Econometrica Supplementary Material

SUPPLEMENT TO "IDENTIFICATION IN AUCTIONS WITH SELECTIVE ENTRY": NUMERICAL ILLUSTRATIONS OF IDENTIFIED SETS

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This supplement presents numerical illustrations of the pointwise identified and pointwise sharp identified sets derived in Section 3 and Appendix A of our main paper. Consistent with the paper's focus on identification, these should be interpreted as describing information in principle recoverable from a large auction sample.

WE ASSUME THAT THE JOINT DISTRIBUTION $F(v, s)$ follows a Gaussian copula $C_\rho(F_v, s)$, where the marginal distribution of values $F_v(\cdot)$ is Normal($\mu = 100$, $\sigma = 10$) and the entry cost is $c = 2$. The correlation parameter ρ measures the precision of S as a measure of V, with $\rho = 0$ generating the LS case and $\rho \to 1$ approaching the S case. In what follows, we present results for $\rho = 0.2$, $\rho = 0.75$, and $\rho = 0.95$, representing minimally, moderately, and highly selective entry processes, respectively. Except where noted otherwise, we assume potential competition N varies exogenously on the set $\mathcal{N} = \{2, 3, \dots, 16\}$.

Pointwise Identified Set

We first illustrate the bounds derived in Section 2. Using the equilibrium characterization in Theorem 1, it is straightforward to calculate the set of equilibrium entry thresholds $\mathcal{S} = \{s_2^*, \dots, s_{16}^*\}$ at each value of ρ considered. These and the corresponding ex post distributions $F^*(v; s_N^*)$ for each N are the objects identified by a standard $(N, n, \mathbf{b})$ sample.

We next use Proposition 3 to obtain identified bounds $F^+(\cdot|s)$ and $F^-(\cdot|s)$ on $F(\cdot|s)$. These bounds imply a pointwise identified set $\mathcal{F}_0$ in three-dimensional space, for which we provide two sets of graphical representations below. First, Figure S.1 illustrates the S dimension of our bounds for two values of v across ρ. The "stairstep" nature of these bounds in s follows from lack of information on $F^*(v; s)$ at points outside the identified set $\mathcal{S}$. Meanwhile, Figure S.2 illustrates the V dimension of our bounds at a selection of thresholds s_N^* in $\mathcal{S}$. Note the lack of informative upper (lower) bounds at the minimum (maximum) of $\mathcal{S}$, a point to which we return when discussing sharpness below.

Finally, we translate bounds on $F(\cdot|\cdot)$ derived from Proposition 3 into sharp bounds on c as in Proposition 4. In the examples considered here, these turn out to be quite tight: $c \in [1.990, 2.010]$ when $\rho = 0.2$, $c \in [1.977, 2.024]$ when $\rho = 0.75$, and $c \in [1.985, 2.016]$ when $\rho = 0.95$, where as above true $c = 2$.

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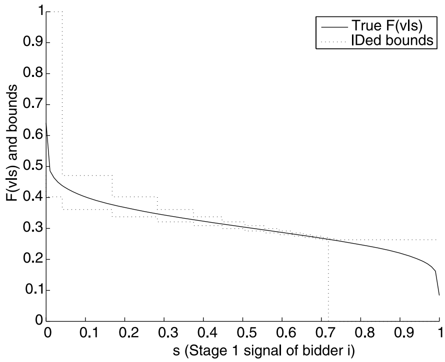
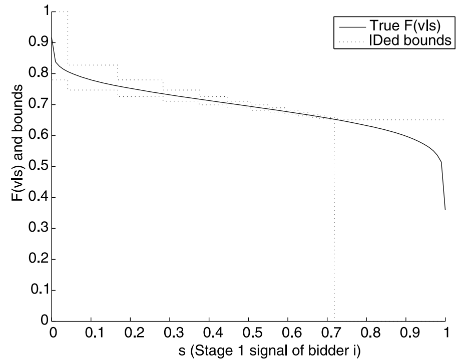
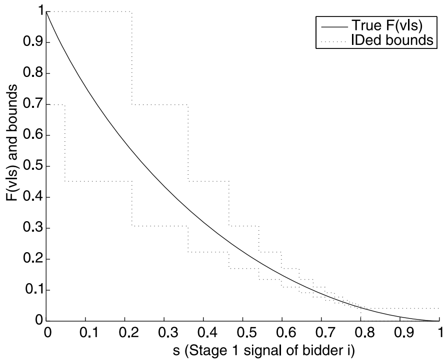


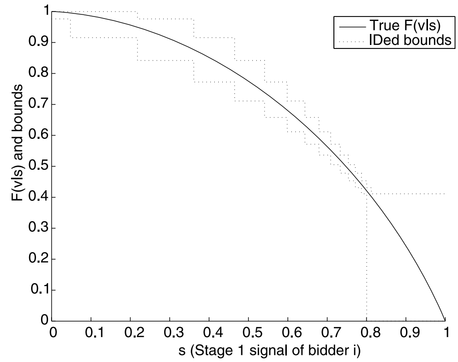
(a) Bounds at $v = 95$, $\rho = 0.2$



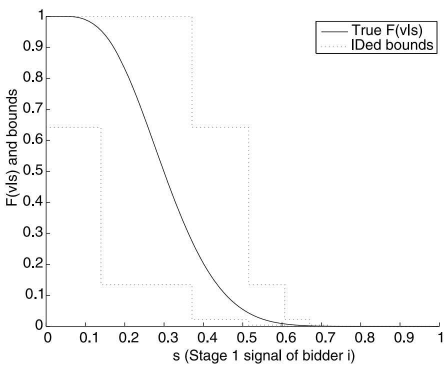
(b) Bounds at $v = 105$, $\rho = 0.2$



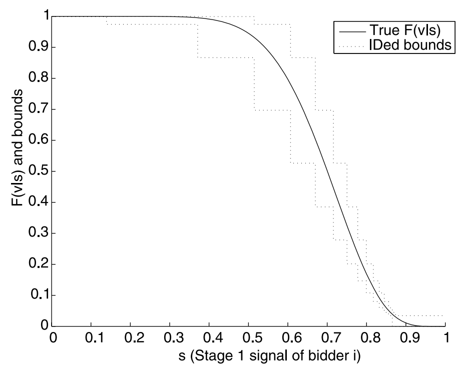
(c) Bounds at $v = 95$, $\rho = 0.75$



(d) Bounds at $v = 105$, $\rho = 0.75$

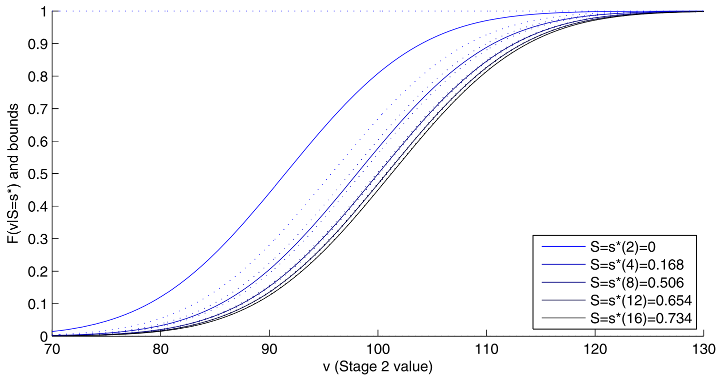


(e) Bounds at $v = 95$, $\rho = 0.95$

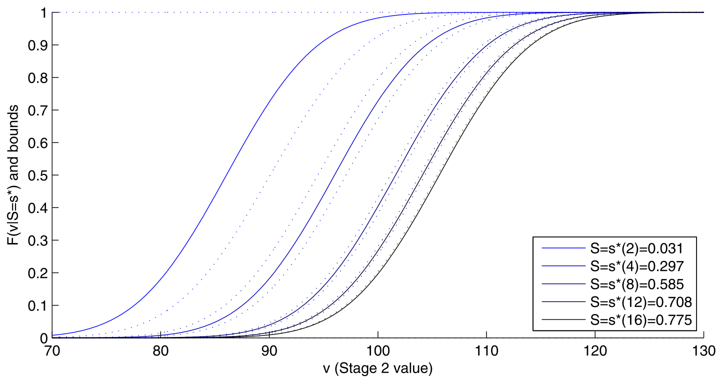


(f) Bounds at $v = 105$, $\rho = 0.95$

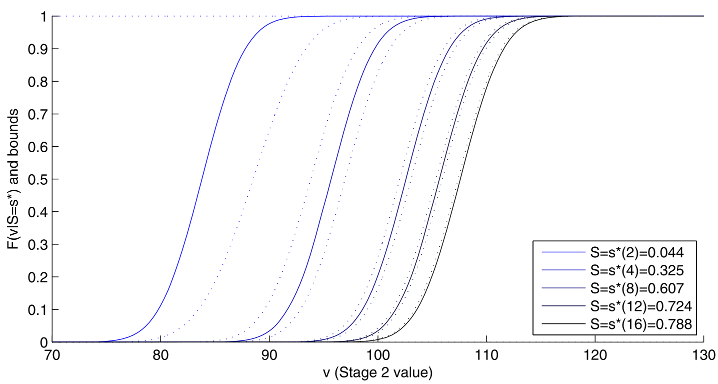
FIGURE S.1.—Bounds on $F(v|s)$ across S, $\mathcal{N} = \{2, \ldots, 16\}$.



(a) Bounds on $F(v|s_N^*)$ for selected N, $\rho = 0.2$

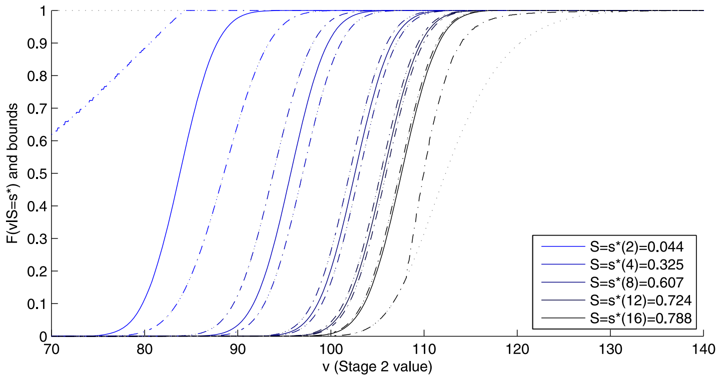


(b) Bounds on $F(v|s_N^*)$ for selected N, $\rho = 0.75$

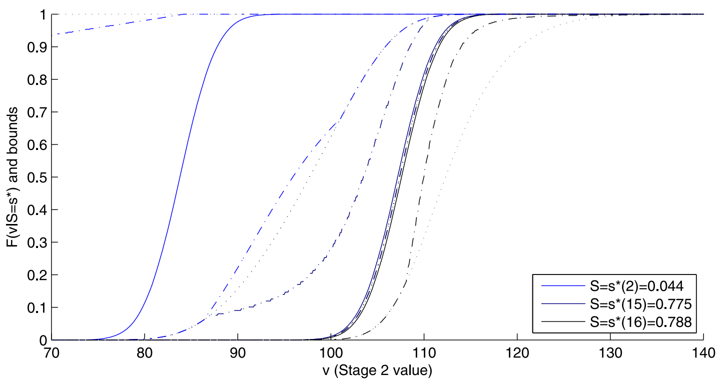


(c) Bounds on $F(v|s_N^*)$ for selected N, $\rho = 0.95$

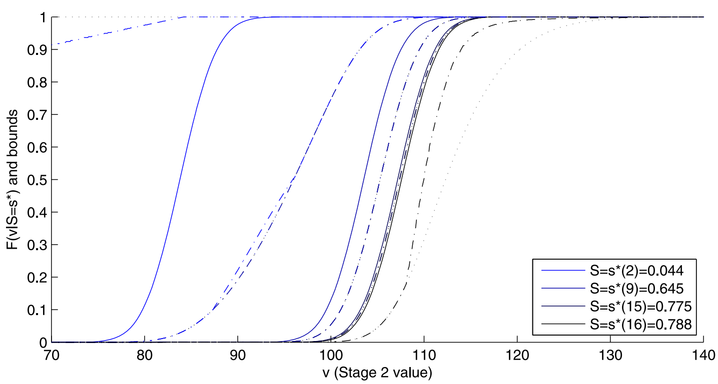
FIGURE S.2.—Bounds on $F(v|s)$ across $\mathcal{V}$, $\mathcal{N} = \{2, \ldots, 16\}$.



(a) Sharp bounds on $F(v|s_N^*)$ at selected N, $\mathcal{N} = \{2, 3, \dots, 16\}$



(b) Sharp bounds on $F(v|s_N^*)$ at selected N, $\mathcal{N} = \{2, 15, 16\}$



(c) Sharp bounds on $F(v|s_N^*)$ at selected N, $\mathcal{N} = \{2, 9, 15, 16\}$

FIGURE S.3.—Pointwise sharp bounds on $F(v|s)$, $\rho = 0.95$, various $\mathcal{N}$.

Pointwise Sharp Identified Set

We next apply the refinement in Appendix A to the initial identified sets $\mathcal{F}_0$ derived above. In particular, we first apply the test of sharpness in Proposition 5 on a grid in $\mathcal{V}$. Where appropriate, we then iterate as in Proposition 6 to obtain the pointwise sharp identified set.

Results of this procedure are encouraging: in all three examples, our initial bounds are pointwise sharp everywhere they are informative. Refinement is possible at some points with uninformative initial bounds, and in these cases Proposition 6 yields the pointwise sharp identified set in a single iteration. But on balance, the numerical results presented here suggest that the refinement in Appendix A will not be critical in applications.

We conclude this section with a counterexample: a data generating process (DGP) in which initially informative bounds can be refined. This example is of necessity somewhat artificial: bounds on c contain relatively little information on $F(\cdot|\cdot)$, so to induce refinement we need both tight bounds on c and a large gap in identified entry thresholds. Such a pattern is most likely to obtain when the DGP involves a large gap in $\mathcal{N}$ at low competition levels, so for clarity we consider the extreme case $\mathcal{N} = \{2, 15, 16\}$. Figure S.3(b) plots the results, which, as expected, show substantial gains in the sharp identified set. In practice, however, it is difficult to envision applications where such large gaps in $\mathcal{N}$ arise naturally, and even adding a single intermediary point ($N = 9$, Figure S.3(c)) is sufficient to dissipate most gains. This, in turn, reinforces our assessment that the initial bounds in Proposition 3 are likely to be sufficient in most applications.

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