List of equation for Smets and Wouters (2007) model

some additional parameter definitions:

$$\tilde{\pi} = 1 + \bar{\pi}/100 \tag{1}$$

$$\tilde{\gamma} = 1 + \bar{\gamma}/100 \tag{2}$$

$$\beta = \frac{1}{\bar{R}^f \exp\left(\sigma_m^2/2\right) \tilde{\gamma}^{-\sigma_c}} \tag{3}$$

$$\bar{R}^n = \frac{\tilde{\pi}}{\beta \exp\left(\sigma_m^2/2\right) \tilde{\gamma}^{-\sigma_c}} \tag{4}$$

$$\bar{R}^k = \bar{R}^f \exp\left(-\sigma_{mR^k}\right) - (1-\delta) \tag{5}$$

$$\bar{w} = \left(\frac{\alpha^\alpha \left(1-\alpha\right)^{1-\alpha}}{\theta_p \left(\bar{R}^k\right)^\alpha}\right)^{\frac{1}{1-\alpha}} \tag{6}$$

$$\tilde{wc} = \frac{1}{\theta_w} \frac{(1-\alpha)}{\alpha} \bar{R}^k \frac{\bar{k}}{\bar{y}} \frac{\bar{y}}{\bar{c}} \tag{7}$$

$$\frac{\bar{i}}{\bar{k}} = \left(1 - \frac{1-\delta}{\tilde{\gamma}}\right) \tilde{\gamma} \tag{8}$$

$$\frac{\bar{l}}{\bar{k}} = \frac{1-\alpha}{\alpha} \frac{\bar{R}^k}{\bar{w}} \tag{9}$$

$$\frac{\bar{k}}{\bar{y}} = \Phi \left(\frac{\bar{n}}{\bar{k}}\right)^{\alpha-1} \tag{10}$$

$$\frac{\bar{i}}{\bar{y}} = \frac{\bar{i}}{\bar{k}} \frac{\bar{k}}{\bar{y}} \tag{11}$$

$$\frac{\bar{c}}{\bar{y}} = 1 - \frac{\bar{g}}{\bar{y}} - \frac{\bar{i}}{\bar{y}} \tag{12}$$

exogenous shocks:

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{a,t} \tag{13}$$

$$\hat{b}_t = \rho_b \hat{b}_{t-1} + \epsilon_{b,t} \tag{14}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t} \tag{15}$$

$$\hat{q}_t = \rho_q \hat{q}_{t-1} + \epsilon_{q,t} \tag{16}$$

$$\hat{m}_t = \rho_m \hat{m}_{t-1} + \epsilon_{m,t} \tag{17}$$

$$\hat{s}_{p,t} = \rho_p \hat{s}_{p,t-1} + \epsilon_{p,t} - \mu_p \epsilon_{p,t-1} \tag{18}$$

$$\hat{s}_{w,t} = \rho_w \hat{s}_{w,t-1} + \epsilon_{w,t} - \mu_w \epsilon_{w,t-1} \tag{19}$$

flexible economy:

$$\hat{a}_t = \alpha \hat{rk}_t^f + (1-\alpha)\hat{w}_t^f \tag{20}$$

$$\hat{u}_t^f = \left(\frac{1}{\Psi/(1-\Psi)}\right)\hat{rk}_t^f \tag{21}$$

$$\hat{rk}_t^f = \hat{w}_t^f + \hat{L}_t^f - \hat{k}_t^f \tag{22}$$

$$\hat{k}_t^f = \hat{\bar{k}}_{t-1}^f + \hat{u}_t^f \tag{23}$$

$$\hat{x}_t^f = \left(\frac{1}{1+\bar{\beta}\tilde{\gamma}}\right)\left(\hat{x}_{t-1}^f + \bar{\beta}\tilde{\gamma}\hat{x}_{t+1}^f + \frac{1}{\tilde{\gamma}^2\varphi}\hat{pk}_t^f\right) + \hat{q}_t \tag{24}$$

$$\hat{pk}_t^f = -\hat{rr}_t^f + \frac{\frac{1}{1-h/\tilde{\gamma}}}{\sigma_c\left(1+h/\tilde{\gamma}\right)}\hat{b}_t + \frac{\bar{R}^k}{\bar{R}^k + (1-\delta)}\hat{rk}_{t+1}^f + \frac{(1-\delta)}{\bar{R}^k+(1-\delta)}\hat{pk}_{t+1}^f \tag{25}$$

$$\hat{c}_t^f = \frac{h/\tilde{\gamma}}{1+h/\tilde{\gamma}}\hat{c}_{t-1}^f + \frac{1}{1+h/\tilde{\gamma}}\hat{c}_{t+1}^f + \frac{(\sigma_c-1)\,\tilde{wc}}{\sigma_c\left(1+h/\tilde{\gamma}\right)}\left(\hat{l}_t - \hat{l}_{t+1}\right) - \frac{1-h/\tilde{\gamma}}{\sigma_c\left(1+h/\tilde{\gamma}\right)}\hat{rr}_t^f + \hat{b}_t \tag{26}$$

$$\hat{y}_t^f = \frac{\bar{c}}{\bar{y}}\hat{c}_t^f + \frac{\bar{i}}{\bar{y}}\hat{x}_t^f + \hat{g}_t + \bar{R}^k\frac{\bar{k}}{\bar{y}}\hat{u}_t^f \tag{27}$$

$$\hat{y}_t^f = \Phi\left(\alpha\hat{k}_t^f + (1-\alpha)\,\hat{l}_t^f + \hat{a}_t\right) \tag{28}$$

$$\hat{w}_t^f = \sigma_l\hat{l}_t^f + \frac{1}{1-h/\tilde{\gamma}}\hat{c}_t^f - \frac{h/\tilde{\gamma}}{1-h/\tilde{\gamma}}\hat{c}_{t-1}^f \tag{29}$$

$$\hat{\bar{k}}_t^f = \left(1-\frac{\bar{i}}{\bar{k}}\right)\hat{\bar{k}}_{t-1}^f + \frac{\bar{i}}{\bar{k}}\hat{x}_t^f + \frac{\bar{i}}{\bar{k}}\tilde{\gamma}^2\varphi\hat{q}_t \tag{30}$$

measurement equations:

$$\Delta\hat{y}_t = \hat{y}_t - \hat{y}_{t-1} + \tilde{\gamma} \tag{31}$$

$$\Delta\hat{c}_t = \hat{c}_t - \hat{c}_{t-1} + \tilde{\gamma} \tag{32}$$

$$\Delta\hat{i}_t = \hat{i}_t - \hat{i}_{t-1} + \tilde{\gamma} \tag{33}$$

$$\Delta\hat{w}_t = \hat{w}_t - \hat{w}_{t-1} + \tilde{\gamma} \tag{34}$$

$$\pi_t^{obs} = \hat{\pi}_t + \tilde{\pi} \tag{35}$$

$$R_t^{obs} = \hat{r}_t + \left(\bar{R}^n - 1\right)\cdot 100 \tag{36}$$

$$L_t^{obs} = \hat{l}_t + \bar{l} \tag{37}$$

$$R_t^q = (\omega\cdot\sigma_{r^q}\cdot 100) + \frac{1}{1-\Omega}\left(\hat{R}_t^k - \hat{r}_t + \hat{\pi}_{t+1}\right) + \epsilon_{t,Q}\cdot 10 \tag{38}$$

sticky wage and price economy:

$$\hat{mc}_t = \alpha \hat{rk}_t + (1-\alpha)\hat{w}_t - \hat{a}_t \tag{39}$$

$$\hat{rk}_t = \hat{w}_t - \hat{k}_t + \hat{l}_t \tag{40}$$

$$\hat{k}_t = \hat{\bar{k}}_{t-1} + \hat{u}_t \tag{41}$$

$$\hat{x}_t = \left(\frac{1}{1+\bar{\beta}\tilde{\gamma}}\right)\left(\hat{x}_{t-1} + \bar{\beta}\tilde{\gamma}\hat{x}_{t+1} + \frac{1}{\tilde{\gamma}^2\varphi}\hat{pk}_t\right) + \hat{q}_t \tag{42}$$

$$\hat{\lambda}_t = \frac{\sigma_c \cdot h/\tilde{\gamma}}{1-h/\tilde{\gamma}}\hat{c}_{t-1} - \frac{\sigma_c}{1-h/\tilde{\gamma}}\hat{c}_t + \frac{(\sigma_c - 1)\,\tilde{wc}}{1-h/\tilde{\gamma}}\hat{l}_t \tag{43}$$

$$\hat{M}_t = \hat{\lambda}_t - \hat{\lambda}_{t-1} \tag{44}$$

$$0 = \hat{M}_{t+1} + \hat{r}_t - \hat{\pi}_{t+1} - \frac{\sigma_c\,(1+h/\tilde{\gamma})}{1-h/\tilde{\gamma}}\hat{b}_t \tag{45}$$

$$0 = \hat{M}_{t+1} + \hat{R}^k_{t+1} \tag{46}$$

$$\hat{R}^k_t = \frac{\bar{R}^k}{\bar{R}^k + (1-\delta)}\hat{rk}_t + \frac{(1-\delta)}{\bar{R}^k + (1-\delta)}\hat{pk}_t - \hat{pk}_{t-1} \tag{47}$$

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{i}}{\bar{y}}\hat{x}_t + \hat{g}_t + \bar{R}^k\frac{\bar{k}}{\bar{y}}\hat{u}_t \tag{48}$$

$$\hat{y}_t = \Phi\left(\alpha\hat{k}_t + (1-\alpha)\,\hat{l}_t + \hat{a}_t\right) \tag{49}$$

$$\hat{\bar{k}}_t = \left(1 - \frac{\bar{i}}{\bar{k}}\right)\hat{\bar{k}}_{t-1} + \frac{\bar{i}}{\bar{k}}\hat{x}_t + \frac{\bar{i}}{\bar{k}}\tilde{\gamma}^2\varphi\hat{q}_t \tag{50}$$

$$\hat{r}_t = \rho\hat{r}_{t-1}(1-\rho)\left(r_{pi}\hat{\pi}_t + r_y\left(\hat{y}_t - \hat{y}^f_t\right) + r_{\Delta y}\left(\hat{y}_t - \hat{y}^f_t - \hat{y}_{t-1} + \hat{y}^f_{t-1}\right)\right) + \hat{m}_t \tag{51}$$

$$\hat{\pi}_t = \frac{1}{1+\bar{\beta}\tilde{\gamma}\iota_p}\left(\bar{\beta}\tilde{\gamma}\hat{\pi}_{t+1} + \iota_p\hat{\pi}_{t-1} + \frac{\left((1-\xi_p)\,\frac{\left(1-\bar{\beta}\tilde{\gamma}\xi_p\right)}{\xi_p}\right)}{(\Phi-1)\,\varepsilon_p + 1}\hat{mc}_t\right) + \hat{s}_{p,t} \tag{52}$$

$$\begin{aligned}\hat{w}_t = {} & \frac{1}{1+\bar{\beta}\tilde{\gamma}}\hat{w}_{t-1} + \frac{\bar{\beta}\tilde{\gamma}}{1+\bar{\beta}\tilde{\gamma}}\hat{w}_{t+1} + \frac{1+\bar{\beta}\tilde{\gamma}\iota_w}{1+\bar{\beta}\tilde{\gamma}}\hat{\pi}_t + \frac{\iota_w}{1+\bar{\beta}\tilde{\gamma}}\hat{\pi}_{t-1} + \frac{\bar{\beta}\tilde{\gamma}}{1+\bar{\beta}\tilde{\gamma}}\hat{\pi}_{t+1} \\ & + \frac{(1-\xi_w)\left(1-\bar{\beta}\tilde{\gamma}\xi_w\right)}{\left(1+\bar{\beta}\tilde{\gamma}\right)\xi_w\left((\theta_w-1)\,\varepsilon_w + 1\right)}\Big(\sigma_l\hat{l}_t + \\ & \frac{1}{1-h/\tilde{\gamma}}\hat{c}_t - \frac{h/\tilde{\gamma}}{1-h/\tilde{\gamma}}\hat{c}_{t-1} - \hat{w}_t\Big) + \hat{s}_{w,t}\end{aligned} \tag{53}$$

References

Smets, F. and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.