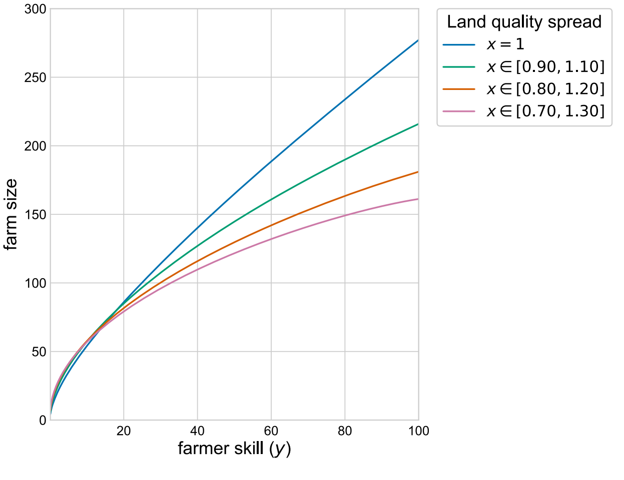

FIGURE 1.—Firm size distribution for different dispersion in x (x is log-normally distributed LN(0, 0.2), i.e., $\log(x)$ is normally distributed with mean 0 and variance 0.2, truncated at the bounds indicated in the legend, with the measure of the truncated distribution normalized to 1).

Taking derivatives of this production function and substituting the parameter values for this application reveals that (1) is satisfied.[3] Using this new production function, Figure 4 on page 105 in Eeckhout and Kircher (2018) becomes Figure 1. This yields qualitatively simlar results, and the discussion in Eeckhout and Kircher (2018) applies unchanged to this new example. A mean-preserving spread in input heterogeneity reduces heterogeneity in the distribution of land holdings across farms, as better firms buy less but better land.

REFERENCES

ADAMOPOULOS, T., AND D. RESTUCCIA (2014): "The Size Distribution of Farms and International Productivity Differences," *American Economic Review*, 104 (6), 1667–1697. [3]

EECKHOUT, J., AND P. KIRCHER (2018): "Assortative Matching With Large Firms," *Econometrica*, 86 (1), 85–132. [1-4]

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[3]Substituting the derivatives of (8) into (2) and substituting the parameter values of the U.S. calibration, we obtain

$$\begin{aligned}
&-0.003508r^2x\left(y\left(\frac{l}{r}\right)^x\right)^{0.5}\left(0.00134143k^{0.25}+0.000496347\left(y\left(\frac{l}{r}\right)^x\right)^{0.25}\right)\\
&\quad\times\left(0.8902045k^{0.25}+0.1097955\left(y\left(\frac{l}{r}\right)^x\right)^{0.25}\right)^6\\
&\quad\Big/\left[k^{1.75}ly\left(0.00471307k^{0.25}\left(y\left(\frac{l}{r}\right)^x\right)^{0.75}+0.05731921k^{0.5}\left(y\left(\frac{l}{r}\right)^x\right)^{0.5}\right.\right.\\
&\quad\left.\left.+0.3098232k^{0.75}\left(y\left(\frac{l}{r}\right)^x\right)^{0.25}+0.62799915k+0.000145325\left(y\left(\frac{l}{r}\right)^x\right)\right)\right]\\
&<0.
\end{aligned}$$

Using the parameter values from the developing country calibration, the inequality (2) becomes

$$\begin{aligned}
&-0.00099679r^2x\left(y\left(\frac{l}{r}\right)^x\right)^{0.5}\left(0.00134143k^{0.25}+0.00049635\left(y\left(\frac{l}{r}\right)^x\right)^{0.25}\right)\\
&\quad\times\left(0.8902045k^{0.25}+0.1097956\left(y\left(\frac{l}{r}\right)^x\right)^{0.25}\right)^6\\
&\quad\Big/\left[k^{1.75}ly\left(0.00471307k^{0.25}\left(y\left(\frac{l}{r}\right)^x\right)^{0.75}+0.05731921k^{0.5}\left(y\left(\frac{l}{r}\right)^x\right)^{0.5}\right.\right.\\
&\quad\left.\left.+0.3098232k^{0.75}\left(y\left(\frac{l}{r}\right)^x\right)^{0.25}+0.62799915k+0.00014532\left(y\left(\frac{l}{r}\right)^x\right)\right)\right]\\
&<0.
\end{aligned}$$

Both inequalities are satisfied as long as x, y, l, r, k are positive.