

# Supplement to “The aggregate effects of labor market frictions”

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In this Online Appendix, we provide further detail on additional sensitivity analyses conducted relative to the benchmark models described in the paper (Section D), as well as information on the numerical methods used to compute the quantitative illustrations summarized in the paper (Section E).

## APPENDIX D: SENSITIVITY ANALYSES

This Appendix describes our parameterization of the process for idiosyncratic shocks, contrasts it with alternatives in the literature, and performs sensitivity analyses on our main results to reasonable changes in the parameters of the shock process.

*Idiosyncratic shock process* Recall that we specify the process of idiosyncratic productivity as a geometric AR(1),

$$\ln x_{t+1} = \rho_x \ln x_t + \varepsilon_{xt+1}, \quad (1)$$

where  $\varepsilon_{xt+1} \sim N(0, \sigma_x^2)$  and  $t$  denotes *quarters*. Note that  $\ln x$  is implicitly demeaned here. In what follows, we infer  $\rho_x$  and  $\sigma_x$  from estimates of the persistence and volatility of idiosyncratic productivity in data of *different* frequencies.

*Low(er) frequency data* Our baseline parameterization is informed in large part by Abraham and White (2006), who measure plant-level total factor productivity (TFP) using annual microdata for the US manufacturing sector. We interpret their measurement of annual log TFP as being  $X_s \equiv \sum_{t=1}^4 \ln x_{st}$ , where  $\ln x_{st}$  is the realization in quarter  $t$  of year  $s$ . In other words, the measurement of annual log TFP reflects the accrual of log points (in excess of mean) throughout the four quarters of the year.

Abraham and White run least squares regressions of  $X_s$  on  $X_{s-1} \equiv \sum_{t=1}^4 \ln x_{s-1t}$ . An average of estimated slope coefficients from their weighted and unweighted regressions

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is

$$\frac{\text{cov}(X_s, X_{s-1})}{\text{var}(X_{s-1})} = 0.39. \quad (2)$$

Assuming that the annual data are generated from the quarterly process (1), one can relate the covariance and variance terms in (2) to  $\rho_x$  and  $\sigma_x$  as follows:

$$\text{cov}(X_s, X_{s-1}) = [\rho_x + 2\rho_x^2 + 3\rho_x^3 + 4\rho_x^4 + 3\rho_x^5 + 2\rho_x^6 + \rho_x^7] \frac{\sigma_x^2}{1 - \rho_x^2}, \quad (3)$$

and,

$$\text{var}(X_{s-1}) = [4 + 6\rho_x + 4\rho_x^2 + 2\rho_x^3] \frac{\sigma_x^2}{1 - \rho_x^2}. \quad (4)$$

Solving for the implied quarterly persistence yields  $\rho_x = 0.68$ . To infer  $\sigma_x$ , we use Abraham and White's estimate of  $\text{var}(X_{s-1}) = 0.21$ , which in turn implies that  $\sigma_x = 0.1034$ .

Foster, Haltiwanger, and Syverson (2008) also provide estimates of the productivity process based on plant-level measurements of TFP. Unlike Abraham and White, they restrict their sample to several industries for which real output can be most credibly measured. This mitigates measurement error at the expense of a more representative panel. In addition, Foster et al.'s data are not annual, but are drawn instead from the quinquennial Census of Manufacturers. Nonetheless, we can convert their estimates of the persistence and volatility of productivity to a quarterly frequency by following the same steps above. We find that  $\rho_x = 0.943$  and  $\sigma_x = 0.022$ .<sup>1</sup>

*Quarterly frequency* Cooper, Haltiwanger, and Willis (2015) estimated a dynamic labor demand model using quarterly data. They specify a geometric AR(1) for idiosyncratic productivity, as in (1), and parameterize the process in order to replicate certain moments concerning labor inputs. They estimate a persistence parameter of  $\rho_x = 0.4$  and an innovation standard deviation of  $\sigma_x = 0.5$ .

*Monthly frequency* Cooper, Haltiwanger, and Willis (2007) estimated a search and matching model using monthly data. Accordingly, they specify an idiosyncratic productivity process that follows a geometric AR(1) at a *monthly* frequency. Their estimates of the monthly analogues to  $\rho_x$  and  $\sigma_x$  are  $\rho_x^m = 0.395$  and  $\sigma_x^m = 0.212$ .<sup>2</sup>

Now taking their monthly AR(1) as the underlying driving force, we can calculate the implied persistence and volatility of quarterly data. Specifically, letting  $x_{mt}$  denote the realization in month  $m \in \{1, 2, 3\}$  of quarter  $t$ , we compute the persistence and volatility

<sup>1</sup>These estimates correspond to their measure of "physical TFP"

<sup>2</sup>These average Cooper et al.'s results for two specifications of adjustment costs that seem most germane to our application: (i) a fixed cost to adjust and linear cost to hire, and (ii) a fixed cost to adjust and linear cost to fire.

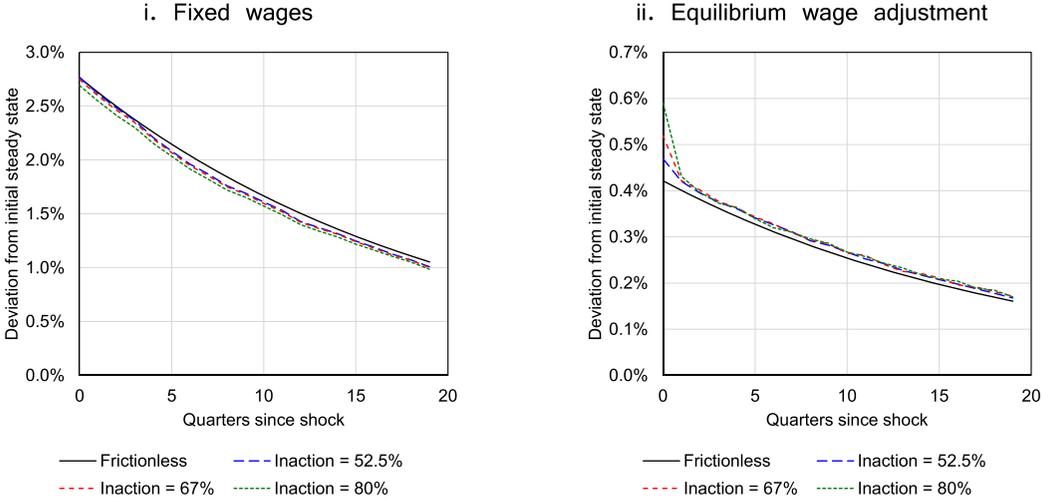


FIGURE A. Impulse responses of mandated and frictionless employment: fixed costs. *Notes:* Each panel plots the impulse response of aggregate frictionless employment in contrast to the impulse responses of mandated employment for each stated average quarterly inaction rate.

of  $\sum_{m=1}^3 \ln x_{mt}$ . This leads to

$$\rho_x = \frac{\rho_x^m + 2(\rho_x^m)^2 + 3(\rho_x^m)^3 + 2(\rho_x^m)^4 + (\rho_x^m)^5}{3 + 4\rho_x^m + 2(\rho_x^m)^2} = 0.194, \quad (5)$$

and

$$\sigma_x^2 = (1 - \rho_x^2) \frac{3 + 4\rho_x^m + 2(\rho_x^m)^2}{1 - (\rho_x^m)^2} (\sigma_x^m)^2 = 0.251. \quad (6)$$

This implies a standard deviation  $\sigma_x = 0.501$ .

*Mandated vs. frictionless employment* In Section 1.1 of the main text, we noted that, strictly speaking,  $n^*$  should be interpreted as *mandated employment*—that is, the level of employment that the firm would choose if the adjustment friction were suspended *in the current period only*. This is conceptually distinct from *frictionless employment*—the level of employment the firm would choose if the adjustment friction were suspended *indefinitely*.

We claimed that the quantitative distinction between these is minor, however. Figure A provides some justification for that claim. It depicts the impulse responses of both mandated and frictionless employment implied by the quantitative investigations of the fixed costs model underlying Figure 2 in the main text, with and without equilibrium wage adjustment. With fixed wages, the dynamics of mandated and frictionless employment are almost indistinguishable. With equilibrium wage adjustment, some small differences appear, with mandated employment responding a little more on impact, and the more so the greater the fixed cost. In comparison to the overshooting of flow-balance

employment apparent in Figure 2 of the main text, however, these differences between mandated and frictionless employment are very small. Thus, the message of Figure 2 is essentially unimpaired.

*Robustness of results* We now probe the robustness of our results to variation in the parameterization of the productivity process. To conserve space, we look at two models: one includes a fixed cost of adjusting, and the other includes symmetric per-worker costs to hire and fire.

We conduct four simulations of each model. In one,  $\rho_x = 0.9$ , which places it in the neighborhood of the estimate in Foster et al. In another,  $\rho_x = 0.3$ , which is the midpoint between the two estimates in Cooper et al. For both of these simulations,  $\sigma_x$  is set to its baseline value of 0.15. In the third and fourth simulations,  $\rho_x$  is returned to its baseline value (of 0.7), and  $\sigma_x$  is varied. In the third, the baseline value of  $\sigma_x$  is halved, which reduces it in the direction of Foster et al. In the fourth,  $\sigma_x$  is doubled, which raises it in the direction of Cooper et al. (For results for still larger  $\sigma_x$ s, see the simulations of the search model in Appendix C.) In *every* parameterized version of each model, the cost of adjusting is set to induce an inaction rate of 67%, which is in the middle of the range we consider in the main text.

Figures B and C present the results. Figure B illustrates the impulse responses from models with a fixed cost of adjusting. The top panel varies  $\rho_x$ , whereas the bottom panel varies  $\sigma_x$ . These results are difficult to distinguish from the baseline case presented in the main text (Figure 2.B), with one exception, namely the case where  $\rho_x = 0.3$ . The rise in flow-balance employment on impact is smaller when  $\rho_x$  is smaller, though still substantially larger than that of its frictionless counterpart.

Figure C presents results from models with a (symmetric) linear cost of adjusting. Again, flow-balance employment rises slightly less in the case where  $\rho_x$  is small. But, the impulse responses are very similar to what we present in the main text (Figure 3.B).

*Matching the establishment-size distribution* It is difficult for adjustment cost models to replicate the empirical establishment-size distribution when calibrated with Gaussian idiosyncratic shocks, as we and many others do. Specifically, the empirical firm-size distribution is well known to have a Pareto right tail, something that is necessarily missed with Gaussian shocks.

We have explored the robustness of our results to this issue in an extended quantitative exercise that augments the model underlying Figure 10 in the paper to accommodate the empirical establishment-size distribution. We do this by adding fixed productivity differences to firms in the model. Specifically, we assume that a firm's idiosyncratic productivity is now given by  $\varphi \cdot x$ , where  $x$  is a geometric Gaussian AR(1) (as before), and  $\varphi$  is a time-invariant firm fixed-effect distributed according to a Pareto( $m, s$ ) distribution. In the latter,  $m$  is the minimum and  $s$  is the "shape" that controls the mass in the right tail of firm productivities.

Starting from the model underlying Figure 10, we calibrate  $m$  and  $s$  in the distribution of fixed effects to match the establishment-size distribution in the QCEW data. Specifically, we use  $m$  and  $s$  to target the share of establishments with 10 employees or fewer (73.7% in the QCEW), and the share with 100 employees or more (2.4%). The

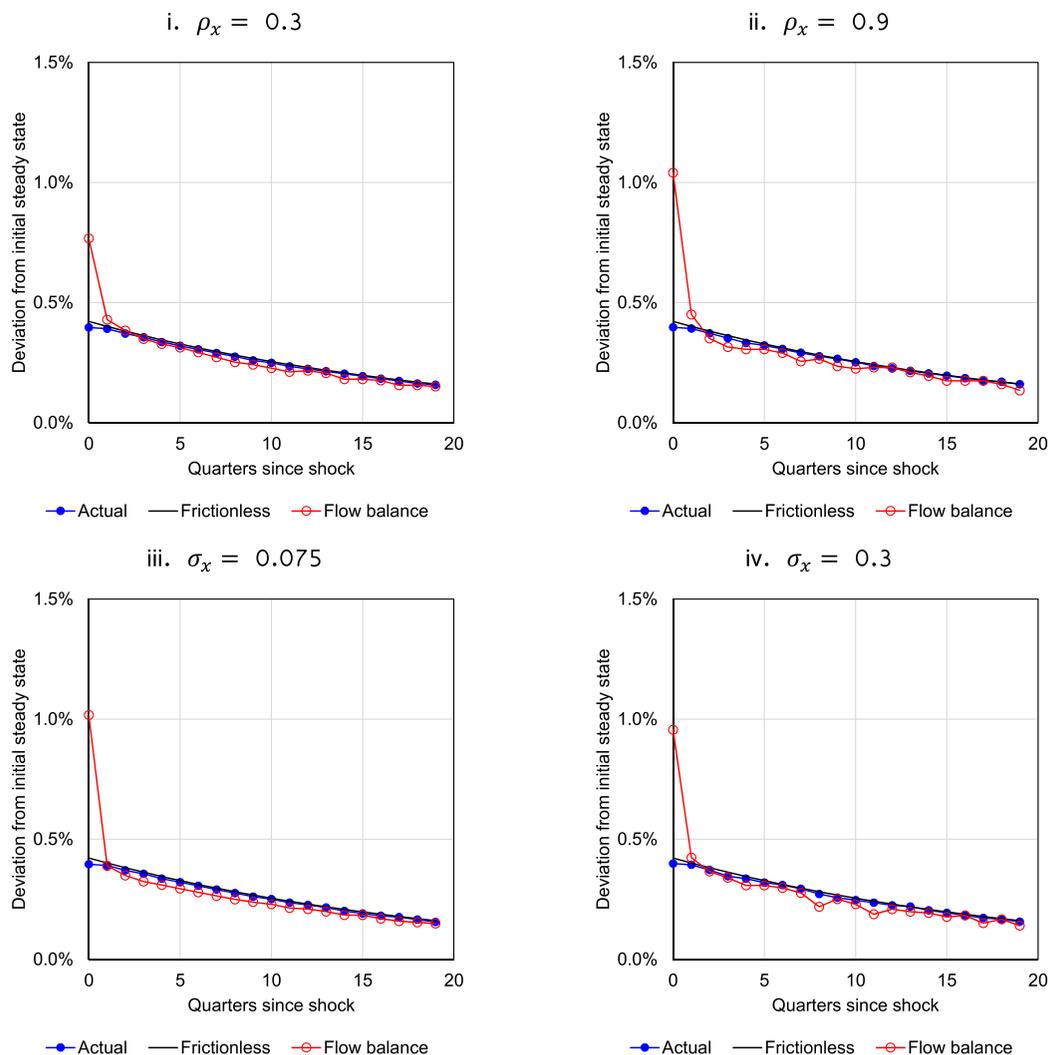


FIGURE B. Impulse responses of aggregate employment under alternative calibrations: fixed costs.

model also matches the (non-targeted) share of establishments over 200 employees (0.9%), but slightly undershoots the (nontargeted) share above 500 employees (0.1% versus 0.3%). As before, we calibrate the linear adjustment cost to maintain the same rate of inaction (86%) as in Figure 10.

Figure D plots the impulse responses of actual employment and flow-balance employment in the augmented model that matches the establishment-size distribution, and compares it with the original case depicted in Figure 10B. Qualitatively, the results are very similar in the augmented model that matches the size distribution. Thus, matching the establishment-size distribution appears to have little impact on the implications for our flow-balance statistic. This makes sense: one would expect that Proposition 2 would hold for each fixed effect. Since we find that a case with fixed wages is most

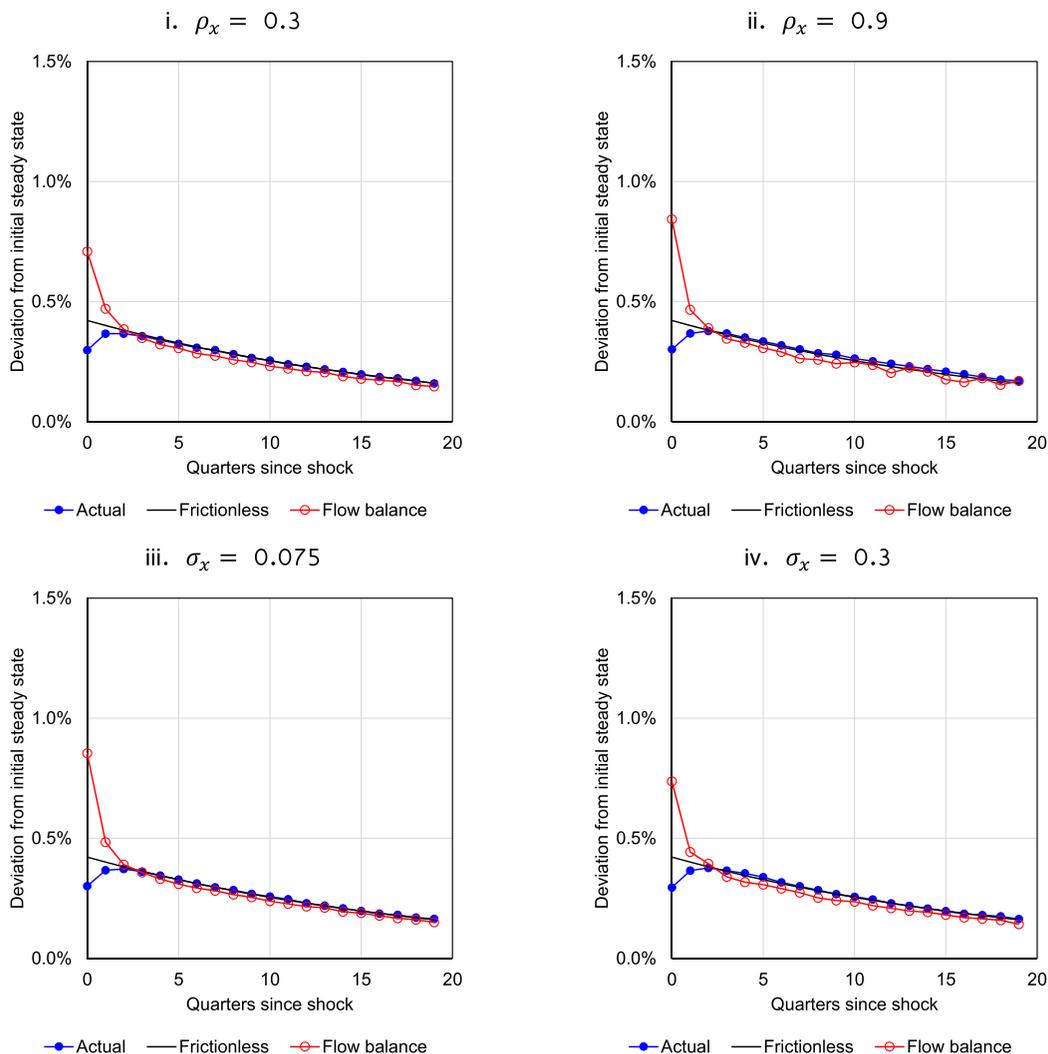


FIGURE C. Impulse responses of aggregate employment under alternative calibrations: linear costs.

able to match the dynamics of actual employment, there are in turn no equilibrium feedback effects on wages in the model. Nonetheless, it is reassuring that this makes little difference to the overall message of the paper.

#### APPENDIX E: DETAILS OF NUMERICAL METHODS

We detail how we solve the labor market equilibrium models in the main text. As in many other applications of heterogeneous agent models, our state space is, in principle, infinite dimensional. The reason for this is that firms forecast the future wage to make their labor demand decision. The future wage is, in turn, jointly determined with aggregate employment, which is a function of the full distribution of firm size. We follow [Krusell](#)

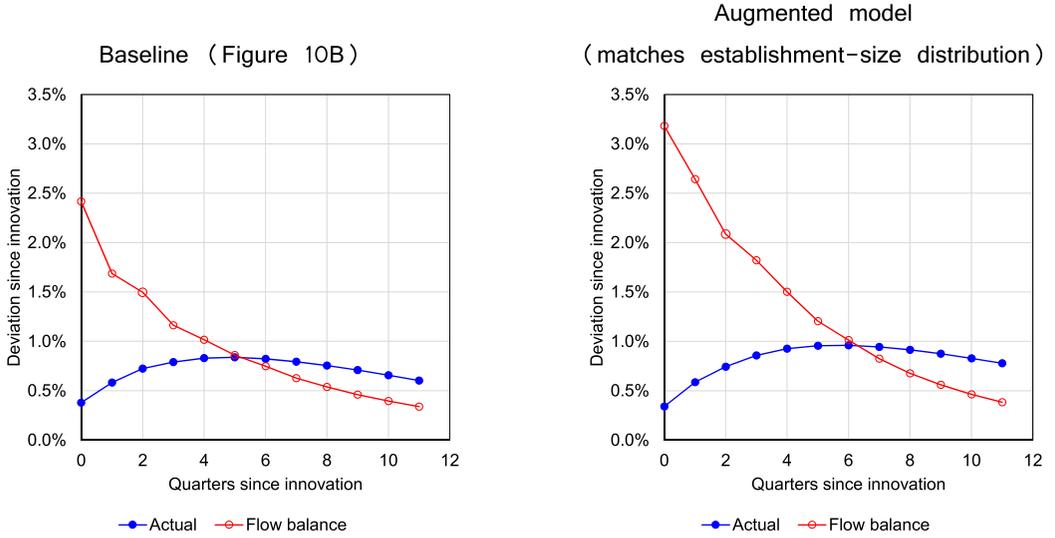


FIGURE D. Impulse responses of aggregate employment under alternative calibrations: linear costs.

and Smith’s (1998) bounded rationality approach to prune the state space. Specifically, we find that firms are able to make a very accurate projection of aggregate employment one period ahead,  $N'$ , based only on knowledge of the current mean of the firm size distribution,  $N$ , as well as current productivity  $p$ ,

$$\ln N' = \nu_0 + \nu_N \ln N + \nu_p \ln p. \quad (7)$$

This is the forecast rule for aggregate employment that firms use in all of our models of Section 1.

In the search model, firms have to forecast one more price, namely, market tightness,  $\theta$ . This, again, is a function of the distribution of firm size as well as productivity. Fortunately, we find that variation in tightness can be almost entirely accounted for (in a statistical sense) by variation in mean firm size  $N$  and  $p$ . Therefore, we assume that firms anticipate that tightness obeys the relation,

$$\ln \theta = \theta_0 + \theta_N \ln N + \theta_p \ln p. \quad (8)$$

The coefficients in (7) and (8) solve a fixed-point problem. Our general approach is to, first, conjecture the coefficients and solve the firms’ optimal labor demand policies. We then simulate the decision rules for 250,000 firms over 200 quarters (the latter excludes the “burn-in” time).<sup>3</sup> This yields a time series for  $N$ , and we run the regression (7). If the implied coefficients differ from our initial conjecture, we update and repeat. We implement this routine for the models of Sections 1.3 and 1.4.

<sup>3</sup>Note that (7) and (8) are not perfect forecasts, and thus we are required to solve for a fixed point in  $N$  (and  $\theta$ , where applicable) at every period of the simulation.

TABLE 2. Estimated parameters of Krusell–Smith forecast equations.

Adjustment Cost	Inaction Rate	Employment Regression				Tightness Regression			
		$\nu_0$	$\nu_N$	$\nu_p$	$R^2$	$\Theta_0$	$\Theta_N$	$\Theta_p$	$R^2$
Fixed	52.5%	2.951	0.013	0.417	0.9999	–	–	–	–
	67%	2.875	0.036	0.406	0.9999	–	–	–	–
	80%	2.711	0.087	0.383	0.9998	–	–	–	–
Linear	52.5%	2.411	0.174	0.350	0.9999	–	–	–	–
	67%	2.007	0.308	0.289	0.9999	–	–	–	–
	80%	1.431	0.505	0.200	0.9998	–	–	–	–
Search	52.5%	0.509	0.824	0.098	0.9987	–53.161	18.159	11.211	0.9994
	67%	0.417	0.857	0.074	0.9992	–68.639	23.247	12.494	0.9996
	80%	0.315	0.892	0.055	0.9995	–96.489	32.918	11.951	0.9994

Our approach to the search model (Section 1.5) differs in two respects. First, time aggregation is a more acute problem in the search model because of the worker flows. As noted in the main text, we solve and simulate the model at a bi-weekly frequency. Second, the size of the idiosyncratic shocks ( $\sigma_x$ ) is much larger. As a result, the law of large numbers does not “kick in” even if we use a very large number of firms, which means the simulated paths of  $N$  and  $\theta$  are noisy. We thus switch to using Young’s (2010) nonstochastic simulator, which constructs the joint distribution of  $n$  and  $x$  in order to aggregate up to  $N$ . This yields smoother time series. For this simulator, we need at least 500 grid points in the productivity ( $x$ ) dimension.

Table 2 summarizes the estimates of (7) and (8). We make two observations. First, the pattern in the coefficient estimates is reasonably intuitive. For instance, when adjustment frictions are higher, the coefficient  $\nu_p$  on  $\ln p$  in (7) is typically lower and the coefficient  $\nu_N$  on lagged employment is typically higher. Second, the goodness of fit, as summarized by the  $R^2$ , is especially excellent in the models of fixed and time-invariant linear costs. As den Haan (2010) has noted, though,  $R^2$  measures only the quality of the one-step ahead forecast. But, one may use the estimates in the table to verify, as we have, that the entire impulse response of aggregate employment implied by (7) also closely matches the impulse response simulated from the heterogeneous-agent models and shown in Figures 2 through 4. The fit of the regressions in the search model is slightly worse, and as a result, there is a slight bit of “daylight” between the impulse responses simulated from the model and those implied directly from the forecast equations. For instance, for the baseline parameterization (of 52.5% inaction), the model-generated IRF of aggregate employment peaks after 15 fortnights at 0.446%, whereas the IRF implied by (7) peaks after 18 fortnights at 0.441%.

The last item to address is the simulation of the impulse response of flow-balance employment,  $\hat{N}$ . As noted in the main text, we track the migration of firms into and out of each integer-valued employment level,  $n$ , to calculate the two ingredients of the flow-balance density, namely, the share of firms that flow into each  $n$  and the probability of outflow from each  $n$ . This mimics what we do in the data, with one slight exception. When we work with the data, we noted that, to reduce noise in our estimates of the

flows at high levels of employment, we compute the flows within bins that pool together a range of employer sizes (i.e., 501–510). We do not have to implement these bins in our model-generated data. Bins are necessary only if we aim to replicate the share of very large establishments (i.e., with over 500 workers), which can be done if we introduce a distribution of time-invariant productivity into the model. This is computationally cumbersome, but we have done it for the baseline parameterization, in which inaction is 52.5% per quarter. The impulse responses are virtually identical to what we presented in the main text, where we omit a fixed productivity distribution.

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