Not-For-Publication Appendix to:

A New Approach to Measuring Economic Policy Shocks, with an Application to Conventional and Unconventional Monetary Policy

Atsushi Inoue†
Vanderbilt University

Barbara Rossi*
ICREA-Univ. Pompeu Fabra, Barcelona GSE, and CREI

April, 2021

1 Details on Monetary Policy Announcements Since 2008:11

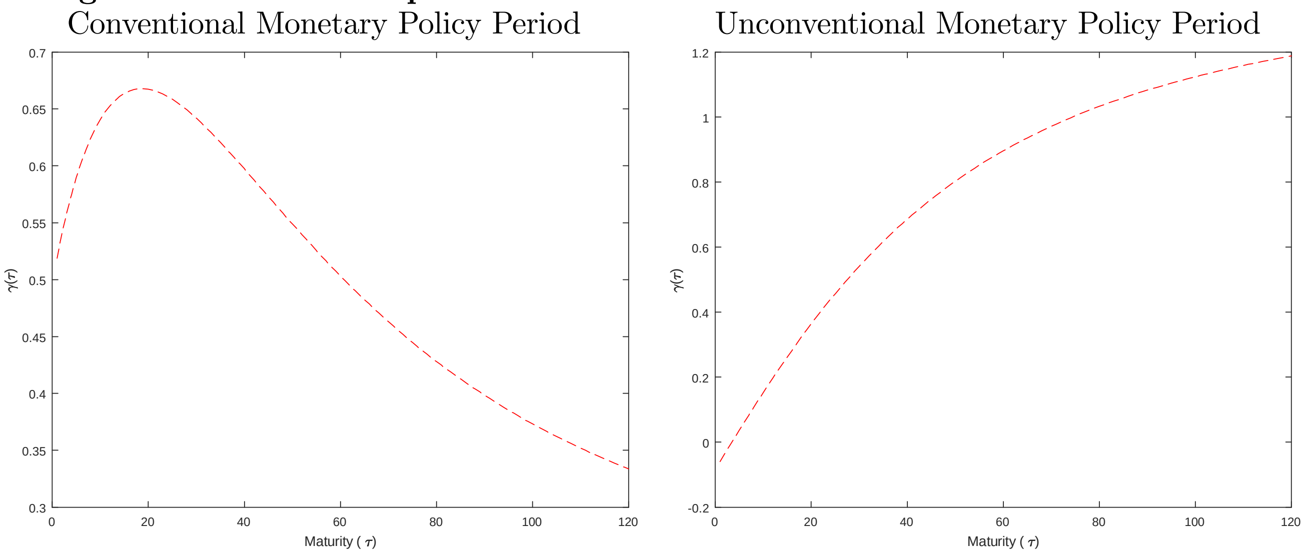
Table A1. Unconventional Monetary Policy Announcements

Date	Announcement
25 11 2008	Start of LSAP-I
01 12 2008	Treasury securities purchases
16 12 2008	FFR lowered to zero lower bound
28 1 2009	Disappointing FOMC meeting
18 3 2009	Additional Treasury purchases
10 8 2010	Start of LSAP-II
21 9 2010	Confirming existing reinvestments
3 11 2010	Additional Treasury purchases
9 8 2011	Decided to maintain a highly accommodative stance of monetary policy
21 9 2011	Maturity Extension Program
25 1 2012	Decided to maintain a highly accommodative stance of monetary policy
20 6 2012	Continuation of Maturity Extension Program
13 9 2012	Start of LSAP-III
12 12 2012	Decided to maintain a highly accommodative stance of monetary policy
19 6 2013	Decided to maintain a highly accommodative stance of monetary policy
18 9 2013	Continuation of QE
18 12 2013	Begin tapering QE in January
19 3 2014	Possibility of raising the fed funds rate 6 months after the end of QE and decided to make a reduction in the pace of its asset purchases
18 6 2014	Cut another $10 billion from its purchases of Treasury assets and mortgages and decided to make a further reduction in the pace of its asset purchases
17 9 2014	Reduced its QE bond purchase by another $10 billion and decided to make a further reduction in the pace of its asset purchases
17 12 2014	Prepared to raise interest rates only when the economy improves enough and reaffirmed the current target range for the FFR remains appropriate
18 3 2015	Reaffirmed the current target range for the federal funds rate remains appropriate
17 6 2015	Signaled the possibility of a rate increase in three to six months
17 9 2015	Left the fed funds rate unchanged
16 12 2015	Raised the fed funds rate a quarter point to 0.5 percent
16 3 2016	Kept the interest rate unchanged
15 6 2016	Voted against raising rates

Notes to the table. The dates include the most important dates from Wright (2012).

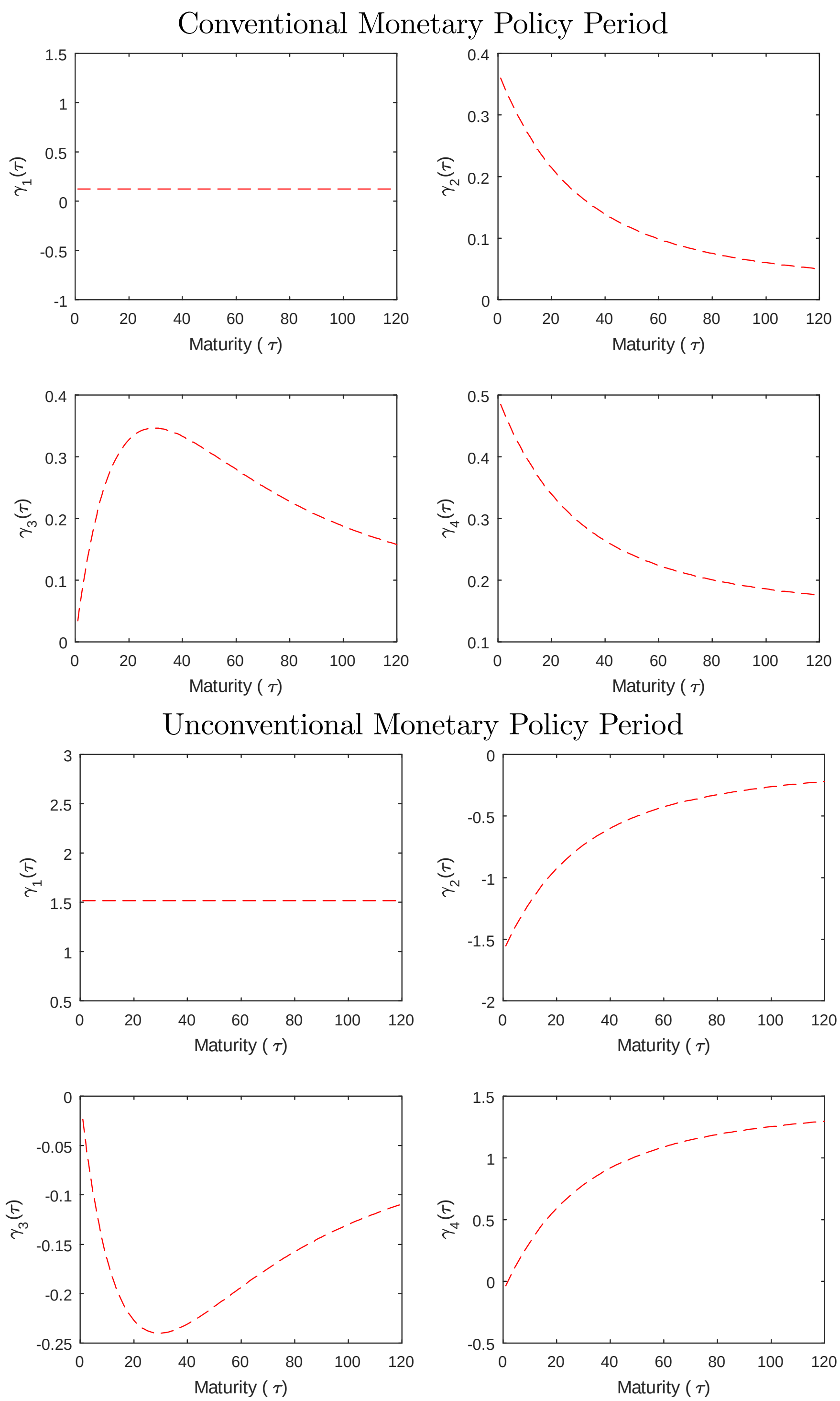
2 Relationship Between Our Shock and Other Monetary Policy Shocks

Figure A1. Relationship Between Our Shock Nakamura and Steinsson's



Notes. The figure depicts the coefficient $\gamma(\tau)$ in the regression of our functional monetary policy shock, $\varepsilon_t^f(\tau)$, on the Nakamura and Steinsson's (2017) monetary policy shock.

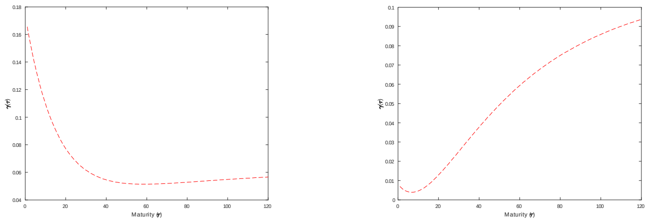
Figure A2: Relationship Between the Components of Our Shock and Nakamura and Steinsson's



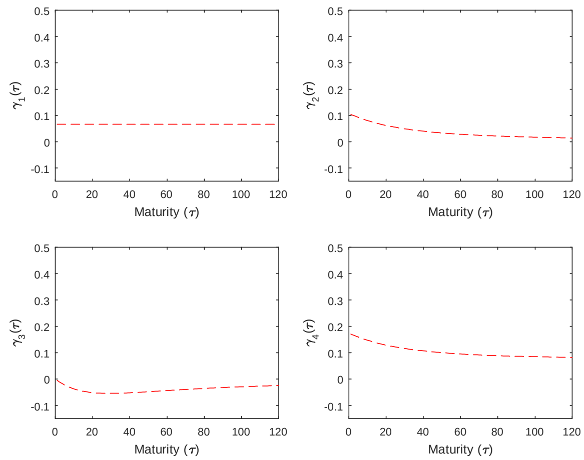
Notes. The figure depicts the correlation relationship the components of our functional monetary policy shock and Nakamura and Steinsson's (2017) (narrative) monetary policy shock.

Figure A3: Panel A: Our Shock vs. Krippner (2015)

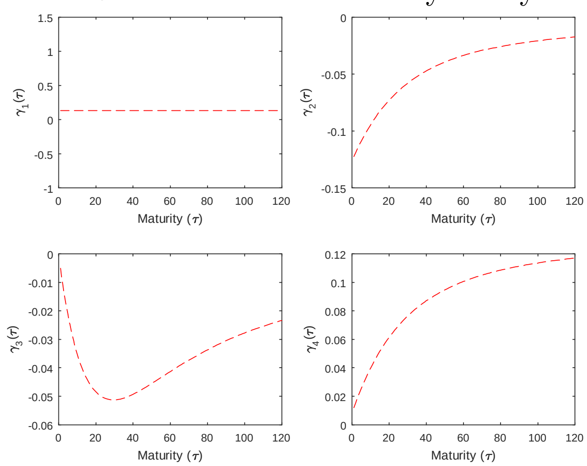
A. Conventional Period   B. Unconventional Period



Panel C. Conventional Monetary Policy Period



Panel D. Unconventional Monetary Policy Period



Notes. Panels A-B in the figure depicts the relationship between our functional monetary policy shock, $\varepsilon_t^f(\tau)$, and Krippner (2015) in the bottom panels. Panels C-D depicts the correlation between the components of our functional monetary policy shock and Krippner's (2015) monetary policy shock.

3 Robustness to Adjusting the Shock for the Number of Days in the Month

In this robustness exercise, we re-scale the shocks depending on the day of the month it took place. The weight equals the number of days from the day of the shock until the end of the month, divided by the number of days in the month.

Figure B1. Output Response in Conventional Times

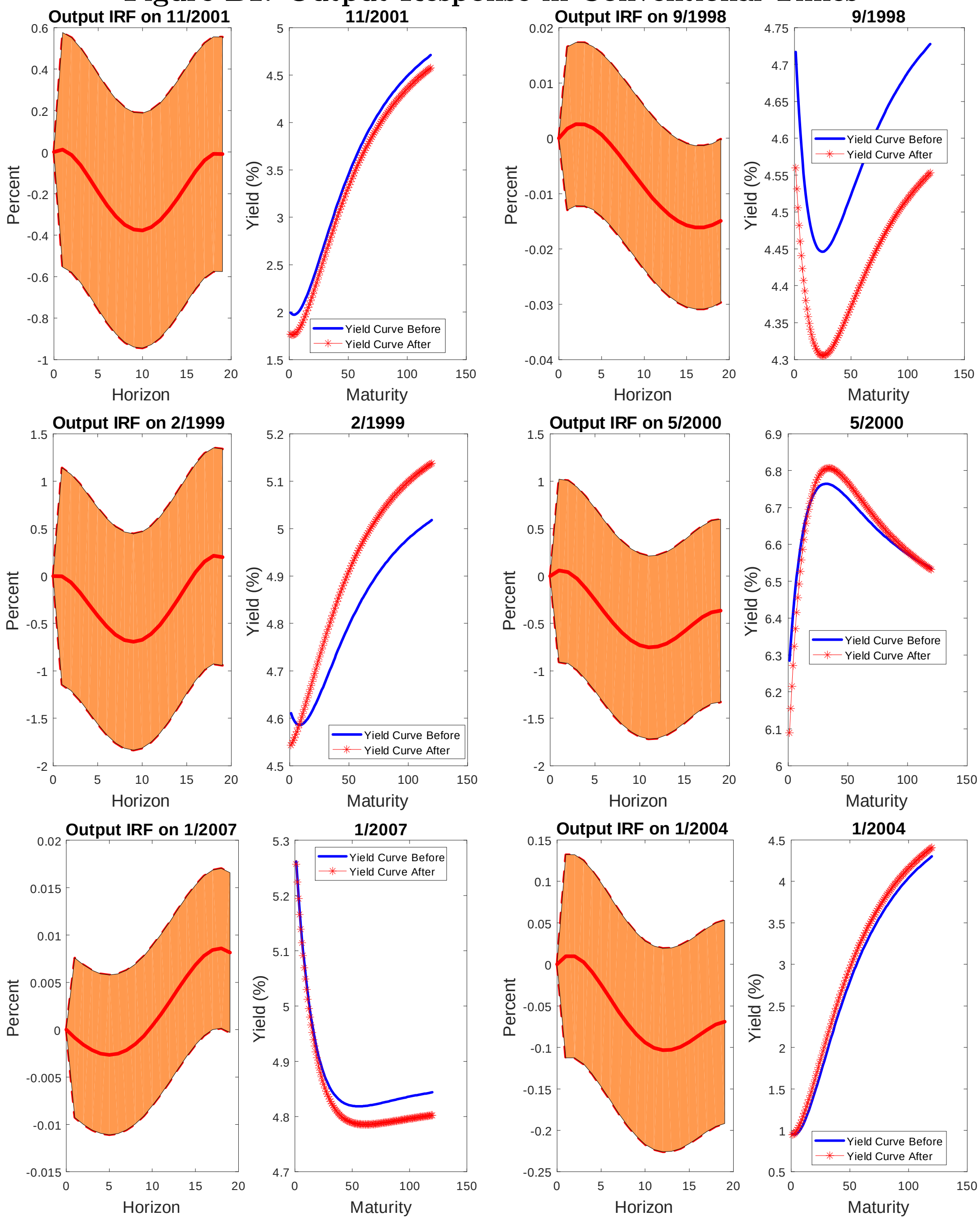


Figure B2. Inflation Response in Conventional Times

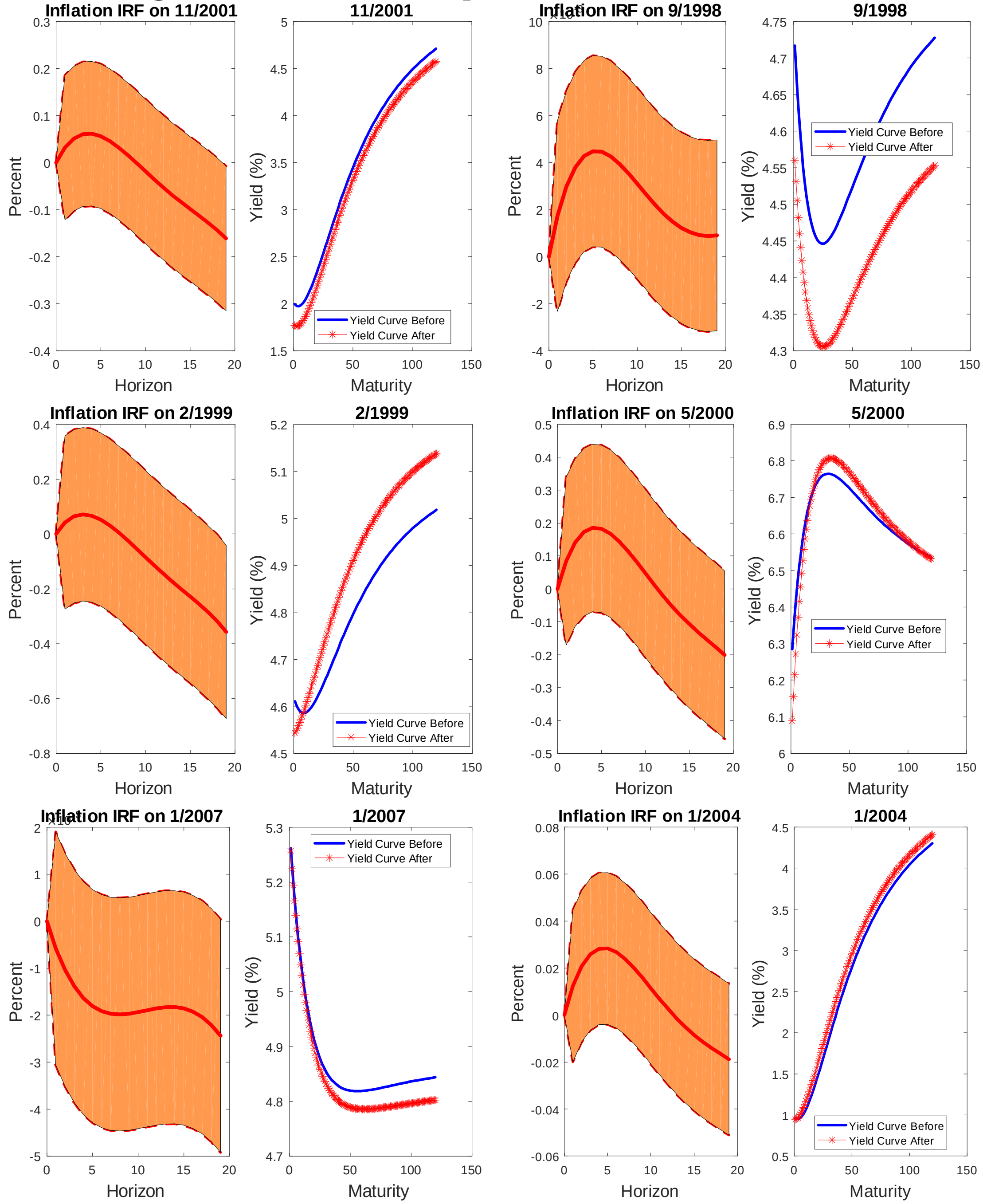


Figure B3. Output Response in Unconventional Times

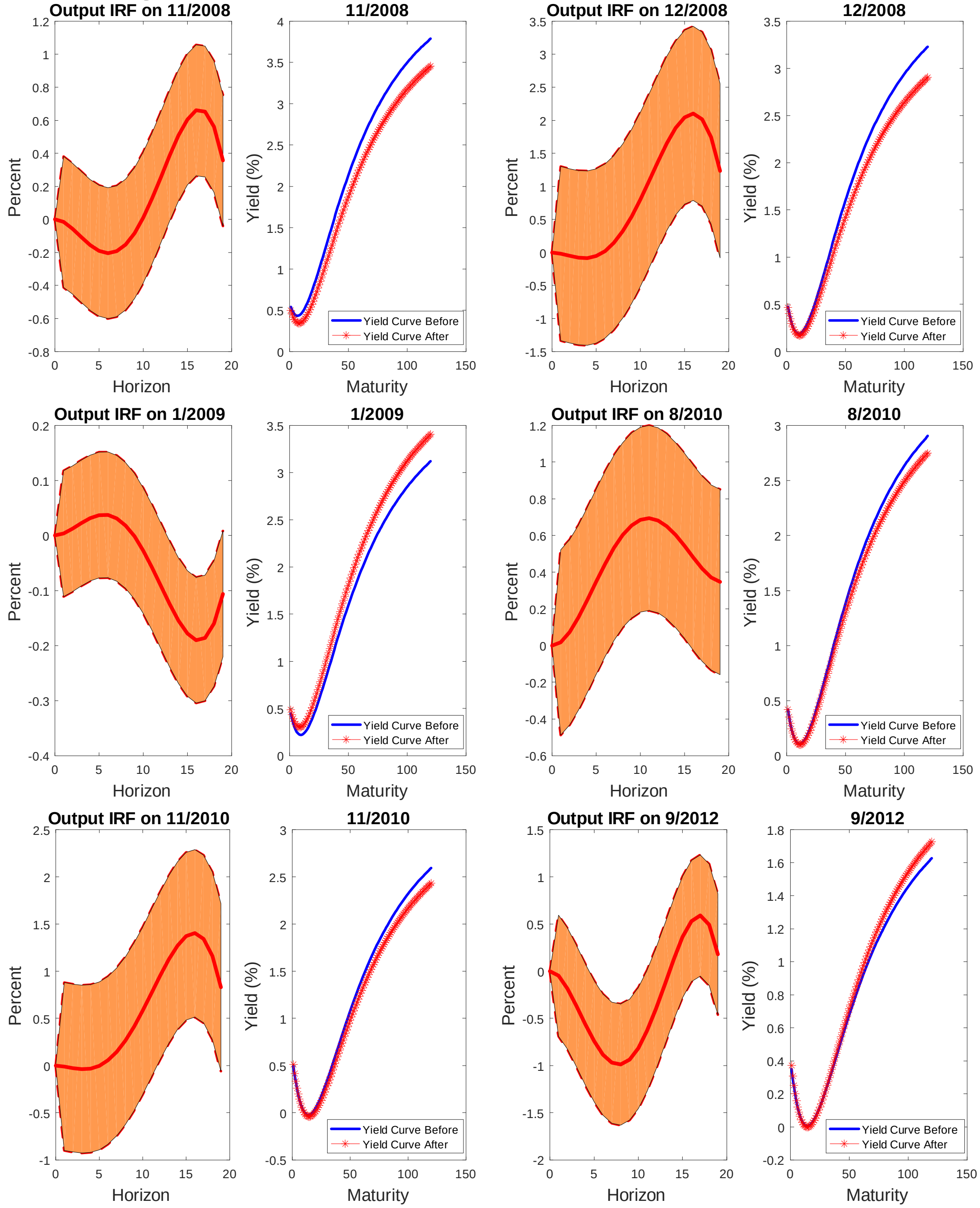
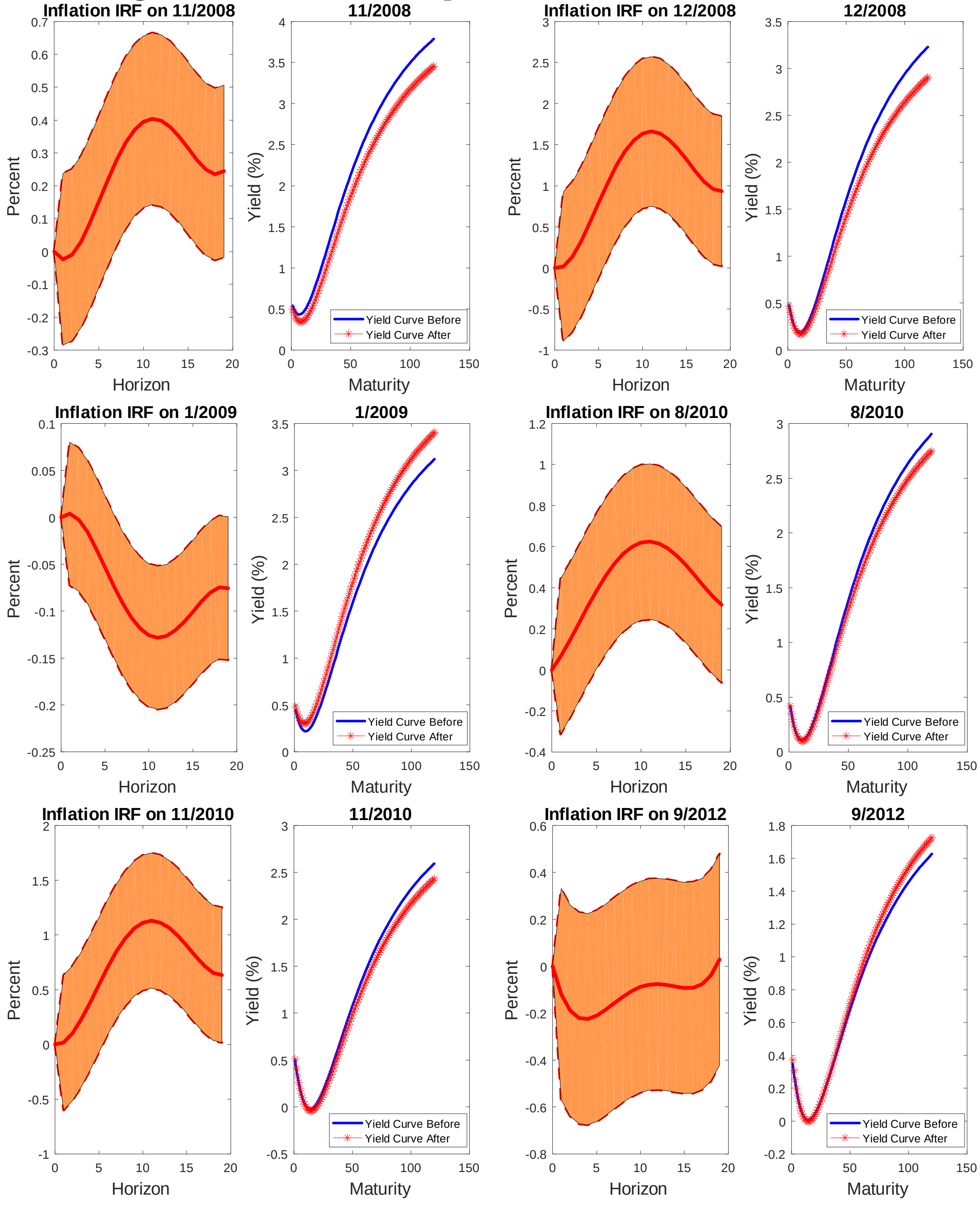


Figure B4. Inflation Response in Unconventional Times



4 Robustness to Cumulating Multiple Shocks that Appear in the Same Month

In this robustness exercise, we sum multiple monetary policy shocks when they appear in the same month.

Figure C1. Output Response in Conventional Times

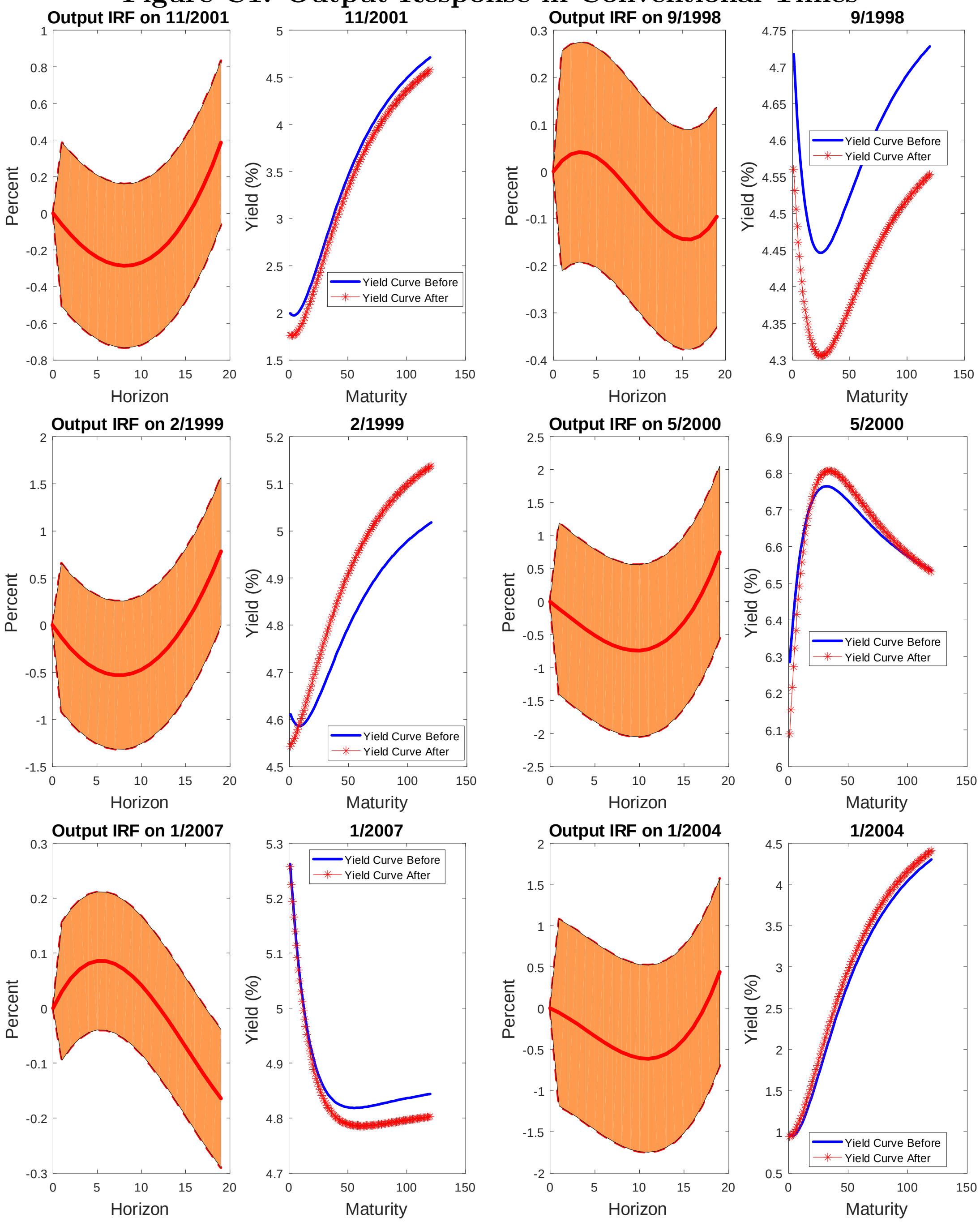


Figure C2. Inflation Response in Conventional Times

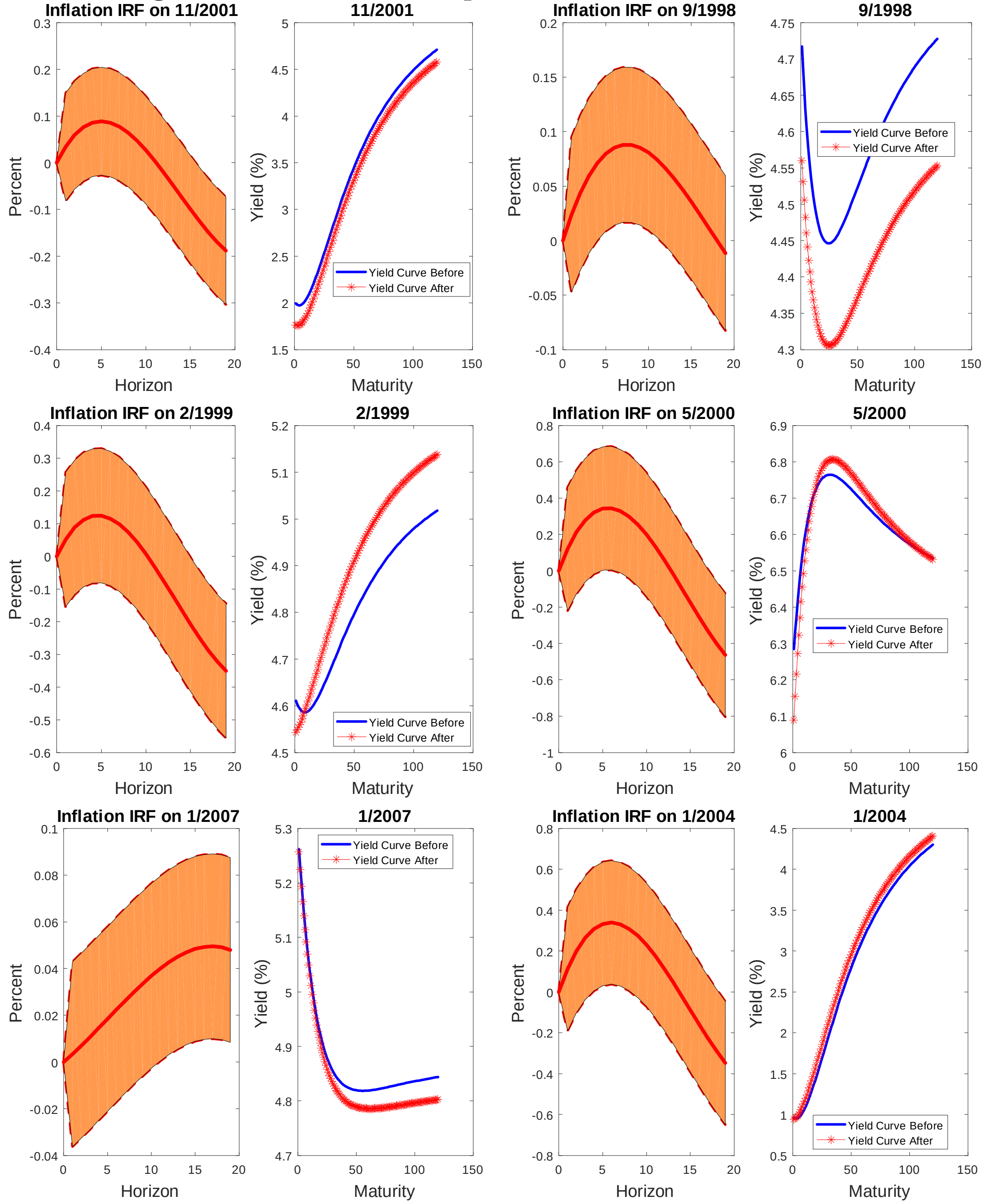


Figure C3. Output Response in Unconventional Times

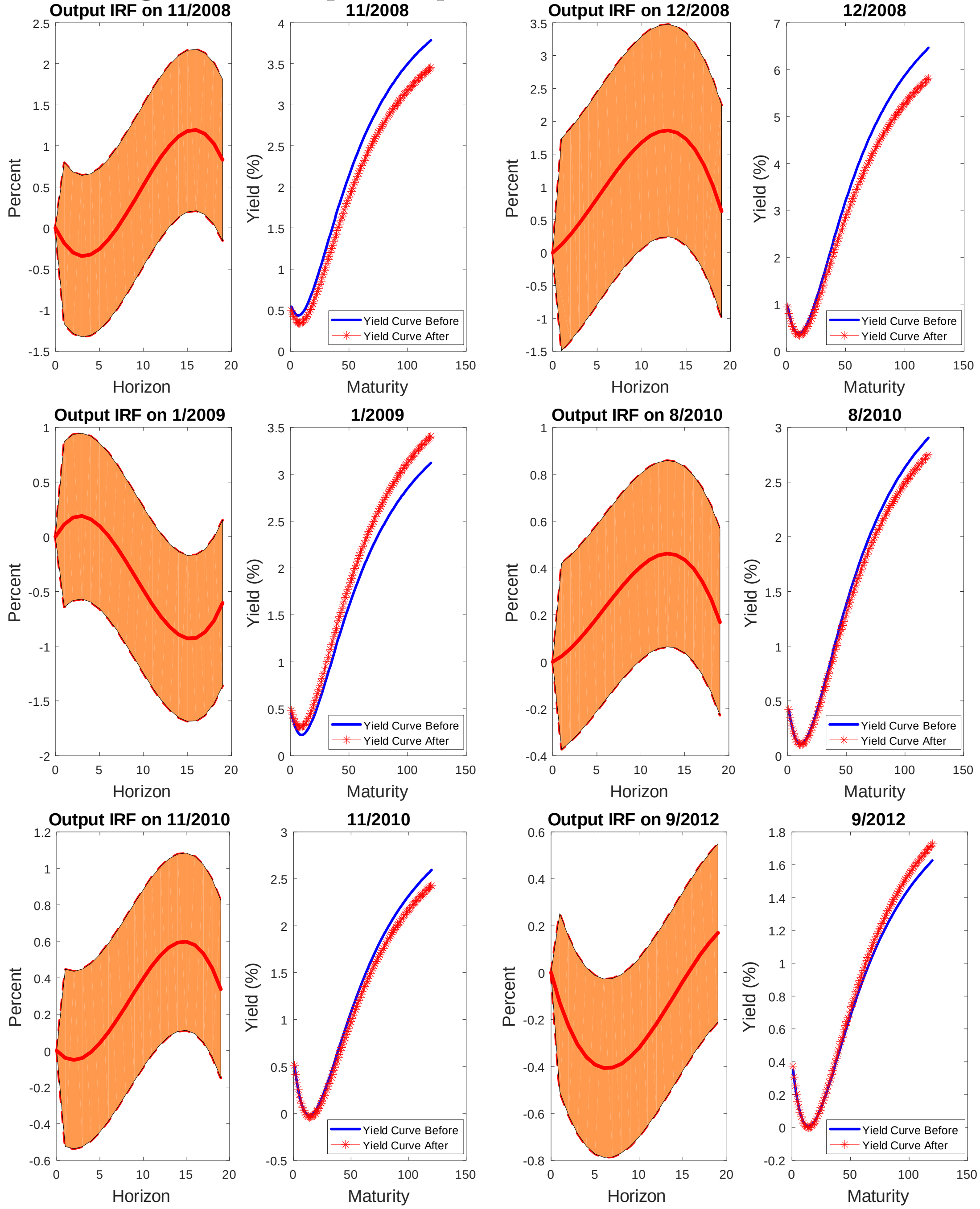
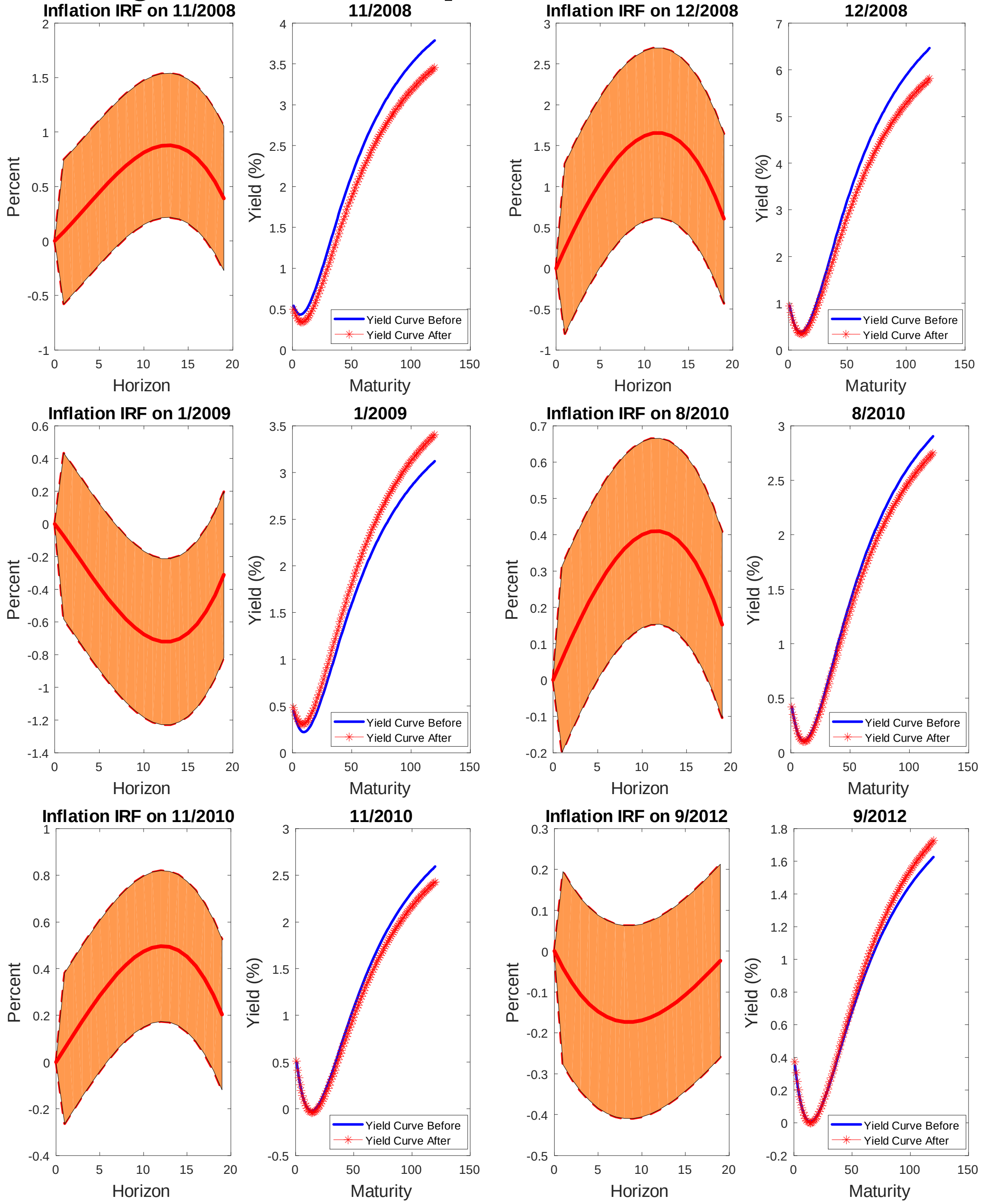


Figure C4. Inflation Response in Unconventional Times



5 The Special Case of the Nelson and Siegel (1987) Model and Univariate X_t

We report the calculations for the case of the Nelson and Siegel (1987) model, focusing on the case where X_t is a scalar univariate autoregressive process ($n = p = 1$). In the Nelson and Siegel (1987) model, $q = 3$. Please note that the notation is specific to this section. We shall prove that:

$$\begin{bmatrix} X_t \\ \beta_{1,t} \\ \dots \\ \beta_{3,t} \end{bmatrix} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,2} & \phi_{1,2} \\ \tilde{\phi}_1 & \phi_{2,2} & 0 & 0 \\ \tilde{\phi}_2 & 0 & \phi_{2,2} & 0 \\ \tilde{\phi}_q & 0 & 0 & \phi_{2,2} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \beta_{1,t-1} \\ \beta_{2,t-1} \\ \beta_{3,t-1} \end{bmatrix} + \begin{bmatrix} u_{X,t} \\ \tilde{u}_{1,t} \\ \tilde{u}_{2,t} \\ \tilde{u}_{3,t} \end{bmatrix}. \quad (1)$$

and that the VAR has a vector moving average representation:

$$\begin{aligned} X_t &= \sum_{i=0}^{\infty} \theta_{1,i} u_{X,t-i} + \psi_{1,1} \left(\sum_{i=1}^{3} I_i \tilde{u}_{i,t-1} \right) + \psi_{1,2} \left(\sum_{i=1}^{3} I_i \tilde{u}_{i,t-2} \right) + \dots & (2) \\ \beta_{1,t} &= \theta_{2,1} u_{X,t-1} + \theta_{2,2} u_{X,t-2} + \psi_{2,1} \tilde{u}_{1,t} + \psi_{2,2} \tilde{u}_{1,t-1} + \cdots & (3) \\ &\cdots \\ \beta_{3,t} &= \theta_{q+1,1} u_{X,t-1} + \theta_{3+1,2} u_{X,t-2} + \psi_{q+1,1} \tilde{u}_{q,t} + \psi_{3+1,2} \tilde{u}_{3,t-1} + \cdots & (4) \end{aligned}$$

In the Nelson and Siegel (1987) model, $g_1(\tau; \lambda) = 1$, where

$$\begin{aligned} c_2(\tau; \lambda) &= \tilde{c}_1 + \tilde{c}_2 g_2(\tau; \lambda) + \tilde{c}_3 g_3(\tau; \lambda), \\ \phi_{21}(\tau; \lambda) &= \tilde{\phi}_1 + \tilde{\phi}_2 g_2(\tau; \lambda) + \tilde{\phi}_3 g_3(\tau; \lambda), \\ u_{f,t}(\tau; \lambda) &= \tilde{u}_{1,t} + \tilde{u}_{2,t} g_2(\tau; \lambda) + \tilde{u}_{3,t} g_3(\tau; \lambda). \quad (5) \end{aligned}$$

Repeated substitutions of (1) into itself yield:

$$
\begin{aligned}
X_t &= c_1 + \phi_{1,1}(c_1 + \phi_{1,1}X_{t-2} + \phi_{1,2}\int w(\tau)f_{t-2}(\tau;\lambda)d\tau + u_{X,t-1}) \\
&\quad + \phi_{1,2}\int w(\tau)(c_2(\tau;\lambda) + \phi_{2,1}(\tau;\lambda)X_{t-2} + \phi_{2,2}f_{t-2}(\tau;\lambda) + u_{f,t-1}(\tau;\lambda))d\tau + u_{Xt} \\
&= (1+\phi_{1,1})c_1 + \phi_{1,2}\int w(\tau)c_2(\tau;\lambda)d\tau + u_{X,t} + \phi_{1,1}u_{X,t-1} + \phi_{1,2}\int w(\tau)u_{f,t-1}(\tau;\lambda)d\tau \\
&\quad + (\phi_{1,1}^2 + \phi_{1,2}\int w(\tau)\phi_{2,1}(\tau;\lambda)d\tau)X_{t-2} + \phi_{1,2}(\phi_{1,1}+\phi_{2,2})\int w(\tau)f_{t-2}(\tau;\lambda)d\tau \\
&= (1+\phi_{1,1}+\phi_{1,1}^2 + \phi_{1,2}\int w(\tau)\phi_{2,1}(\tau;\lambda)d\tau)c_1 \\
&\quad + (1+\phi_{1,1}+\phi_{2,2})\phi_{1,2}\int w(\tau)c_2(\tau;\lambda)d\tau \\
&\quad + u_{X,t} + \phi_{1,1}u_{X,t-1} + (\phi_{1,1}^2 + \phi_{1,2}\int w(\tau)\phi_{2,1}(\tau;\lambda)d\tau)u_{X,t-2} \\
&\quad + \underbrace{\phi_{1,2}}_{=\psi_{1,1}}\int w(\tau)u_{f,t-1}(\tau;\lambda)d\tau + \underbrace{\phi_{1,2}(\phi_{1,1}+\phi_{2,2})}_{=\psi_{1,2}}\int w(\tau)u_{f,t-2}(\tau;\lambda)d\tau + \cdots, \qquad (6) \\
f_t(\cdot;\lambda) &= c_2(\cdot;\lambda) + \phi_{2,1}(\cdot;\lambda)(c_1 + \phi_{1,1}X_{t-2} + \phi_{1,2}\int w(\tau)f_{t-2}(\tau;\lambda)d\tau + u_{X,t-1}) \\
&\quad + \phi_{2,2}(c_2(\cdot;\lambda) + \phi_{2,1}(\cdot;\lambda)X_{t-2} + \phi_{2,2}f_{t-2}(\cdot;\lambda) + u_{f,t-1}(\cdot;\lambda)) + u_{f,t}(\cdot;\lambda), \\
&= (1+\phi_{2,2})c_2(\cdot;\lambda) + \phi_{2,1}(\cdot;\lambda)c_1 + \phi_{2,1}(\cdot;\lambda)u_{X,t-1} + u_{f,t}(\cdot;\lambda) + \phi_{2,2}u_{f,t-1}(\cdot;\lambda) \\
&\quad + (\phi_{1,1}+\phi_{2,2})\phi_{2,1}(\cdot;\lambda)X_{t-2} + \phi_{21}(\cdot;\lambda)\phi_{1,2}\int w(\tau)f_{t-2}(\tau;\lambda)d\tau + \phi_{2,2}^2 f_{t-2}(\cdot;\lambda) \\
&\quad + \cdots. \qquad (7)
\end{aligned}
$$

Then, using eqs. (5) and (6), the differential[1] of X_{t+h} in the direction

$$u^*_{f,t}(\tau;\lambda) \;=\; \tilde{u}^*_{1,t} + \tilde{u}^*_{2,t}g_2(\tau;\lambda) + \tilde{u}^*_{3,t}g_3(\tau;\lambda)$$

is

$$\psi_{1,h}\int w(\tau)u^*_{f,t}d\tau \;=\; \psi_{1,h}(I_1\tilde{u}^*_{1,t} + I_2\tilde{u}^*_{2,t} + I_3\tilde{u}^*_{3,t}). \qquad (8)$$

Because (1) holds for every τ, this model can be written as a four-variable VAR model:

$$\begin{bmatrix} X_t \\ \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \end{bmatrix} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2}I_1 & \phi_{1,2}I_2 & \phi_{1,3}I_3 \\ \tilde{\phi}_1 & \phi_{2,2} & 0 & 0 \\ \tilde{\phi}_2 & 0 & \phi_{2,2} & 0 \\ \tilde{\phi}_3 & 0 & 0 & \phi_{2,2} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \beta_{1,t-1} \\ \beta_{2,t-1} \\ \beta_{3,t-1} \end{bmatrix} + \begin{bmatrix} u_{X,t} \\ \tilde{u}_{1,t} \\ \tilde{u}_{2,t} \\ \tilde{u}_{3,t} \end{bmatrix}, \qquad (9)$$

[1] As we discuss in the Not-for-Publication Appendix, the differential we define here is a Gateaux differential.

where the intercept terms are omitted for simplicity. Similarly, because (7) holds for each τ, we have a vector moving average representation:

$$X_t = u_{X,t} + \underbrace{\phi_{1,1}}_{\equiv\theta_{1,1}} u_{X,t-1} + \underbrace{(\phi_{1,1}^2 + \phi_{1,2}\int w(\tau)\phi_{2,1}(\tau)d\tau)}_{\equiv\theta_{1,2}} u_{X,t-2} + \underbrace{\phi_{1,2}}_{\equiv\psi_{1,1}}(I_1\tilde{u}_{1,t-1} + I_2\tilde{u}_{2,t-1} + I_3\tilde{u}_{3,t-1}) \tag{10}$$

$$+\underbrace{\phi_{1,2}(\phi_{1,1} + \phi_{2,2})}_{\equiv\psi_{1,2}}(I_1\tilde{u}_{1,t-2} + I_2\tilde{u}_{2,t-2} + I_3\tilde{u}_{3,t-2}) + ... \tag{11}$$

$$\beta_{1t} = \tilde{\phi}_1 u_{X,t-1} + (\phi_{1,1} + \phi_{2,2})\tilde{\phi}_1 u_{X,t-2} + \tilde{u}_{1t} + \phi_{22}\tilde{u}_{1,t-1} + (\phi_{1,2}\tilde{\phi}_1 I_1 + \phi_{22}^2)\beta_{1,t-2} + \cdots \tag{12}$$

$$\beta_{2t} = \tilde{\phi}_2 u_{X,t-1} + (\phi_{1,1} + \phi_{2,2})\tilde{\phi}_2 u_{X,t-2} + \tilde{u}_{2t} + \phi_{22}\tilde{u}_{2,t-1} + (\phi_{1,2}\tilde{\phi}_2 I_1 + \phi_{22}^2)\beta_{2,t-2} + \cdots \tag{13}$$

$$\beta_{3t} = \tilde{\phi}_3 u_{X,t-1} + (\phi_{1,1} + \phi_{2,2})\tilde{\phi}_3 u_{X,t-2} + \tilde{u}_{3t} + \phi_{22}\tilde{u}_{3,t-1} + (\phi_{1,2}\tilde{\phi}_3 I_1 + \phi_{22}^2)\beta_{3,t-2} + \cdots \tag{14}$$

i.e., using a more general notation:

$$X_t = u_{X,t} + \theta_{1,1}u_{X,t-1} + \theta_{1,2}u_{X,t-2} + \psi_{1,1}(\sum_{j=1}^{q} I_j\tilde{u}_{j,t-1}) \tag{15}$$

$$+\psi_{12}(\sum_{j=1}^{q} I_j\tilde{u}_{j,t-2}) + ...$$

$$\beta_{1,t} = \theta_{2,1}u_{X,t-1} + \theta_{2,2}u_{X,t-2} + \tilde{u}_{1,t} + \psi_{2,2}\tilde{u}_{1,t-1} + \cdots \tag{16}$$

$$...$$

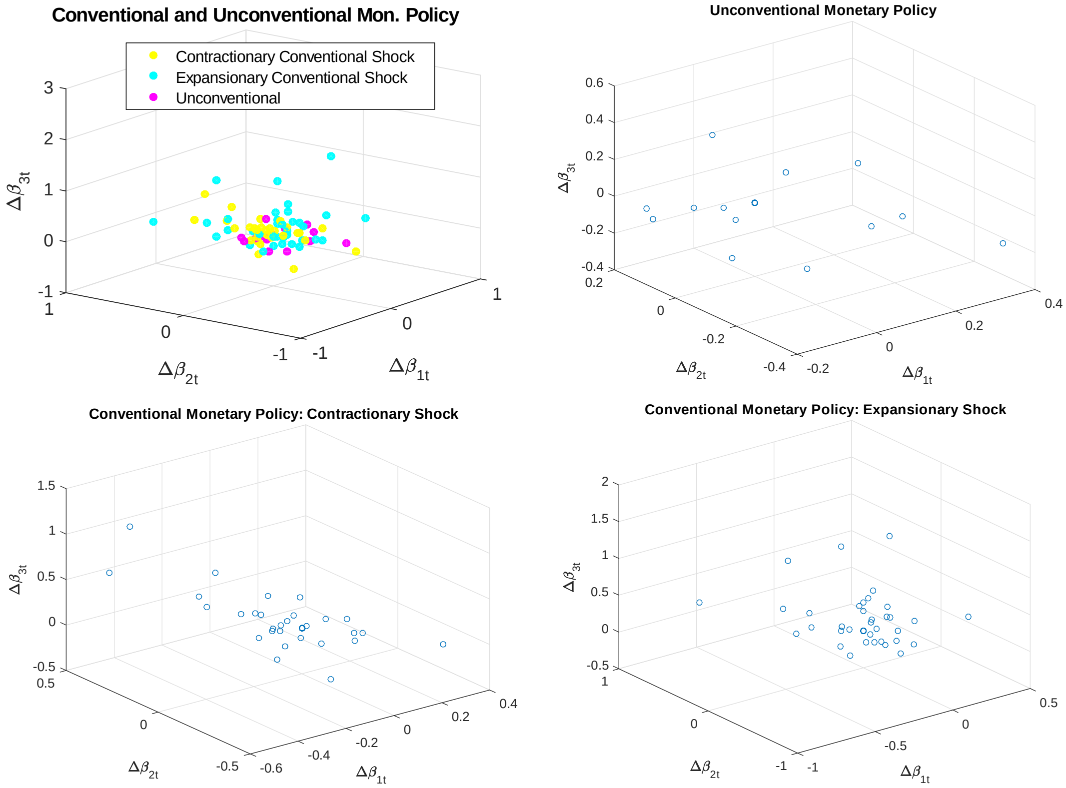
$$\beta_{q,t} = \theta_{q+1,1}u_{X,t-1} + \theta_{q+1,2}u_{X,t-2} + \tilde{u}_{q,t} + \psi_{q+1,2}\tilde{u}_{q,t-1} + \cdots , \tag{17}$$

where $\theta_{1,1} = \phi_{1,1}$, $\theta_{1,2} = (\phi_{1,1}^2 + \phi_{1,2}\int w(\tau)\phi_{2,1}(\tau)d\tau)$, $\theta_{2,1} = \tilde{\phi}_1$, $\theta_{2,2} = (\phi_{1,1} + \phi_{2,2})\tilde{\phi}_1$, $\psi_{1,1} = \phi_{1,2}$, $\psi_{1,2} = \phi_{1,2}(\phi_{1,1} + \phi_{2,2})$, etc.

6 Additional Evidence on the Multi-Dimensionality of the Monetary Policy Shock

Can monetary policy be fully summarized by movements in short-term interest rates (a situation which we refer to as "one-dimensional monetary policy", following Gürkaynak et al., 2005a), or is monetary policy operating in other ways as well? We investigate this issue by plotting the monetary policy shocks in Figure D. If monetary policy shocks were "one-dimensional" then all the shocks should line up along one dimension, that is, they should belong to the same line. The figure visually suggests that this is not the case. To control for the possible asymmetry of monetary policy shocks, we consider expansionary and contractionary shocks separately, and we also distinguish between conventional and unconventional monetary policy periods. In particular, both unconventional and expansionary conventional monetary policy shocks, depicted in the graphs on the right, seem scattered along more than two dimensions. The contractionary shocks instead, depicted on the bottom left graph, visually appear to be lying on a plane.

Figure D. Monetary Policy Shocks in the Nelson and Siegel Model



Notes. The scatterplots depict the monetary policy shocks as a function of the factors $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$.

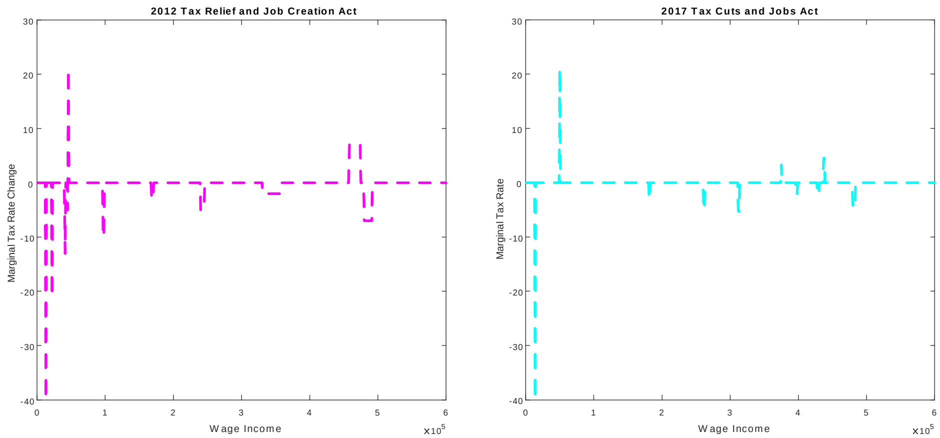
7 Additional Examples of Functional Shocks

This section discusses two additional examples of functional shocks. The first example is a functional tax policy shock. We focus on two episodes: the American Taxpayer Relief Act and Middle Class Tax Relief and Job Creation Act in 2012 and the Tax Cuts and Jobs Act in 2017. Panel 1 in Figure E plots the changes in the marginal tax rates in 2008 and in 2018 as a function of wage income. The calculations are based on a representative household that consists of a 38-year-old taxpayer, a 38-year-old spouse and two children under 13 and are done using Internet TAXSIM (version 27). In the simulations, we assume a representative adult who, in both years, earned the same salary, received \$100 dividend income, \$100 interest income, no capital gains or losses other than wage income and no other income, and spent \$3000 on real estate taxes, \$10000 on child care expenses and \$10000 on mortgage. The pictures show the functional shocks in the two years: the change in the marginal tax schedule as a function of wage income. Clearly, the change in the tax schedule affects individuals differently, depending on their wage income, and our functional shock can describe such heterogeneity.

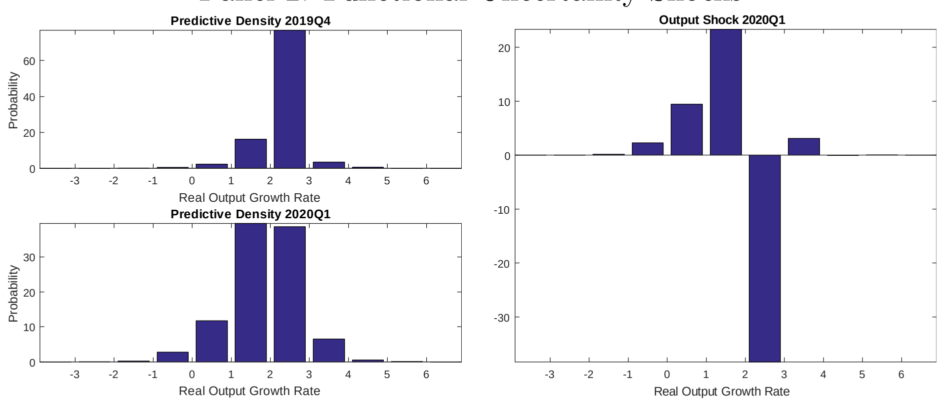
The second example is a functional uncertainty shock. The coronavirus COVID-19 pandemic created a huge uncertainty shock. There are several measures of uncertainty; one such measure (ex-ante uncertainty) is the change in the predictive density (see e.g. Rossi, Sekhposyan and Soupre, 2016). The Survey of Professional Forecasters publishes forecast densities of several macroeconomic variables, including real output growth. The graph on the left of Figure E, Panel 2, shows the forecast densities in the fourth quarter of 2019 and the first quarter of 2020, before and after the COVID-19 pandemic shock. The functional uncertainty shock is the downward shift in the predictions of future output growth, and is depicted on the panel on the right.

Figure E. Monetary Policy Shocks in the Nelson and Siegel Model

Panel 1. Functional Tax Policy Shocks



Panel 2. Functional Uncertainty Shocks



References

Krishnamurthy, A. and A. Vissing-Jorgensen (2011), "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy", *Brookings Papers on Economic Activity* 42(2), 215-287.

Nelson, C.R. and A.F. Siegel (1987), "Parsimonious Modeling of Yield Curve", *Journal of Business* 60, 473–489.

Rossi, B., T. Sekhposyan and M. Soupre (2016), "Understanding the Sources of Macroeconomic Uncertainty", *CEPR Discussion Paper* 11415.