General Financial Economic Equilibria

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Abstract

Uncertain demands and supplies, given prices, may not be equated to define an equilibrium. New concepts of equilibria are then formulated by modeling markets as an abstract agent absorbing the clearing risk. The new equilibria invoke the theory of acceptable risks to define a two price equilibrium termed a general financial economic equilibrium (GFEE). The market sets two prices for each commodity, one at which it buys and the other at which it sells. The two prices are determined by targeting the aggregate random net inventory and net revenue exposures to be acceptable risks. For an $n-$ commodity economy there are then $2n$ equilibrium equations for the $2n$ prices. However, as all the equations cannot be simultaneously satisfied in a general equilibrium, an equilibrium approximation is defined and illustrated on minimizing the deviation from equilibrium in determining the $2n$ prices. The introduction of a two price labor market naturally leads to the concept of both an equilibrium unemployment rate and an equilibrium unemployment insurance rate. It is shown that the unemployment rate rises with the productivity of the economy and can be mitigated by expanding the number of products. Technological innovation accompanied by product expansion is observed to be employment neutral and socially acceptable. Similarly redistributive strategies from the upper end of the income scale towards the middle or lower end can lower equilibrium unemployment levels via their effects on aggregate demand. Productivity shocks like COVID lead to higher equilibrium unemployment support levels measured by the income ratios of the unemployed to the employed. The magnitude of the increase depends on the levels of labor market risk acceptability. The analysis of a skill differentiated labor market makes the case for income and aggregate demand support via unemployment compensations in a GFEE.

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JEL Classification: D50, D51, D58

1 Introduction

Equilibrium theory in economics seeks to determine prices that clear markets. The theory may be seen as beginning with the scissors of Marshall (1890) illustrating market clearing for a single good. The general equilibrium for an n good economy was formulated by Walras (1874). Arrow and Debreu (1954) provides a proof establishing conditions under market clearing price systems exist. The literature has since seen many applications, made by the construction and analysis of particular macro, international, and computable general equilibria. Additionally, there are results on the social optimality of such equilibria.

At the core of the underlying equilibrium theory, beginning with Marshall (1890), lies the idea of determining prices by equating demands to supplies or excess demands to zero. It is then critical that demands, supplies or excess demands thereof, be deterministic functions of prices, as they are in the models. In actual economies, however, both demands and supplies of goods are uncertain and the difference is at best a random quantity. It can then not be equated to zero. A way around this problem, followed in Chapter 7 of Debreu (1959), is to treat random demands and supplies as deterministic functions of prices contingent on the random states. State contingent prices can then be used to clear state contingent markets. However, if prices have to be set first and cannot be made state contingent, as all the relevant states are too many to be described or even enumerated, then the excess demand is random at best and the equilibrium equations are rendered ineffective. Demand and supply are exact outcomes only after the specification of a large number of inputs and it may not be practical for any real economy to deliver such state contingent price systems.

Given a positive exposure in the economy to the absence of clearing, every price system has to deal with the risk of exposure to unsold goods and unmet demand. In this paper we seek manage the clearing risk exposure by introducing the market as the sole risk absorbing agent in the economy. As a result individual economic participants are taken to be immune to this clearing risk and continue to form their economic plans without making any behavioral adjustments on account of the additional clearing risk. The result is a new two price equilibrium theory.

In our formulation the demands and supplies remain functions of prices faced by market participants, as it is in the existing theory, but after allowing for randomness to be accomodated into the specification. Importantly, market participants will not face the risks of unsold goods or unmet demands. They are made good and they know they will be made good. Hence the risks associated with the absence of clearing are of no concern to them. Consequently their behavior need not be modified in response to such exposures. In this regard one may view the market as the clearing agent with enough inventory in place to make good on the resulting discrepancies.

In formulating the new equilibrium we fall back on a lesson to be taken from the Arrow and Debreu (1954) theory. The existence proof introduced the market formally as an abstract price setting agent with the only objective being that of attempting to clear markets. Here the market will be viewed as a financially well endowed agent absorbing all the risks from the absence of clearing and making all the other participants good. The market is then a financial risk absorbing agent. The resulting equilibrium reflects market level risk attitudes and is termed a General Financial Economic Equilibrium (GFEE).

With a view towards being financially safe we introduce a two price economy. The market will determine two prices for each commodity, one at which it sells to participants demanding goods and the other a lower price that it offers to those selling goods to the market. All participants trade with the market and not with each other. For an n good economy one would then require $2n$ equations for the $2n$ prices of the GFEE. The theory of acceptable risks of Artzner, Delbaen, Eber and Heath (1999) is employed to formulate the new $2n$ equations and the new equilibrium. We shall observe that not all $2n$ equations may be simultaneously satisfied and an approximate equilibrium is defined by minimizing the deviations from zero for the stated equations. The equilibrium sought is then found on minimizing, for the set of two prices, a deviation metric bounded below by zero. The resulting equilibrium need not be unique but it is expected that it will be locally be unique.

The literature in general equilibrium theory has reported on random demands and supplies and has considered stochastic equilibria. Examples include Green and Majumdar (1975), Hildenbrand and Radner (1979), Bhattacharya and Majumdar (2004), and Toda (2010). However the focus of attention has been on the process of price adjustment in a presumed equilibrium under the law of one price. Toda (2010) is further directed towards the properties of the equilibrium distributions that result.

The general theory of a two price GFEE is first developed and presented. The theory is illustrated with many, numerically solved, examples that shed light on many matters of economic concern. As a broad policy prescription the theory presents the foundation for the development of a widespread system of income support for economic agents commensurate with their skill levels as all labor must be bought by the market at income support prices to be then partially sold to production units at higher prices. The price differentials determine the structure of unemployment insurance by category of skill in labor markets.

For a first example we recast the scissors of Marshall (1890) in a two price one good GFEE. From a general equilibrium perspective, we formulate next a full employment n good two price general equilibrium. In this model the labor market clears at one price but all the other markets are two price markets. The two good case is numerically solved for a variety inputs and the dependence of the two price equilibrium on the inputs is investigated and reported on. Some n good full employment cases are then considered.

A GFEE is next constructed with a two price labor market. A consequence of which is the formulation of an equilibrium unemployment rate and an equilibrium unemployment insurance level in a general equilibrium economy. The equilibrium unemployment rate rises with increased productivity and falls with an expansion of the number of products in the economy. It also rises with an increase in the elasticity of substitution in preferences for a constant elasticity of substitution utility function. Furthermore, income redistributons from the upper end of the income scale towards the middle or lower end can lower equilibrium unemployment levels via their effects on aggregate demand. Productivity shocks like COVID can lead to rises in the unemployment support levels as measured by the income ratios of the unemployed to the employed. The magnitude of this rise can depend on attitudes to risk prevailing in the clearing of labor markets. Finally we consider examples of two price labor markets that are differentiated by skill levels in the labor market. In this context we comment on the equilibrium levels of unemployment insurance as they apply to the prevailing skill differentials.

There is an extensive literature modeling bid and ask prices as abberations that prevent the market's convergence to the ideal formulation of the law of one price. The departures from a fundamental law of one price are explained in different ways. They could result as a consequence of transactions costs as studied in Constantinedes (1986), Jouini and Kallal (1995), Lo, Mamaysky and Wang (2004). The classical model is free of such costs. Alternatively they could reflect the impact of informed traders on market makers (Copeland and Galai (1983), Easley and O'Hara (1987), Glosten and Milgrom (1985), Back and Baruch (2004)). In the classical model all traders are equally and fully informed. Transactions costs were expanded to include order processing and inventory costs in Demsetz (1968), Stoll (1978), Ho and Stoll (1981, 1983). Mean returns and bid-ask spreads are related via liquidity considerations in Ahimud and Mendelson (1986). Once again the classical equilibrium model is free of these considerations.

In yet another approach to two prices, Madan (2015) approaches the two prices from a no arbitrage perspective, as an alternative to an equilibrium solution. The two prices arise naturally as nonlinear functions of promised outcomes when the set of zero cost traded claims are not closed under negation.

The two price literature in the form of bid and ask prices for financial securities that trade at some frequency is only formally related to the two prices of a GFEE. The latter may more accurately be compared to the two prices for commodities represented by the wholesale and retail markets for goods and services. The market represented by wholesalers buys at wholesale prices from producers in what are large quantities. The goods and sevices then make their way into the retail market in smaller sizes and at higher retail prices. The gaps between the retail and wholesale prices are in general greater than financial security bid ask spreads and probably more akin to the two prices of a GFEE.

Here the validity of the classical equilibrium model is directly questioned on recognizing that demands, supplies and excess demands at any price are random variables that cannot be equated to zero to solve for an equilibrium price. The market is then modeled as the single agent absorbing all the clearing risk. Importantly, market participants are modeled with immunity to this risk and hence their actions are consistent with this risk being ignored. The market modeled as a clearing agent is solely subject to risk and operates with a degree of conservatism. There are then two equations targeting conservatively positive excess supplies and net revenues. These two sets of equations, when simultaneously minimized for deviations from zero, deliver a two price equilibrium approximation. There are no transactions or processing costs and no ill informed market participants.

The outline of the paper is as follows. Section 2 addresses the policy analyses opened up by the new model. Section 3 presents the general theory of two price GFEE. Section 4 revises the scissors model of Marshall (1890) to accommodate a two price equilibrium with uncertainty. A two price approximate general equilibrium with multiple commodities with full employment is formulated in Section 5. Equilibrium unemployment is introduced in Section 6 for a two price labor market. Section 7 reports on a number of applications addressing

- Effects of productivity on employment in the economy.
- Implications of technological differentiation between sectors and consumer preferences.
- Employment benefits of redistributive strategies.
- Effects of productivity shocks like COVID on equilibrium levels of unemployment support.
- Implications of combining productivity shocks with an unequal income distributions.
- Skill differentiation in labor markets and its effects on the levels of unemployment insurance.

Section 8 concludes.

2 Policy Implications

An important aspect of economic policy are its effects on employment in the aggregate, across the sectors of economy, and between the varied classes of the labor market. This is particularly true in times of crises accompanied by greater economic uncertainty. These effects are not open to modeling in the classical model as the entire impact is on wages with market clearing delivering full employment for all labor types. The two price equilibrium formulated here delivers equilibrium levels of unemployment in all sectors and labor types. It also delivers equilibrium levels of income support for the unemployed across all sectors and for all types of labor. We present results of experiments on these matters.

First we note that the effects on equilibrium employment of increased productivity can be to raise the levels of unemployment. This is mitigated by the expansion in the number of production activities, each one of which is highly productive. The expansion in the number of internet based activities accompanying the growth of the internet being a case in point.

The employment effects of rising income inequality can also be detrimental. Experiments are presented showing how income redistribution strategies can help mitigate these effects in a static two price equilibrium.

When income inequality and productivity shocks are put together the effects on unemployment and the income support for the unemployed are observed to rise substantially.

Finally experiments address unemployment support levels by skill categories in the labor market with higher support levels for skilled labor.

The experiments presented are a beginning into the study of static two price economic equilibria. The study of dynamic models in the same direction is an open research agenda. The model delivers excess supply levels in all sectors of the economy some of which are considered briefly in stylized models that could be subjected to a deeper analysis.

3 The General Theory of a GFEE

Consider a classical partial or general equilibrium with excess demand functions determined as functions of prices that we may write as functions $z(p)$ from R^n to R^n for n prices p mapped to n excess demands z. The functions $z(p)$ are the difference between demand functions $D(p)$ and supply functions $S(p)$. The equilibrium is defined by the equation

$$z(p) = 0.$$

In the classical Arrow Debreu theory of general equilibrium the excess demand functions are homogeneous of degree zero in prices and hence only relative prices are determined. They are then $n-1$ in number. The excess demands are also not independent but satisfy via budget constraints the condition

$$p^T z(p) = 0.$$

Hence they are also $n-1$ in number. The theory provided conditions under which one has a proof for the existence of an equilibrium.

Suppose now that there is randomness in the demand and supply functions and for events ω_D and ω_S the demand and supply functions are

$$\begin{aligned} D &= D(p, \omega_D), \\ S &= S(p, \omega_S). \end{aligned}$$

These are deterministic functions reflecting randomness in preferences and technologies. The classical equilibrium theory was extended to accomodate such uncertainty by introducing state contingent price systems $p(\omega_D, \omega_S)$ to be solved for the state contingent equilibrium prices.

We wish to consider equilibria with uncertain demands and supplies in which prices are not to be made state contingent and the economic system must then accomodate itself to the prevailing uncertainties. In principle there are too many events determining demands and supplies and requiring prices to depend on all these events is not practical.

For this purpose we introduce a new concept of a risk absorbing market that allows for stochasticity in clearing accompanied with a new associated two price equilibrium theory. The market buys all supplies from suppliers at prices p_L and sells all goods to consumers at prices p_U. The market then experiences stochastic excess supplies of

$$z(p_L, p_U, \omega_D, \omega_S) = S(p_L, \omega_S) - D(p_U, \omega_D)$$

and stochastic net revenues

$$R(p_L, p_U, \omega_D, \omega_S) = p_U D(p_U, \omega_D) - p_L S(p_L, \omega_S)$$

The market as a clearing agent has an interest in the positivity of these random entities in all components. Hence positive outcomes are market acceptable but negative outcomes must be tolerated. For the associated two price equilibrium theory we adopt the theory of acceptable risks by requiring excess supplies and revenues be individually acceptable in all components. Adopting the theory of acceptable risks of Artzner, Delbaen, Eber and Heath (1999), there must exist sets of test measures $\mathcal{M}_i, \mathcal{N}_i$ for the i^{th} component defining acceptable excess supplies and net revenues by

$$\begin{aligned} \inf_{Q \in \mathcal{M}_i} E^Q \left[z_i(p_L, p_U, \omega_D, \omega_S) \right] &= 0 \\ \inf_{Q \in N_i} E^Q \left[R_i(p_L, p_U, \omega_D, \omega_S) \right] &= 0 \end{aligned}$$

These are $2n$ equilibrium equations that may possibly be employed for the determination of the $2n$ prices p_L, p_U.The lack of clearing even as an expectation makes it possible for us to discuss concepts like equilibrium unemployment and equilibrium unemployment insurance that may also be differentiated by skill levels in the labor market.

If risk acceptability is modeled to be law invariant and comonotone additive as described in Kusuoka (2001) then one may define two price equilibria using concave distortions of probability and write the equilibrium equations as

$$\mathcal{E}_i \left(z_i(p_L, p_U, \omega_D, \omega_S) \right) = 0 \tag{1}$$

$$\widetilde{\mathcal{E}}_i \left(R_i(p_L, p_U, \omega_D, \omega_S) \right) = 0, \tag{2}$$

where $\mathcal{E}_i$, and $\widetilde{\mathcal{E}}_i$ are distorted expectation operators.

Proposition 1 *Two price equilibria deliver positive excess supplies with the percentage excess supply dominated by the price markup. Specifically*

$$\frac{E^P \left[S_i(p_L, \omega_S) \right] - E^P \left[D_i(p_U, \omega_D) \right]}{E^P \left[S_i(p_L, \omega_S) \right]} \leq \frac{\eta}{1+\eta} \tag{3}$$

$$\eta = \frac{p_{U_i} - p_{L_i}}{p_{L_i}} \tag{4}$$

Proof. As expectations are above distorted expectations the equation of distorted expectations to zero implies that

$$E^P\left[S_i(p_L,\omega_S)\right] \geq E^P\left[D_i(p_U,\omega_D)\right] \tag{5}$$

and

$$p_{U_i}E^P\left[D_i(p_U,\omega_D)\right] - p_{L_i}E^P\left[S_i(p_L,\omega_S)\right] \geq 0 \tag{6}$$

Division of equation (6) by p_{L_i} times $E^P\left[D_i(p_U,\omega_D)\right]$ yields that

$$\frac{p_{U_i}}{p_{L_i}} \geq \frac{E^P\left[S_i(p_L,\omega_S)\right]}{E^P\left[D_i(p_U,\omega_D)\right]}$$

Algebraic manipulations deliver the result (3). ■

In particular if the markup tends to zero then the markets approaching clearing.The greater the concavity of the distortions employed in the definition of the market equilibrium the sharper the inequalities (5) and (6) and the greater the spread between the lower and upper prices for an fixed level of uncertainty in the economy. Additionally we may observe that as the uncertainty goes to zero distorted expectations converge to expectations that converge to the certain outcome. The result is then the classical outcome of market clearing under the law of one price. The role of the two prices is thereby inherently tied to the clearing risk embedded by the uncertainties.

There are, however, some general equilibrium issues with requiring all net revenues to be individually acceptable or equivalently, when acceptability is defined using distortions, that all distorted net revenues be equated to zero. Aggregate general equilibrium balance equations between incomes and expenditures force aggregate expected net revenue to be zero. This is the case in the general equilibrium models employed in the paper. For a common distortion for all markets the sum of distorted expectations of net revenues is dominated by the distorted expectation of aggregate net revenues that is itself dominated by the expected net revenue which is zero. As a consequence one cannot require all net revenue distorted expectations to be nonnegative or zero as this would, in most cases, force aggregate expected net revenue to be positive. Such an outcome is inconsistent with the general equilibrium income expenditure balance equations. Given that we wish to target a zero level for the distorted expectations and recognize that the income expenditure balance equations imply their impossibility the equilibriums sought minimize the distance of the vector of distorted expectations from zero. The $2n$ equilibrium prices are then found by minimizing a nonnegative distance bounded below by zero. Given the general equilibrium balance equations the resulting solution may be seen as a best approximation to the desired equilibrium.

The welfare tradeoffs for a GFEE are more complicated. One may consider the expectation of utilities across the random components of preferences but even then the economic equilibrium attained is dependent on the market's attitude or the risk attitudes of the risk absorbing agent to the embedded clearing risks. In Section 12 of the paper where we have a single aggregate consumer we comment on the tradeoffs in expected utility and the related shortfall risks that arise. Financial economies are more complex structures than the deterministic economies devoid of risk exposures.

4 Marshall's Scissors in a GFEE

Marshall (1890) introduced the demand supply model for the equilibrium price of a single good. He even commented that to ask whether it was demand or supply considerations that determined price was akin to asking which of the two blades of a scissor were cutting a piece of paper. The demand and supply functions depicted by Marshall were deterministic functions of the price. If they moved because of random effects then the market clearing price would move and become random reflecting the influences on the demand and supply curves. One may imagine suppliers bringing small quantities to market to meet buyers wishing to buy such small quantities and the two groups haggle with each over price to determine the terms at which transactions occur.

Another view of a more modern economy is that of suppliers planning the delivery of large quantities to market on promised terms. The market then organizes the partitioning of these large quantities into smaller packages of interest to buyers who pick them up from the market at terms of sale offered to them by the market. The market offers suppliers a possibly lower price p_L at which it absorbs or takes delivery of the quantity supplied q_S. The buyers are offered the possibly higher price p_U for which they take up the quantity demanded q_D. Suppliers and demanders are modeled as delivering and taking up random quantities that are planned quantities subject to shocks to be absorbed by the market as the risk absorbing agent. The market takes up supplies and delivers demanded amounts on the prices it has set. However, the market faces two aggregate uncertainties. The first is that of excess supplies $q_S - q_D$ being positive or negative and taken up or delivered from reserved inventories. The second is that of net revenues $p_U q_D - p_L q_S$. The market has an interest in keeping both entities positive.

The explicit modeling of a risk absorbing market then depends on the definition of acceptable risks. For this purpose, we adopt the general two price equilibrium theory modeling acceptable risks as described in Artzner, Delbaen, Eber and Heath (1999). Acceptable risks are there defined as convex cones of random outcomes that contain the default cone of nonnegative outcomes. Convex combinations and scalar multiples of nonnegative outcomes are themselves nonnegative. These properties are preserved in the definition of acceptable risks and convex combinations and scalar multiples of acceptable risks are also taken to be acceptable. In addition nonnegative outcomes are acceptable.

Whether arbitrary scaling of an acceptability risk should be acceptable has been widely discussed and questioned. Arbitrary multiples can of course be so huge that their acceptability may easily be questioned as reasonable. The view taken here is that the market is large relative to the risks under consideration and for the scalings that are relevant acceptability is maintained. For the relatively small marginal scalings acceptability is assumed and it is formally extended to arbitrary scalings as a mathematically simplifying convenience.

Artzner, Delbaen, Eber and Heath (1999) show that the acceptable risks reduce to those having a positive or nonnegative expectation under a set of test probabilities or scenario measures. Additionally, asking for a nonnegative expectation, leads to the original probability being one of the test probabilities. On the other hand, if only a positive expectation under the original probability is required then the set of acceptable risks is a very wide convex set, and no bigger set is convex. Acceptable risks are then being defined very generously. For a conservative and market appropriate definition of risk acceptability, the set of test probabilities should be larger than just the single original probability. These are the sets of test measures $\mathcal{M}_i$, $\mathcal{N}_i$ referred to earlier. An explicit definition of market risk acceptability then requires a definition of all the test probabilities or measures.

Kusuoka (2001) proposes a practical solution to this question. It is based on two additional assumptions to be satisfied by the concept of risk acceptability. The first is that acceptability be defined in terms of the distribution function of a risk without any regard for how it arises. In this connection it may be noted that if one were defining acceptability of a risk to a person then it would matter if the risk of losing a million dollars occurred when the person was a billionaire or just a millionaire. The former may be acceptable and the latter possibly not. However, such considerations are less relevant when modeling acceptability for an abstract risk absorbing market. Here we take market acceptability to be defined solely by the risk distribution function.

The second assumption is about comonotone additivity. The conservative valuation of risk may be defined as the infimum of its expectations under all test probabilities. In general the value of the sum of two risks is greater than the sum of the values taken individually. If one offsets the risk of the other the sum can even be riskless via diversification benefits in the package. Now two risks are said to be comonotone if they have no negative comovements. Hence there are no diversification benefits possible. For comonotone risks we ask that the value of the sum be the sum of its values or that conservative valuation satisfy the property of comonotone additivity.

Under these two assumptions, Kusuoka (2001), shows that there must exist a concave distribution function on the unit interval, $\Psi(u)$, $0 \leq u \leq 1$, with the property that a risk X with distribution function $F_X(x)$ is acceptable just if

$$\mathcal{E}\left(X\right) = \int_{-\infty}^{\infty} x d\Psi\left(F_X(x)\right) \geq 0. \tag{7}$$

Observe that for any distribution function $F(x)$, that $\Psi(F(x))$ is another distribution function and $\mathcal{E}\left(X\right)$ is the expectation conducted under this distorted distribution function. One may also rewrite equation (7) as

$$\mathcal{E}\left(X\right) = \int_{-\infty}^{\infty} x\Psi'(F_X(x))dF_X(x), \tag{8}$$

to observe that the probabilities of loss events with $F_X(x)$ near zero are being inflated by $\Psi'(F_X(x))$ while the opposite occurs for gains with $F_X(x)$ near unity. The operation $\mathcal{E}(X)$ of equations (7) or (8) are referred to as distorted expectations that lie below the expectation and may be viewed as conservative or risk adjusted expectations. They provide us with a nonlinear risk valuation function with many applications explored in Madan and Schoutens (2016, 2022). The set of test probabilities supporting acceptability defined by a nonnegative distorted expectation is all probabilities Q with the property that for events A, $Q(A) \leq \Psi(P(A))$ for an original probability P.

For acceptability of excess supplies and net revenues by the market, the targeted equilibrium equations defining the two price p_L, p_U are

$$\begin{aligned} \mathcal{E}(q_S - q_D) &= 0 && (9) \\ \mathcal{E}(p_U q_D - p_L q_S) &= 0. && (10) \end{aligned}$$

The equations (9) and (10) are the equilibrium excess supply and equilibrium net revenue equations defining the equilibrium prices p_L, p_U. They are special cases for the general equations (1) and (2) for the special case of a single commodity partial equilibrium.

The application of these equations requires the choice of a concave distortion Ψ. For this we follow Cherny and Madan (2009) who introduced the parametric distortions *minvar*, *maxvar* and a combination *minmaxvar*. For γ an integer a risk was acceptable under the *minvar* distortion just if the expectation of the minimum of γ independent draws from the distribution was positive. Under *maxvar* one draws from such a bad distribution that the maximum of γ independent draws matches the original distribution. Under *maxvar* losses are reweighted upwards towards infinity in equation (8) while under *minvar* gains are similarly reweighted down to zero. To capture both properties the distortion *minmaxvar* was introduced. Under *minmaxvar* for the parameter γ we have

$$\Psi^{\gamma}(u) = 1 - \left(1 - u^{\frac{1}{1+\gamma}}\right)^{1+\gamma}. \qquad (11)$$

The parameter γ is a stress level with the distortion getting more concave as γ is increased, the reweighting getting more severe and the distorted expectation falling further down. In principle one may model equilibrium excess supplies and net revenues with different concave distortions.

4.1 A Simple Toy Model

Consider a simple model for the uncertainties in the post price levels of demand and supply. For a demand curve $D(p_U)$ with uncertainty Z_D the demand is

$$D = D(p_U) + Z_D$$

Similarly for a supply curve $S(p_L)$ and uncertainty Z_S the supply is

$$S = S(p_L) + Z_S$$

The excess supply is then

$$X = S(p_L) - D(p_U) + Z_S - Z_D$$

In a simple model we may take the uncertainties Z_D, Z_S to be normally distributed with zero means variances σ_D^2, σ_S^2 and correlation ρ, ignoring the issues of demand and supply possibly getting negative. The next section takes up log normal models for which the solutions are numerically sought.

The distorted expectation of excess supply is

$$\mathcal{E}\left(X\right) = S(p_L) - D(p_U) + \mathcal{E}\left(Z_S - Z_D\right)$$

For any concave distortion $\Psi(u)$ we have with σ_{XS} the standard deviation of excess supply

$$\begin{aligned}
\mathcal{E}\left(Z_S - Z_D\right) &= \int_{-\infty}^{\infty} x d\Psi\left(N\left(\frac{x}{\sigma_{XS}}\right)\right) \\
&= -\int_{-\infty}^{0} \Psi\left(N\left(\frac{x}{\sigma_{XS}}\right)\right) dx + \int_{0}^{\infty}\left(1 - \Psi\left(N\left(\frac{x}{\sigma_{XS}}\right)\right)\right) dx \\
&= \sigma_{XS}\left[-\int_{-\infty}^{0} \Psi\left(N\left(z\right)\right) dz + \int_{0}^{\infty}\left(1 - \Psi\left(N\left(z\right)\right)\right) dz\right] \\
&= -\theta\sigma_{XS}
\end{aligned}$$

where θ is the negative of the distorted expectation of a standard normal variate. The expectation is zero and the concave distorted expectation is then negative. Equating the distorted expectation of excess supply to zero yields the equation

$$S(p_L) - D(p_U) = \theta\sigma_{XS}. \tag{12}$$

Risk acceptability in excess supply enforces a positive expected excess supply and argues against market clearing on an expected basis. Two price equilibria are organized for positive excess supplies and the organization of positive levels of expected unemployment in labor markets as will be observed later when we take up general equilibria in labor markets.

With positive excess supply and any proximity between the upper and lower prices will generate a negative expected net revenue. The equation setting distorted expectations of net revenue to zero requires expected net revenues to be positive and hence an upper price above the lower price. The net revenue is given by

$$R = p_U D(p_U) + p_U Z_D - p_L S(p_L) - p_L Z_S$$

The distorted expectation of net revenue

$$\mathcal{E}\left(R\right) = p_U D(p_U) - p_L S(p_L) + \mathcal{E}\left(p_U Z_D - p_L Z_S\right).$$

Equating distorted expectations of net revenue to zero yields the equation

$$p_U D(p_U) - p_L S(p_L) = \widetilde{\theta}\sqrt{p_U^2\sigma_D^2 + p_L^2\sigma_S^2 - 2\rho p_U p_L \sigma_D \sigma_S} \tag{13}$$

where $\widetilde{\theta}$ is the distorted expectation of a standard normal variate under a possibly different distortion reflecting the acceptability of net revenue risks.

The equilibrium of this toy model is given by simultaneously solving equations (12) and (13). Differentiating equation (12) we observe that

$$S'(p_L)dp_L - D'(p_U)dp_U = 0$$

or that

$$\frac{dp_U}{dp_L} = \frac{S'(p_L)}{D'(p_U)} < 0$$

and the curve for equilibrium in excess supply has a decreasing relationship between the upper and lower prices.

Differentiating the equation (13) yields

$$\begin{aligned}
& \left[D(p_U) + p_U D'(p_U) - \frac{\widetilde{\theta}\left(\sigma_D^2 p_U - \rho\sigma_D\sigma_S p_L\right)}{\sqrt{p_U^2\sigma_D^2 + p_L^2\sigma_S^2 - 2\rho p_U p_L \sigma_D \sigma_S}}\right] dp_U \\
= & \left[S(p_L) + p_L S'(p_L) + \frac{\widetilde{\theta}\left(\sigma_S^2 p_L - \rho\sigma_D\sigma_S p_U\right)}{\sqrt{p_U^2\sigma_D^2 + p_L^2\sigma_S^2 - 2\rho p_U p_L \sigma_D \sigma_S}}\right] dp_L \qquad (14)
\end{aligned}$$

With positive marginal revenues and costs dominating the risk components we have an increasing relationship between p_U and p_L implied by the condition for zero distorted net revenue expectations. The two price equilibrium is at the intersection of these two curves defining the equilbra for excess supply and net revenue respectively. We may rewrite equation (10) as

$$\begin{aligned}
& D(p_U)\left[1 - \varepsilon_D - \frac{\widetilde{\theta}\left(\sigma_D^2 p_U - \rho\sigma_D\sigma_S p_L\right)}{D(p_U)\sqrt{p_U^2\sigma_D^2 + p_L^2\sigma_S^2 - 2\rho p_U p_L \sigma_D \sigma_S}}\right] dp_U \\
= & \; S(p_L)\left[1 + \varepsilon_S + \frac{\widetilde{\theta}\left(\sigma_S^2 p_L - \rho\sigma_D\sigma_S p_U\right)}{S(p_L)\sqrt{p_U^2\sigma_D^2 + p_L^2\sigma_S^2 - 2\rho p_U p_L \sigma_D \sigma_S}}\right] dp_L
\end{aligned}$$

where $\varepsilon_D, \varepsilon_S$ are the demand and supply elasticities. For demand elasticities below unity the slope is expected to be above unity barring large risk component terms.

Proposition 2 *A solution to the two price equilibrium for the simple toy model is given by solving for p_U in the equation*

$$\begin{aligned}
\frac{p_U D(p_U) - p_L Y}{\sqrt{p_U^2\sigma_D^2 + p_L^2\sigma_S^2 - 2\rho p_U p_L \sigma_D \sigma_S}} &= \widetilde{\theta} \qquad (15) \\
Y &= D(p_U) + \theta\sigma_{XS} \\
p_L &= S^{-1}(Y).
\end{aligned}$$

Proof. The excess supply of model is $\theta\sigma_{XS}$ and hence the supply for any choice of p_U is $Y = D(p_U) + \theta\sigma_{XS}$ which implies that $p_L = S^{-1}(Y)$. For a two price equilibrium the expected net revenue should satisfy

$$p_U D(p_U) - p_L Y = \widetilde{\theta}\sqrt{p_U^2\sigma_D^2 + p_L^2\sigma_S^2 - 2\rho p_U p_L \sigma_D \sigma_S},$$

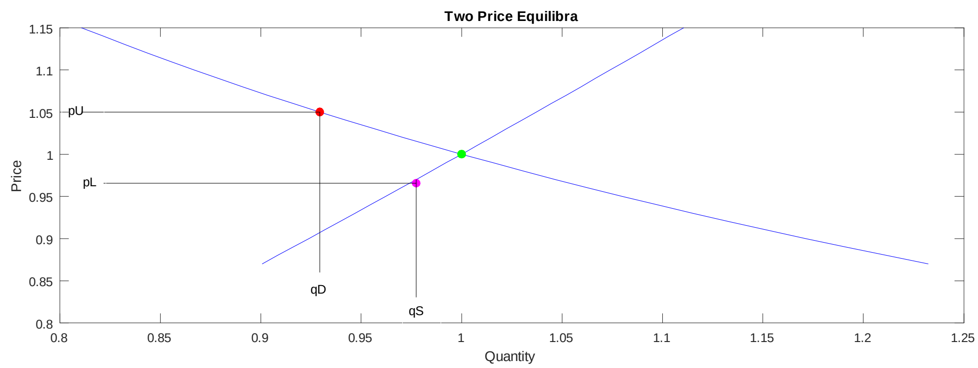

Figure 1: Figure displaying a two price equilibrium for Normally distributed demand and supply uncertainties.

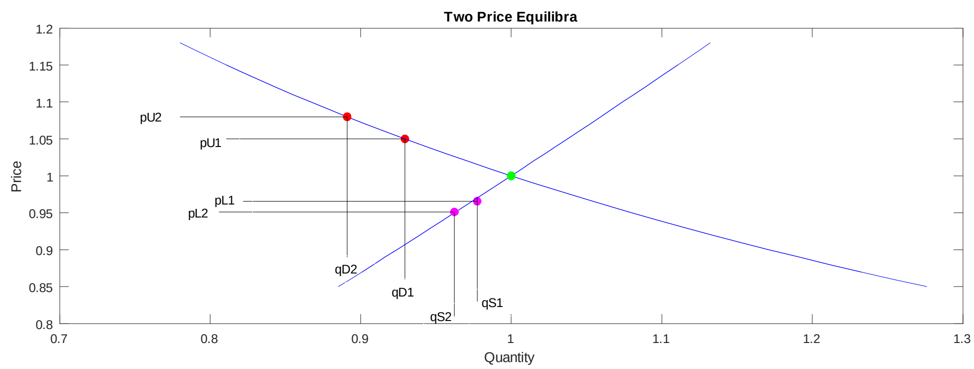

Figure 2: Overlaying a two price equilibrium with greater demand and supply uncertainties.

and the result follows. ■

With constant elasticity demand and supply curves with demand equal to supply equal to unity when the law of one price prevails at a price level of unity the resulting equation in p_U for zero correlation is

$$\frac{p_U^{1-\varepsilon_D} - \left(p_U^{-\varepsilon_D} + \theta\sigma_{XS}\right)^{\frac{1}{\varepsilon_S}} \left(p_U^{-\varepsilon_D} + \theta\sigma_{XS}\right)}{\sqrt{p_U^2\sigma_D^2 + \left(p_U^{-\varepsilon_D} + \theta\sigma_{XS}\right)^{\frac{2}{\varepsilon_S}} \sigma_S^2}} = \widetilde{\theta}.$$

For a demand elasticity of 1.5 and a supply elasticity of 0.75 with $\theta, \widetilde{\theta}$ at $0.2, 0.2$ and σ_D, σ_S at $0.2, 0.1$ Figure 1 displays the two price equilibrium with p_U, p_L at $1.05, 0.97$ and qD, qS at $0.93, 0.98$.

Figure 2 overlays and additional two price equilibrium with higher uncertainties in demand and supply of σ_D, σ_S at $0.3, 0.2$.

Figure 3 illustrates a two price equilibrium where pL and qS exceed the one

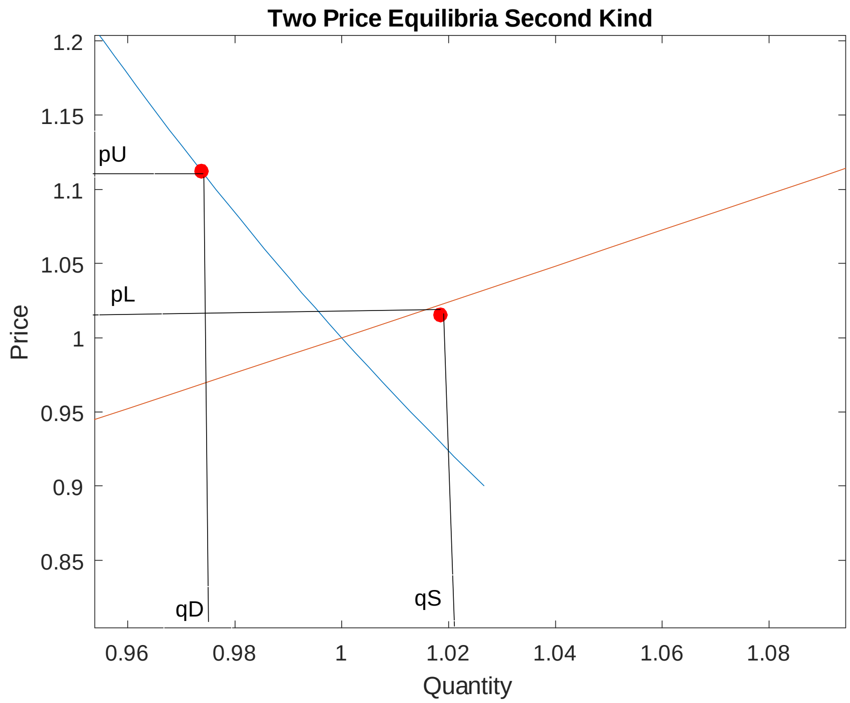

Figure 3: A Two Price Equilibrium where pL and qS exceed the one price levels.

price levels for $\varepsilon_D, \varepsilon_S$ equal to $0.25, 1.2$ respectively.

Involuntary unemployment is traditionally seen to be the result of wages being set above equilibrium levels for a variety of reasons with supply then exceeding demand. In a one price equilibrium one is left with the task of providing rationales for wages above clearing levels. Some reasons relate to wage efficiency or keeping empoyees both productive and happy. Others introduce the effects of Trade Unions, or contract theory considerations whereby labor is reluctant to offer work at lower wages and thereby take work away from colleagues.

The two price model offers a substantially different explanation. With post price setting randomness, in both demands and supplies, the acceptability of this randomness to the clearing agent requires both positive expected supply and positive expected net revenues. These two considerations work towards lifting prices for buyers over what is offered to sellers cutting demand below supply in expectation and raising revenues above costs in expectation. Unemployment is then a natural state as there is always planned excess supply. The magnitude will rise with increases in the uncertainties involved or the stress levels implicit in attitudes towards risk taking in clearing quantities and net revenues. From this perspective, it is full employment that is the anomally to be explained.

A Covid type shock with massive associated uncertainties in all entities will lead to two price equilibria with substantial levels of expected unemployment, matched by income support for unemployed workers and wider spreads for the ratios of employed income to that of the unemployed. Exact market clearing becomes an unattainable myth not relevant to a general financial economic equilibrium where the two price auctioneer must manage non-clearing risks.

In this author's view economic modeling in the absence of risk considerations was an important first step. In particular, it divorced finance from economics. But it is probably time now to recognize the implausibility for economics to exist theoretically without finance. Economics may be in need of a financial reformulation of its core idea, equilibrium. Societies faced with risk exposure have to define its acceptability and live accordingly. Hopefully individual entities can shed their risks to a macro economic risk aggregator and their plans can proceed as absolved from risk considerations. The two price auctioneer manages the aggregated risk in the economy. These considerations motivate the modeling presented in the paper.

4.2 An Explicit GFEE Solution

Consider constant elasticity demand and supply functions with elasticities β, η perturbed by log normal random shocks of volatility σ_D, σ_S as functions of p_U and p_L respectively. Specifically we write

$$q_D(p_U) = A_D p_U^{-\beta} \exp\left(\sigma_D Z_D - \frac{\sigma_D^2}{2}\right) \tag{16}$$

$$q_S(p_L) = A_S p_L^{\eta} \exp\left(\sigma_S Z_S - \frac{\sigma_S^2}{2}\right), \tag{17}$$

with Z_D, Z_S bivariate normal with correlation ρ. Other uncertainty models could also be entertained. One may model the uncertainty by the exponential of one of many infinitely divisible random variables like the variance gamma model of Madan and Seneta (1990), or the normal inverse Gaussian model Barndorff-Nielsen (1998) for other examples. Empirically adequate models could be selected in applying the two price equilibrium theory to data. For an application in this direction we reference Elliott, Madan and Siu (2021). One may solve for the equilibrium two prices as functions of the parameters $A_D, A_S, \beta, \eta, \sigma_D, \sigma_S$, and ρ.

Excess supplies and net revenues are thus given by

$$\begin{aligned} Z &= A_S p_L^{\eta} \exp\left(\sigma_S Z_S - \frac{\sigma_S^2}{2}\right) - A_D p_U^{-\beta} \exp\left(\sigma_D Z_D - \frac{\sigma_D^2}{2}\right) \\ R &= A_D p_U^{1-\beta} \exp\left(\sigma_D Z_D - \frac{\sigma_D^2}{2}\right) - A_S p_L^{1+\eta} \exp\left(\sigma_S Z_S - \frac{\sigma_S^2}{2}\right). \end{aligned}$$

If the uncertainty is ignored the one price equilibrium is obtained as the solution to

$$A_D p^{-\beta} = A_S p^{\eta}$$

or

$$p = \left(\frac{A_D}{A_S}\right)^{\frac{1}{\beta+\eta}}$$

With $A_D = A_S$ the equilbrium price is unity. Assuming $A_D = A_S = 1$, yields

$$\begin{aligned} Z &= p_L^{\eta} \exp\left(\sigma_S Z_S - \frac{\sigma_S^2}{2}\right) - p_U^{-\beta} \exp\left(\sigma_D Z_D - \frac{\sigma_D^2}{2}\right) \\ R &= p_U^{1-\beta} \exp\left(\sigma_D Z_D - \frac{\sigma_D^2}{2}\right) - p_L^{1+\eta} \exp\left(\sigma_S Z_S - \frac{\sigma_S^2}{2}\right) \end{aligned}$$

Evaluating distorted expectations of Z, R at stress levels γ_1, γ_2 deliver the functions

$$\begin{aligned} \mathcal{E}_1(Z) &= \Lambda(p_U, p_L) \\ \mathcal{E}_2(R) &= \Theta(p_U, p_L). \end{aligned}$$

The solution of equating to zero the distorted expectations is then a function of $\beta, \eta, \sigma_D, \sigma_S, \rho$ the elasticities, volatilities, correlation and the distortion parameters γ_1, γ_2.

Consider the base case to have parameters

$$\begin{aligned} \beta &= 2;\ \eta = .5 \\ \sigma_D &= .3;\ \sigma_S = .3 \\ \rho &= .5 \\ \gamma_1 &= .25;\ \gamma_2 = .75. \end{aligned}$$

The solution is given by $p_L = 0.7283$, and $p_U = 1.1513$.

For the sensitivity of the solution to the input parameters we refer the reader to the Appendix.

5 A K good full employment GFEE

Apart from one good partial equilibrium modeling, economic theory has provided us with, the widely studied and applied, general equilibrium model. We now consider such models reformulated as a GFEE. Consider now, to begin with, a two price GFEE generalization of a Walrasian general equilibrium for a K good production economy with full employment. Here we take a two price approach to the markets for goods with the labor market clearing under the law of one price. A later section will relax this assumption and adopt a two price framework in the labor market as well.

Critical to the formulation of a general financial two price economic equilibrium is the specification of the uncertainties faced in the economy after the equilibrium prices have been set. Here we introduce uncertainties in both production and utility functions to generate both uncertain outputs and demands. However individual agents act to maximize expected profits and they take as income the expected earnings that are spent across goods using random utility functions known to consumers but not to the price setting auctioneer. The two price equilibrium is numerically solved by simultaneously attempting to set all distorted expectations for excess supply and net revenue to zero in all markets.

Suppose each of the K goods is produced employing labor in a production technology given by a production function, $f_k(L_k)$ that describes the expected output from employing L_k units of labor. Consider a full employment equilibrium in which the aggregate labor $\overline{L}$ is employed. The actual output of good k, Y_k is however a random outcome subject to a positive unit expectation multiplicative shock M_k. The actual output is then given by

$$Y_k = f_k(L_k)M_k.$$

The output will be supplied to the market at prices offered by the market of p_{L_k} for good k. The suppliers determine employment levels by maximizing expected profits that for a wage rate w are given by

$$\sum_k p_{Lk} f_k(L_k) - wL_k.$$

The supplier's income or actual realized profit is

$$\sum_k p_{Lk} Y_k - wL_k$$

and the entire output can be sold to market at the market offered prices p_{Lk}.There are also m consumers with labor endowments $\overline{L}_j$ and shares in the profits of σ_{jk}. The labor market must clear with full employment and the wage rate is determined to ensure that

$$\sum_k L_k = \sum_j \overline{L}_j = \overline{L}.$$

The actual income of consumer j is then

$$V_j = w\overline{L}_j + \sum_k \sigma_{jk} \left(\sum_k p_{Lk} Y_k - wL_k \right)$$

and reflects the randomness in all the production operations.

The expected income is however,

$$\overline{V}_j = w\overline{L}_j + \sum_k \sigma_{jk} \left(p_{Lk} f_k(L_k) - wL_k \right).$$

The uncertainties associated with output fluctuations are not the concern of the consumer and we take these risks or fluctuations to be financed with the consumer incomes being set at the expected profits. The only critical uncertainties being considered are those associated with market clearing. All the agents of the economy be they producers or consumers determine demands and supplies as deterministic functions of the prices they face. The consumers base their demands on their expected incomes and the prices for the commodities offered by the market.

The demand by consumer j for product k is given by the demand function

$$X_{jk} = D_{jk}\left(p_{U,1}, p_{U,2}, \cdots, p_{U,K}, \overline{V}_j\right).$$

The demand functions are stochastic reflecting random influences on preferences.

The demand for product k is

$$\sum_j X_{jk}$$

while the supply is

$$f_k(L_k)M_k.$$

The excess supply is

$$Z_k = f_k(L_k)M_k - \sum_j X_{jk}$$

The net revenue to the market on account of product k is

$$R_k = \sum_j p_{Uk}X_{jk} - p_{Lk}f_k(L_k)M_k.$$

Given the randomness built into supplies and demands we wish to set p_{Uk}, p_{Lk}, w to clear the labor market and have the distorted expectations of excess supply and net revenue to zero.

5.1 A Particular Equilibrium with CES demand functions

For the generation of particular demands for the K goods consider individuals with constant elasticity of substitution (CES) utility functions. The CES utility function is given by

$$u(x_1, \cdots, x_n) = \left(\sum_k a_k x_k^{\rho}\right)^{1/\rho} \tag{18}$$

Let the elasticity of substitution be σ with $r = 1 - \sigma$ and

$$r = \frac{\rho}{\rho - 1},$$

Proposition 3 *The demand x_k at the prices p_j for good j with income y is given by*

$$x_k = \frac{a_k^{1-r} p_k^{r-1} y}{\sum_j a_j^{1-r} p_j^r}. \tag{19}$$

Proof. See Appendix. ■

To induce stochastic or random demands we take, as an example, the logarithm of the a_k's to be distributed as multivariate normal, and r, the elasticity of substitution to be gamma distributed and independent of the a_k's.

For specific production functions we take

$$Y_k = L_k^{\alpha_k} \exp\left(\sigma_k Z_k - \frac{\sigma_k^2}{2}\right). \tag{20}$$

For lower prices p_{Lk} the expected profit maximizing employment is

$$L_k = \left(\frac{\alpha_k p_{Lk}}{w}\right)^{\frac{1}{1-\alpha_k}} \tag{21}$$

The full employment equation for the wage rate is

$$\sum_k \left(\frac{\alpha_k p_{Lk}}{w}\right)^{\frac{1}{1-\alpha_k}} = \overline{L}. \tag{22}$$

This determines employment and the random supplies. The random profit incomes are

$$\Pi_k = p_{Lk}\left(L_k^{\alpha_k} \exp\left(\sigma_k Z_k - \frac{\sigma_k^2}{2}\right)\right) - wL_k.$$

The aggregate income is

$$Y = w\overline{L} + \sum_k \Pi_k.$$

The expected profits, however, are by commodity

$$\overline{\Pi}_k = p_{Lk} L_k^{\alpha_k} - wL_k.$$

The expected income of individual j is, then

$$\overline{V}_j = w\overline{L}_j + \sum_k \sigma_{jk}\overline{\Pi}_k.$$

The demands for the products by the individuals are then given by

$$X_{jk}\left(p_{U1}, \cdots p_{UK}, \overline{V}_j\right) = \frac{a_{jk}^{1-r_j} p_{Uk}^{r_j-1} \overline{V}_j}{\sum_i a_{jk}^{1-r_j} p_{Uk}^{r_j}} \tag{23}$$

conditional on $r_j, a_{j1}, \cdots, a_{jK}$.

The excess supplies are

$$Z_k = L_k^{\alpha_k} \exp\left(\sigma_k Z_k - \frac{\sigma_k^2}{2}\right) - \sum_j X_{jk}.$$

The sector net revenues are

$$R_k = p_{Uk} \sum_j X_{jk} - p_{Lk} L_k^{\alpha_k} \exp\left(\sigma_k Z_k - \frac{\sigma_k^2}{2}\right).$$

A two price equilibrium in (p_{Lk}, p_{Uk}), $k = 1, \cdots, K$ is one for which

$$\begin{aligned} \mathcal{E}(Z_k) &\geq 0 \\ \mathcal{E}(R_k) &\geq 0. \end{aligned}$$

Setting the distorted expectations equal to zero deliver the $2K$ general equilibrium equations in the $2K$ two price unknowns to be solved for the $GFEE$.

The wage rate may be set to unity as the numeraire with prices reported in the labor numeraire as p_{Lk}/w and p_{Uk}/w. The same goes for profits and the value of output in labor units.

Under this construction the expected value of total demand is the expected value of total income, wage plus profit at evaluated prices p_L, no matter what the prices p_U. Hence in all states expected total revenue equals expected total income. There may be some differences by product but aggregated across products $\sum_k R_k$ has zero expectation at best. The distorted expectation is then always negative. Making it positive is then not a possibility. The market objective is then to keep it from getting too negative. An approximate two price equilibrium is defined by the two price system that minimizes the absolute distorted expectations or their deviation from zero. A numerical analysis of a two and five good model is presented in the Appendix. For a number of products that is small relative to the number of individuals the aggregate discrepancy forced into existence by some distorted net revenue expectations being negative may be accommodated by raising the amount by taxing a large number of individuals an insignificant amount that essentially leaves the demand functions infinitessimally reduced and ignored by the equilibrium solution.

6 Two Price Labor Markets

So far the examples of general equilibrium models we have considered introduced uncertainty into the demand and supply of the commodities produced and consumed. The two price GFEE structure was employed in the product markets to cope with this uncertainty, keeping price systems independent of states driving randomness. The system of two prices was formulated to manage the acceptability of inventory and revenue exposures faced by the market as the risk absorbing agent. It is interesting to enquire into general equilibria with demand and supply uncertainty in the labor market as well, with wages being kept independent of these events as well. We would then have two price labor markets and anticipate positive expected unemployment as an equilibrium outcome. The result stands in sharp contrast to Keynesian involuntary unemployment where one has to construct explanations for why wages are sitting above equilibrium levels.

In principle, such two price labor markets already exist in actual economies. The payroll tax in the US economy essentially differentiates the wage received by workers from what is paid by employers. To the extent the labor market does not clear and there is an excess supply and furthermore the wage received by those looking for work but not employed is reflected in unemployment insurance payments. In a formal two price labor market all suppliers of labor will receive the wage w_L from the market. Those that work will see the employer paying the market the wage w_U. The wage w_L is then income support for all workers, employed or not. For modern economies with fast developing technologies automating many productive tasks, a universal system of income support may be the order of the day. The two price labor market GFEE equilibrium theory delivers such a structure as its equilibrium outcome. The income support also serves as support for aggregate demand. The labor market here will be taken to be homogeneous. In a later section we consider skill differentiation and its consequences for two price GFEE equilibria.

We consider then the introduction of randomized demands and a two price framework for the labor market as well. Here we take the supply of labor to be a unit endowment being sold to market at the price w_L. In a later section we consider random levels of labor supply as well with skill differentiation. The market then sells a quantity L_D at the price w_U to the production units. The market has a net revenue in the labor market of

$$R_L = w_U L_D - w_L L_S. \tag{24}$$

The excess supply in the labor market is

$$Z_L = L_S - L_D. \tag{25}$$

Suppose the production function for product k is

$$Y_k = A_k L_k^{\alpha_k} \exp\left(\sigma_k Z_k - \sigma_k^2/2\right)$$

and employment is determined by maximizing the expected profit given A_k. It follows that

$$L_k = \left(\frac{\alpha_k A_k}{w_U}\right)^{\frac{1}{1-\alpha_k}}. \tag{26}$$

Suppose now that A_k is random and

$$A_k = \exp\left(\beta_k \widetilde{Z}_k - \frac{\beta_k^2}{2}\right). \tag{27}$$

The demand for labor is random at the wage w_U, and the total demand is

$$L_D = \sum_k L_k$$

with the market obtaining the revenue $w_U L_D$. However the supply is $\overline{L}$ and it is purchased by the market for $w_L \overline{L}$. In the labor market risk acceptability is ensured by requiring

$$\begin{aligned} \mathcal{E}\left(\overline{L} - L_D\right) &\geq 0 && (28) \\ \mathcal{E}\left(w_U L_D - w_L \overline{L}\right) &\geq 0, && (29) \end{aligned}$$

for appropriate distorted expectations.

6.1 Solution of a particular case

The economy has five goods with production functions coefficients α_k given by the vector

$$\alpha = (0.8,\ 0.7,\ 0.6,\ 0.7,\ 0.8). \tag{30}$$

The volatilities σ_k are given by the vector

$$\sigma = (0.2,\ 0.18,\ 0.16,\ 0.18,\ 0.25). \tag{31}$$

The scale uncertainties β_k are given by the vector

$$\beta = (0.02,\ 0.01,\ 0.005,\ 0.01,\ 0.02). \tag{32}$$

The elasticity of substitution was gamma distributed with mean 0.2 and variance 0.1. The mean vector for levels of a_k are given by the vector

$$a = (5,\ 4,\ 3,\ 2,\ 1). \tag{33}$$

The corresponding volatilities ζ_k are

$$\zeta = (0.5,\ 0.4,\ 0.3,\ 0.2,\ 0.1). \tag{34}$$

The correlations were 0.75 for each pair uniformly. The stress levels were 0.25 for the excess supplies and 0.5 for the net revenues in all markets.

The equilibrium wage levels were

$$w_L = 2.0562 \tag{35}$$
$$w_U = 2.1799. \tag{36}$$

The price levels for the five goods were

$$p_L = (2.3925,\ 2.0100,\ 1.4364,\ 1.3571,\ 1.4793) \tag{37}$$
$$p_U = (2.9559,\ 2.1499,\ 1.4364,\ 1.3571,\ 1.4793). \tag{38}$$

The expected outputs for the five goods were

$$\overline{Y} = (0.5979,\ 0.3605,\ 0.2489,\ 0.1442,\ 0.0876). \tag{39}$$

The expected employments in the production of the five goods was

$$\overline{L} = (0.5239,\ 0.2324,\ 0.0983,\ 0.0628,\ 0.0474). \tag{40}$$

The equilibrium unemployment rate for the economy was 3.53%. The average demand levels for the five goods were

$$\overline{D} = (0.5372,\ 0.3256,\ 0.2084,\ 0.1014,\ 0.0445). \tag{41}$$

The average net revenues in the five markets for goods were

$$\overline{R} = (0.1577,\ -0.0246,\ -0.0582,\ -.0581,\ -0.0637). \tag{42}$$

The average net revenue in the labor market was 0.0468. The average net revenue across all markets was zero.

If the wage of the employed is taken as the numeraire then the unemployed receive 94.32% of the wage and this is the equilibrium level of unemployment insurance in the equilibrium. The level of unemployment support in the economy may be defined as the income ratio of the unemployed to the employed. Here it is

$$\begin{aligned} UNSL &= \frac{.0353 * 2.0562}{(1 - .0353) * 2.1799} \\ &= 3.45\%. \end{aligned}$$

7 Two Price Equilibrium Applications

A number of applications addressing specific issues are taken up in the subsections of this section.

7.1 Productivity and Employment

The critical parameters of an economy from the perspective of aggregate employment are the productivity of its production sectors. The influence of volatilities and utilities is secondary affecting how matters get distributed across sectors. When the production functions are highly concave the initial labor inputs are highly productive with the productivity falling as employment expends. It may be anticipated that for production coefficients α_k being lower the equilibrium could display higher levels of unemployment. For less productive economies employment would be higher and unemployment rates lower. For productive economies to raise employment levels it then becomes imperative to increase the number of products in the economy.

For an economy with 4, 5, and 6 goods we raised production the coefficients α_k that were uniform across products from the low level of 0.25 to 0.75 in steps of 0.05. The production volatilities were 20% for all products. The other parameters were also uniform across products with β, μ, ζ at .02, 1, and 0.25. The mean levels of the elasticity of substitition were 0.2 with a variance of 0.1. Correlations were 0.75 for all pairs. The stress levels were 0.25 and 0.5 for supplies and net revenues across markets.

Figure 4 presents the curves for equilibrium unemployment as a function of the uniform production coefficient for economies with 4, 5, and 6 goods.

The unemployment rate steadily declines as the economy gets less productive. For productive economies unemployment rates may be lowered by expanding the number of products in the economy. Technological progress accompanied with an expansion in the production activities of the economy is then an adequate way to economically absorb technology. Recent experience with the advances of the digital economy appear to be an innovation in this direction. When the elasticity of substitution is increased to 2.0 for example, this has the effect of fewer goods and raises the unemployment rates. Figure 5 presents

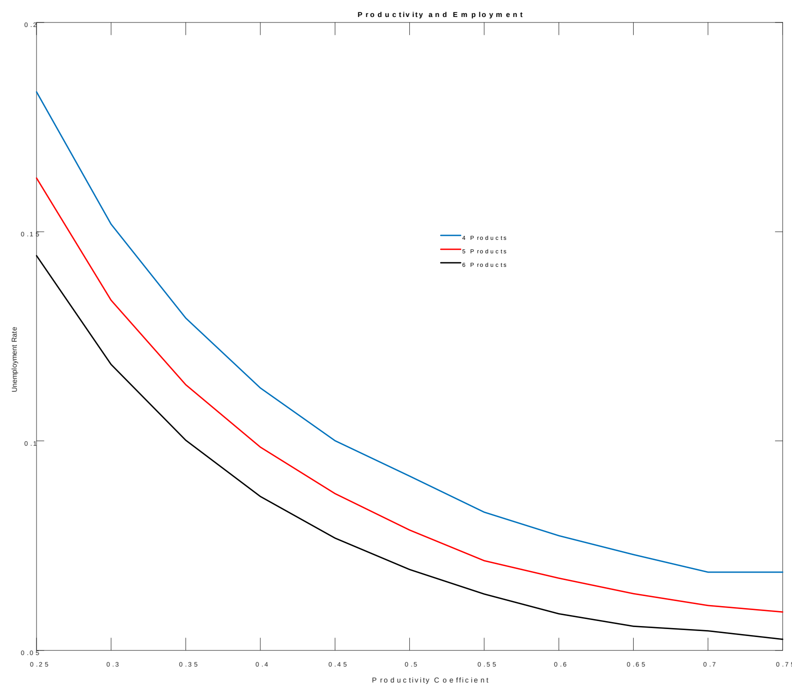

Figure 4: Equilibrium unemployment rates for economies with 4, 5, and 6 uniform products as a function of production coefficients.

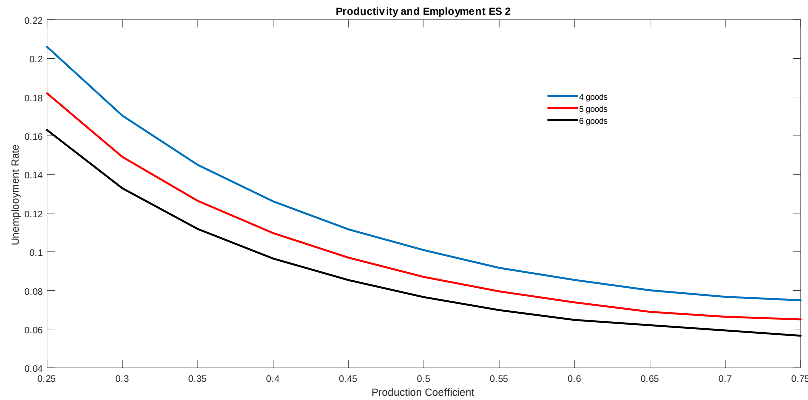

Figure 5: Equilibrium unemployment rates as functions of production coefficients for economies with 4, 5 and 6 products with an elasticity of substitution of 2.0.

the unemployment rates as functions of uniform production coefficients for this higher elasticity of substitution and we observe the increase in the equilibrium unemployment rates.

7.2 Employment and Profits across Economic Sectors

Consider three sectors of an economy. The first is highly capitalized like agriculture for example with a production function coefficient of say 0.25. Less mechanized could be manufacturing with a production coefficient of .5. Finally we take the sevrices sector with a production coefficient of 0.75. The capitalized agricultural sector we take to have the most volatility at 30% with volatilities of 20% and 10% for manufacturing and services. In other respect the sectors are comparable. The elasticity of substitution was taken at 0.2, and the correlations in preference were 0.25 for all pairs.

The equilibrium prices in units of labor for the three sectors were 0.8678, 1.0735, and 1.0957 for the lower prices. The upper prices were 1.1576, 1.2691, and 1.1890. The average employment levels were 0.1304, 0.2882, and 0.4576. The agriculture sector had the lowest employment followed by manufacturing and services. The equilibrium unemployment rate 12.38%. The price markups in the three sectors were 33.39, 18.22, and 8.51 percent respectively. The expected profits in the three sectors were 0.3906, 0.2890 and 0.1506. For rates of return one would need information on capital levels that are exogeneous to the model for now. The average output levels were 0.6003, 0.5377, and 0.5578. The average demand levels were 0.5253, 0.4855, and 0.5122 with comparable standard deviations of 0.0755, 0.0683 and 0.0728.

For the same three good economy, introduce two persons, one with the labor endowment, the other in ownership of all the profits. The first has a preference for agricultural products reflected in the choice of mean values for a_k of 3, 2, and 1 respectively. The second has a preference for services with mean value for the coefficients a_k in the utility function of 1, 2, and 3. The two person three good equilibriun is as follows.

The equilibrium lower prices in units of labor for the three sectors were 0.9828, 0.9837, and 1.1132. The first and third sectors saw an increase while the second fell. The upper prices were 1.3082, 1.1190, and 1.1859. The first rising, the second falling while the third was constant. The average employment levels were 0.1539, 0.2420, and 0.4876 with an increase in the first a decrease in the second and a slight increase in the third. The equilibrium unemployment rate fell to 11.65%. The price markups were 33.11, 13.75, and 6.52 percent. The first was unchanged while the second and third fell. The expected profits in the three sectors in labor units were 0.4611, 0.2427, and 0.1636. The labor income was unity and the profit income was 0.8674. The average demands for the three goods by the first person are 0.4660, 0.2386, and 0.1040. The demands by the second person on average are 0.0843, 0.2094, and 0.4408. They reflect the preferences. The aggregate demands are 0.5503, 0.4480, and 0.5448. The standard deviations dropped to 0.0496, 0.0542, and 0.0506. The average outputs are 0.6257, 0.4927, and 0.5850.The net revenues to the market in aggregate are zero.

7.3 Income Redistribution and Employment

The effects of rising income inequality on aggregate demand and employment have been noted by numerous authors. A recent example is provided by Carvalho and Rezai (2016). Furman and Stiglitz (1999) present a discussion of the issues from a number of perspectives. The analysis is often undertaken in the context of effects on savings rates and economic growth employing the hypothesis that at high incomes the propensity to consume may fall. Such a hypothesis is then questioned empirically.

Within the context of a static equilibrium model one cannot take up dynamic issues of savings and growth. In the models of this paper the propensity to consume is 100% as all incomes are being spent on the goods being produced. However, the two price economy of the labor market as well as other markets delivers an equilibrium level of unemployment and this section reports on the interactions of such with policies of income redistribution. Even in a static model, with all incomes being spent, what matters is the employment effects of what the incomes are being spent on and not on whether they are spent or not. It may well be that lower incomes are spent on mass produced goods with high employment potentials while higher income preferences are for personalized products that generate lower employment opportunities. It is the interaction between production functions and income contingent consumer preferences that deliver the employment effects.

Shifting preferences with income levels necessitates the construction and use of some fairly complicated utility functions capable of displaying product demand functions that are nonlinear in income. With a view towards avoiding these compelxities we consider just a two person two product economy where one person gets all the profit income and the other the wage income. However, the person with the profit income has a preference for the first good while the one with the wage income has a preference for the second good. The first product however has a fast decline in the marginal productivity of labor while the second is closer to a constant marginal productivity of labor. Resdistribution is entertained via a tax on profit incomes that are transferred to the wage earner. The equilibrium is solved for a variety of tax rates and the effects of redistributive policies on the equilibrium economic solution are computed and displayed.

The two goods have production coefficients of 0.2 and 0.8 respectively with volatilities of 20 and 10 percent respectively. The β coefficients are .2 and .02 respectively. The aggregate labor supply is unity. There are two sets of preferences with CES utility functions that have $\mu_1 = (10, 1)$ while $\mu_2 = (1, 10)$ and $\zeta_1 = \zeta_2 = (.1, .1)$. The correlations in preference are 20% for both preferences. The elasticity of substitution is 0.2 with independent gamma distributions with shape and scale parameters 4, and .05 respectively. The stress levels for the two commodities are .25, the two product revenues have stress levels of 0.5. The labor market stress levels are $.2, .4$ for the respective quantity and the revenue levels. Profits were taxed at tax rates of zero to fifty percent in steps of five percent that were redistributed to the wage earners. The equilibrium was solved separately for each tax rate.

Figure 6 presents a graph of the equilibrium unemployment rate as a function of the tax rate. It may be observed that the unemployment rate falls steadily from over seven percent to below five and half percent in response to redistributive tax policies.

The two price markup in the labor market also declines with the tax rate and is displayed in Figure (7).

The corresponding product markups are shown in Figure (8). The first good has a declining markup as income moves to wage earners and the markup on their preferred good rises.

Figure (9) and (10) present the output and employment levels in the two goods.

7.4 Productivity Shocks and Equilibrium Unemployment Support Levels

The COVID-19 crisis may be viewed as a productivity shock. The fact that people have to keep distances between themselves is a natural mechanism for hindering or curtailing outputs in many activities. We may solve two price general equilibria for different productivity levels and evaluate the levels of unemployment support attained in equilibrium. The unemployment support level ($UNSL$) is measured by the ratio of income going to the unemployed in a two price equilibrium relative to the aggregate income of the employed. In equilibrium with the unemployment rate being $unemp$,

$$UNSL = \frac{w_L * unemp}{w_U(1 - unemp)}.$$

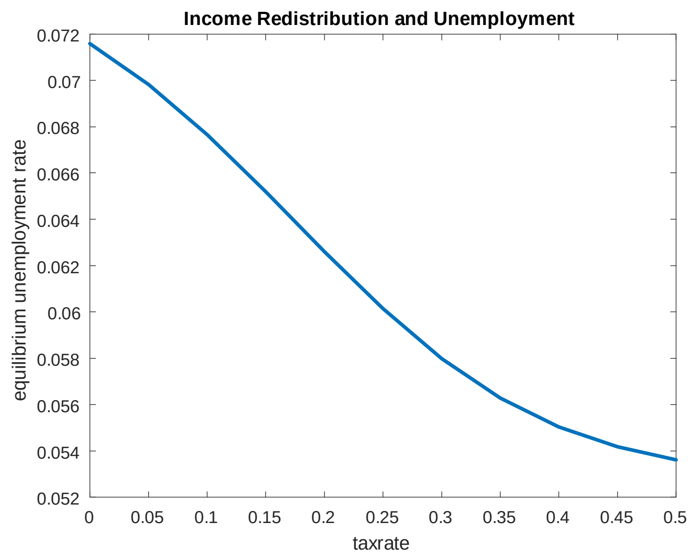

Figure 6: Equilibrium unemployment levels at various tax rates.

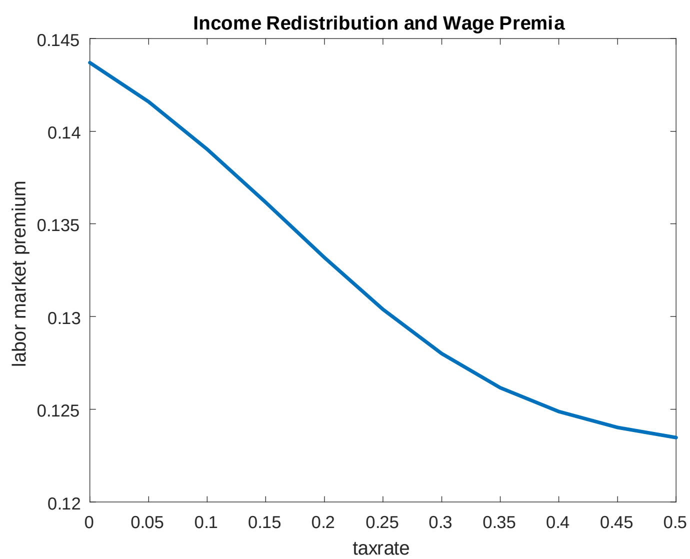

Figure 7: Wage markup to buyers of labor over the seller as function of the tax rate.

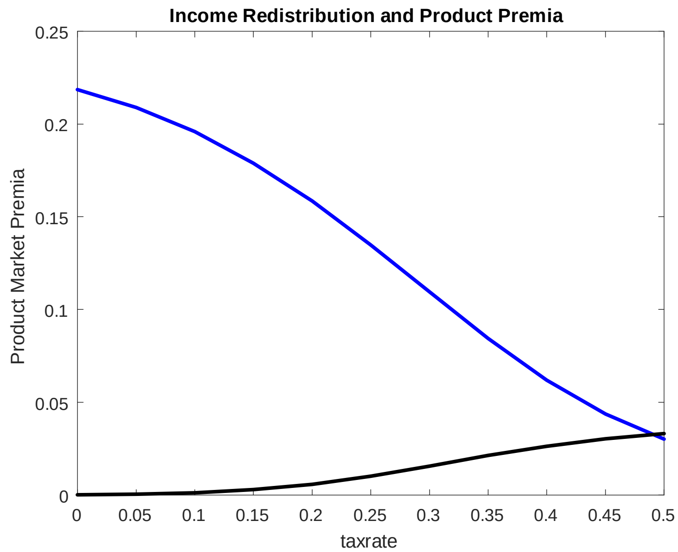

Figure 8: Product market markups for the two goods as a function of tax rates. The markup for the first good is shown in blue. The second good is displayed in black.

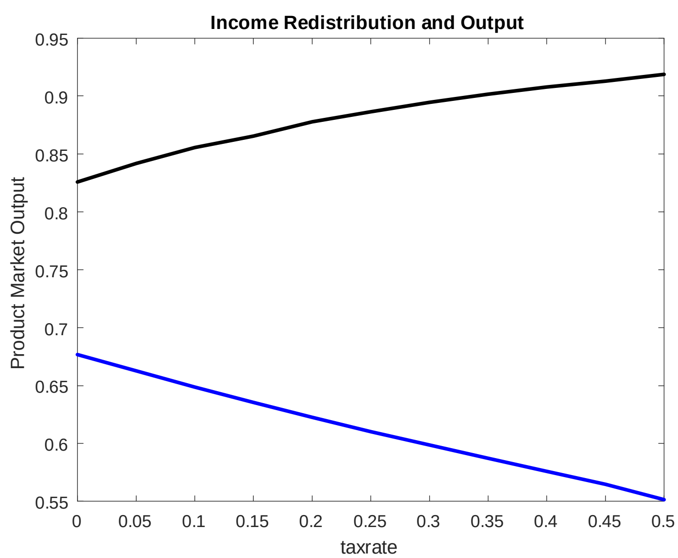

Figure 9: First good in blue and second in black.

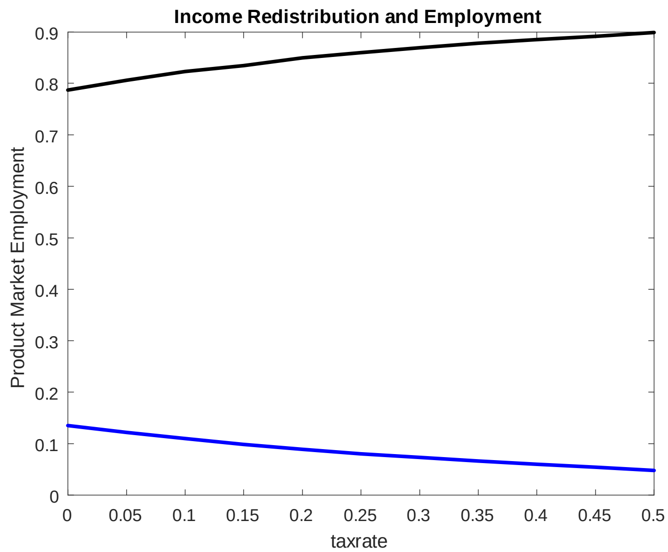

Figure 10: First good in blue and second in black.

The base case reported in Section 6.1 was rerun with prices in labor units or for $w_L = 1$. The value of $UNSL$ in this solution was 3.79%.

Two productivity shocks were introduced by reducing the coefficients α_k across the board by a factor of fifty percent and seventy five percent. For the two shocks that gap between the wage income of the unemployed and employed was 10.39% and 22.61%. These are the premiums w_U/w_L for fifty and seventy five percent productivity shocks. The unemployment rate are also adversely affected with unemployment rates rising from 3.96% to 7.68% and 17.45% for the two productivity shocks respectively. The unemployment support levels rise to 7.56% and 17.24% respectively.

The characteristic feature of a two price economy is the full support of all employable persons at the lower labor market price w_L with those actually at work receiving the premium of $w_U/w_L - 1$. If the employed wage is taken as the numeraire then the levels of unemployment insurance as fraction of the working wage are 92%, in the base case and fall to 90.87% and 81.56 percent in the two productivity shock cases.

The responses to a productivity shock like COVID are also not unique and there is not a single new equilibrium to be considered. If the stress levels for risk acceptability in the labor are increased then the new equilibrium will reflect this change in market risk attitudes. One may solve for the equilibria with labor market stress levels raised from .25, .5 for inventory and revenue to .5 and .75 respectively. When this is done for the first shock the unemployment insurance level falls further from 90.87 to 89.26. For the second shock the insurance level drops even further from 81.56 to 80.45. The unemployment rates also rise further with 7.68% rising to 8.74% and 17.45% for the second shock rising to 18.16%. The unemployment support levels move up from 7.56% to 8.54% and 17.24% rising to 17.85%.

Importantly, one may try to consider the case where unemployment insurance is dropped to zero with no income support for the unemployed. This may be done in the current context by considering very high levels of stress in the labor market. For the second shock with the stress levels raised to 5 for both inventory and revenue, the insurance drops down to 72.53%, the unemployment rate rises to 23.32% and the unemployment support level is up at 22.06. It is probably not a good economic idea to try and drop the unemployment insurance levels down towards zero.

7.5 Combining Productivity Shocks with Income Inequality

Consider now a two good two person economy with profits going to individual one while the second gets all the labor income. The first has a preference for the first good with a low production coefficient while the second prefers the second good with a high production coefficient. We take these to be .25 and .75 respectively with no income redistributions in place. Both production volatilities are 20% and β is set at 0.02. The first person has a preference for first good with $\mu_1 = (.8, .2)$ while $\mu_2 = (.2, .8)$. The demand volatilities or ζ levels are 0.5. The elasticity of substitution is .2 on average and a volatility of 0.1. The correlations in preference are 0.25. Inventory stress levels are 0.25 and revenue stress levels are 0.5.

The level of unemployment insurance is 86.97 cents to the dollar. The unemployment rate is 8.82% and the unemployment support level is at 8.42%. Now subject this economy to a productivity shock reducing production coefficients by a factor of say 50%. The unemployment insurance level drops to 71.43 cents to the dollar. The unemployment rate rises to 27.21% and the unemployment support level rises to 26.71%.

Let us now couple this with a redistributive tax rate of 35%. The unemployment insurance level rises to 76.33 cents in the dollar. The unemployment rate falls to 22.17% and the level of unemployment support falls to 21.74%. There is a case to be made for hightened redistributive efforts in the presence of severe productivity shocks.

7.6 Differentiated Labor Markets

The models considered thus far have had no randomness in the supply of labor. In fact this has been taken to be a fixed constant. In this section we relax this property and allow for a random labor supply. Additionally, given the advent of artificial intelligence, machine learning and the related automation we consider two classes of labor, one more productive on account of skill acquisitions that is also in relatively shorter supply and the other less productive and in greater supply. This allows for the reporting of equilibrium unemployment insurance levels, unemployment rates and unemployment support levels separately for the differentiated labor groups. With the concentration being on labor differentiation, here we aggregate the product market to a one good economy. There is then one output, Y and two types of labor that we denote by L_1 and L_2.

Consider first the classical situation with no uncertainties or random components. Suppose there is a single aggregate utility reflecting preferences for the output and leisure for both labor types. By way of an example consider a Cobb-Douglas utility function with maximal labor supplies of A_1, A_2 for the two groups and utility function

$$u(Y, L_1, L_2) = Y^{\beta}(A_1 - L_1)^{\theta_1}(A_2 - L_2)^{\theta_2}.$$

Suppose further that both types of labor are needed to produce the output and the production fundtion is

$$Y = L_1^{\alpha_1} L_2^{\alpha_2}.$$

The ratio of the productivity for common levels of labor is

$$\pi = L^{\alpha_1 - \alpha_2}$$

and at common employment levels $L < 1$ the first is more productive than the second if $\alpha_1 < \alpha_2$.

Solving for the first best we maximize over choices L_1, L_2 the aggregate utility

$$u(L_1, L_2) = L_1^{\alpha_1\beta} L_2^{\alpha_2\beta}(A_1 - L_1)^{\theta_1}(A_2 - L_2)^{\theta_2}.$$

The solutions are

$$\begin{aligned} L_1 &= \frac{\alpha_1\beta A_1}{\alpha_1\beta + \theta_1} \\ L_2 &= \frac{\alpha_2\beta A_2}{\alpha_2\beta + \theta_2} \end{aligned}$$

For a unit output price the two wages at the value of their marginal products are

$$\begin{aligned} w_1 &= \alpha_1 \left(\frac{\alpha_1\beta A_1}{\alpha_1\beta + \theta_1}\right)^{\alpha_1 - 1} \left(\frac{\alpha_2\beta A_2}{\alpha_2\beta + \theta_2}\right)^{\alpha_2} \\ w_2 &= \alpha_2 \left(\frac{\alpha_1\beta A_1}{\alpha_1\beta + \theta_1}\right)^{\alpha_1} \left(\frac{\alpha_2\beta A_2}{\alpha_2\beta + \theta_2}\right)^{\alpha_2 - 1}. \end{aligned}$$

For a specific numerical example we take $A_1 = 1$ and $A_2 = 2$. The second being in greater supply. We also suppose the marginal utility of leisure is higher for the second group and take $\theta_1 = .25$ and $\theta_2 = .5$. For higher productivity of the first group we take $\alpha_1 = .2$ and $\alpha_2 = .7$. The sum of $\alpha_1 + \alpha_2 < 1$. We take output to be desirable with slowly falling marginal utility and $\beta = .75$;

7.6.1 The Two Price GFEE

We now introduce uncertainty in production with

$$Y = L_1^{\alpha_1} L_2^{\alpha_2} e^{\sigma z - \sigma^2/2}$$

Expected profits are maximized and are given by

$$p_L L_1^{\alpha_1} L_2^{\alpha_2} - w_{U1} L_1 - w_{U2} L_2$$

The solutions for labor demanded in the two groups are then

$$\begin{aligned} L_{D1} &= \left(\frac{p_L \alpha_2^{\alpha_2} \alpha_1^{1-\alpha_2}}{w_{U1}^{1-\alpha_2} w_{U2}^{\alpha_2}} \right)^{\frac{1}{1-\alpha_1-\alpha_2}} \\ L_{D2} &= \left(\frac{p_L \alpha_2^{1-\alpha_1} \alpha_1^{\alpha_1}}{w_{U1}^{\alpha_1} w_{U2}^{1-\alpha_1}} \right)^{\frac{1}{1-\alpha_1-\alpha_2}\prime} \end{aligned}$$

The resulting random supply of output is

$$Y_S = \left(\frac{p_L \alpha_2^{\alpha_2} \alpha_1^{1-\alpha_2}}{w_{U1}^{1-\alpha_2} w_{U2}^{\alpha_2}} \right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}} \left(\frac{p_L \alpha_2^{1-\alpha_1} \alpha_1^{\alpha_1}}{w_{U1}^{\alpha_1} w_{U2}^{1-\alpha_1}} \right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2}} e^{\sigma z - \sigma^2/2}$$

The demand for output and the supply of labor given preference we maximize subject to the budget constraint for which the expected profit is

$$\pi = p_L Y_S - w_{U1} L_{D1} - w_{U2} L_{D2}.$$

The utility maximization problem yields the Lagrangean

$$\mathcal{L} = Y_D^{\beta} (A_1 - L_{S1})^{\theta_1} (A_2 - L_{S2})^{\theta_2} - \lambda (p_U Y_D - w_{L1} L_{S1} - w_{L2} L_{S2} - \pi)$$

and we have the utility maximizing equations

$$\beta \frac{Y_D^{\beta} (A_1 - L_{S1})^{\theta_1} (A_2 - L_{S2})^{\theta_2}}{Y_D} = \lambda p_U \quad (43)$$

$$\theta_1 \frac{Y_D^{\beta} (A_1 - L_{S1})^{\theta_1} (A_2 - L_{S2})^{\theta_2}}{(A_1 - L_{S1})} = \lambda w_{L1} \quad (44)$$

$$\theta_2 \frac{Y_D^{\beta} (A_1 - L_{S1})^{\theta_1} (A_2 - L_{S2})^{\theta_2}}{(A_2 - L_{S2})} = \lambda w_{L2} \quad (45)$$

From the ratio of equation (44) to equation (**??**) we deduce

$$L_{S2} = A_2 - \frac{\theta_2 w_{L1}}{\theta_1 w_{L2}} (A_1 - L_{S1}).$$

The ratio of (43) to (44) yields

$$
\begin{aligned}
A_1 - L_{S1} &= \frac{\theta_1}{\beta}\frac{p_U Y_D}{w_{L1}} \\
&= \frac{\theta_1}{\beta}\frac{w_{L1}L_{S1} + w_{L2}L_{S2} + \pi}{w_{L1}} \\
&= \frac{\theta_1}{\beta}\left(L_{S1} + \frac{w_{L2}}{w_{L1}}\left(A_2 - \frac{\theta_2 w_{L1}}{\theta_1 w_{L2}}(A_1 - L_{S1})\right) + \frac{\pi}{w_{L1}}\right) \\
&= \frac{\theta_1}{\beta}L_{S1} + \frac{\theta_1}{\beta}\frac{w_{L2}}{w_{L1}}A_2 - \frac{\theta_1}{\beta}\frac{w_{L2}}{w_{L1}}\frac{\theta_2 w_{L1}}{\theta_1 w_{L2}}A_1 + \frac{\theta_1}{\beta}\frac{w_{L2}}{w_{L1}}\frac{\theta_2 w_{L1}}{\theta_1 w_{L2}}L_{S1} + \frac{\theta_1}{\beta}\frac{\pi}{w_{L1}} \\
&= \frac{\theta_1}{\beta}\frac{w_{L2}}{w_{L1}}A_2 - \frac{\theta_2}{\beta}A_1 + \frac{\theta_1 + \theta_2}{\beta}L_{S1} + \frac{\theta_1}{\beta}\frac{\pi}{w_{L1}}
\end{aligned}
$$

It follows that

$$
\begin{aligned}
L_{S1} &= \frac{\beta + \theta_2}{\beta + \theta_1 + \theta_2}A_1 - \frac{\theta_1}{\beta + \theta_1 + \theta_2}\left(\frac{w_{L2}}{w_{L1}}A_2 + \frac{\pi}{w_{L1}}\right) \\
L_{S2} &= \frac{\beta + \theta_1}{\beta + \theta_1 + \theta_2}A_2 - \frac{\theta_2}{\beta + \theta_1 + \theta_2}\left(\frac{w_{L1}}{w_{L2}}A_1 + \frac{\pi}{w_{L2}}\right)
\end{aligned}
$$

We may write

$$
\begin{aligned}
w_{L1}L_{S1} &= \frac{\beta + \theta_2}{\beta + \theta_1 + \theta_2}w_{L1}A_1 - \frac{\theta_1}{\beta + \theta_1 + \theta_2}\left(w_{L2}A_2 + \pi\right) \\
w_{L2}L_{S2} &= \frac{\beta + \theta_1}{\beta + \theta_1 + \theta_2}w_{L2}A_2 - \frac{\theta_2}{\beta + \theta_1 + \theta_2}\left(w_{L1}A_1 + \pi\right) \\
Y_D &= \frac{\beta}{\beta + \theta_1 + \theta_2}\frac{w_{L1}A_1 + w_{L2}A_2 + \pi}{p_U}.
\end{aligned}
$$

7.6.2 A Numerical Solution

We now introduce uncertainty in preferences by taking β to be gamma distributed with a mean of .75 and a volatility of .5. θ_1, θ_2 are gamma distributed with mean of .25 and .5 with volatilities of .15 and .3. There are six distorted expectations that are to equated to zero to solve for $w_{L1}, w_{L2}, w_{U1}, w_{U2}, p_L, p_U$. The stress levels in all cases were set at 0.2.

The solution gave working wages for the two labor types in units of output consumption with deflator p_U, of $w_{U1}/p_U = 0.3981$ and $w_{U2}/p_U = 0.2235$ or a 78% premium for the skilled group. The unemployment insurance payment in the two groups was 86.67% of the working wage for the skilled group and 83.5% for the other group. The unemployment rates in the two groups were 16 and 51 percent respectively. The unemployment payments in the two groups as a proportion of average wage income of both groups was 12.47 and 21.38 percent. The average labor supplied by the was 0.5696 and 0.4951 for groups one and two respectively.

8 Conclusion

For uncertain demands and supplies at any price, equilibrium cannot be defined by equating demands to supplies. Viewing the market as an abstract agent that is the counterparty for all transactions by economic participants, the market is modeled as an agent that absorbs the clearing risk of the economy. For risks to be held by the market they must be acceptable. The theory of acceptable risks is applied to for this purpose. The market is modeled as setting two prices for each commodity, one at which it buys and the other at which it sells. The two prices are determined by attempting to ensure the acceptability of both random net inventory and net revenue. For an $n-$ commodity economy there are then $2n$ equilibrium equations for the $2n$ prices. Not all equations can be simultaneously satisfied and an approximate equilibrium is defined by minimizing the departure from the zero for the required equations. The approximate equilibria are illustrated using numerical solutions for two and five good full employment economies.

The introduction of a two price labor market leads to the concept of an equilibrium unemployment rate for an economy as well as an equilibrium level of unemployment insurance. It is shown that the unemployment rate rises with the productivity of the economy. The rise can be mitigated by expanding the number of products in the economy. Technological innovation accompanied by product expansion can therefore be employment neutral and socially acceptable. Other examples illustrate the effects of sector differentiations in technology and preferences. The benefits for unemployment rates associated with income redistribution policies are presented. COVID is modeled as a productivity shock raising equilibrium unemployment support levels measured by the unemployed to employed ratio of income levels. The shocks also reduce the equilibrium unemployment insurance levels. The magnitude of these effects depend on risk attitudes in the clearing of labor markets. A final section of the paper reports on a one good economy with the two price labor markets skill differentiated. Equilibrium unemployment metrics are then obtained for the two groups separately.

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Appendix

Sensitivities of Solutions to Inputs

For an analysis of the sensitivities of the two price solution in the Toy model of Section 4.2 to the inputs we present five graphs for the equilibrium lower and upper prices p_L, p_U as functions of $\beta, \eta, \sigma_D, \sigma_S$, and ρ in Figures 11 13 14 15 and 16 respectively.

Figure 12 shows numerical approximations to the curves for acceptable excess supply and acceptable net revenue as functions of the upper and lower prices.

The sensitivities of solutions to the inputs defining the equilibria are covered in the appendix.

Numerical Analysis of a Two Good Model

Specific solutions are reported for the simple case of a two good, one consumer economy GFEE with CES utility for the consumer. The production functions for the two goods have $\alpha_k = .5$ and $\sigma_k = .2$ for $k = 1, 2$. Suppose the labor endowment of the one consumer is unity. For the utility function

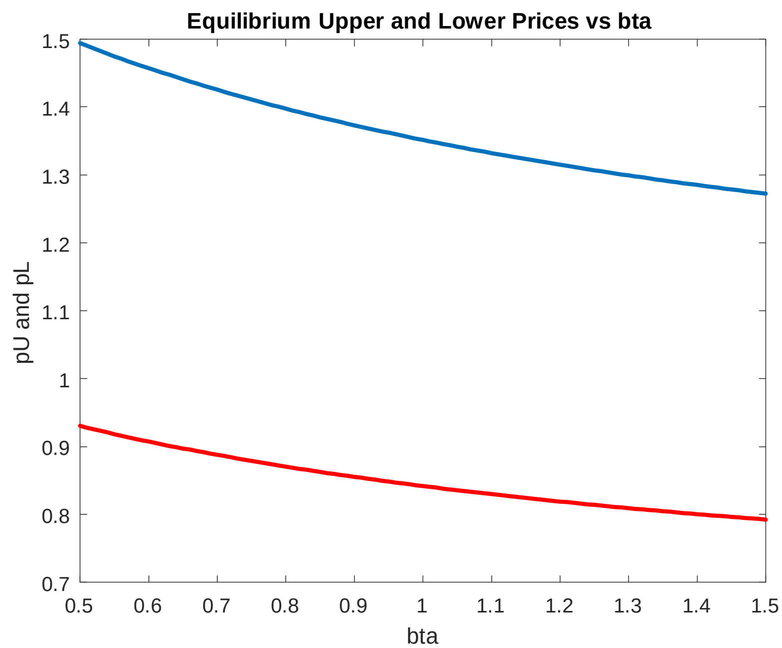

Figure 11: pU and pL as functions of the demand elasticity,

Figure 12: Curves for acceptability of excess supply and net revenue as functions of the upper and lower prices. The curve in blue graphs points that are just acceptable levels of excess supply while the curve in red graphs just acceptability for net revenues.

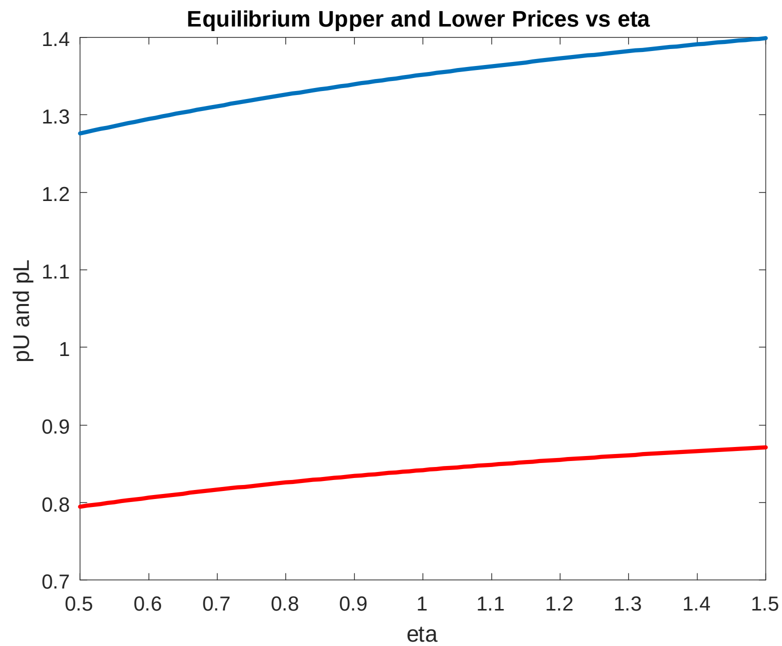

Figure 13: pU and pL as functions of the supply elasticity.

generating random demands it is supposed that the elasticity of substitution is gamma distributed with a mean of 1.2 and a volatility of 0.6. The coefficients a_k, $k = 1, 2$ are log mormally distributed with mean levels of unity for the coefficients a_k and volatilities for the logarithm of the coefficients of 0.2 for both of them. The correlation between the logarithm of these coefficients is 0.5. The stress levels for the distorted expectations of excess supplies and net revenues are the same for all four components and are varied in steps of 0.01 starting at 0.01 and ending at 0.5.

As the stress levels are raised reflecting more conservative levels of risk acceptance on the part of the risk taker in the economy we observe the following patterns in the two price equilibrium solutions. First, employment and output levels for the two goods are unaffected. The employment levels are consistently .5 in the two sectors and the expected outputs are .7. There is also not much movement in the lower prices for the two goods. They are at 1.41 units of labor at all stress levels. This is not the case for the upper prices charged to the consumer. They start at around 1.42 units of labor at the stress level of 0.01. This is close to a law of one price solution. However, at the other end, for a stress level of 0.5, the prices for the two goods are as high as 1.6 units of labor. This has implications for the levels of expected demands or consumption for the consumer. The expected demands fall from 0.70 for each good down to 0.60 for the two goods. The expected net revenues are maintained at zero across the

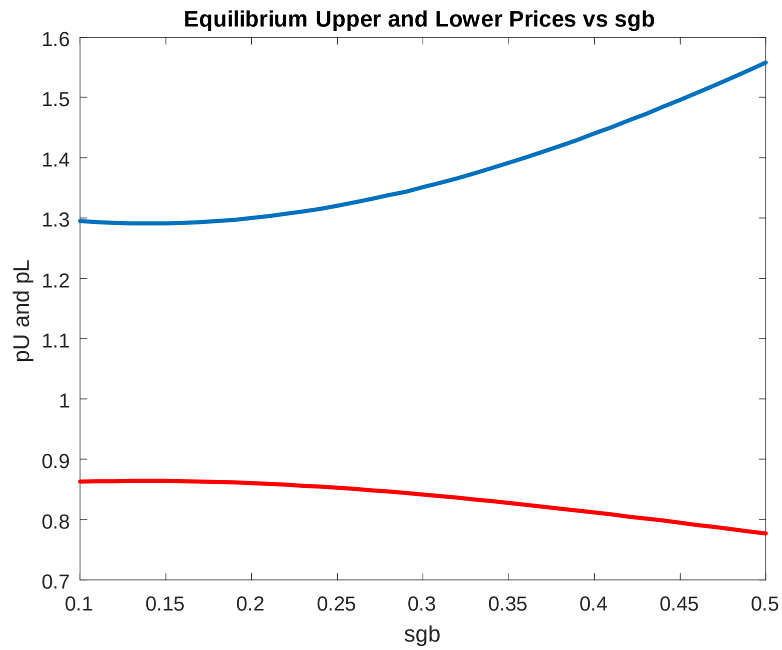

Figure 14: pU and pL as functions of the demand volatility.

stress levels. The effect of conservatism in risk exposure is to lift the prices of goods and decrease consumption. The price increases matched the decrease in consumption leaving expected net revenues at zero and employment and output unaffected.

For an analysis of the relationship between equilibrium solutions and the economic inputs we solve for the equilibrium at a variety of input points. There were three sets of production functions, three production volatility sets, two mean levels for the elasticity of substitution with volatility at half the mean, three correlations between the two goods in the CES utility function, three levels for mean levels for a_k and their volatilities and three sets of stress levels. All combinations of these choices were run to get a total of 1458 equilibrium solutions for the two good economy.

Nine equilibrium solution outputs were regressed on the inputs. The equilibrium solution outputs regressed were the markup of the upper to the lower price for the two goods, the demand shortfall relative to the output for the two goods, the relative employment and output for the first good relative to the second, the relative expected profit for the first to second, and the expected net revenues in the two goods measured in units of labor as the numeraire.

The inputs for the regressions were the production coefficients α_k and their volatilities, the mean level for the elasticity of substitution in the utility function, the correlation in the utility function coefficients a_k, their means and volatilities,

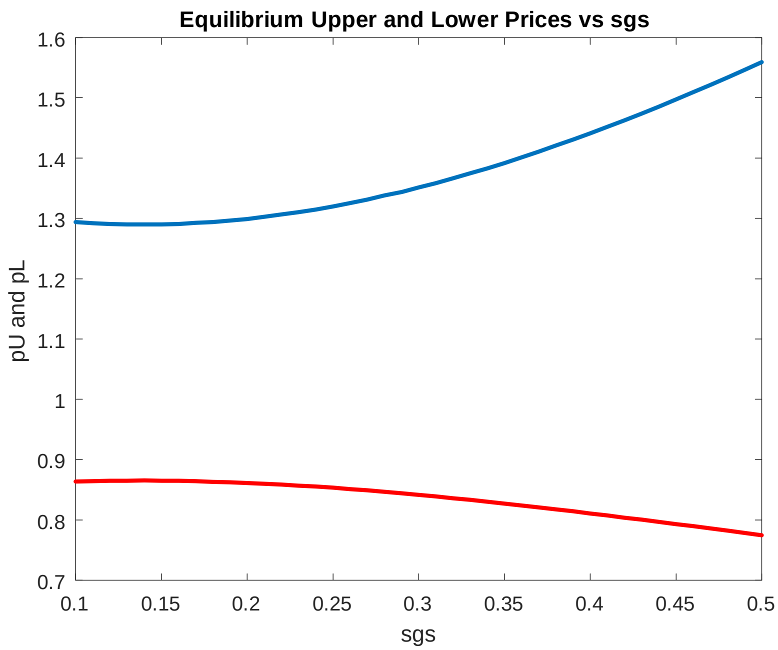

Figure 15: pU and pL as functions of the supply volatility.

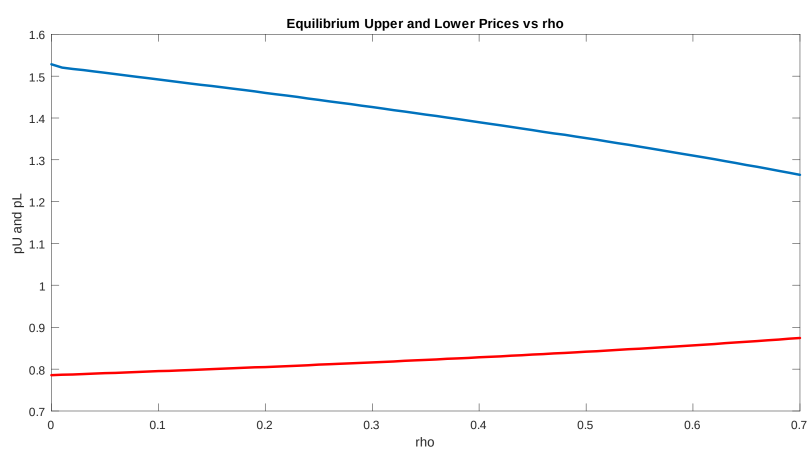

Figure 16: pU and pL as functions of the demand supply correlation.

and the two stress levels for the two excess supplies and the two net revenues. Both goods had the same stress level for the excess supplies and the net revenues. There were twelve explanatory variables for the nine equilibrium outputs. The results for these regressions are presented in Table 17.

It may be observed that the price markups are positively related to their own production coefficients and negatively to the other while they are positively related to their own production volatility. They are positively related to the elasticity of substitution between the goods but unaffected by correlations in preferences. They are positively related to their means for the coefficients a_k and negatively to the opposite a_k. With regards to volatilities in the $a_k's$ they are positively related to the opposite volatility. They are strongly related to stress in excess supply but less so to net revenue stress.

The demand shortfalls are negatively related to their own production coefficients and positively to the opposites. They are also negatively related to all the production volatilities. They are negatively related to the elasticity of substitution and unresponsive to correlations in preference. They are negatively related to mean levels for their $a_k's$ and positively to the opposite. They are negatively related to the opposite volatility in the $a_k's$. They are also negatively related to stress levels in excess supply.

Relative employment is positive in its own production coefficient, negative in the opposite and the other way around for the volatilities. The elasticity of substitution lowers relative employment. They are positive in their own a_k and negative in the opposite. Relative output is comparable to relative employment. Relative profitability behaves comparably to output and employment except it is negative in the own production coefficient and positive in the opposite. Net revenues are positive in own production volatility and negative in the opposite. They are also positive in the own mean levels for a_k and negative in the opposite.

The coefficients of multiple correlation ($R^2\prime s$) for the regressions lie between 60 and 80 percent with a mean around 70%.

Proof of Proposition 3

To derive the demand function we formulate the Lagrangean for utility maximization subject to the budget constraint with

commodity prices p_k and income y as

$$\mathcal{L}(x,\lambda)=\left(\sum_k a_k x_k^{\rho}\right)^{1/\rho} - \lambda\left(\sum_k p_k x_k - y\right)$$

The first order conditions yield

$$\left(\sum_k a_k x_k^{\rho}\right)^{1/\rho-1} a_k x_k^{\rho-1} = \lambda p_k.$$

Multiplication by x_k and summing over k yields

$$\left(\sum_k a_k x_k^{\rho}\right)^{1/\rho-1} = (\lambda y)^{1-\rho}$$

					TABLE 1				
			Equilibrium Solutions Regressed on Economy Inputs						
				Dependent Variables					
Explanatory Variable	Markup 1	Markup 2	Demand Shortfall 1	Demand Shortfall 2	Rel. Labor.	Rel. Output	Rel. Profit	Net Rev. 1	Net Rev. 2
Constant	0.7054	0.6330	1.1934	1.2117	0.6053	0.9806	-0.2301	0.0052	-0.0068
	31.60	23.68	79.42	79.50	11.43	30.29	-1.71	0.79	-1.00
Prod. Coeff. 1	0.1690	-0.1554	-0.1315	0.1032	2.0072	0.3185	-1.8576	-0.0070	0.0078
	10.07	-7.73	-11.65	9.01	50.45	13.09	-18.33	-1.40	1.53
Prod. Coeff. 2	-0.0551	0.0458	0.0534	-0.0530	-1.2308	-0.1815	4.4350	0.0142	-0.0129
	-3.28	2.28	4.73	-4.63	-30.94	-7.46	43.75	2.87	-2.52
Prod. Vol. 1	0.5058	0.0173	-0.0991	-0.2438	-0.4908	-0.3427	-1.1416	0.2253	-0.2440
	24.12	0.69	-7.02	-17.03	-9.87	-11.27	-9.01	36.35	-38.16
Prod. Vol. 2	-0.0817	0.7012	-0.1913	-0.2287	0.2281	0.1535	0.5736	-0.2647	0.2726
	-3.90	27.92	-13.55	-15.97	4.59	5.05	4.53	-42.70	42.64
Elas. Subst.	0.1738	0.5265	-0.1168	-0.2474	-0.3165	-0.1904	-0.7846	-0.0409	0.0425
	7.61	19.26	-7.60	-15.87	-5.85	-5.75	-5.69	-6.06	6.11
Corr.	-0.0082	-0.0119	0.0061	0.0080	0.0072	0.0046	0.0158	0.0006	-0.0006
	-0.97	-1.18	1.08	1.40	0.36	0.38	0.31	0.22	-0.23
Mean a1	0.1386	-0.0843	-0.0713	0.0193	0.3703	0.2112	0.7794	0.0414	-0.0429
	33.06	-16.78	-25.26	6.75	37.23	34.72	30.76	33.35	-33.51
Mean a2	-0.0623	0.1494	0.0108	-0.0713	-0.2103	-0.1526	-0.4649	-0.0332	0.0347
	-14.86	29.75	3.81	-24.90	-21.14	-25.09	-18.34	-26.75	27.11
Vol. a1	-0.0027	0.0638	-0.0074	-0.0346	-0.1055	-0.0656	-0.2176	-0.0124	0.0129
	-0.13	2.54	-0.52	-2.42	-2.12	-2.16	-1.72	-2.00	2.02
Vol. a2	0.0485	0.0119	-0.0288	-0.0169	0.0611	0.0401	0.1354	0.0078	-0.0082
	2.31	0.48	-2.04	-1.18	1.23	1.32	1.07	1.26	-1.28
Stress ES	0.4918	0.5277	-0.3562	-0.3524	-0.0261	-0.0221	-0.0561	-0.0037	0.0036
	29.32	26.27	-31.54	-30.77	-0.66	-0.91	-0.55	-0.75	0.70
Stress NR	-0.0029	0.0896	0.0138	-0.0635	-0.0682	-0.0582	-0.1587	0.0060	-0.0047
	-0.17	4.46	1.22	-5.54	-1.71	-2.39	-1.57	1.20	-0.92
RSQ	72.55	74.42	63.36	63.78	72.21	61.09	70.49	80.33	80.93

Figure 17: Results of regressing equilibrium solution outputs on the inputs defining the economy.

Hence it follows that

$$
\begin{aligned}
(\lambda y)^{1-\rho} a_k x_k^{\rho-1} &= \lambda p_k \\
x_k^{1-\rho} &= \frac{(\lambda y)^{1-\rho} a_k}{\lambda p_k} \\
x_k &= y \left(\frac{a_k}{p_k}\right)^{\frac{1}{1-\rho}} \left(\frac{1}{\lambda}\right)^{\rho}.
\end{aligned}
$$

Multiplying by p_k and summing yields

$$\left(\frac{1}{\lambda}\right)^{\rho} = \frac{1}{\sum_k a_k^{\frac{1}{1-\rho}} p_k^{\frac{\rho}{\rho-1}}}.$$

Hence

$$x_k = \frac{a_k^{\frac{1}{1-\rho}} p_k^{-\frac{1}{1-\rho}} y}{\sum_k a_k^{\frac{1}{1-\rho}} p_k^{\frac{\rho}{\rho-1}}}$$

The result follows on substituting for ρ in terms of r.

Solution of a five good example

A five good example for a full employment economy with unit labor endowment is illustrated. The economy is specified by the production functions, α, v, the distribution for the elasticity of substitution, the correlations in the logarithms of the coefficients a_k their means and volatilities, and the stress levels for excess supply and net revenue. The elasticity of substitution is taken to be gamma distributed with a mean of 0.2 and a variance of 0.1. The other parameters for the five goods were as follows.

$$
\begin{aligned}
\alpha &= [.3\ .4\ .5\ .6\ .7] \\
v &= [.25\ .23\ .21\ .19\ .17] \\
\mu &= [5\ 4\ 3\ 2\ 1] \\
\zeta &= [.5\ .4\ .3\ .2\ .1].
\end{aligned}
$$

The stress levels for all five excess supplies were 0.25 and net revenues were 0.5.

For this economy the shortfalls in demand relative to output, the markups for the upper price relative to the lower price and the expected revenues by sector in labor units were as follows.

	1	2	3	4	5
shortfall	0.1152	0.1379	0.1923	0.3620	0.5752
markup	0.2973	0.1921	0	0	0
Exp. Revenue	0.3193	0.0311	−0.1076	−0.1149	−0.1269

The markups were concentrated in the sectors with higher productivity and volatility where demand was high and volatile. These sectors had the lower shortfalls.