

February 12, 2002

Comparison of Model Specification Methods¹

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Abstract

The objective of this study is to compare alternative computerized model-selection strategies in the context of the vector autoregressive (VAR) modeling framework. The focus is on a comparison of subset modeling strategies with the general-to-specific reduction approach automated by PcGets. Different measures of the possible gains of model selection are considered: (i) the chances of finding the ‘correct’ model, that is, a model which contains all necessary right-hand side variables and is as parsimonious as possible, (ii) the accuracy of the implied impulse-responses and (iii) the forecast performance of the models obtained with different specification algorithms. In the Monte Carlo experiments, the procedures recover the DGP specification from a large VAR with anticipated size and power close to commencing from the DGP itself when evaluated at the empirical size. We find that subset strategies and PcGets are close competitors in many respects. Only the forecast comparison indicates a clear advantage of the PcGets algorithm.

JEL classification: C32, C51.

Keywords: Model selection; Vector autoregression; Subset model; Lag order determination; Data mining.

¹We are grateful to David Hendry for valuable comments and suggestions. Financial support by the Deutsche Forschungsgemeinschaft, SFB 373, and the European Commission under the Training and Mobility of Researchers Programme (contract No. ERBFMRXCT980213) is gratefully acknowledged by the first and the third author. Financial support from the UK Economic and Social Research Council under the grant L138251009 is gratefully acknowledged by the second author. The computations reported in this paper were carried out with PcGets1, see Hendry & Krolzig (2001), Ox3, see Doornik (2001), and Gauss 3.2.32.

1 Introduction

Specifying a parsimonious model for the data generation process (DGP) of a set of variables is a crucial step in econometric analysis. Typically a general model or model class is chosen at the initial stage of the modeling exercise. If the model is found to be general enough to describe the relevant features of the DGP well, i.e., the model is congruent, usually restrictions are imposed to avoid overparameterization which in turn may result in inefficient estimates and forecasts as well as poor structural analysis. Therefore many tools have been suggested which can be used in this important part of the modeling task. Examples of such tools are statistical tests and model selection criteria. Moreover, different algorithms have been proposed which specify the sequence in which the tools should be used, in order to arrive at a useful model. In the context of econometric time series models so-called subset procedures are available, for example. One of their features is that they reduce the model along a specific path which is determined by a variable selection criterion or a sequence of statistical tests. As an extension of these procedures, expert system type algorithms have been developed recently. They take advantage of the currently available substantial computing power and consider various reduction paths using a range of alternative model selection and model checking procedures.

The objective of this study is to compare straightforward subset modeling strategies with the currently most sophisticated expert system PcGets in the context of a vector autoregressive (VAR) modeling framework. The question of primary interest is to determine whether there is any gain from and, if so, how much can be gained by using the more sophisticated apparatus incorporated in PcGets rather than just some standard statistical procedure. In this comparison different measures of the possible gains will be used. Specifically, we investigate the chances of finding the ‘correct’ model, that is, a model which contains all necessary right-hand side variables and is as parsimonious as possible. In addition, we check the accuracy of the implied estimated impulse responses and compare the forecast performance of the models obtained with different specification algorithms. Our comparison is based on Monte Carlo simulations and thereby it is in particular relevant for gaining information of the relative performance of the considered procedures in small samples.

The procedures to be compared are simple model selection strategies as discussed in Brüggemann & Lütkepohl (2001) on the one hand and the PcGets system on the other hand, as mentioned previously. The subset modeling procedures are based on sequential t-tests or model selection criteria such as Akaike’s (1974) AIC, Hannan & Quinn’s (1979) HQ or Schwarz’s (1978) SC. Starting from a sufficiently large VAR model, zero restrictions are placed on the coefficients sequentially where a search for the best restriction is performed in each step. The subset procedures differ in the way they perform the sequential search for suitable restrictions. Because Brüggemann & Lütkepohl (2001) found little gains from considering the full system in each step we will focus on single equation search procedures in this study. In contrast, PcGets is a more sophisticated computer-automated approach to econometric modeling which achieves the model reduction by a

joint selection and diagnostic testing process comparing different model reduction paths. Detailed descriptions of all procedures included in the comparison will be given in the following section. Section 3 presents the Monte Carlo setup and discusses the results. Finally, conclusions are drawn in Section 4.

2 Model Selection Procedures

The comparison will be performed in the framework of standard finite order VAR(p) processes,

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \quad (2.1)$$

where $y_t = (y_{1t}, \dots, y_{Kt})'$ is a K -dimensional vector of observable time series variables, A_1, \dots, A_p are $(K \times K)$ VAR parameter matrices, ν is a $(K \times 1)$ intercept vector and u_t is a white noise process with zero mean and nonsingular, time invariant covariance matrix $E(u_t u_t') = \Sigma_u$.

The subset procedures as well as PcGets work on the individual equations of the model to determine possible zero restrictions, i.e., to eliminate lags of variables. If the restricted model is then used for impulse response analysis or prediction, a feasible GLS procedure is applied for estimating the full system at once. In the following we first present the subset procedures and then provide a brief description of the main features of the PcGets approach.

2.1 Subset Modeling Procedures

Individual parameter restrictions are placed on each system equation separately. Therefore, we write the k -th equation of (2.1) as

$$y_{kt} = x_{1t}\theta_1 + \cdots + x_{Nt}\theta_N + u_t, \quad t = 1, \dots, T. \quad (2.2)$$

All right-hand side variables are denoted by x_{nt} including the constant. The variable selection criteria are of the form

$$CR(i_1, \dots, i_n) = \log(SSE(i_1, \dots, i_n)/T) + c_T n/T, \quad (2.3)$$

where $SSE(i_1, \dots, i_n)$ is the sum of squared errors obtained by including $x_{i_1 t}, \dots, x_{i_n t}$ in the regression model (2.2) and c_T is a sequence indexed by the sample size T . The following variable elimination strategies have, for instance, been considered in the literature (see, e.g., Lütkepohl (1991) and Brüggemann & Lütkepohl (2001)).

Full Search (FS)

Choose the regressors which minimize $CR(i_1, \dots, i_n)$ for all subsets $\{i_1, \dots, i_n\} \subset \{1, \dots, N\}$ and $n = 0, \dots, N$. □

This procedure requires a substantial computational effort if N is large because the set $\{1, \dots, N\}$ has 2^N subsets. Consequently, 2^N models have to be compared. A computationally more efficient procedure sequentially eliminates one variable in each step as follows. For simplicity we assume that the remaining variables are renumbered after each step such that in step j , $N - j + 1$ regressors are under consideration.

Sequential Elimination of Regressors (SER)

Sequentially delete those regressors which lead to the largest reduction of the given criterion until no further reduction is possible. Formally:

Step j: Delete x_{nt} if

$$CR(1, \dots, n-1, n+1, \dots, N-j+1) \\ = \min_{l=1, \dots, N-j+1} CR(1, \dots, l-1, l+1, \dots, N-j+1)$$

and

$$CR(1, \dots, n-1, n+1, \dots, N-j+1) \leq CR(1, \dots, N-j+1). \quad \square$$

It is also possible, of course, to choose individual zero coefficients on the basis of the t-ratios of the parameter estimators. In that case, the regressors with the smallest absolute values of t-ratios are eliminated until they are all greater than some threshold value γ . A formal description of this procedure follows.

Testing Procedure (TP)

Let $t_n^{(j)}$ be the t-ratio from an OLS estimation associated with θ_n in the j -th step of the procedure.

Step j: Delete x_{nt} if $|t_n^{(j)}| = \min_{i=1, \dots, N-j+1} |t_i^{(j)}|$ and $|t_n^{(j)}| \leq \gamma$. Stop if all $|t_n^{(j)}| > \gamma$. \square

In each step of this procedure a single regressor is eliminated. Then new t-ratios are computed for the reduced model. It is shown by Brüggemann & Lütkepohl (2001) that this strategy is equivalent to the sequential elimination based on model selection criteria for a suitably chosen threshold value γ . More precisely, if $\gamma = \{[\exp(c_T/T) - 1](T - N + j - 1)\}^{1/2}$ in the j th step of the elimination procedure, the same final model is obtained that also results by sequentially minimizing the selection criterion defined by the penalty term c_T . Therefore it is apparent that the threshold value depends on the sample size, the number of regressors in the model and the selection criterion through c_T . The threshold values for the t-ratios correspond to the critical values of the tests. The well-known criteria AIC, HQ and SC may be used in these procedures. Their c_T values are $c_T = 2$, $c_T = 2 \log \log T$ and $c_T = \log T$, respectively. Hence, for an equation with 20 regressors and a sample size of $T = 100$, choosing a model by AIC, HQ or SC roughly corresponds to eliminating all regressors with t-values which are not significant at the 15% - 20%, 10% or 2% - 3% level, respectively (see Brüggemann & Lütkepohl (2001)). Because SER and TP are equivalent, we only use TP in the Monte Carlo experiments.

An alternative subset modeling strategy proceeds by checking the variables sequentially from the last to the first. Formally it may be described in the following way.

Top Down Procedure (TD)

For $n = N, N - 1, \dots, 1$, delete x_{nt} if

$$CR(1, \dots, n - 1, n + 1, j_n, \dots, N_n) \leq CR(1, \dots, j_n, \dots, N_n),$$

where N_n is the largest lag remaining in the equation when x_{nt} is checked and x_{jt} is the last previously checked regressor which remains in the equation. \square

Notice that the final outcome of this procedure may depend on the order in which the regressors are included in the model. Clearly, this introduces some arbitrariness into the procedure. In the present time series context it makes sense to check the higher order lags first before proceeding to smaller lags. Of course, there may be good arguments for alternative orderings in specific situations. For example, one may check the lags of one of the variables first and then turn to another variable and so on. In fact, this type of procedure was proposed by Hsiao (1979, 1982) for causality analysis. All subset procedures used in the Monte Carlo study have been implemented using Gauss and a convenient graphical user interface is provided by the software JMulTi.²

2.2 PcGets

PcGets, see Hendry & Krolzig (1999), Krolzig & Hendry (2001) and Hendry & Krolzig (2001) implements automatic general-to-specific (*Gets*) modeling for linear, dynamic, regression models based on the principles discussed in Hendry (1995). First, an initial statistical general unrestricted model (GUM) is tested for the absence of mis-specification (denoted congruence), which is then maintained throughout the selection process by diagnostic checks, thereby ensuring a congruent final model. The diagnostic tests require careful choice to ensure they characterize the salient attributes of congruency, are correctly sized, and do not overly restrict reductions. Next statistically-insignificant variables are eliminated by selection tests, both in blocks and individually. Many reduction paths are searched, to prevent the algorithm from becoming stuck in a sequence that inadvertently eliminates a variable which matters, and thereby retains other variables as proxies. Path searches in PcGets terminate when no variable meets the pre-set criteria, or any diagnostic test becomes significant. Non-rejected models are tested by encompassing: if several remain acceptable, so are congruent, undominated, mutually-encompassing representations, the reduction process recommences from their union. If no unique outcome is obtained or if all selected simplifications re-appear, the search is terminated using the Schwarz (1978) information criterion. Lastly, sub-sample insignificance seeks to identify ‘spuriously significant’ regressors.

²The software package JMulTi can be downloaded at: <http://ise.wiwi.hu-berlin.de/oekonometrie>.

An overview of the algorithm is shown in Table 1, see Hendry & Krolzig (2001) for details. In the following we briefly discuss the econometrics of the different stages of the *PcGets* model-selection algorithm relevant for VAR modeling.

2.2.1 The GUM and pre-search tests (Stage 0)

The starting point for *Gets* model-selection is the GUM, so the key issues concern its specification and congruence. In the case of the VAR, the researcher has to specify the order and the dimension of the process. An overall F-test of all regressors checks that there is something to model, misspecification tests check the congruence of the model. *PcGets* then undertakes various ‘pre-search’ simplification F-tests to exclude variables from the GUM. Since variables found to be irrelevant on such tests are excluded from later analysis, this step uses a loose significance level (such as 50%). The lag order pre-selection consists of F-tests on the longest-lag blocks till the null is rejected. The next step consists of block (F) tests of groups of variables, ordered by their t^2 -values in the GUM. In the *top-down* reduction sequence the t^2 -test statistics are ordered from the smallest up, with cumulative F-tests on increasing block sizes till the null is rejected; the model size decreases until rejection. The *bottom-up* reduction sequence involves F-tests on decreasing block sizes from the largest t^2 -tests down till the model is congruent. The model size increases until no misspecifications are found. According to the outcome of a block F-test, *PcGets* will continue to work with one of the reductions.

2.2.2 Multi-stage multi-path search (Stages I/II)

The *PcGets* reduction path relies on a classical, sequential simplification and testing approach designed to reduce the complexity of the model by ensuring the congruency of the reduction. Many possible paths from that GUM are investigated: reduction paths considered include both multiple deletions as well as single, so t- and/or F-test statistics are used as simplification criteria. Along each path the least significant variable having a t-value less than the critical value is eliminated. If any diagnostic tests fail, that path is terminated, and the algorithm returns to the last accepted model of the search path: if the last accepted model cannot be further reduced, it becomes the terminal model of the particular search path; otherwise, the last removed variable is re-introduced, and the search path continues with a new reduction by removing the next least-significant variable of the last accepted model. If all tests are passed, but one or more variables are insignificant, the least significant variable of those is removed. If that specification has already been tested on a previous path, the current search path is terminated. Finally, if all diagnostic tests are passed, and all variables are significant, the model is the terminal model of that search path. Should multiple congruent contenders eventuate after a reduction round, encompassing can be used to test between them, with only the surviving non-nested specifications retained. If multiple models survive the ‘*testimation*’ process, their union forms a new general model, and selection path searches recommence. Such a process repeats till a unique contender emerges, or the previous union is reproduced,

then stops. In the latter case a final selection is made using information criteria, otherwise a unique congruent and encompassing reduction has been located.

2.2.3 Sub-sample evaluation (Stage III)

As a check for potential over-selection in *Stage II*, *PcGets* exploits sub-sample information by investigating split samples for significance (as against constancy). This mimics the idea of recursive estimation: Since non-central ‘t’-values diverge with increasing sample size, whereas central ‘t’s fluctuate around zero, the latter have a low probability of exceeding any given critical value in two sub-samples, even when those sample overlap. Thus, adventitiously-significant variables may be revealed by their insignificance in one or both of the sub-samples. Consequently, a progressive research strategy can gradually eliminate ‘adventitiously-significant’ variables and tilt the size-power balance favorably. The sub-sample information is used to accord a ‘reliability’ score to variables, which investigators may use to guide their model choice.

2.2.4 Calibration

Balancing the objectives of small size and high power still involves a trade-off, but one that is dependent on the algorithm. The ‘*testimation*’ process of *PcGets* depends on a number of decisions regarding the specification of the algorithm. Krolzig & Hendry (2001) investigate the calibration of *PcGets* with regard to the operational characteristics of the diagnostic tests, the selection probabilities of DGP variables, and the deletion probabilities of non-DGP variables. Based on intensive Monte-Carlo studies, Hendry & Krolzig (2001) propose a ‘*liberal*’ and a ‘*conservative*’ strategy which aim to provide maximum power at a controlled empirical size of 1% and 5%. These will be used in the following Monte Carlo experiments.

3 Simulation Comparison

3.1 Monte Carlo Design

We have considered the following DGPs:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} + \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \alpha_{21,2} & 0 \end{bmatrix} \begin{bmatrix} y_{1,t-2} \\ y_{2,t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \quad (3.1)$$

with covariance matrix $\Sigma_u = I_2$ and

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} \nu_1 \\ \nu_2 \\ 0 \end{bmatrix} + \begin{bmatrix} \alpha_{11,1} & 0 & 0 \\ 0 & \alpha_{22,1} & 0 \\ 0 & 0 & \alpha_{33,1} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11,2} & \alpha_{12,2} & 0 \\ \alpha_{21,2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{1,t-2} \\ y_{2,t-2} \\ y_{3,t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \quad (3.2)$$

with white noise covariance matrix

$$\Sigma_u = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The ν and α coefficients are nonzero and assume values to be discussed shortly. A process similar to (3.1) was also used in simulations by Lütkepohl (1991), Brüggemann & Lütkepohl (2001) and Krolzig (2001). It is very simple and may in fact be too simple for our purposes. It is expected to give an advantage to the TD strategy because all nonzero coefficients are concentrated at the lowest lags. Whenever a nonzero coefficient appears, there is no zero coefficient at a corresponding lower lag. This feature contrasts with DGP (3.2), where nonzero coefficients appear, e.g., at lag 2 when the corresponding lag 1 coefficient is zero (see $\alpha_{12,i}$ and $\alpha_{21,i}$). The process may also be more difficult to specify correctly due to its higher dimension.

We generated $M = 1000$ sets of time series and applied the single equation strategies from Section 3 to the generated time series. To be more precise, we fitted VAR(4) models and then applied the model specification strategies. Of course, the coefficient matrices A_3 and A_4 of the true DGPs contain zeros only. We are interested in determining the ability of different procedures to detect the zero coefficients correctly without replacing too many parameters by zero. In other words, we are interested in the size and power of the procedures. Therefore, controlling the t-ratios is useful in our simulation comparison because a coefficient with a small population t-ratio is more likely to be restricted to zero than one with a larger t-ratio, given that the choice of restrictions in all model selection procedures to be compared is based to some extent on these quantities. For this reason we have chosen parameter values with preassigned absolute t-ratios.

Notice that in a regression model $y = X\theta + u$, the t-ratio of the k th coefficient is

$$t_k = \frac{\hat{\theta}_k}{\hat{\sigma}_{\theta_k}} = \frac{\sqrt{T}\hat{\theta}_k}{\hat{\sigma}_u \sqrt{(T^{-1}X'X)^{-1}_{kk}}}.$$

The corresponding population statistic is

$$t_k^* = \frac{\theta_k}{\sigma_{\theta_k}} = \frac{\sqrt{T}\theta_k}{\sigma_u \sqrt{Q_{kk}^{-1}}},$$

where $Q = \text{plim } T^{-1}X'X$ and Q_{kk}^{-1} is the k th diagonal element of the inverse of Q . For our VAR(2) processes it can be shown that

$$Q = \begin{bmatrix} 1 & \tilde{\mu}' \\ \tilde{\mu} & \Gamma + \tilde{\mu}\tilde{\mu}' \end{bmatrix},$$

where $\tilde{\mu} = (1, 1)' \otimes \mu$. Here $\mu = (I_K - A_1 - \dots - A_p)^{-1}\nu$ is the mean of the VAR and Γ is the covariance matrix of (y_t, y_{t-1}) . Hence, it is easy to work out the population t-ratios of the parameters in our DGPs. We have chosen the parameter values such that prespecified t-ratios of the nonzero

coefficients are ensured. The actual parameter values for the DGPs are given in Tables 2 and 3. We also give the corresponding moduli of the nonzero reverse characteristic roots of the autoregressive polynomials in the tables. For example, for each of the DGPs (3.1) there are two roots with the largest moduli and there is one root which is zero. Obviously, the larger the t-ratios, the closer the roots are to the unit circle. As these processes are closer to the nonstationarity region, we cannot expect that the selection properties of the reduction approaches will be monotonous functions of the underlying t-values. These and other characteristics of the DGPs will be of importance for the outcome of our experiment.

3.2 Evaluation of the Monte Carlo Results

3.2.1 Selection properties

The results for the DGPs (3.1) and (3.2) are summarized in Figures 1 - 6. In Figures 1 and 2 the average size and power of the different procedures are depicted. Here the average size is determined as the average relative frequency of including a zero coefficient. That is, the average rejection frequency of all coefficients which are actually zero is determined. The average power is the corresponding probability of including the nonzero coefficients. The average size is plotted on the horizontal axis whereas the power is shown on the vertical axis. For instance, in the upper left-hand panel of Figure 1, which shows results for the first equation of the DGP, the FS procedure combined with the SC criterion is seen to have a size of about 4% and a power of about 40%. For each of the procedures using model selection criteria, SC results in the smallest and AIC in the largest sizes whereas HQ is in between. To be able to assess the results, the theoretical power of a t-test is also shown in the figures by a solid line. That line is a power envelop because we cannot hope to do better than the nonsequential t-tests with our procedures. It is obvious that all the procedures are doing quite well in this respect. There are, however, some notable differences with respect to the empirical size. For example, subset procedures using the AIC criterion typically have a size around 20%, the size of HQ based algorithms is around 10% while the size of SC based strategies is between 4 and 8%. The difference between FS, TP and TD is very small when the same selection criterion is used. The size of the conservative PcGets (PcGets-CON) algorithm is smallest of all selection methods, while the size of the liberal variant (PcGets-LIB) is typically very close to the SC based subset procedures.

In Figures 3 and 4 the overall size and power of the procedures are shown. The horizontal axis shows the estimated probability that at least one of the zero coefficients is included whereas the vertical axis shows the probability that all nonzero coefficients are included. The solid line is based on selecting the nonzero coefficients by multiple t-tests. Thus, it may again be regarded as a power envelop. In this case all procedures are a bit away from this envelop for DGPs with t-ratios less than 5. For the two-dimensional DGP the TD strategy is slightly superior to the other subset modeling procedures which may be a consequence of the specific type of DGP, as mentioned earlier. Consequently, the ordering inverses in case of the three-dimensional DGP. In addition,

for t-ratios less than 5 there is a clear trade-off between the probability of retaining irrelevant variables and selecting DGP variables. The liberal strategies based on AIC and HQ have the highest probability of including DGP variables but at the same time the probability of including non-DGP variables is also very high. In contrast, for $|t| > 5$ moving from AIC to SC based algorithms is associated with almost no loss in power (selection probability) while the probability of including non-DGP variables decreases drastically.

Finally, Figures 5 and 6 show the relative frequencies of finding precisely the true DGP, that is, all zero coefficients are deleted and all nonzero coefficients are included. For this process type, the subset procedures are doing quite well relative to PcGets for processes with t-values smaller than 4. In fact, there are many cases when the simple subset strategies have a higher probability of finding the true DGP, however, the chance to find the DGP is very limited in any case. This has to be expected given the probability to find the DGP if one starts with it, which is represented by the horizontal lines (at significance levels of 1 and 5 percent). For t-values larger than 4, the conservative variant of PcGets consistently selects the true DGP with highest probability, although some subset procedures, especially the parsimonious SC based methods, are close competitors.

3.2.2 Impulse response analysis

We now check how the selection properties of the proposed selection strategies are translated into the accuracy of the impulse responses implied by the empirical model.

The infinite-order vector moving-average representation of the VAR model in (2.1) is

$$y_t = \mu + \sum_{j=0}^{\infty} \Psi_j \varepsilon_{t-j}, \quad (3.3)$$

where $\mu = (I_K - \sum_{i=1}^p A_i)^{-1} \nu$ and $\Psi_0 = I_K$. $\Psi(L)$ is the inverse lag polynomial to $A(L)$, $(I_K - \sum_{j=1}^p A_j L^j)^{-1}$. The (k, l) -th element $\psi_{kl,j}$ of the MA matrix Ψ_j can be interpreted as the reaction of variable k in response to a unit shock in variable l , j periods ago.

Figure 7 displays the mean squared error (MSE) of the impulse responses of the system for the 11 reduction approaches and the true VAR relative to the MSE for the full VAR(4). Let $\psi_j = \text{vec}(\Psi_j)$ collect the response of the system to shocks j periods ago, then the graphs report for the various DGPs, each associated with a certain t-value, the relative MSE:

$$\frac{\text{MSE}_{\mathcal{M}}(j)}{\text{MSE}_{(4)}(j)}$$

for $j = 1$ and 5 , where \mathcal{M} is the reduction method, ψ_j is the theoretical response of the system given the specification of the DGP, and $\hat{\psi}_j$ is the estimated response of the system given the estimated VAR: $\hat{\Psi}(L) = \hat{A}(L)^{-1}$. The MSE is defined as

$$\text{MSE}(j) = \frac{1}{M} \sum_{m=1}^M \left(\psi_j - \hat{\psi}_{j,m} \right)' \Sigma_{\psi_j}^{-1} \left(\psi_j - \hat{\psi}_{j,m} \right)$$

where Σ_{ψ_j} has been estimated using the simulated impulse-responses for the full VAR(4) and M is the number of replications in our simulation experiments.

By definition, the true model has always the smallest MSE. Overall, the reduction procedures do quite well for $j = 5$, but are hurt by small t-values for $j = 1$. In this case, the reduction procedures suffer from imposing wrong null-restrictions when the true autoregressive coefficient is non-zero but hard to estimate. It is worth noting that the reduction methods work like shrinkage methods and therefore the Litterman prior (see Doan, Litterman & Sims (1984)) would suffer in the same way.

3.2.3 Forecasting

To assess the forecasting power of the selected models relative to the full VAR(4) model we compare MSPEs of 1- and 5-step ahead out-of-sample forecasts. More precisely, we compute the MSPE as

$$\text{MSPE}(h) = \frac{1}{M} \sum_{m=1}^M (y_{T+h,m} - \hat{y}_T(h)_m)' \Sigma_y(h)^{-1} (y_{T+h,m} - \hat{y}_T(h)_m) \quad (3.4)$$

where $y_{T+h,m}$ is the generated vector of the m -th K -variate time series for which the forecast is made, $\hat{y}_T(h)_m$ denotes the corresponding h step ahead forecast at origin T from model specification under consideration and $\Sigma_y(h)$ is the theoretical covariance matrix of the corresponding h -step prediction error (see Lütkepohl (1991, Sec. 3.5) for precise expressions). Now we can express the percentage improvement of the forecast precision relative to the full VAR(4) as

$$\frac{\text{MSPE}_{(4)}(h) - \text{MSPE}_{\mathcal{M}}(h)}{\text{MSPE}_{(4)}(h)} \times 100, \quad (3.5)$$

where $\text{MSPE}_{(4)}$ denotes the MSPE of the full VAR(4) model and $\text{MSPE}_{\mathcal{M}}$ is the MSPE of the selected model.

For convenience of the reader, we present the forecast comparison in graphical form. In Figure 8 and 9 we give the percentage improvement in 1- and 5-step ahead forecast MSPEs relative to the full VAR(4) model according to equation (3.5). From results for the bivariate DGPs (3.1) in Figure 8 we find for the 1-step forecast the largest gain if the DGP is a white noise process ($|t| = 0$). This is not surprising since it should pay to delete nuisance regressors in this case and it does: The gain in forecast precision from model selection is between 4% and 12%. In contrast, for DGPs with population t-values 2 and 3 the subset procedures FS, TP and TD are associated with a loss in forecasting performance of up to 2%, while the PcGets selection gives a gain of 3.9-6.5%. With increasing population t-values the gain in forecasting performance increases only slightly but for $|t| > 3$ all model selection methods lead to improved forecasts relative to the full VAR(4) model. A similar pattern can be observed for the 5-step forecasts. For $|t| = 2$ all subset procedures and the conservative PcGets method are associated with a small loss ($< 1\%$) in forecasting power. Only the liberal variant of PcGets improves the forecasting power by 1.5%. Again, for $|t| > 3$ model

selection pays off, although gains are typically much smaller than for the 1-step ahead forecast ($< 3.7\%$). In all considered cases, PcGets selected models are better at forecasting than the subset procedures, although in some cases the differences are not very large. The difference between the subset strategies FS, TP and TD is usually fairly small. In fact, we often observe that for a given information criterion, FS, TP and TD lead to improvements with the same order of magnitude. Moreover, we typically observe that using the most parsimonious criterion (i.e. SC) leads to the largest improvement, especially for large population t-values.

In Figure 9 we give results for the forecasting experiment for the 3-dimensional DGP (3.2). Similar to the results in the bivariate case, we find that PcGets consistently leads to the largest improvements in forecasting performance. At best the improvement is 14.2% for $h = 1$ and 9.5% for $h = 5$, which compares to 10% ($h = 1$) and 5.8% ($h = 5$) for the best subset method available. From all subset methods the most parsimonious ones (with SC as an information criterion) work best. Consistent with the size and power results, TD often seems to have the edge when compared to FS and TP. As before, the gain for 1-step ahead forecasts is larger than for 5-step forecasts. In contrast to the bivariate case, all selected models now lead to improved forecast MSPEs, which might be an indication that model selection is especially useful in models with larger dimensions. Overall the forecast comparison indicates a clear advantage of the PcGets algorithm.

4 Conclusions

In this paper we have evaluated alternative computerized model selection strategies for VAR models. In particular, we compared single path subset strategies based on information criteria (or sequential testing procedures) with the more sophisticated PcGets algorithm. The latter provides a computer automation of general-to-specific (*Gets*) modeling of linear, dynamic, regression models using multiple search paths along which the reduction of the model is conducted.

To assess the performance of alternative reduction methods we compared different measures of possible gains associated with model selection. Using Monte Carlo experiments, we first investigated the size and power properties. Moreover, we analyzed the chances of finding the ‘correct’ model, that is, a model which contains all necessary right-hand side variables and is as parsimonious as possible. Then the accuracy of the implied impulse-responses and the forecast performance of the models obtained with different specification algorithms were compared.

One aim of the paper was to evaluate alternative reduction strategies, to see if they worked well, indifferently, or failed badly. The results come much closer to the first. In fact, results from our Monte Carlo experiments show that the procedures recover the DGP specification from a large VAR with anticipated size, and power close to commencing from the DGP itself when evaluated at the empirical size. Results for the impulse response accuracy indicate many situations where model selection can successfully increase the precision relative to the full VAR model. However, there are also some cases (when the population t-value is small), where model selection strategies delete to many relevant variables and hence, adversely affect the impulse response accuracy. We

find a similar picture for the forecasting exercise, where the largest gain from model selection is found in the trivariate systems.

The comparison of subset methods with PcGets revealed some interesting results: The simple subset methods are close competitors to the more sophisticated PcGets algorithm. In fact, in some cases a simple top down procedure has the best power-size trade-off. Moreover, for $t < 4$ there are some cases when subset strategies have the highest probability of finding the true DGP. On the other hand, for t -values larger than 4, the conservative variant of PcGets consistently selects the true DGP with highest probability. Overall the differences between PcGets and SC based subset methods are surprisingly small given the fact that PcGets is based on multiple search paths. But searching over multiple search paths obviously pays in some situations: The forecast comparison indicates a clear advantage of the PcGets algorithm.

In this study we only considered stationary DGPs. A natural extension will therefore include the investigation of different model selection methods in the presence of integrated and cointegrated time series.

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Table 1: The *PcGets* algorithm

Stage 0. Pre-search reductions

1. Estimation and test of the GUM;
2. Outlier correction;
3. Adjust significance level of diagnostics;
4. Lag order pre-selection;
5. Sort variables in order of their t^2 values
 - Two-step top-down reduction;
 - Bottom-up reduction;
 - Encompassing.

Stage I. Multiple model reduction paths:

1. Sequential estimation and test of reductions
 - (a) Remove insignificant variables.
 - (b) Model reductions are subjected to a wide range of diagnostic tests:
 - Chow tests for structural stability;
 - residual autocorrelation;
 - ARCH effects in the residuals;
 - normality;
 - heteroscedasticity.
2. Encompassing

Stage II. Union testing

1. Estimation and test of the new GUM;
2. Multiple model reduction paths;
3. Encompassing and final model selection.

Stage III. Sub-sample evaluation

1. Test the significance of every selected variable in two overlapping sub-samples;
2. Penalize variable accordingly.

Table 2: Parameters for the 2-dimensional DGP (3.1) and T = 100

	ν_1	ν_2	$a_{11,1}$	$a_{21,1}$	$a_{12,1}$	$a_{22,1}$	$a_{21,2}$	Σ_u	modulus ^a
$ t = 0$	0	0	0	0	0	0	0	I_K	
$ t = 2$	0.213	0.214	0.192	0.196	-0.185	0.191	0.202	I_K	.43; .20
$ t = 3$	0.352	0.353	0.276	0.289	-0.250	0.267	0.307	I_K	.57; .23
$ t = 4$	0.531	0.538	0.348	0.379	-0.286	0.323	0.419	I_K	.68; .26
$ t = 5$	0.748	0.787	0.407	0.467	-0.298	0.358	0.538	I_K	.76; .28
$ t = 6$	0.990	1.096	0.455	0.556	-0.293	0.377	0.663	I_K	.81; .29
$ t = 8$	1.493	1.846	0.526	0.737	-0.262	0.389	0.926	I_K	.88; .31
$ t = 10$	1.995	2.741	0.573	0.922	-0.228	0.387	1.198	I_K	.92; .32

^aModulus of nonzero reverse characteristic roots of VAR process.

Table 3: Parameters for the 3-dimensional DGP (3.2) and T = 100

	ν_1	ν_2	$a_{11,1}$	$a_{22,1}$	$a_{33,1}$	$a_{11,2}$	$a_{21,2}$	$a_{12,2}$	σ_1^2	modulus ^a
$ t = 0$	0	0	0	0	0	0	0	0	1	
$ t = 2$	0.212	0.214	0.193	0.192	0.196	0.193	0.192	-0.187	0.951	.53; .36; .20
$ t = 3$	0.320	0.360	0.279	0.275	0.287	0.280	0.299	-0.237	0.745	.65; .41; .29
$ t = 4$	0.502	0.575	0.357	0.346	0.371	0.363	0.366	-0.287	0.706	.75; .43; .37
$ t = 5$	0.598	0.930	0.429	0.403	0.447	0.448	0.535	-0.244	0.389	.82; .45; .44
$ t = 6$	1.045	1.625	0.495	0.448	0.514	0.540	0.523	-0.272	0.417	.87; .51; .43
$ t = 7$	3.384	3.987	0.557	0.485	0.573	0.642	0.348	-0.401	0.835	.90; .57; .41

^aModulus of nonzero reverse characteristic roots of VAR process.

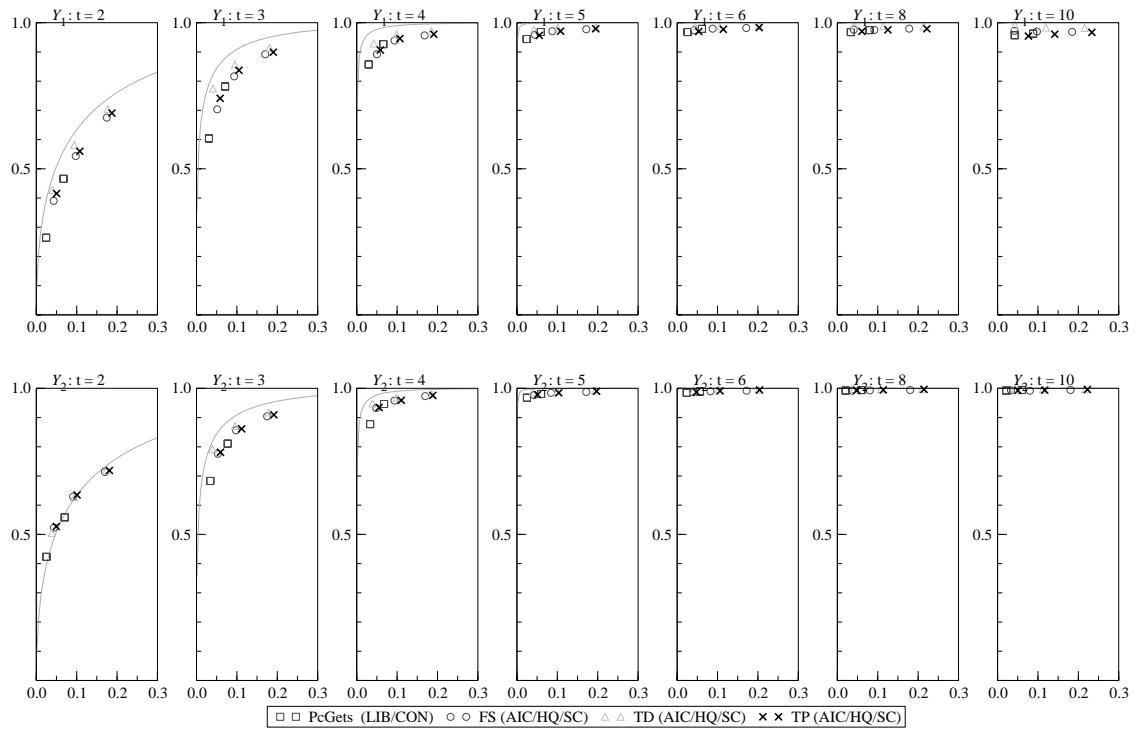


Figure 1: DGP (3.1): Power – size trade-off

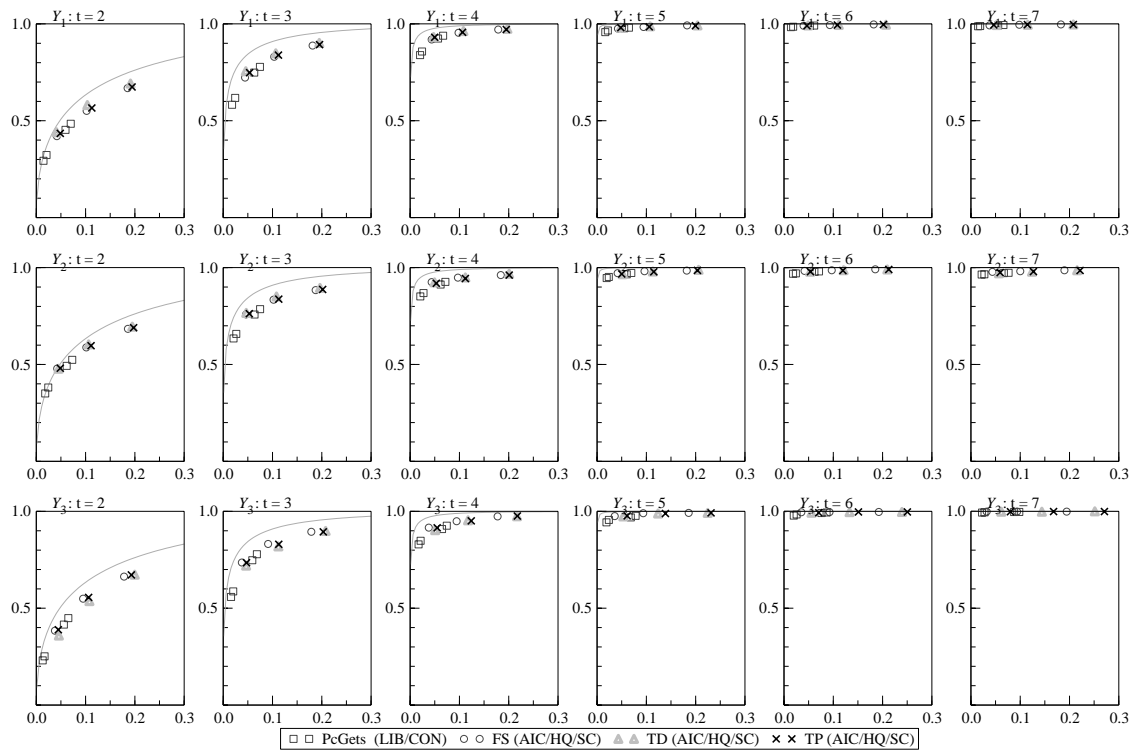


Figure 2: DGP (3.2): Power – size trade-off

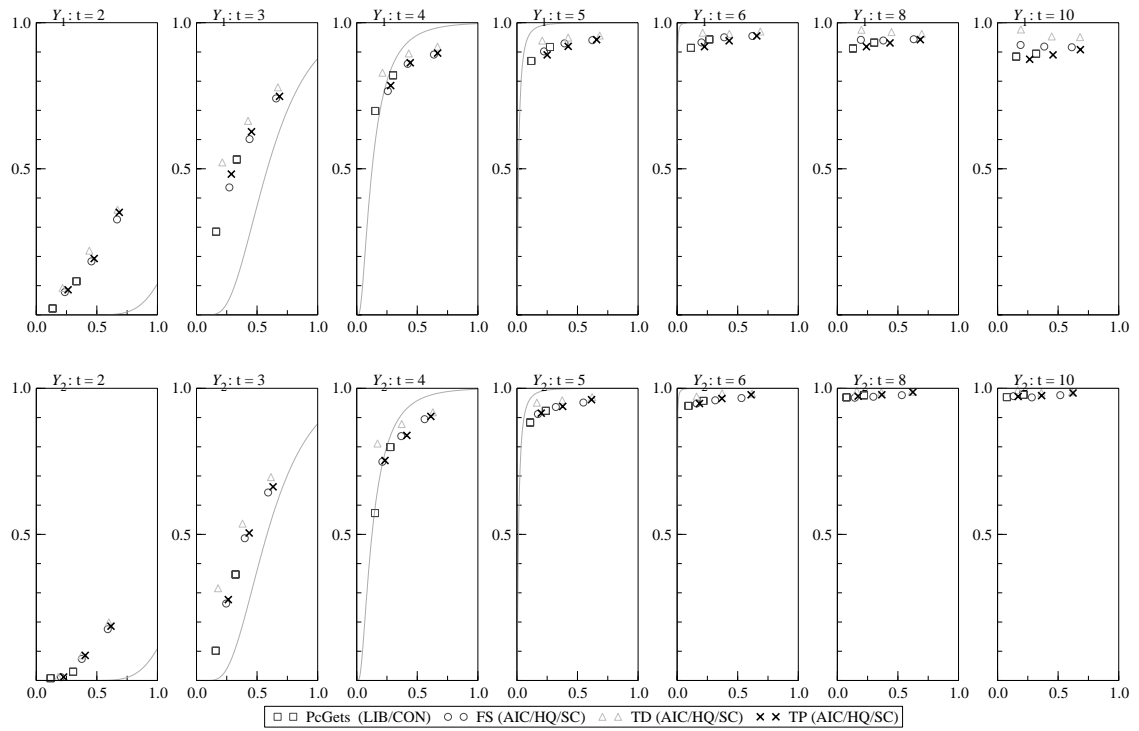


Figure 3: DGP (3.1): Selection versus non-deletion probabilities

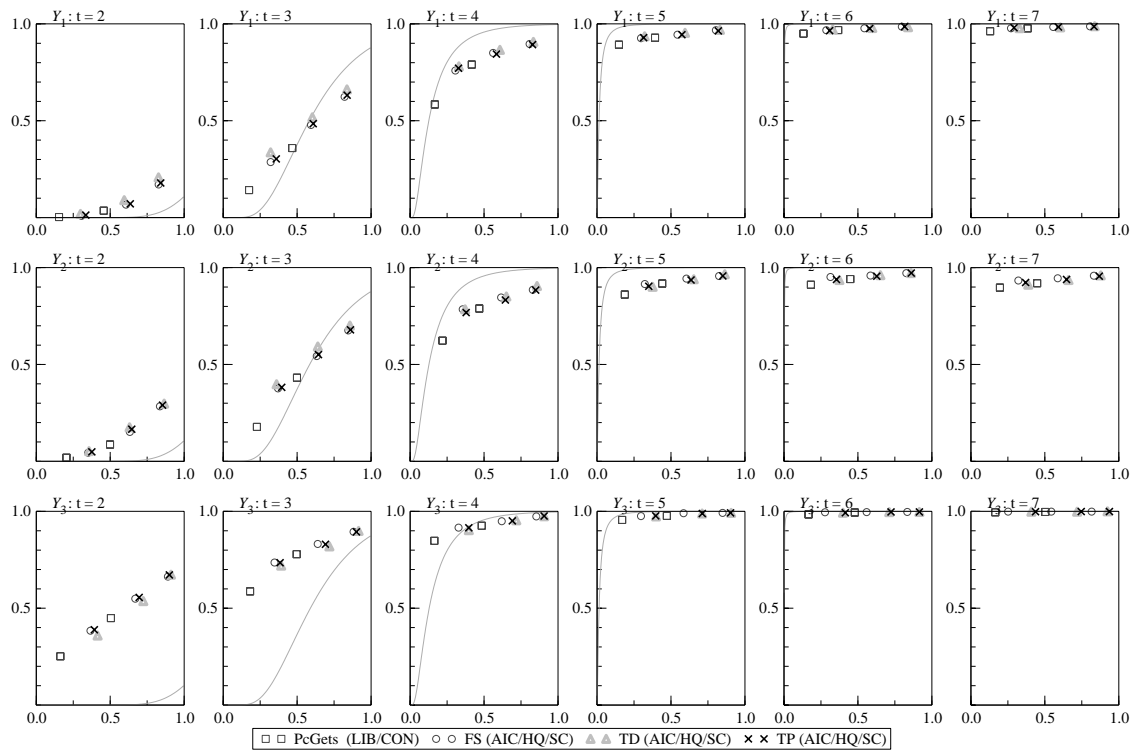


Figure 4: DGP (3.2): Selection versus non-deletion probabilities

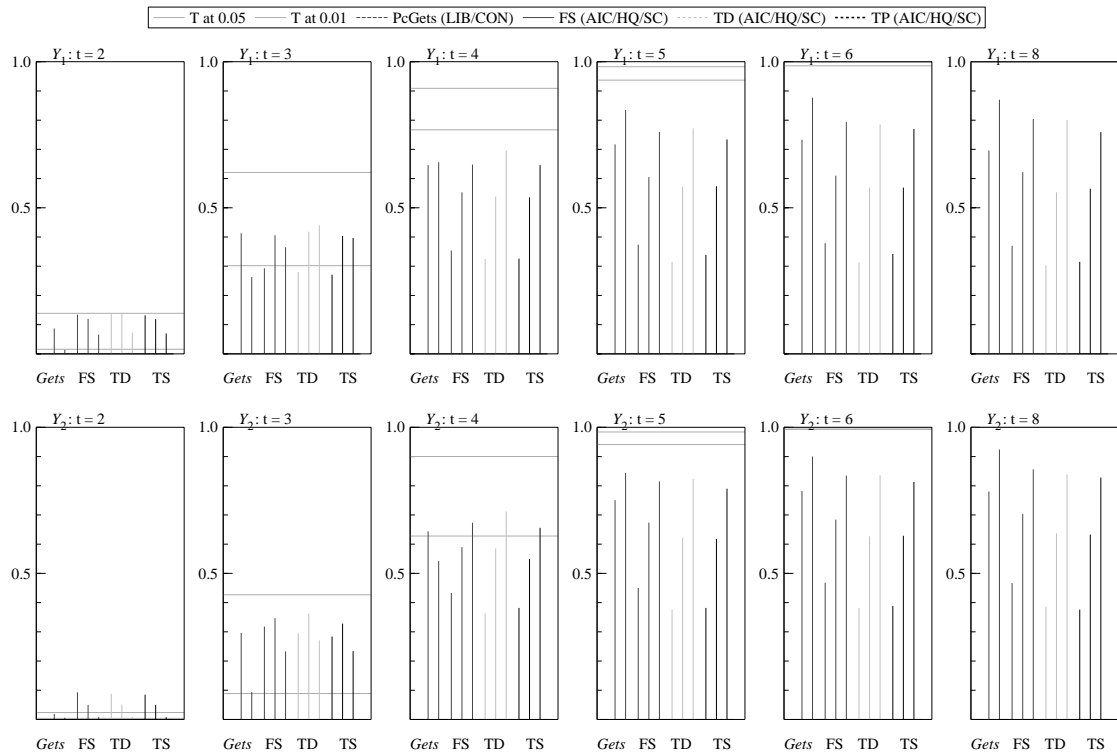


Figure 5: Probability to find the DGP (3.1)

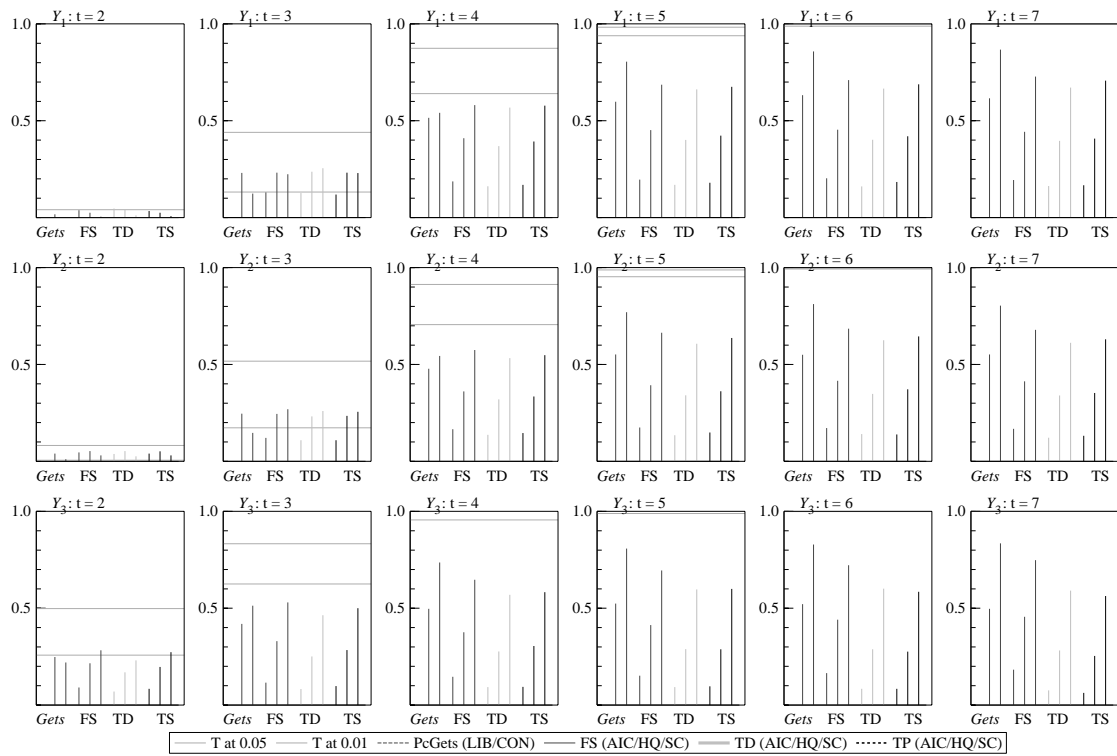


Figure 6: Probability to find the DGP (3.2)

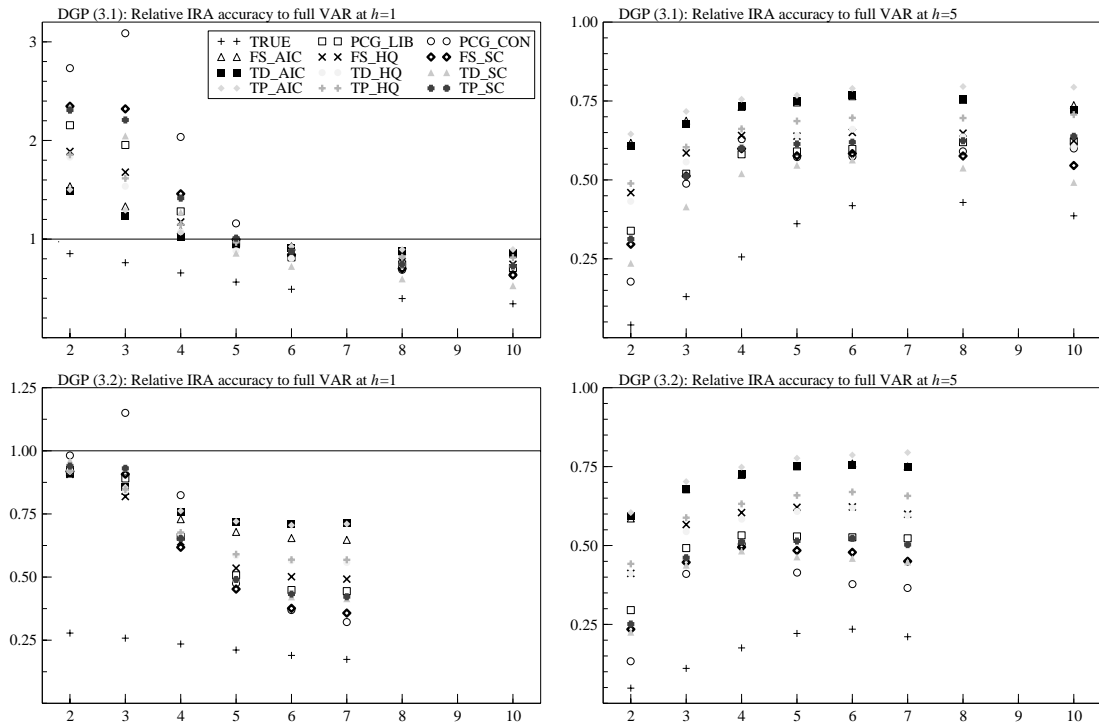


Figure 7: Impulse-response accuracy relative to the VAR(4)

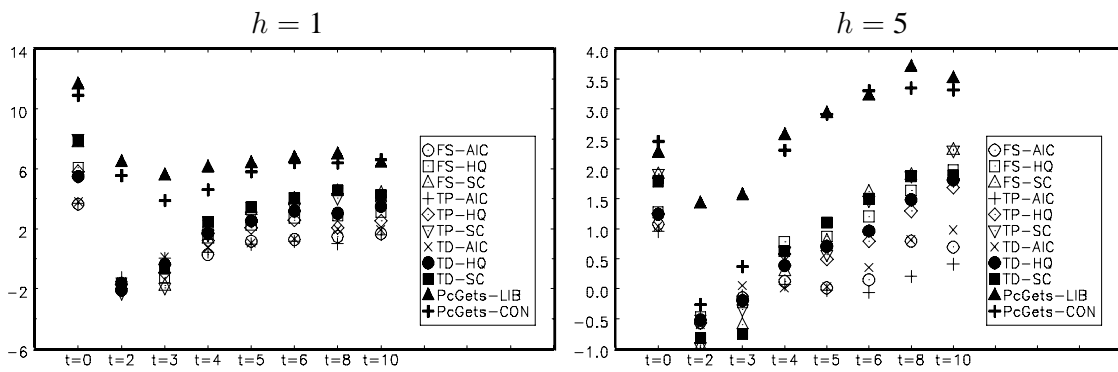


Figure 8: Percentage Improvement of Normalized MSPE Relative to VAR(4), DGP (3.1).

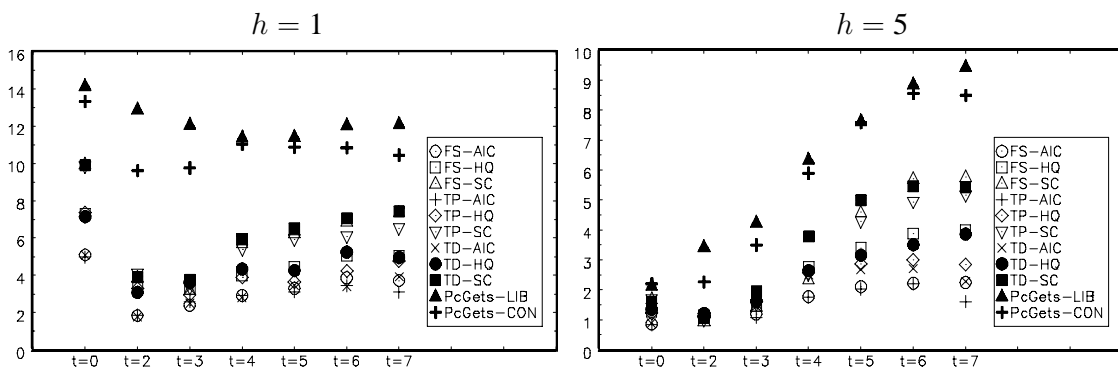


Figure 9: Percentage Improvement of Normalized MSPE Relative to VAR(4), DGP (3.2).