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THE NONPARAMETRIC IDENTIFICATION OF TREATMENT EFFECTS IN DURATION MODELS: CORRIGENDUM

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In Abbring and Van den Berg (2003b) we investigated the identification of causal multivariate duration models. We focused on the case in which one duration concerns the point in time a treatment is initiated (S) and we were interested in the effect of this treatment on some outcome duration (Y). Our basic model (Model 1A on p. 1503) specified the joint distribution of these two durations, conditional on observed covariates, as a bivariate mixed proportional hazards model augmented with an effect of the treatment on the outcome hazard. This model allowed for two dependent unobservable factors, one (V_S) in the treatment hazard and one (V_Y) in the outcome hazard. Proposition 3 (p. 1506) established that, under some conditions, this model's primitives, including the joint distribution G of $(V_Y, V_S)'$, are identified. This result is correct as stated in the paper.

However, our Propositions 4 (p. 1508) and 5 (p. 1510), which made similar claims about variants of Model 1A, need to be qualified. The consequents of Propositions 4 and 5 for variants that allow the treatment effect on the outcome hazard to depend on a third unobservable factor only hold under the additional condition that $\Pr(V_S = 0) = 0$.

We will first discuss Proposition 4 (the case of Proposition 5 is very similar). This proposition covers Model 1B, which is a variant of Model 1A with a third unobservable (V_Δ) that only affects the outcome hazard after the start of the treatment. Proposition 4 (p. 1508) stated that, under some assumptions, Model 1B's primitives, including the joint distribution G_Δ of $(V_Y, V_\Delta, V_S)'$, are identified. Indeed, Proposition 4's proof established identification of all primitives except G_Δ. However, the proof's last sentence (p. 1515), which claimed identification of G_Δ, is incorrect. It sought to identify G_Δ by constructing its Laplace transform $\mathcal{L}_{G_\Delta}$ from $\mathcal{L}^{(S)}_{G_\Delta}$, the partial derivative of $\mathcal{L}_{G_\Delta}$ with respect to its third argument, which was already shown to be identified (p. 1514). This requires a boundary condition that fixes $\mathcal{L}_{G_\Delta}(z_1, z_2, z_3)$ for all $(z_1, z_2) \in [0, \infty)^2$ and some (limiting) value of z_3. If it is not ruled out that $\Pr(V_S = 0) > 0$ then the one used in the proof, $\mathcal{L}_{G_\Delta}(0, 0, 0) = 1$, does not suffice. We show that this can easily be resolved.

Let G^+_Δ be the (sub-)distribution of $(V_Y, V_\Delta, V_S)'$ on $\{V_S > 0\}$. Note that its Laplace transform $\mathcal{L}_{G^+_\Delta}$ is given by

$$\mathcal{L}_{G^+_\Delta}(z_1, z_2, z_3) = \int_{(0,\infty)} \int_{[0,\infty)} \int_{[0,\infty)} \exp\left(-z_1 v_Y - z_2 v_\Delta - z_3 v_S\right) G_\Delta(dv_Y, dv_\Delta, dv_S).$$

By bounded convergence, this gives the boundary condition

$$\lim_{z_3 \to \infty} \mathcal{L}_{G^+_\Delta}(z_1, z_2, z_3) = 0, \tag{1}$$

which suffices to determine $\mathcal{L}_{G_\Delta^+}$ from its identified derivative $\mathcal{L}^{(S)}_{G_\Delta^+} = \mathcal{L}^{(S)}_{G_\Delta}$:

$$\mathcal{L}_{G_\Delta^+}(z_1, z_2, z_3) = -\int_{z_3}^{\infty} \mathcal{L}^{(S)}_{G_\Delta}(z_1, z_2, \xi) d\xi. \tag{2}$$

Consequently, under Proposition 4's conditions, $\mathcal{L}_{G_\Delta^+}$ and, by the uniqueness of the multivariate Laplace transform, G_Δ^+ are identified. Moreover, Proposition 2 directly ensures that the distribution G of $(V_Y, V_S)'$ is identified. If G_Δ^+ is a proper (non-defective) distribution— that is, if the additional condition that $\Pr(V_S = 0) = 0$ holds— then $G_\Delta = G_\Delta^+$ is identified, as in Proposition 4's consequent.

This result can be contrasted with Proposition 3 for Model 1A, which established full identification without further conditions. The key difference with Model 1B is that Model 1A only contains unobserved factors V_Y and V_S that affect outcome and treatment hazards from the start, so that their distribution G can be identified from "pre-treatment data," *i.e.* data on the identified minimum of Y and S. Specifically, Proposition 3's proof applied Proposition 2, which is a result from Abbring and Van den Berg's (2003a) for the competing risks model, to establish identification of, among other things, G.

As noted above, the same argument can be used to establish identification of G from pre-treatment data in Model 1B. However, the third unobserved factor in Model 1B, V_Δ, only affects the post-treatment outcome hazard. Therefore, the identification analysis of its joint distribution G_Δ with V_Y and V_S necessarily involves data on post-treatment outcomes. Specifically, Proposition 4's proof relied on Equation (12) on p. 1514, which expressed the (sub-)density of S at a point s on $\{Y > y\}$ for $y > s$ in terms of $\mathcal{L}^{(S)}_{G_\Delta}$ to identify the latter. We have seen that, in turn, $\mathcal{L}^{(S)}_{G_\Delta}$ can be used to identify G_Δ^+, but not necessarily G_Δ. This is intuitive: From data on the sub-populations that are treated at specific times we cannot possibly identify the distribution of (V_Y, V_Δ) on the sub-population with $V_S = 0$, which is never treated.

Now consider the first part of Proposition 5. This covered Model 2A (p. 1510), which is a multiple-spell extension of Model 1A. Like Model 1A, it only involves two unobserved factors, V_Y and V_S, which affect the outcome and treatment hazards from the start. Consequently, as in the analysis of Model 1A, data on the identified minima of outcome and treatment durations are informative on their distribution G. However, Proposition 5's proof did not appeal to an identification result for the multiple-spell competing risks model to identify G from such pre-treatment data only. Instead, much like Proposition 4's proof, it used post-treatment data to establish identification of the partial derivative $\mathcal{L}^{(S)}_G$ of the Laplace transform of G with respect to its second argument and then erroneously states that $\mathcal{L}_G(0, 0) = 1$ suffices to identify G from this. We can easily rectify this along the lines of our above discussion of Proposition 4. The sub-distribution G^+ of $(V_Y, V_S)'$ on $\{V_S > 0\}$ can be identified without further conditions. If G^+ is a

proper distribution — that is, if $\Pr(V_S = 0) = 0$ — then $G = G^+$ is identified.

The identification analysis of G_Δ in Model 1B required post-treatment data because pre-treatment data carry no information on V_Δ. In contrast, our use of post-treatment data to identify G in Model 2A was and is a matter of choice. We can alternatively mimic the argument for Model 1A and use Abbring and Van den Berg's (2003a) identification result for the multiple-spell competing risks model to identify G from pre-treatment data, provided that the additional condition spelled out in their Proposition 3(b) holds. To sum up, in Model 2A we can fully identify G if either G^+ is a proper distribution — $\Pr(V_S = 0) = 0$ — or the condition in Abbring and Van den Berg (2003a, Proposition 3(b)) holds.

Finally, consider Proposition 5's second part. This covered Model 2B (p. 1510), a multiple-spell extension of Model 1B. Like Model 1B, it involves a third unobserved factor V_Δ that only affects post-treatment outcome hazards. Just like Proposition 4's proof, part (ii) of Proposition 5's proof erroneously states that $\mathcal{L}_{G_\Delta}$ can be determined from $\mathcal{L}^{(S)}_{G_\Delta}$ and $\mathcal{L}_{G_\Delta}(0,0,0) = 1$. As in our analysis of Model 1B, we can show that G^+_Δ is identified under the conditions of Proposition 5. Again, $G_\Delta = G^+_\Delta$ follows if $\Pr(V_S = 0) = 0$. As in our analysis of Model 2A, if the condition in Abbring and Van den Berg (2003a, Proposition 3(b)) holds, then the marginal distribution G of $(V_Y, V_S)'$ can be identified from pre-treatment data only.

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