Econometrica Supplementary Material

SUPPLEMENT TO "PRODUCTION AND LEARNING IN TEAMS"

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A. STATIONARY DISTRIBUTIONS

LET $e(k, x)$ DENOTE THE DISTRIBUTION OF WORKERS across employment states at the beginning of the search-and-matching stage in the current period. In order to formally express how this distribution evolves over time, we need to introduce some notation to describe the equilibrium policy functions. Let $h_y(k, x)$ denote the probability that a firm in state $y \in Y$ hires a worker of type $k \in K$ in state $x \in X$. The probability $h_y(k, x)$ is equal to 1 if the marginal value of the worker to the firm, $v_k(y)$, is greater than the worker's outside option $z_k(x)$. Otherwise, $h_y(k, x)$ is equal to 0. Let $r_{(i,j)}(k, i)$ denote the probability that, conditional on hiring a worker of type k, a firm in state $(i, j) \in K \times K$ replaces employee i. The probability $r_{(i,j)}(k, i)$ is equal to 1 if $\hat{V}_{k,j} + U_i > \hat{V}_{k,i} + U_j$, it is equal to 0 if $\hat{V}_{k,j} + U_i < \hat{V}_{k,i} + U_j$, and it is equal to $1/2$ if $\hat{V}_{k,j} + U_i = \hat{V}_{k,i} + U_j$. Lastly, let $d_y(k)$ denote the probability that a firm in state $y \in Y$ fires an employee of type k.

Let $e_1(k, x)$ denote the distribution of workers after the search-and-matching process has taken place, but before the firms have had the chance of firing their employees. The measure $e_1(k, u)$ of unemployed workers is given by

$$\begin{aligned} &e_1(k, u) \\ &\quad = e(k, u)\left\{(1-\lambda_u) + \left[\sum_y \lambda_u p_y\big(1 - h_y(k, u)\big)\right]\right\} \\ &\qquad + e(k, 0)\delta + \sum_\ell e(k, \ell)\left\{\delta + \left[\sum_{i,x} q_i(x) h_{k,\ell}(i, x) r_{k,\ell}(i, k)\right]\right\}. \qquad (12) \end{aligned}$$

The first term on the right-hand side is the measure of unemployed workers at the beginning of the search-and-matching stage who either did not contact a firm or contacted a firm but were not hired. The second term is the measure of workers who were employed

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without a coworker at the beginning of the search-and-matching stage and lost their job for exogenous reasons. The last term is the measure of workers who were employed with a coworker of type ℓ at the beginning of the search-and-matching stage and either lost the job for exogenous reasons or were replaced by a new hire.

The measure $e_1(k,0)$ of workers employed without a coworker is given by

$$\begin{aligned}
&e_1(k,0)\\
&\quad = e(k,u)\{\lambda_u p_{0,0} h_{0,0}(k,u)\} + e(k,0)\left\{\left[\sum_y \lambda_e p_y \big(1-h_y(k,0)\big)\right]\right\}\\
&\qquad + e(k,0)\left\{\left[\sum_{i,x} q_i(x)\big(1-h_{k,0}(i,x)\big)\right] + \left[1-\delta-\lambda_e-\sum_{i,x} q_i(x)\right]\right\}\\
&\qquad + \sum_\ell e(k,\ell)\left\{\lambda_e p_{0,0} h_{0,0}(k,\ell) + \left[\sum_y \lambda_e p_y h_y(\ell,k)\right] + \delta\right\}.
\end{aligned} \tag{13}$$

The first term on the right-hand side is the measure of workers who were unemployed at the beginning of the search-and-matching stage and who were hired by a firm in state $(0,0)$. The second, third, and fourth terms are the measure of workers who were employed on their own at the beginning of the search-and-matching stage and remained in the same employment position. The last term is the measure of workers who were employed with a coworker of type ℓ at the beginning of the search-and-matching stage and who were either hired by a firm in state $(0,0)$ or who lost their coworker to a poaching firm or to unemployment.

The measure $e_1(k,\ell)$ of workers employed with a coworker of type ℓ is given by

$$\begin{aligned}
&e_1(k,\ell)\\
&\quad = e(k,u)\bigg\{\lambda_u p_{\ell,0} h_{\ell,0}(k,u)\\
&\qquad + \lambda_u\left[\sum_s p_{\ell,s} h_{\ell,s}(k,u) r_{\ell,s}(k,s) + p_{s,\ell} h_{s,\ell}(k,u) r_{s,\ell}(k,s)\right]\bigg\}\\
&\qquad + e(k,0)\left\{\lambda_e\left[\sum_s p_{\ell,s} h_{\ell,s}(k,0) r_{\ell,s}(k,s) + p_{s,\ell} h_{s,\ell}(k,0) r_{s,\ell}(k,s)\right]\right\}\\
&\qquad + e(k,0)\left\{\lambda_e p_{\ell,0} h_{\ell,0}(k,0) + \left[\sum_x q_\ell(x) h_{k,0}(\ell,x)\right]\right\}\\
&\qquad + e(k,\ell)\left\{\lambda_e\left[\sum_y p_y\big(1-h_y(k,\ell)\big)\right] + \lambda_e\left[\sum_y p_y\big(1-h_y(\ell,k)\big)\right]\right\}\\
&\qquad + e(k,\ell)\left\{\left[\sum_{i,x} q_i(x)\big(1-h_{k,\ell}(i,x)\big)\right] + \left[1-2\delta-2\lambda_e-\sum_{i,x} q_i(x)\right]\right\}\\
&\qquad + \sum_s e(k,s)\bigg\{\lambda_e p_{\ell,0} h_{\ell,0}(k,s)
\end{aligned}$$

$$+\lambda_e\left[\sum_t p_{\ell,t}h_{\ell,t}(k,s)r_{\ell,t}(k,t)+p_{t,\ell}h_{t,\ell}(k,s)r_{t,\ell}(k,t)\right]\Bigg\}$$
$$+\sum_s e(k,s)\left\{\sum_x q_\ell(x)h_{k,s}(\ell,x)r_{k,s}(\ell,s)\right\}. \tag{14}$$

The right-hand side includes all the ways in which a worker finds himself employed with a coworker of type ℓ at the production stage. First, the worker could have been unemployed and been hired by a firm in state $(\ell,0)$ or hired by a firm in state (ℓ,s) as a replacement for s. Second, the worker could have been employed by himself and been hired by a firm in state $(\ell,0)$, by a firm in state (ℓ,s) as a replacement for s, or his employer could have hired a worker of type ℓ. Third, the worker could have been employed with a coworker of type ℓ and the team had survived. Fourth, the worker could have been employed with a coworker of type s and ended up, because he moved or the firm replaced his coworker, with a coworker of type ℓ.

Let $e_2(k,x)$ denote the distribution of workers at the beginning of the production stage (i.e., after the firm has had the option of firing some of its employees). The measures $e_2(k,u)$, $e_2(k,0)$, and $e_2(k,\ell)$ are given by

$$\begin{aligned}
e_2(k,u)&=e_1(k,u)+e_1(k,0)d_{k,0}(k)+\sum_\ell e_1(k,\ell)d_{k,\ell}(k),\\
e_2(k,0)&=e_1(k,0)\big(1-d_{k,0}(k)\big)+\sum_\ell e_1(k,\ell)\big(1-d_{k,\ell}(k)\big)d_{k,\ell}(\ell),\\
e_2(k,\ell)&=e_1(k,\ell)\big(1-d_{k,\ell}(k)\big)\big(1-d_{k,\ell}(\ell)\big).
\end{aligned}\tag{15}$$

The measure of workers of type k in state u is equal to the measure of k-workers who were unemployed plus the measure of workers who were employed and were fired by their employer. The measure of workers of type k in state 0 is equal to the measure of k-workers who were employed without a coworker and were not fired plus the measure of workers who were employed with a coworker who was fired. The measure of workers of type k in state ℓ is equal to the measure of k-workers who were employed with a coworker of type ℓ and whose employer did not fire anyone.

Let $e_3(k,x)$ denote the distribution of workers at the beginning of the entry-and-exit stage of next period (i.e., after the workers' human capital accumulation process is completed). The measures $e_3(k,u)$, $e_3(k,0)$, and $e_3(k,\ell)$ are given by

$$\begin{aligned}
e_3(k,u)&=\sum_s g_s(k|u)e_2(s,u),\\
e_3(k,0)&=\sum_s g_s(k|0)e_2(s,0),\\
e_3(k,\ell)&=\sum_{s,t} g_s(k|t)g_t(\ell|s)e_2(s,t).
\end{aligned}\tag{16}$$

The measure of unemployed workers of type k at this stage is equal to the sum of the measures of unemployed workers of type s at the previous stage whose human capital type goes from s to k. The measure of workers of type k employed on their own is the sum of the measures of workers of type s employed on their own whose human capital type goes from s to k. The measure of workers of type k employed with a coworker of type ℓ is the sum of the measures of workers of type s employed with a coworker of type t who transitions from s to k and from t to ℓ.

Finally, let $e_+(k,x)$ denote the distribution of workers at the beginning of the search-and-matching stage of next period (i.e., after the entry-and-exit process is completed). The measures $e_+(k,u)$, $e_+(k,0)$, and $e_+(k,\ell)$ are given by

$$\begin{aligned} e_+(k,u) &= (1-\sigma)e_3(k,u) + \sigma\pi_k, \\ e_+(k,0) &= (1-\sigma)e_3(k,0) + \sum_\ell \sigma(1-\sigma)e_3(k,\ell), \\ e_+(k,\ell) &= (1-\sigma)^2 e_3(k,\ell). \end{aligned} \tag{17}$$

The distribution $e(k,x)$ is stationary if and only if

$$e_+(k,x) = e(k,x). \tag{18}$$

Given the stationary distribution of workers $e(k,x)$, we can derive the stationary distribution of firms as

$$\begin{aligned} n_{k,0} &= e_{k,0}, \\ n_{k,\ell} + n_{\ell,k} &= (e_{k,\ell} + e_{\ell,k})/2, \\ n_{0,0} &= n - \sum_k n_{k,0} - \sum_{k,\ell} n_{k,\ell}. \end{aligned} \tag{19}$$

The measure $n_{k,0}$ of firms with only one employee of type k is equal to the measure of workers of type k who are employed without a coworker. The measure $n_{k,\ell} + n_{\ell,k}$ of firms with employees (k,ℓ) or (ℓ,k) is equal to half of the sum between the measure of workers of type k employed with a coworker of type ℓ and the measure of workers of type ℓ employed with a coworker of type k. The remaining firms are idle.

B. WAGES

Let $\tilde{W}_{k,x}(w)$ denote the worker's value from being in state x at the wage w at the beginning of the search-and-matching stage. Let $\hat{W}_{k,x}(w)$ denote the worker's value from being in state x at the wage w after the search-and-matching stage is completed but before the worker's employer has had the option of firing some of its employees. Lastly, let $W_{k,x}(w)$ denote the worker's value from being in state x at the wage w at the beginning of the production stage.

At the beginning of the production stage, the worker's value of being in state x at the wage w is given by

$$\begin{aligned} W_{k,0}(w) &= w + \beta\mathbb{E}_{k_+}\{(1-\sigma)\tilde{W}_{k_+,0}(w)\}, \\ W_{k,\ell}(w) &= w + \beta\mathbb{E}_{k_+,\ell_+}\{\sigma(1-\sigma)\tilde{W}_{k_+,0}(w) + (1-\sigma)^2\tilde{W}_{k_+,\ell_+}(w)\}. \end{aligned} \tag{20}$$

Consider the expression for $W_{k,0}(w)$. In the current period, the worker is paid the wage w. In the next period, the worker exits the labor market with probability σ, in which case his continuation value is 0, and remains in the labor market with probability $1-\sigma$, in which case his continuation value is $\tilde{W}_{k_+,0}(w)$. Now, consider the expression for $W_{k,\ell}(w)$.

In the current period, the worker is paid the wage w. In the next period, the worker exits the labor market with probability σ, in which case the worker's continuation value is 0. The worker stays in the labor market but his coworker exits with probability $\sigma(1-\sigma)$, in which case the worker's continuation value is $\tilde{W}_{k_+,0}(w)$. The worker and his coworker stay in the labor market with probability $(1-\sigma)^2$, in which case the worker's continuation value is $\tilde{W}_{k_+,\ell_+}(w)$.

At the beginning of the search-and-matching stage, the worker's value of being in state 0 at the wage w is given by

$$\begin{aligned}\tilde{W}_{k,0}(w) = {} & \hat{W}_{k,0}(w) + \delta\big[U_k - \hat{W}_{k,0}(w)\big] + \sum_{i,x} q_i(x)\big[h_{k,0}(i,x)\big[\hat{W}_{k,i}(w) - \hat{W}_{k,0}(w)\big]\big] \\ & + \sum_y \lambda_e p_y\big[\max\big\{\hat{W}_{k,0}(w), \min\big\{v_k(y), \gamma v_k(y) + (1-\gamma)v_k\big((0,0)\big)\big\}\big\} \\ & - \hat{W}_{k,0}(w)\big]. \end{aligned} \tag{21}$$

The worker moves into unemployment with probability δ. In this case, the worker's continuation value is U_k. The firm contacts a worker of type i in state x with probability $q_i(x)$ and hires him with probability $h_{k,0}(i,x)$. In this case, the worker's continuation value is $\hat{W}_{k,i}(w)$. The worker contacts a poaching firm in state y with probability $\lambda_e p_y$. In this case, three different situations may arise depending on the value of the worker to the poaching firm, $v_{k_+}(y)$, the value of the worker to the incumbent firm, $v_k((0,0))$, and the value to the worker of being employed at the incumbent firm at the wage w, $\hat{W}_{k,0}(w)$. If $v_k(y) \leq \hat{W}_{k,0}(w)$, the worker remains with the incumbent at the wage w. Hence, his continuation value is $\hat{W}_{k,0}(w)$. If $v_k(y) > \hat{W}_{k,0}(w)$ and $v_k(y) \leq v_k((0,0))$, the worker remains with the incumbent but his wage is increased to match his outside option. Hence, his continuation value is $v_k(y)$. If $v_k(y) > v_k((0,0))$, the worker moves to the poacher and captures a fraction γ of the gains from trade. Hence, his continuation value is $v_k((0,0)) + \gamma[v_k(y) - v_k((0,0))]$.

Similarly, the worker's value from being in state $x = \ell$ at the wage w is given by

$$\begin{aligned}\tilde{W}_{k,\ell}(w) = {} & \hat{W}_{k,\ell}(w) + \delta\big[U_k - \hat{W}_{k,\ell}(w)\big] \\ & + \Big(\delta + \sum_y \lambda_e p_y h_y(\ell,k)\Big)\big[\hat{W}_{k,0}(w) - \hat{W}_{k,\ell}(w)\big] \\ & + \sum_{i,x} q_i(x)\big[h_{k,\ell}(i,x)\big[r_{k,\ell}(i,k)U_k + \big(1 - r_{k,\ell}(i,\ell)\big)\hat{W}_{k,i}(w) - \hat{W}_{k,\ell}(w)\big]\big] \\ & + \sum_y \lambda_e p_y\big[\max\big\{\hat{W}_{k,\ell}(w), \min\big\{v_k(y), \gamma v_k(y) + (1-\gamma)v_k\big((\ell,0)\big)\big\}\big\} \\ & - \hat{W}_{k,\ell}(w)\big]. \end{aligned} \tag{22}$$

The above expression is similar to the previous one. Therefore, we shall only point out the differences. First, the coworker may move into unemployment. In this case, the worker's continuation value is $\hat{W}_{k,0}(w)$. Second, the coworker may be hired by a poaching firm. In this case, the worker's continuation value is also $\hat{W}_{k,0}(w)$. Third, when a firm hires a new employee, the continuation value of the worker depends on whom the firm replaces.

If the firm replaces the coworker, the worker's continuation value is $\hat{W}_{k,i}(w)$. If the firm replaces the worker, his continuation value is U_k.

After the search-and-matching process is complete but before the firm has had the opportunity of firing some of its employees, the worker's value from being in state 0 at the wage w is given by

$$\hat{W}_{k,0}(w) = \max\{U_k, \min\{d_{k,0}(k)U_k + (1 - d_{k,0}(k))W_{k,0}(w), v_k((0,0))\}\}. \tag{23}$$

Similarly, the worker's value from being in state ℓ at the wage w is given by

$$\begin{aligned} &\hat{W}_{k,\ell}(w) \\ &\quad = \max\{U_k, \min\{d_{k,\ell}(k)U_k + (1 - d_{k,\ell}(k))[d_{k,\ell}(\ell)W_{k,0}(w) \\ &\qquad + (1 - d_{k,\ell}(\ell))W_{k,\ell}(w)], v_k((\ell,0))\}\}. \end{aligned} \tag{24}$$

In words, $\hat{W}_{k,0}(w)$ and $\hat{W}_{k,\ell}(w)$ are the maximum between the value of unemployment to a worker of type k and the lowest between the value to the worker of staying with his current employer at the wage w and the marginal value of the worker to the production unit operated by the firm.

The Bellman equations (20)–(24) can be solved for the employment value functions $W_{k,x}$, $\tilde{W}_{k,x}$, and $\hat{W}_{k,x}$. Given a history of a worker's lifetime utilities and employment states at the production stage, the value function $W_{k,x}$ can be inverted to recover the history of the worker's wages.

C. IDENTIFICATION

Figure 8 illustrates the key elements of the model's Jacobian matrix. Panels (A) through (F) plot moments with respect to deviations in key parameters, holding all other parameters constant. We fit a Gaussian process regression and quadratic polynomial to the moments as a function of the parameter varying on the x-axis.

D. PLANNER'S PROBLEM AND OPTIMAL SUBSIDIES

D.1. *Planner's Problem*

At the beginning of the production stage, the problem of a utilitarian social planner is

$$\begin{aligned} &S(e_2) \\ &\quad = \max_{h,r,d} \sum_k \Big[e_2(k,u)b_k + e_2(k,0)f(k,0) + \sum_\ell e_2(k,\ell)f(k,\ell)/2 \Big] \\ &\qquad + \beta S(e_2^+), \end{aligned} \tag{25}$$

subject to the law of motions (12)–(19). The state of the planner's problem is the distribution $e_2(k,x)$ of workers across types k and across employment states x at the beginning of the production stage. The choices in the planner's problem are the probability $h_y(k,x)$ with which a firm in state y hires a worker of type k in employment state x, the probability $r_y(k,i)$ with which a firm of type y replaces the employee i with a new hire of type k, and the probability $d_y(i)$ with which a firm of type y fires an employee of type i. The

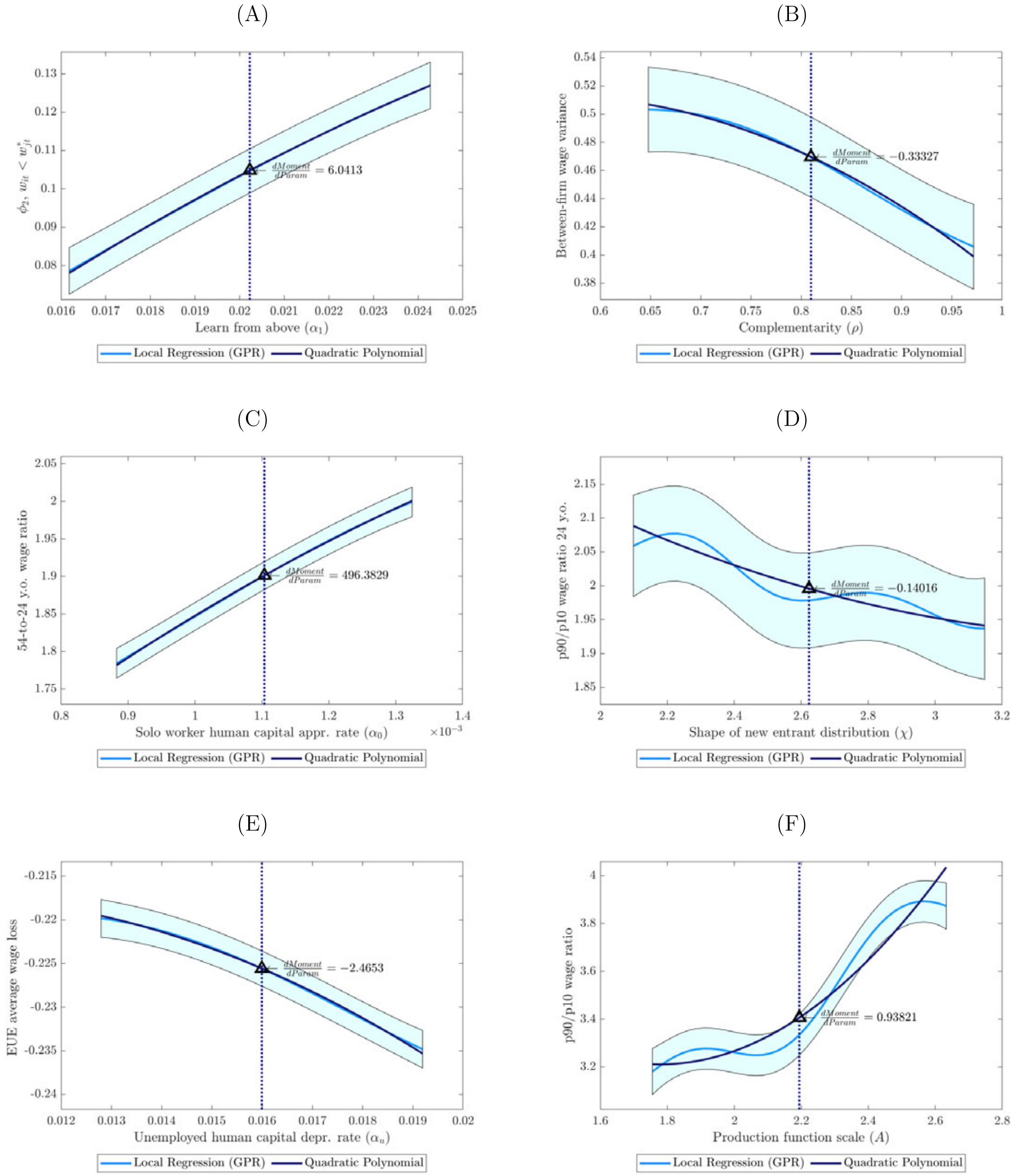

FIGURE 8.—Selected elements of Jacobian matrix.

objective of the planner's problem is to maximize the present value of income generated by unemployed workers and employed workers, discounted at the factor β. The state of the planner's problem $e_2(k, x)$ and the planner's choices determine the planner's state $e_2^+(k, x)$ at the beginning of the next production stage.

We want to characterize the steady state of the planner's problem and compare it with the equilibrium steady state. To this aim, we denote as U_k^* the derivative of the planner's value function with respect to the measure $e_2(k, u)$ of unemployed workers of type k, evaluated at the steady-state distribution of workers e_2^*. Similarly, we denote as $S_{k,0}^*$ the derivative of the planner's value function with respect to the measure $e_2(k,0)$ of workers of type k who are employed without a coworker, and as $S^*_{k,\ell}$ the derivative of the planner's value function with respect to the measure $e_2(k,\ell)$ of workers of type k who are employed with a coworker of type ℓ.

In order to compare the steady state of the social planner's problem with the equilibrium steady state, it is convenient to define

$$\begin{aligned} V^*_{k,0} &= S^*_{k,0} + V^*_{0,0}, \\ V^*_{k,\ell} &= S^*_{k,\ell} + S^*_{k,\ell} + V^*_{0,0}, \end{aligned} \tag{26}$$

where $V^*_{0,0}$ represents the value to the planner of an additional production unit in state $y=(0,0)$. Then, $V^*_{k,0}$ denotes the sum between the marginal value of an additional worker of type k who is employed without a coworker and the value of an additional idle firm. This is the value of an additional production unit in state $y=(k,0)$. Similarly, $V^*_{k,\ell}$ denotes the sum between the marginal value of an additional worker of type k who is employed with a coworker of type ℓ, the marginal value of an additional worker of type ℓ who is employed with a coworker of type k, and the value of an additional idle firm. This is the value of an additional production unit in state $y=(k,\ell)$. Moreover, as in the analysis of equilibrium, it is useful to denote as $\tilde{V}^*_y$ the marginal value of a production unit at the beginning of the search-and-matching stage, and as $\hat{V}^*_y$ the marginal value of a production unit after the search-and-matching process is completed, but before the option of firing workers.

Using the above notation, it is easy to show that the marginal values $V^*_{0,0}$, $V^*_{k,0}$, and $V^*_{k,y}$ at the beginning of the production stage are

$$\begin{aligned} V^*_{0,0} &= 0 + \beta \tilde{V}^*_{0,0}, \\ V^*_{k,0} &= f(k,0) + \beta \mathbb{E}\{\sigma \tilde{V}^*_{0,0} + (1-\sigma)\tilde{V}^*_{k_+,0}\}, \\ V^*_{k,\ell} &= f(k,\ell) + \beta \mathbb{E}\{\sigma(1-\sigma)(\tilde{V}^*_{k_+,0} + \tilde{V}^*_{\ell_+,0}) + \sigma^2 \tilde{V}^*_{0,0} + (1-\sigma)^2 \tilde{V}^*_{k_+,\ell_+}\}. \end{aligned} \tag{27}$$

Using the optimality of h and r, it is easy to show that the marginal value $\tilde{V}^*_{0,0}$ of a production unit in state $(0,0)$ at the beginning of the search-and-matching stage is

$$\tilde{V}^*_{0,0} = \hat{V}^*_{0,0} + \left[\sum_{i,x} q^*_i(x)\big(v^*_i((0,0)) - z^*_i(x)\big)^+\right]. \tag{28}$$

As in Section 2, $q^*_i(x)$ denotes the probability that a firm meets a worker of type i in state x, which equals $\lambda_u e^*_{i,u}/n$ for $x=u$, $\lambda_e e^*_{i,0}/n$ for $x=0$, and $\lambda_e e^*_{i,j}/n$ for $x=j$, where $e^*_{k,x}$ denotes the steady-state distribution of workers across types and employment states at the beginning of the search-and-matching stage. As in Section 2, $v^*_i(y)$ denotes the marginal value of a firm in state y from hiring a worker of type i, which is $\hat{V}^*_{i,0} - \hat{V}^*_{0,0}$ for $y=(0,0)$, $\hat{V}^*_{k,i} - \hat{V}^*_{k,0}$ for $y=(k,0)$, and $\max\{\hat{V}^*_{k,i} + U^*_\ell, \hat{V}^*_{\ell,i} + U_k\} - \hat{V}^*_{k,\ell}$ for $y=(k,\ell)$. As in Section 2, $z^*_i(x)$ denotes the marginal cost of moving a worker of type i away from his employment status x, which is equal to U^*_i for $x=u$, $\hat{V}^*_{i,0} - \hat{V}^*_{0,0}$ for $x=0$, and $\hat{V}^*_{i,j} - \hat{V}^*_{j,0}$ for $x=j$.

Using again the optimality of h and r, it is easy to show that the marginal value $\tilde{V}^*_{k,0}$ of a production unit in state $(k,0)$ at the beginning of the search-and-matching stage is

$$\tilde{V}^*_{k,0} = \hat{V}^*_{k,0} + \delta\big(\hat{V}^*_{0,0} + U^*_k - \hat{V}^*_{k,0}\big)$$

$$+\sum_{i,x} q_i^*(x)\big(v_i^*\big((k,0)\big)-z_i^*(x)\big)^+ + \sum_y \lambda_e p_y^* \max\big(v_k^*(y)-z_k^*(0)\big)^+. \tag{29}$$

Analogously, the marginal value $\tilde{V}_y^*$ of a production unit in state (k,ℓ) is

$$\begin{aligned}\tilde{V}_{k,\ell}^* &= \hat{V}_{k,\ell}^* + \delta\big(\hat{V}_{k,0}^* + U_\ell^* - \hat{V}_{k,\ell}^*\big) + \delta\big(\hat{V}_{\ell,0}^* + U_k^* - \hat{V}_{k,\ell}^*\big) \\ &\quad + \sum_y \lambda_e p_y^*\big(v_k^*(y) - z_k^*(\ell)\big)^+ + \sum_y \lambda_e p_y^*\big(v_\ell^*(y) - z_\ell^*(k)\big)^+ \\ &\quad + \sum_{i,x} q_i^*(x)\big(v_i^*\big((k,\ell)\big) - z_i^*(x)\big)^+. \end{aligned} \tag{30}$$

Using the optimality of d, it is easy to show that the marginal values $\hat{V}_{0,0}^*$, $\hat{V}_{k,0}^*$, and $\hat{V}_{k,y}^*$ after the search-and-matching process is completed are

$$\begin{aligned}\hat{V}_{0,0}^* &= V_{0,0}^*, \\ \hat{V}_{k,0}^* &= \max\big\{V_{0,0}^* + U_k^*, V_{k,0}^*\big\}, \\ \hat{V}_{k,\ell}^* &= \max\big\{V_{k,0}^* + U_\ell^*, V_{\ell,0}^* + U_k^*, V_{0,0}^* + U_k^* + U_\ell^*, V_{k,\ell}^*\big\}.\end{aligned} \tag{31}$$

Lastly, the marginal value of an unemployed worker of type k is

$$U_k^* = b_k + \beta\mathbb{E}\Big\{U_{k_+}^* + \sigma\big[0 - U_{k_+}^*\big] + (1-\sigma)\Big[\sum\nolimits_y \lambda_u p_y^*\big(v_{k_+}^*(y) - U_{k_+}^*\big)^+\Big]\Big\}. \tag{32}$$

The equations for the marginal value of unemployed workers and production units in the planner's problem are very similar to the equations for the value of unemployment and production units in the stationary equilibrium. The sole difference is that the equations in the planner's problem do not depend on the bargaining powers γ and $1-\gamma$. In fact, the marginal value of an unemployed worker to the planner includes the full value of the meetings between that worker and firms (rather than a fraction γ). The marginal value of a production unit to the planner includes the full value of the meetings between the firm associated with that production unit and outside workers (rather than a fraction $1-\gamma$), and the full value of the meetings between the workers associated with that production unit and poaching firms (rather than a fraction γ).

It is easy to understand the difference between the marginal values in the planner's problem and the private values in the equilibrium. At the margin, the value to the planner of an additional unemployed worker or production unit must include the full value of the meetings generated by these entities. In equilibrium, the private value of an unemployed worker, idle firm, or production unit only includes the fraction of the value of a meeting that is captured by these entities. For the private values in equilibrium to coincide with the marginal values in the planner's problem, it would have to be the case that—in a meeting between two parties—both parties are rewarded with the entire value of the meeting. Clearly, this is not possible as it requires distributing more resources than are available.

The above observation implies that the equilibrium is inefficient. Formally, let $\{U_k^*, V_y^*, \hat{V}_y^*\}$ and $\{e_{k,x}^*, n_y^*\}$ denote the steady-state value functions and distributions of the planner's problem and let $\{U_k, V_y, \hat{V}_y\}$ and $\{e_{k,x}, n_y\}$ denote the steady-state value functions and distributions in equilibrium. To see that the equilibrium is not efficient, it is sufficient to note that—even if $\{e_{k,x}, n_y\} = \{e_{k,x}^*, n_y^*\}$—the equilibrium values $\{U_k, V_y, \hat{V}_y\}$ are different from the social values $\{U_k^*, V_y^*, \hat{V}_y^*\}$ as they solve a different system of Bellman equations. Moreover, since $\{U_k, V_y, \hat{V}_y\}$ are different from $\{U_k^*, V_y^*, \hat{V}_y^*\}$, the equilibrium policies will be typically different[10] from the social planner's choices and, hence, $\{e_{k,x}, n_y\}$ will also be different from $\{e_{k,x}^*, n_y^*\}$.

We have thus established the following result.

PROPOSITION 1—Inefficiency of Equilibrium: *Any stationary equilibrium* $\{U_k, V_y, \hat{V}_y\}$ *and* $\{e_{k,x}, n_y\}$ *is inefficient. That is,* $\{U_k, V_y, \hat{V}_y\}$ *and* $\{e_{k,x}, n_y\}$ *is different from the planner's steady state* $\{U_k^*, V_y^*, \hat{V}_y^*\}$ *and* $\{e_{k,x}^*, n_y^*\}$.

D.2. *Optimal Subsidies*

We now want to find a system of subsidies that implements the efficient allocation—in the sense that, given such a system of subsidies, there exists a stationary equilibrium that coincides with the steady state of the social planner's problem. Formally, let $s_{k,u}$ denote a subsidy paid by the government to a worker of type k who is unemployed, let $s_{0,0}$ denote a subsidy paid to a production unit in state $(0,0)$, $s_{k,0}$ a subsidy paid to a production unit in state $(k,0)$, and $s_{k,\ell}$ a subsidy paid to a production unit in state (k,ℓ). The system of subsidies $s_{k,u}$, s_y implements the efficient allocation if it makes the equilibrium value functions U_k, V_y, and V_y coincide with the social planner's marginal values U_k^*, V_y^*, and V_y^*, when U_k, V_y, and V_y are evaluated at the stationary distribution $\{e_k^*, n_y^*\}$ of the solution of the planner's problem. In fact, when this condition is satisfied, the equilibrium policy functions coincide with the planner's policy functions and, hence, there is a stationary equilibrium that coincides with the steady state of the planner's problem.

The optimal subsidy $s_{k,u}$ is such that the equilibrium value of unemployment to a worker of type k is U_k^*. That is, $s_{k,u}$ is such that

$$U_k^* = b_k + s_{k,u} + \beta\mathbb{E}\Big\{(1-\sigma)\Big[U_{k_+}^* + \sum\nolimits_y \lambda_u p_y^* \gamma\big(v_{k_+}^*(y) - U_{k_+}^*\big)^+\Big]\Big\}. \tag{33}$$

The optimal subsidy s_y is such that the equilibrium value of a production unit in state y is V_y^*. That is, s_y is such that

$$\begin{aligned} V_{0,0}^* &= s_{0,0} + \beta\tilde{V}_{0,0}, \\ V_{k,0}^* &= f(k,0) + s_{k,0} + \beta\mathbb{E}\big\{\sigma\tilde{V}_{0,0} + (1-\sigma)\tilde{V}_{k_+,0}\big\}, \\ V_{k,\ell}^* &= f(k,\ell) + s_{k,\ell} + \beta\mathbb{E}\big\{\sigma(1-\sigma)(\tilde{V}_{k_+,0} + \tilde{V}_{\ell_+,0}) + \sigma^2\tilde{V}_{0,0} + (1-\sigma)^2\tilde{V}_{k_+,\ell_+}\big\}. \end{aligned} \tag{34}$$

In the above expressions, the value $\tilde{V}_{0,0}$ is given by

$$\tilde{V}_{0,0} = \hat{V}_{0,0}^* + \Big[\sum_{i,x} q_i^*(x)(1-\gamma)\big(v_i^*\big((0,0)\big) - z_i^*(x)\big)^+\Big]. \tag{35}$$

[10] Of course, it may be possible that—even though the equilibrium values are different from the social values—their ranking is the same so that the planner's steady-state distribution is a steady state of the market economy. In our numerical examples, we find that this is not the case.

Similarly, the value $\tilde{V}_{k,0}$ is given by

$$\begin{aligned}\tilde{V}_{k,0} = {}& \hat{V}^*_{k,0} + \delta\big[\hat{V}^*_{0,0} + U^*_k - \hat{V}^*_{k,0}\big] \\ & + \sum_{i,x} q^*_i(x)(1-\gamma)\big(v^*_i\big((k,0)\big) - z^*_i(x)\big)^+ \\ & + \sum_{y} \lambda_e p^*_y \gamma \max\big(v^*_{k_+}(y) - z^*_{k_+}(0)\big)^+. \end{aligned} \tag{36}$$

Lastly, the value $\tilde{V}_{k,\ell}$ is given by

$$\begin{aligned}\tilde{V}_{k,\ell} = {}& \hat{V}^*_{k_+,\ell_+} + \delta\big(\hat{V}^*_{k_+,0} + U^*_{\ell_+} - \hat{V}^*_{k_+,\ell_+}\big) + \delta\big(\hat{V}^*_{\ell_+,0} + U^*_{k_+} - \hat{V}^*_{k_+,\ell_+}\big) \\ & + \sum_{y} \lambda_e p^*_y \gamma\big(v^*_{k_+}(y) - z^*_{k_+}(\ell_+)\big)^+ + \sum_{y} \lambda_e p^*_y \gamma\big(v^*_{\ell}(y) - z^*_{\ell}(k)\big)^+ \\ & + \sum_{i,x} q^*_i(x)(1-\gamma)\big(v^*_i\big((k,\ell)\big) - z^*_i(x)\big)^+. \end{aligned} \tag{37}$$

Given the solution to the planner's problem, equations (35)–(37) can be used to solve for $\tilde{V}_y$. Given $\tilde{V}_y$ and the solution to the planner's problem, equation (33) can be used to solve for the optimal subsidy $s_{k,u}$, and equation (34) can be used to solve for the optimal subsidy s_y. By construction of the optimal subsidies $s_{k,u}$ and s_y, it follows that $\{U^*_k, V^*_y, \hat{V}^*_y\}$ and $\{e^*_{k,x}, n^*_y\}$ satisfy all the conditions for a stationary equilibrium. Notice that the optimal subsidies need not balance the budget of the government. As usual, however, the budget of the government can be rebalanced through a lump-sum tax T collected from all workers and firms that does not affect the equilibrium policy functions.

We have thus established the following result.

PROPOSITION 2—Optimal Subsidies: *Given the system of subsidies s in* (33)–(34), *the steady state of the planner's problem*, $\{U^*_k, V^*_y, \hat{V}^*_y\}$ *and* $\{e^*_{k,x}, n^*_y\}$, *is a stationary equilibrium*.

E. ADDITIONAL TABLES AND FIGURES

E.1. *Robustness*

We carry out an extensive robustness analysis of regression (7). In Table X, we run a version of regression (7) in which we define $w^*_{j,t}$ as the median log wage among the stable coworkers of individual i at firm j in year t. The estimates of the coefficient ϕ_2 are essentially the same as when $w^*_{j,t}$ is defined as the average log wage among the stable coworkers of individual i at firm j in year t. Specifically, the estimate of the regression coefficient ϕ_2 remains 0.12 for individuals who earn less than their coworkers, and the estimate falls from 0.04 to 0.03 for individuals who earn more than their coworkers.

In Table XI, we consider a version of regression (7) in which we explore the role of firm dynamics. Specifically, we construct a dummy variable for each firm j in year t that takes the value 1 if employment at the firm is non-decreasing in every quarter of year t, and 0 otherwise. We then add to the right-hand side of (7) the "firm growth" dummy and interaction terms between the individual's wage, the coworkers' wage, and the "firm growth" dummy. We find that, for non-growing firms, the estimate of the regression coefficient

TABLE X

INDIVIDUAL WAGE REGRESSION: MEDIAN WAGE.

Dependent Variable	(1) Wage at $t+2$	(2) Wage at $t+2$
Sample	$w_{i,t} < w^*_{j,t}$	$w_{i,t} \geq w^*_{j,t}$
Median coworker wage at t	0.127	0.0280
	(0.00343)	(0.00520)
Individual wage at t	0.512	0.788
	(0.00351)	(0.00527)
R-squared	0.366	0.520
Round N	244,000	100,000

Note: SE clustered at SEIN level. Controls include: State, 1-digit SIC, race and gender dummies, and year fixed effects. Median coworker wage is computed among stable coworkers at t.

ϕ_2 is 0.13 for individuals who earn less than their coworkers, and 0.05 for individuals who earn more than their coworkers. For growing firms, the estimate of the regression coefficient ϕ_2 is 0.10 for individuals who earn less than their coworkers, and 0.035 for individuals who earn more than their coworkers. Overall, firm dynamics appear to play a very limited role.

In Table XII, we consider a version of regression (7) in which we explore the role of firm size. Specifically, we divide firms into 10 groups based on the number of the stable coworkers on the individual (less than 10, 11 to 20, ..., 91 to 100). We then run regression (7) with a firm-size fixed effect, as well as interaction terms between the individual's wage, the coworkers' wage, and the firm size, using the smallest size as the baseline. We find that the estimates of ϕ_2 for workers who earn less than their coworkers slightly increase with the size of the firm, ranging from 0.11 for the smallest firms to 0.15 for the largest firms. For workers who earn more than their coworkers, the estimates of ϕ_2 range from 0.06 for the smallest firms to -0.03 for the largest firms. Many of the estimates are not statistically significant and, overall, we do not see any clear role for firm size.

TABLE XI

INDIVIDUAL WAGE REGRESSION: FIRM GROWTH.

Dependent Variable	(1) Wage at $t+2$	(2) Wage at $t+2$
Sample	$w_{i,t} < w^*_{j,t}$	$w_{i,t} \geq w^*_{j,t}$
Coworker wage at t	0.130	0.0499
	(0.00364)	(0.00727)
Individual wage at t	0.507	0.776
	(0.00376)	(0.00696)
Coworker wage at t × Positive firm employment growth	−0.0351	−0.0158
	(0.00789)	(0.0151)
Individual wage at t × Positive firm employment growth	0.0303	−0.0152
	(0.00790)	(0.0145)
R-squared	244,000	100,000
Round N	0.366	0.520

Note: SE clustered at SEIN level. Controls include: State, 1-digit SIC, race and gender dummies, and year fixed effects. Positive firm employment growth equals 1 if the firm grew in each consecutive quarter in year t.

TABLE XII

INDIVIDUAL WAGE REGRESSION: FIRM SIZE.

Dependent Variable	(1) Wage at $t+2$	(2) Wage at $t+2$
Sample	$w_{i,t} < w^*_{j,t}$	$w_{i,t} \geq w^*_{j,t}$
Coworker wage at t	0.113	0.0611
	(0.00477)	(0.00881)
Coworker wage at t × 11 to 20 coworkers	0.0146	0.0723
	(0.00882)	(0.0194)
Coworker wage at t × 21 to 30 coworkers	0.0389	0.0598
	(0.0109)	(0.0244)
Coworker wage at t × 31 to 40 coworkers	0.0130	0.0304
	(0.0130)	(0.0272)
Coworker wage at t × 41 to 50 coworkers	0.0336	−0.0270
	(0.0151)	(0.0309)
Coworker wage at t × 51 to 60 coworkers	0.0528	−0.00590
	(0.0167)	(0.0325)
Coworker wage at t × 61 to 70 coworkers	0.0274	−0.0225
	(0.0189)	(0.0382)
Coworker wage at t × 71 to 80 coworkers	0.0593	0.0179
	(0.0190)	(0.0428)
Coworker wage at t × 81 to 90 coworkers	0.0470	−0.0682
	(0.0213)	(0.0441)
Coworker wage at t × 91 to 100 coworkers	0.0438	−0.0902
	(0.0236)	(0.0497)
R-squared	0.369	0.523
Round N	244,000	100,000

Note: SE clustered at SEIN level. Controls include: State, 1-digit SIC, race and gender dummies, and year fixed effects. Firm size measured using stable coworkers in year t as defined in the text.

In Table XIII, we consider a version of regression (7) in which we explore the role of unemployment duration. Specifically, we construct a dummy variable that takes the value 1 if the worker goes through an EUE spell with two or more quarters of unemployment, and the value 0 otherwise. We then add to regression (7) a dummy for "long unemployment spell" and interaction terms between the individual's wage, the coworkers' wage, and the "long unemployment spell" dummy. The estimates of ϕ_2 are the same for individuals who go through a single quarter of unemployment and for those who go through multiple quarters of unemployment. The estimates of ϕ_1 are slightly lower for individuals who go through multiple quarters of unemployment (0.47 rather than 0.54 for individuals earning less than their coworkers, 0.75 rather than 0.79 for individuals earning more than their coworkers).

E.2. *Freund Calibration*

Freund (2022) showed that a between-firm share of wage variance of 0.4 for large firms (what we see in the data) is equivalent to a between-firm share of wage variance of 0.6 for firms with two workers. Even though Freund's correction does not directly apply to our model, in this appendix, we calibrate the model targeting a 60% between-firm share of the wage variance share. Table XIV contains the value of the calibrated parameters. Table XV contains the calibration targets and their model counterparts.

TABLE XIII

INDIVIDUAL WAGE REGRESSION: UNEMPLOYMENT DURATION.

Dependent Variable	(1) Wage $t+2$	(2) Wage $t+2$
Sample	$w_{i,t} < w^*_{j,t}$	$w_{i,t} \geq w^*_{j,t}$
Coworker wage at t	0.127	0.0460
	(0.00404)	(0.00751)
Individual wage at t	0.537	0.789
	(0.00417)	(0.00726)
Coworker wage at t × EUUE	−0.00236	−0.00724
	(0.00650)	(0.0136)
Individual wage at t × EUUE	−0.0732	−0.0386
	(0.00654)	(0.0130)
R-squared	0.372	0.526
Round N	244,000	100,000

Note: SE clustered at SEIN level. Controls include: State, 1-digit SIC, race and gender dummies, and year fixed effects. EUUE is an indicator for whether an individual in the original EUE sample had two or more consecutive quarter of non-employment in year $t+1$.

E.3. *Policy Functions for Counterfactuals*

We report the policy functions for the NLC model (Fig. 9), for the HSM model (Fig. 10), and for the utilitarian social planner (Fig. 11).

TABLE XIV

CALIBRATED PARAMETER VALUES.

Parameter	Description	Freund	Baseline
α_0	Learning by doing	0.001	0.001
α_1	Learning from coworkers	0.025	0.020
α_u	Human capital depreciation	0.017	0.016
ρ	Production complementarity	0.521	0.810
A	Production efficiency	2.334	2.194
χ	Entrant distribution	1.421	2.623
λ_u	Meeting rate, unemployment	0.307	0.340
λ_e	Meeting rate, employed	0.222	0.238
δ	Separation rate	0.008	0.009
b	Flow value of unemployment	1.133	0.976

TABLE XV

CALIBRATION TARGETS AND MODEL FIT.

Target	Source	Data	Freund	Baseline
ϕ_2, $w_{it} \leq w^*_{jt}$	LEHD	0.12	0.14	0.12
ϕ_2, $w_{it} > w^*_{jt}$	LEHD	0.05	0.01	0.03
ϕ_1, $w_{it} \leq w^*_{jt}$	LEHD	0.51	0.53	0.52
ϕ_1, $w_{it} > w^*_{jt}$	LEHD	0.77	0.76	0.78
Between firm wage variance	Song, Price, Guvenen, Bloom, and von Wachter (2019)	$\{0.6, 0.4\}$	0.55	0.44
v_2, $w_{it} \leq w^*_{jt}$	LEHD	0.30	0.23	0.12
v_2, $w_{it} > w^*_{jt}$	LEHD	0.39	0.02	0.02
v_1, $w_{it} \leq w^*_{jt}$	LEHD	0.25	0.32	0.32
v_1, $w_{it} > w^*_{jt}$	LEHD	0.17	0.30	0.22
EUE average wage loss	LEHD	−0.18	−0.28	−0.22
54-to-24 y.o. wage ratio	CPS	1.88	2.01	1.93
Mean wage growth	CPS	0.02	0.03	0.02
p90/p10 wage ratio	CPS	4.23	4.14	3.50
p90/p10 wage ratio 24 y.o.	CPS	2.77	2.48	2.07
UE Rate	CPS	0.22	0.22	0.24
EE Rate	CPS	0.02	0.01	0.01
EU Rate	CPS	0.01	0.01	0.01
Flow unemployment value	Hall and Milgrom (2008)	0.71	0.71	0.73

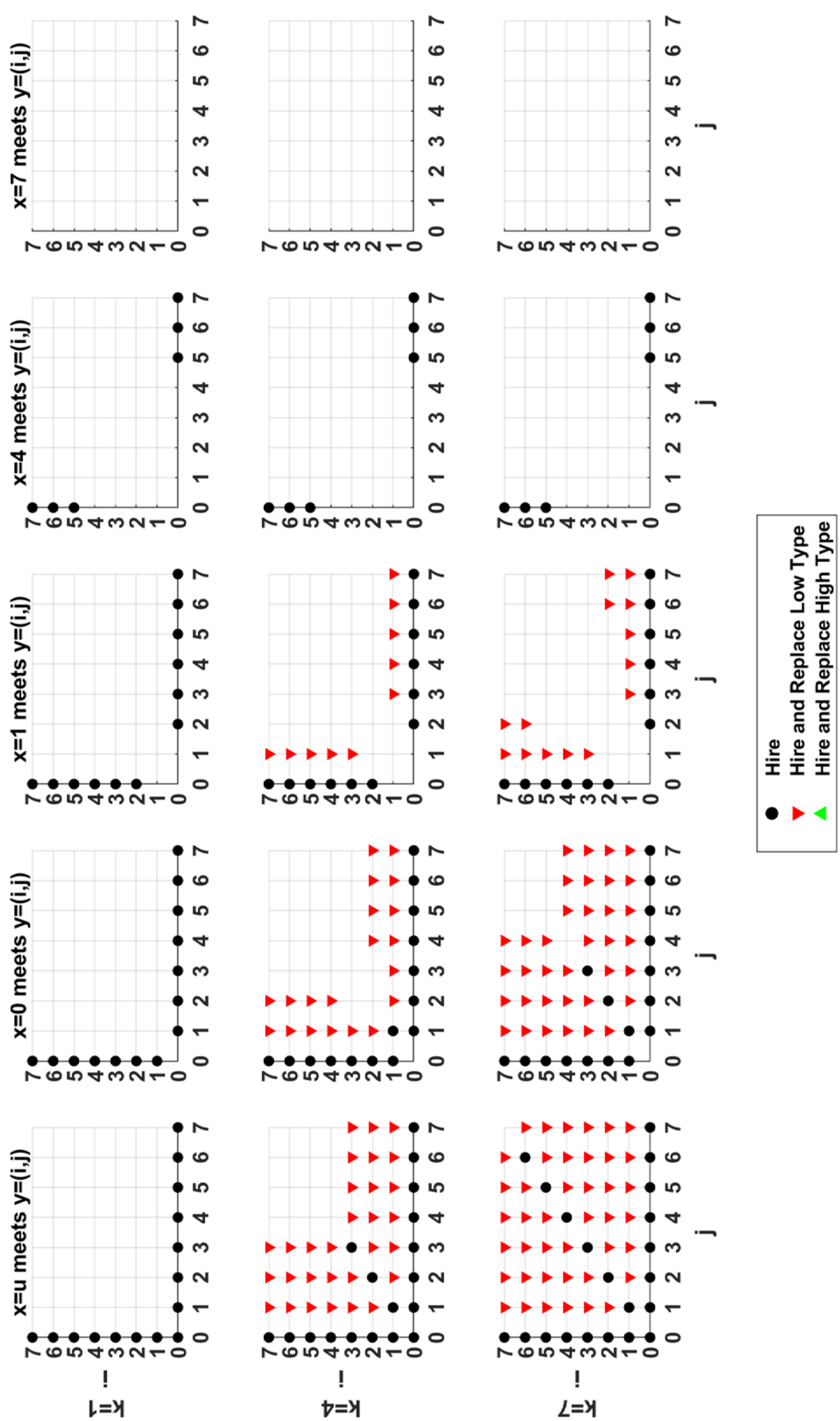

FIGURE 9.—Equilibrium policy function NLC.

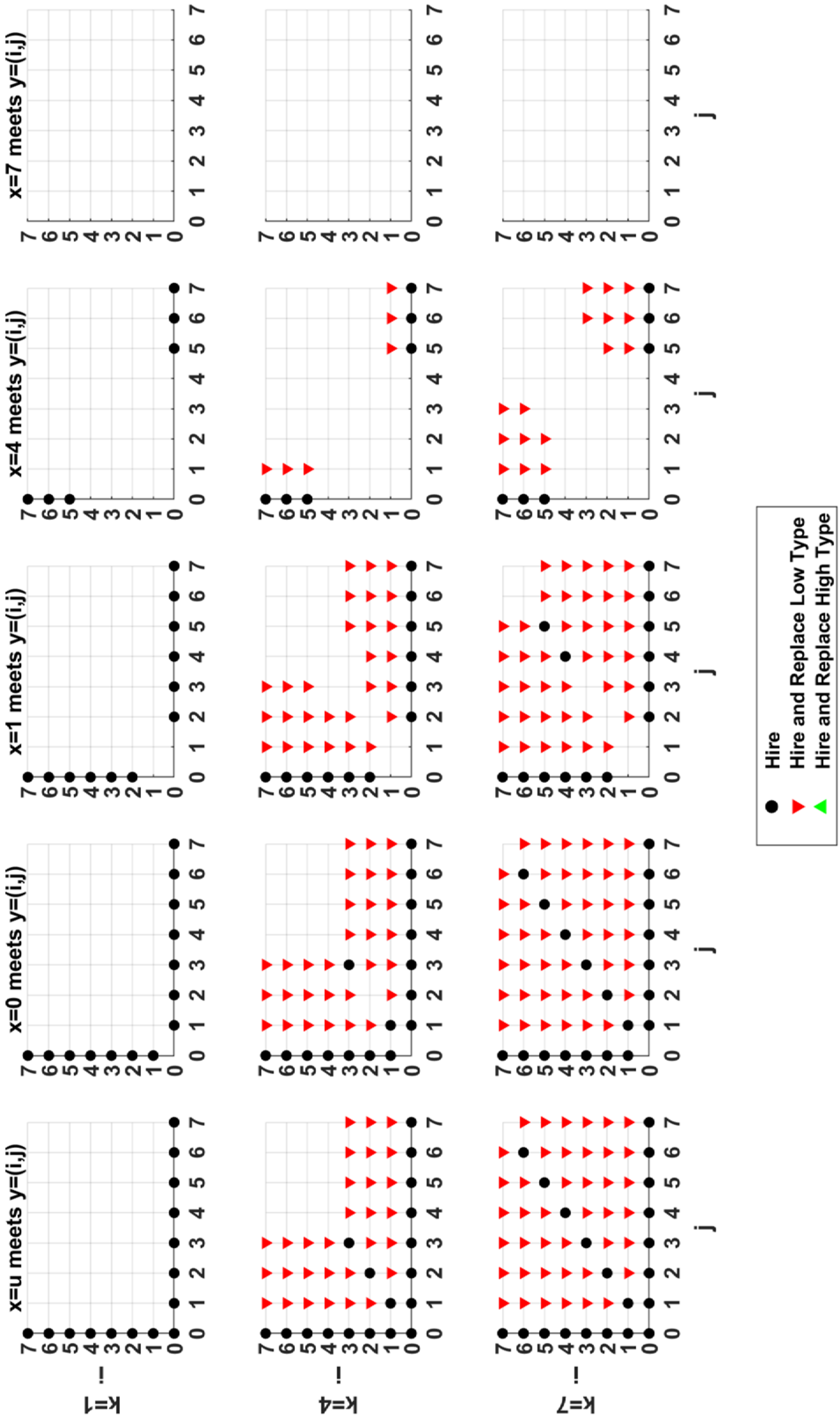

FIGURE 10.—Equilibrium policy function HSM.

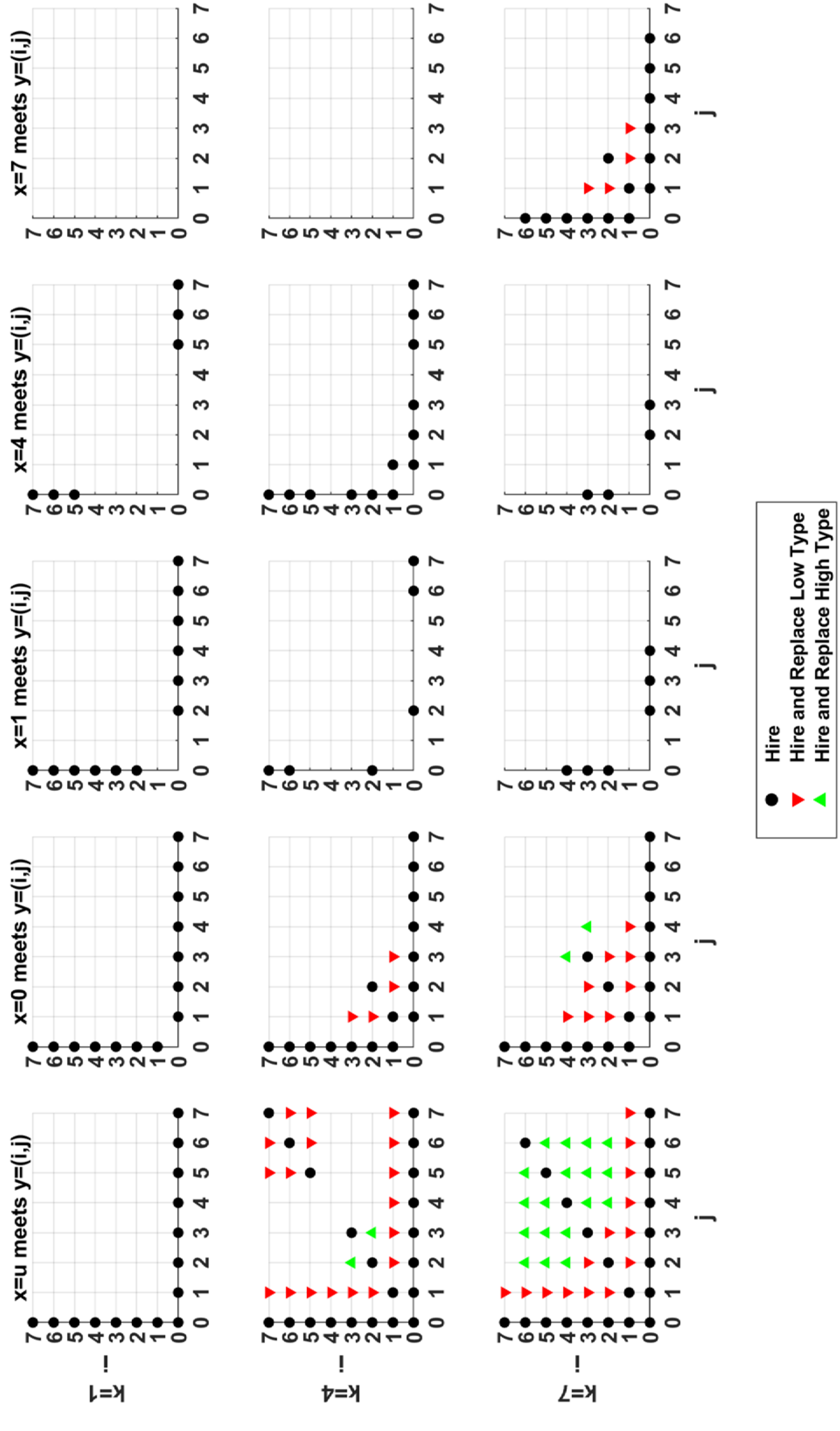

FIGURE 11.—Equilibrium policy function SP.

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