Platforms and Policies

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Abstract

It is frequently believed, and empirically observed, that the outcome of an election determines government action to at least some extent. However, the dominant two-party spatial model of political competition trivializes any post-electoral policy-making process by adopting a winner-takes-all assumption, that is, the party that obtains more than fifty percent of the vote implements the policy announced during the electoral campaign.

This paper differentiates between policies and platforms at two levels: it incorporates a non-trivial policy-setting process, and it models voters who care not only about the implemented policy but also about the platform they support with their vote. We show that platform convergence is a non-robust feature created by the winner-takes-all assumption. The lightest influence of the opposition in the policy-making process provokes a divergent tendency in platform writing. Platforms can be even more extreme than the candidates' ideal policies. With purely pragmatic voters, parties radicalize their positions at equilibrium but the implemented policy is moderate and consistently differs from the median voter ideal policy. When voters care about both the policy and the platforms, parties announce differentiated yet non-extreme policies, voters concentrate around the platforms, and substantial turnout rates are generically obtained even with positive costs of voting. Abstention occurs among voters with extreme views as well as with moderate views. Our results are consistent with the observation of polarized platforms and moderate policies.

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JEL classification: D72

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1 Introduction

It is frequently believed, and empirically observed, that the outcome of an election determines government action. This paper differentiates between policies and platforms at two levels: it incorporates a non-trivial policy-setting process, and it models voters who care not only about the implemented policy but also about the platform they support with their vote. We will show that the lightest influence of the opposition in the policy-making process provokes a divergent tendency in platform writing.

In the relationship between party platforms and policies, papers like Poole and Rosenthal (1984a,b, 1991, 1997), Fiorina (1974), Poole and Daniels (1985), Snyder (1996), and Alesina and Rosenthal (1995) compare the perceived location of platforms and implemented policies, and conclude that while platforms tend to be polarized, policies are moderate or centrist.[1] Likewise, Budge and Hofferbert (1990, 1992), Hofferbert and Klingemann (1990), Klingermann et al. (1994), and King et al. (1993) find that while there exist links between platforms and governmental outcomes, they differ systematically.

Despite this evidence, the dominant two-party spatial models of political competition do not differentiate between platforms and policies. They assume that the party that obtains more than fifty percent of the vote adopts the policy announced during the electoral campaign. As a consequence, any post-electoral policy-making process becomes a triviality. And this is true not only for the most classical representations of spatial competition (Downs, 1957; Black, 1958)), but also for more sophisticated models incorporating uncertainty into the analysis (Wittman, 1973, 1990; Roemer, 1999).[2]

In this paper we depart from the dominant literature by letting the margin of victory play a role in the policy setting process. That is, in the spirit of the empirical results, we let the implemented policy differ to some extent from the electoral platform of the winning party. In particular, the implemented policy will depend on the voting outcome and on the platforms announced by each party, in such a way that the higher the fraction of votes obtained by a party, the closer the implemented policy is to its proposal.[3]

Modifying the policy-setting process has two important implications on voting behavior often overlooked by the literature. First, unlike in the winner-takes-all case, if electoral platforms and implemented policies differ, then voting for the preferred candidate is not always a dominant strategy. Consider, for instance, a voter with

[1] Poole and Rosenthal (1984b) use thermometer scores to measure the perceived location of candidates and policies; Fiorina (1974), Poole and Rosenthal (1984a), Poole and Daniels (1985) use interest groups ratings; Poole and Rosenthal (1991, 1997), Snyder (1996), Alesina and Rosenthal (1995) discover the same pattern using roll call votes in congress.

[2] See Roemer (2001) for an extended analysis of Downs and Wittman models.

[3] It will become clear later in the paper that the gap between policies and platforms does not need to be large for the results to obtain. Thus, in principle, the implemented policy and the platform announced by the winning party may be quite close to each other.

moderate views who finds the proposal of the left party (L) more appealing than the one from the right party (R). Suppose also that she expects, due to an overwhelming support for the left party, that the implemented policy will be 'too lefty' for her. Then, she may decide to vote for the right party in order to 'moderate' the policy implemented by the left. That is, we may find *strategic voting* in the sense that a voter who prefers the alternative offered by L to the one announced by party R may, nevertheless, vote for R to moderate L's policy.

Second, since platforms and policies do not necessarily coincide, we cannot ignore that voters care not only about the implications over final policies, but also about the platform they support with their vote. For example, a conservative voter may feel reluctant to vote for an extreme-right party, regardless of the impact that a larger support for that party would have on the implemented policy. Her reluctance would come not from the impact over final policies, but from the ideology represented by such a party. If the extreme-right party were the only alternative to obtain a more conservative policy, we may expect this voter to feel alienated and abstain.[4]

Therefore, we will distinguish two components in voting behavior. On the one hand, insofar as voters care about the policy outcome, they will choose to vote for one party or the other based on the effect that a larger support for each party has over the implemented policy. We refer to this component as the *pragmatic* component of preferences, and we may think of it as representing the policy-oriented side of voters. On the other hand, voters will consider the platform they support with their vote when deciding for whom to vote. This is the *ideological* component of preferences, representing the platform-oriented side of voters. Because platforms and policies do not necessarily coincide, voters who would have supported a party from a purely pragmatic point of view may decide not to do it because they strongly disagree with the party's platform. Hence, the final voting decision will depend on both the pragmatic and the ideological components of preferences.

Summarizing, we study a unidimensional political game with a dynamic structure. At period one, parties announce their campaign platforms. At period two, citizens observe the platforms and decide whether to vote for one of the two candidates, or to abstain. Their decisions are based on their preferences over policies and platforms, and their expectations about the outcome of the election. Finally, after the election, a policy-setting process effects a policy as a function of both the platforms of each party and their electoral support.

We find that convergence of parties' platforms, traditionally associated with political competition, is an artifact of the winner-takes-all assumption. If the opposition party intervene (to some extent) into policy-setting, a divergence tendency forces parties to take radical positions. Observe that there is no uncertainty about the distribution of the electorate in this model.[5] Thus, we find not only party dif-

[4]See Rabinowitz and Macdonald (1989) for a similar voting behavior in a different context.

[5]Since Wittman (1973, 1983, 1990) and Calvert (1985), it is known that some form of incomplete information (uncertainty) and parties with policy preferences may explain platform differentiation in a traditional two-party model (see Roemer (2001) for an extended analysis of uncertainty in

ferentiation in a complete information setting, but more importantly a polarization tendency that drives parties towards radical positions.[6] On the other hand, it is the fact that voters are concerned with not only policy but with voting for a close platform as well what explains some convergence, as extreme platforms can alienate large portions of the electorate.

The remainder of the paper is organized as follows. Section 2 describes the model and introduces the equilibrium concept. The reader interested in an overview of the main results can look first at the examples worked out in Section 3. The following two sections derive the formal results. Section 4 presents a comparative statics exercise where we study the case of an electorate only concerned about the implemented policy. Section 5 studies the full model, where the electorate cares about both platforms and policies. Finally, Section 6 concludes.

2 The Model

We model the electoral process as a political game between parties and voters with a dynamic structure. Two parties, L and R, announce their campaign platform by choosing a location on the policy space. Citizens observe the platforms and vote for one of the two candidates, or abstain. Finally, after the election, a policy formation process takes place during which both parties play a role in shaping the implemented policy: one as the governing party, the other as the opposition.

The following sections describe the three steps in the political game: platform announcement, voting, and policy formation. Following the standard *backward induction* methodology, we start from the last stage, and then work backward to describe the behavior of the players before.

The electoral arena is assumed to be represented by a unidimensional policy space $T = \left[\underline{t}, \bar{t}\right] \subset \mathbb{R}$. We will denote $T^2 = T \times T$.

2.1 Policy Formation

A key feature of the present model is that the implemented policy may differ from the electoral platform of the winner. Policy is assumed to be the result of a post-electoral process where both parties play a role relative to their electoral support. We may think, for instance, that a democratic society is integrated by different institutions and groups. Some of them will favor the policies announced by the winning party, some will favor the opposition's platforms, but all may have some influence on

political competition models.) However, unlike in our model, competition forces parties to partially converge towards the (expected) median voter.

[6] In a different setup, Ingberman and Villani (1993) and Alesina and Rosenthal (2000) also incorporate institutional constraints in policymaking and find platform polarization. However, in their setting, "if the distribution of voter ideal points were common knowledge, both parties would, as in the traditional model, continue to choose platforms at the median" (Alesina and Rosenthal, 2000, p.6).

the actions taken by the government, and their influence will be a non-decreasing function of the support received by the corresponding party (Ortuño-Ortín, 1997). Alternatively, we may assume that the ruling party knows that its term will reach an end and it expects that the other party will be in power sometime in the future. Then, this party may go to some extent towards the interests of the opposition when setting the policy, expecting the same treatment when it itself becomes the opposition party. That is, the adopted policy would represent the outcome of a tacit collusion between forward-looking parties in a plurality political environment (Dixit et al., 2000).[7] For the purpose of this paper, it will be convenient to model any policy setting story in a reduce form by directly specifying a map from platforms and voting outcomes to final policies.

Thus, we take the *weight function*, $g : [0, 1] \to [0, 1]$ as a *datum* of the model. If ν is the fraction of active voters that vote for L, then $g(\nu)$ captures the weight of party L's proposal in the implemented policy. The following assumption describes the conditions imposed on g.

Assumption 1 (A1) a) $g \in C^1$;
b) $g(0) = 0,\ g(1) = 1$;
c) $g'(\nu) > 0, \quad \forall \nu \in (0, 1)$.

Parts (*b*) and (*c*) are very natural assumptions. (*b*) says that if one of the parties obtains all the votes, its platform is then adopted. By (*c*), the larger the fraction of the vote a party receives, the stronger its influence in the determination of the policy. Finally, (*a*) introduces continuity: small perturbations in the voting outcome cannot produce large changes in the implemented policy.

The relevant properties of the outcome mapping are captured by an implemented policy function, which associates to each voting result a final policy outcome. Formally:[8]

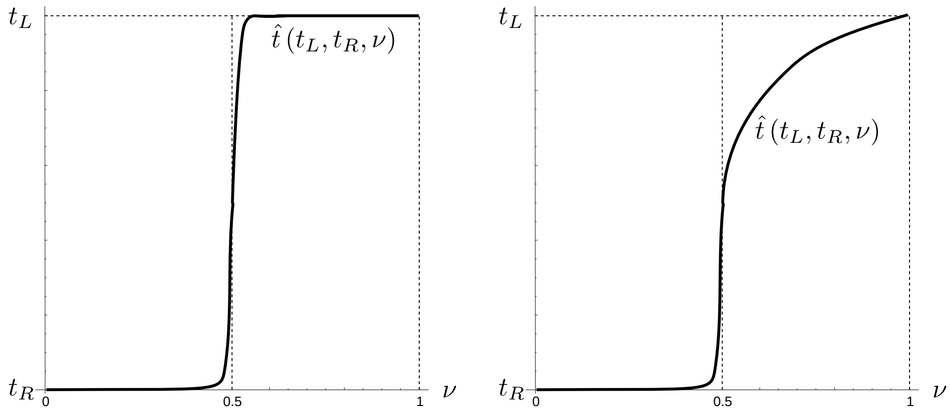
Definition 2.1 *Define the* ***implemented policy function*** $\hat{t} : T^2 \times [0, 1] \to T$ *as*

$$\hat{t}\,(t_L, t_R, \nu) = g\,(\nu)\, t_L + (1 - g(\nu))\, t_R \tag{2.1}$$

Observe that, because the conditions in Assumption 1 are quite undemanding, the implemented policy function $\hat{t}$ is general enough to represent a wide variety of scenarios. We want to highlight, in particular, that we have not imposed any restriction on the weight given to the opposition's platform in policy making (beyond that it has to be positive.) Thus, the implemented policy does not need to be far from the platform of the winner. For instance, as Figure 1(a) shows, it is easy to find a function g that satisfies A1 and for which the winner party adopts a policy

[7]See also Austen-Smith and Banks (1998); Alesina and Rosenthal (1995, 1996, 2000); Gerber and Ortuño-Ortín (1998); Grossman and Helpman (1996).

[8]The implemented policy function described here is equivalent to the *legislative outcome function* in Austen-Smith (1989), the *implemented policy function* in Ortuño-Ortín (1997), and the *outcome function* F in Gerber and Ortuño-Ortín (1998).



(a) The "almost" winner-takes-all function: the party with more than $(50+\epsilon)\%$ of the votes adopts a policy arbitrarily close to its electoral platform.

(b) A non-symmetric function: the left party is more sensitive to the voting outcome than the right party.

Figure 1: Examples of implemented policy functions. Given, t_L and t_R, the function $\hat{t}$ assigns to the share of votes received by party L the policy finally adopted. Assumption 1 allows for (a) *almost* discontinuous functions and (b) non-symmetric functions.

arbitrarily close to its electoral platform if it obtained a plurality larger than just an *epsilon*.

Furthermore, as represented in Figure 1(b), the implemented policy function need not be symmetric either and still satisfies Assumption 1. That is, one of the parties may be more inclined to compromise than the other one.

2.2 Voting

In this section we describe the actual voting process: we model voting behavior and define the set of possible voting outcomes.

Consider a continuum of citizens distributed according to their ideal policy. Let $F : \mathbb{R} \rightarrow [0,1]$ describe the distribution of citizens, and let μ be the probability measure associated to T.[9]

Citizens observe the platforms announced by the parties, t_L and t_R in T, and anticipate the policy-making process (i.e. they know the implemented policy function $\hat{t}$ and they have expectations regarding the support that each party will receive.) With this information in mind, each citizen has to decide between voting for L, voting for R, or abstaining. Let $S = \{L, R, A\}$ be the set of actions for a particular

[9]Since F is a distribution function, there exists a unique probability measure μ such that $\mu((a,b]) = F(b) - F(a)$ for all a, b (see Durrett (1996), Corollary 1.3, p.6).

citizen, with typical element s: $s = L$ and $s = R$ denote that the citizen votes for party L and party R, respectively; $s = A$ when the citizen abstains. Therefore, a citizen takes t_L, t_R, and ν as given and chooses an action $s \in S$ in order to maximize her utility. Let $v : S \times T^2 \times [0,1] \times T \to \mathbb{R}$, where $v(s; t_L, t_R, \nu, \tau)$ is the utility of a citizen with ideal policy τ who chooses s when the platforms are t_L and t_R, and she expects a fraction ν of the electorate to support L.

2.2.1 The Pragmatic Component and the Ideological Component of Preferences

As explained in the Introduction, a citizen has two concerns when casting her vote: the impact on the policy, and the platform that she is supporting with her vote. Hence, the utility function v will be defined as the sum of two components, one for each of the two concerns.

The first component is derived as follows. Insofar a voters care about the policy outcome, they will choose to vote for one party or the other based on the effect that a larger support for that party has on the implemented policy.[10] We refer to this component as *pragmatic voting*. Thus, purely pragmatic voting must be derived from the preferences of the voter over policies.[11] Let preferences over policies be represented by the single-peaked utility $u : T \times T \to \mathbb{R}$, where $u(t; \tau)$ represents the utility that a citizen τ obtains when t is implemented. Let $\tau = \arg\max \{u(t;\tau) : t \in T\}$, i.e. τ represents the ideal policy of citizen τ. Because the policy is a function of the electoral platforms and the allocation of votes, it is convenient to define the reduced form of the utility over implemented policies as

$$\hat{u}(t_L, t_R, \nu; \tau) = u\left(\hat{t}(t_L, t_R, \nu); \tau\right). \tag{2.2}$$

Thus, citizen τ's *pragmatic* component of voting for L is the utility change implied by a larger support for party L. More precisely, for each pair of policies (t_L, t_R) and given the expected voting outcome ν, τ's pragmatic utility of voting for L will be the partial derivative of τ's utility over policies with respect to a change in the fraction of the vote received by party L.[12] Observe that, because the fraction of voters for R is one minus the fraction of voters for L, an increase in the support for

[10] Recall that policies are determined not only by the platforms announced but also by the relative support that each party receives.

[11] Because, unlike under the winner-takes-all assumption, the implemented policy will generally differ from the platforms, it is natural to differentiate between voting preferences and the utility from implemented policies. Perhaps a similar distinction should be incorporated in winner-takes-all models when uncertainty is present. In those cases, the *expected* policy consistently differs from the platform of the winning party.

[12] The use of the derivative comes from the assumption that there exists a continuum of citizens. However, our results do not depend on that assumption. We could consider instead a sufficiently large number of citizens whose distribution is represented by a density function. In that case, votes would have a positive, but tiny, effect on the implemented policy, and the pragmatic component of voting would be defined by the change in the utility of the voter implied by the effect of his action on the implemented policy. The use of the continuum simplifies the presentation and is standard

R is equivalent to a decrease in the support for L. Finally, if a citizen abstains, we set the utility equal to zero. Consequently, we define the **pragmatic component of voting**, $w : S \times T^2 \times [0,1] \times T \to \mathbb{R}$, as[13]

$$w(s; t_L, t_R, \nu, \tau) = \begin{cases} \frac{\partial \hat{u}(t_L, t_R, \nu; \tau)}{\partial \nu} & \text{if } s = L \\ -\frac{\partial \hat{u}(t_L, t_R, \nu; \tau)}{\partial \nu} & \text{if } s = R \\ 0 & \text{if } s = A \end{cases} \tag{2.3}$$

The second component of voting reflects the concern of a voter with the platform she supports with her vote, beyond the impact on the implemented policy. Let $\hat{\eta} : S \times T^2 \times T \to \mathbb{R}$ be the **ideological component of voting**, such that $\hat{\eta}(L; t_L, t_R, \tau) \equiv \eta(t_L; \tau)$ represents the utility of agent τ from voting for the platform t_L, and, similarly, $\hat{\eta}(R; t_L, t_R, \tau) \equiv \eta(t_R; \tau)$ is the utility of voting for platform t_R. Observe that non-pragmatic utility depends only on the platform supported by the vote, and not on the implemented policy. In particular, it does not involve any expectations about what the rest of the electorate will choose to do. We normalize the ideological utility of abstention to zero. Then,

$$\hat{\eta}(s; t_L, t_R, \tau) = \begin{cases} \eta(t_L; \tau) & \text{if } s = L \\ \eta(t_R; \tau) & \text{if } s = R \\ 0 & \text{if } s = A \end{cases} \tag{2.4}$$

Finally, let the pragmatic and the non-pragmatic components of voting enter additively into the utility function:

$$v(s; t_L, t_R, \nu, \tau) = w(s; t_L, t_R, \nu, \tau) + \hat{\eta}(s; t_L, t_R, \tau). \tag{2.5}$$

It follows that a citizen will abstain if she obtains negative utility from voting. If she votes, she will support the party that reports her higher utility.

2.2.2 Consistent Vote

Therefore, given (t_L, t_R, ν), we can partition the electorate into those citizens who vote for L, those who vote for R, and those who abstain according to

$$\begin{aligned} L(t_L, t_R, \nu) &= \{\tau \in T : v(L; t_L, t_R, \nu, \tau) \geq \max\{0, v(R; t_L, t_R, \nu, \tau)\}\}, \\ R(t_L, t_R, \nu) &= \{\tau \in T : v(R; t_L, t_R, \nu, \tau) > \max\{0, v(L; t_L, t_R, \nu, \tau)\}\}, \\ A(t_L, t_R, \nu) &= \{\tau \in T : \max\{v(L; t_L, t_R, \nu, \tau), v(R; t_L, t_R, \nu, \tau)\} < 0\}. \end{aligned}$$

in the literature of mass elections.
Likewise, we could replace ν, the fraction of active voters who vote for L, by the fraction of citizens who vote for L without affecting the results.

[13]The pragmatic component of voting is similar to what Alesina and Rosenthal (1995) define as voting with *conditional sincerity*. They say that a voter votes with conditional sincerity if he prefers an increase in the expected vote for the party he votes for.

Then, a fraction $V_L(t_L, t_R, \nu) = \mu(L(t_L, t_R, \nu))$ of the citizenry votes for L, while party R receives $V_R(t_L, t_R, \nu) = \mu(R(t_L, t_R, \nu))$. Finally, a fraction $\mu(A(t_L, t_R, \nu))$ of the citizenry abstains.

We have described how citizens vote (or abstain) as a function of the electoral platforms and the expected support for each party. However, our goal is to find a correspondence that assigns to each pair of platforms the set of all *consistent* or *rational-expectations* voting outcomes, according to the following definition.

Definition 2.2 *We say that ν is a* ***consistent vote*** *for the pair of policies t_L, t_R in T if*

$$\frac{V_L(t_L, t_R, \nu)}{V_L(t_L, t_R, \nu) + V_R(t_L, t_R, \nu)} = \nu, \tag{2.6}$$

that is, if the allocation of votes implied when ν is expected gives rise to a fraction of votes for L equal to ν.

Thus, we can define the **voting outcome correspondence** $\chi : T^2 \rightrightarrows 2^{[0,1]}$ as,

$$\chi(t_L, t_R) = \{\nu \in [0,1] : \nu \text{ is a } \textit{consistent vote} \text{ for } (t_L, t_R)\}.$$

Perhaps abusing language, and without further specifications, this definition admits the possibility that $\chi(t_L, t_R) = \emptyset$ for some t_L and t_R. However, in what follows, we will restrict the analysis to environments where $\chi(t_L, t_R) \neq \emptyset$, $\forall (t_L, t_R)$.

2.3 Political Equilibrium

Parties are the actual players of the political game. There are two parties that run for election and have single-peaked preferences over policies represented by the utility function $\pi_J : T \to \mathbb{R}$, $J = L, R$. Let $\tau_J = \arg\max\{\pi_J(t) : t \in T\}$ be the ideal policy of party J.[14] Assume, without loss of generality, that $\tau_L < \tau_R$.

It is convenient to write the utility of a party as a function of the platforms and the vote allocation by using the definition of the implemented policy function. Define $\hat{\pi}_J : T^2 \times [0,1] \to \mathbb{R}$ as

$$\hat{\pi}_J(t_L, t_R, \nu) = \pi_J\left(\hat{t}(t_L, t_R, \nu)\right).$$

For illustrative purposes, assume that there exists a unique consistent voting outcome for each pair of policies, that is, χ is a function. Then we can write $\tilde{\pi}_J(t_L, t_R) = \hat{\pi}_J(t_L, t_R, \chi(t_L, t_R))$ and a political equilibrium is simply a Nash equilibrium of the two-party game where parties choose platforms in T to maximize their payoffs $\tilde{\pi}_J$. In general, however, χ may not be single-valued and non-empty for all pair of policies. Therefore we introduce the following more general definition of a political equilibrium:

[14] Observe that the preferences of a party do not need to coincide with those of a particular voter, even if they share the same ideal point. Also, it is worth emphasizing that we do not need to assume that voters know the parties' ideal points.

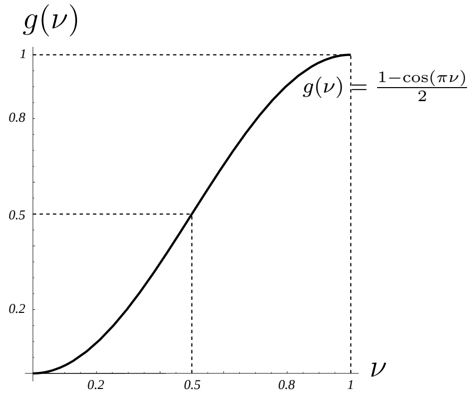


Figure 2: Weight of Votes Function.

Definition 2.3 *We say that* $(t_L, t_R, \bar{\nu})$ *is a* ***political equilibrium*** *if*

(i) $\bar{\nu} \in \chi(t_L, t_R)$,

(ii) $\hat{\pi}_L(t_L, t_R, \bar{\nu}) \geq \hat{\pi}_L(t, t_R, \nu)$ $\forall t \in T$ *and* $\forall \nu \in \chi(t, t_R)$, *and*

(iii) $\hat{\pi}_R(t_L, t_R, \bar{\nu}) \geq \hat{\pi}_R(t_L, t, \nu)$ $\forall t \in T$ *and* $\forall \nu \in \chi(t_L, t)$.

That is, at equilibrium, there is no possible voting outcome for which a party can improve by announcing a different platform. Therefore, the definition represents situations where no party has an incentive to deviate from its equilibrium strategy.

3 Example

Before presenting the formal analysis, let us anticipate the main results with an illustrative example.

Consider a political process where two ideological parties, L and R, compete over a single issue, a tax rate for example. Let $T = [0, 1]$ represent the policy space. The weight-of-votes function takes the following form (see Figure 2):

$$g(\nu) = \frac{1 - \cos(\pi \nu)}{2}. \tag{3.1}$$

Because g is concave for $\nu > \frac{1}{2}$ and convex for $\nu < \frac{1}{2}$, winning the election makes a difference: the weight of the platform of the winner party in the policy-setting process is more than proportional to the share of votes received by that party.

Voters care about the implemented policy (the tax rate). Let the preferences of voters over policy outcomes be represented by the following Euclidean utility function:

$$u(t; \tau) = -\frac{1}{2}(t - \tau)^2 \tag{3.2}$$

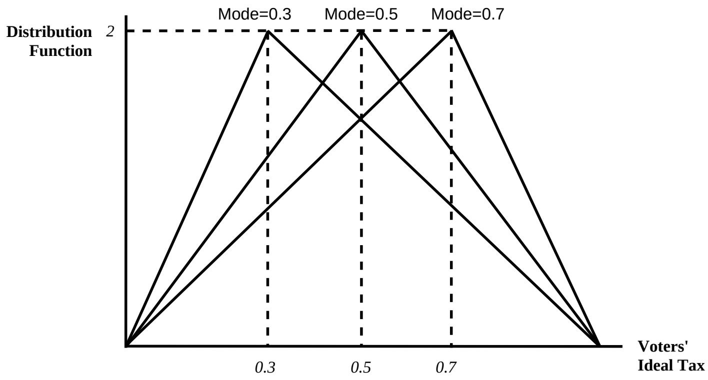


Figure 3: Family of Triangular Density Functions.

Then, the pragmatic component of voting is given by (see(2.3))

$$w\left(L;t_L,t_R,\nu,\tau\right)=\left(\hat{t}\left(t_L,t_R,\nu\right)-\tau\right)g'(\nu)\left(t_R-t_L\right)$$
$$w\left(R;t_L,t_R,\nu,\tau\right)=\left(\tau-\hat{t}\left(t_L,t_R,\nu\right)\right)g'(\nu)\left(t_R-t_L\right)$$

As argued before, voters also care about the platform they are supporting with their vote. Let the ideological component of voting represent the dis-utility of supporting a platform far from one's ideal policy. In particular, we take ideological voting to be a linear function of the distance between the "ideology" of the voter and the platform of the party:

$$\eta\left(t;\tau\right)=-2\left|t-\tau\right|.$$

Finally, if a citizen abstains, her utility of voting is zero, since both the pragmatic component and the ideological component are zero.

In summary, the utility of voting (the sum of the pragmatic and the ideological components of voting) of a citizen τ is

$$\begin{aligned}
v\left(L;t_L,t_R,\nu,\tau\right)&=\left(\hat{t}\left(t_L,t_R,\nu\right)-\tau\right)g'(\nu)\left(t_R-t_L\right)-2\left|t_L-\tau\right|,\\
v\left(R;t_L,t_R,\nu,\tau\right)&=\left(\tau-\hat{t}\left(t_L,t_R,\nu\right)\right)g'(\nu)\left(t_R-t_L\right)-2\left|t_R-\tau\right|,\\
v\left(A;t_L,t_R,\nu,\tau\right)&=0.
\end{aligned}$$

Let the electorate be distributed according to a triangular distribution with mode m (see Figure 3 for a few members of this family of density functions).

Finally, let parties have Euclidean preferences represented by

$$\pi_J(t)=-\frac{1}{2}\left(t-\tau_J\right)^2.$$

We take the ideal policy of the left party to be $\tau_L=0.2$, while the right party's ideal policy is $\tau_R=0.75$.

After several manipulations, it is easy to show that, given (t_L, t_R, ν), voting occurs on two intervals around the electoral platforms: $[\lambda_1, \lambda_2] \ni t_L$ and $[\rho_1, \rho_2] \ni t_R$. That is,[15]

$$V_L(t_L, t_R, \nu) = F(\lambda_2) - F(\lambda_1),$$
$$V_R(t_L, t_R, \nu) = F(\rho_2) - F(\rho_1).$$

We study two cases: *(a)* a symmetrically distributed electorate (i.e. $m = \frac{1}{2}$), and *(b)* a right-skewed distribution ($m = 0.7$.) We have represented the political equilibria for these cases in Figure 4. Comparing a symmetric example, an asymmetric example, and the median voter, we can offer some intuition about the main results of the paper.

(i) Parties announce differentiated, non-extreme platforms. In the symmetric case, they locate symmetrically with respect to the mean ($t_L^* = .25$ and $t_R^* = 0.75$); while, in the non-symmetric case, they shift to the right ($t_L^* = .28$ and $t_R^* = .8$) responding to a right-skewed electorate. Also, equilibrium platforms may be more moderate or more radical than parties' ideal policies.

(ii) The equilibrium policy does not need to coincide with the ideal policy of the median voter, except when the electorate is symmetrically distributed. In that case, $t^* = 0.5 =$ median $=$ mean. In the other case, without symmetry, $t^* = 0.58 <$ median $= 0.61$. Observe that the implemented policy also responds in the right direction to the skewness of the population.

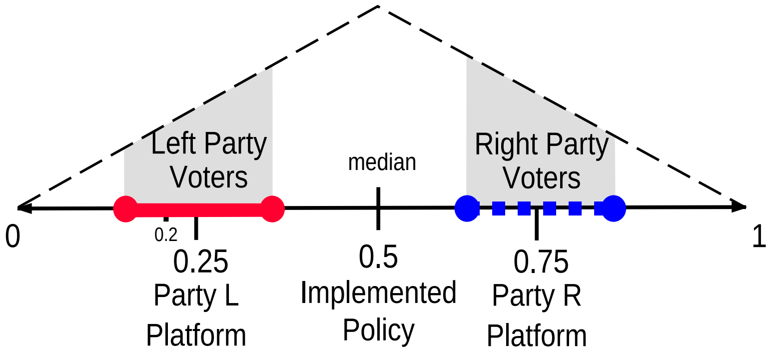
(iii) While both parties receive 50% of the vote under symmetry, the Right party wins with 55% of the votes when the electorate is right-skewed. Therefore, we can explain why parties with zero probability of winning still participate in elections. This is in contrast with dominant models, with or without uncertainty, that need a positive probability of winning for a party to participate in elections.

(iv) Although there is a cost associated to voting, a substantial fraction of the population prefers voting to abstaining. Turnout rates are around 37% and 40.5%, respectively. Furthermore, turnout tends to occur in "bands" around the party platforms. Hence, abstention occurs among voters with extreme views as well as with moderate views.

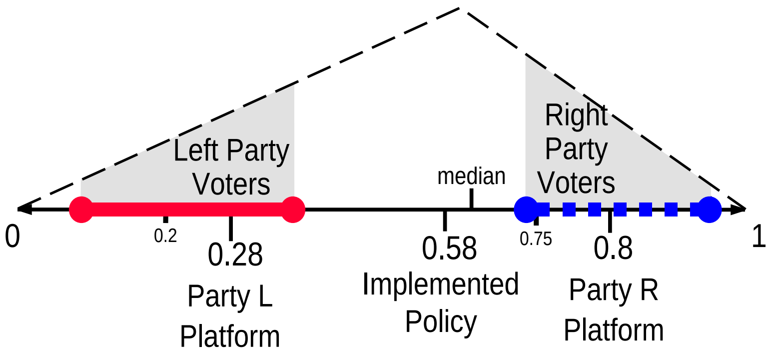
[15] Of course, λ_1, λ_2, ρ_1, and ρ_2 are functions of t_L, t_R, and ν. In particular, for $t_L < t_R$,

$$\lambda_2 = \alpha\,\hat{t} + (1-\alpha)\,t_L; \quad \lambda_1 = \max\left\{\beta\,\hat{t} + (1-\beta)\,t_L, 0\right\};$$
$$\rho_1 = \alpha\,\hat{t} + (1-\alpha)\,t_R; \quad \rho_2 = \min\left\{\beta\,\hat{t} + (1-\beta)\,t_R, 1\right\};$$

where, $\alpha = \frac{g'(\chi(t_L,t_R))(t_R-t_L)}{g'(\chi(t_L,t_R))(t_R-t_L)+2}$, $\beta = \frac{g'(\chi(t_L,t_R))(t_R-t_L)}{g'(\chi(t_L,t_R))(t_R-t_L)-2}$, and $\hat{t} = g(\nu)\,t_L + (1-g(\nu))\,t_R$.



(a) Symmetric distribution.



(b) Right-skewed distribution.

Figure 4: Typical Equilibria for Symmetric and Asymmetric Distributions of the Electorate.

Finally, observe that the symmetric features of the equilibrium in case (a) are a consequence of a symmetrically distributed electorate, and do not depend on any other symmetry restriction (like symmetric preferences.) It is also worth pointing out that, as we will show in Section 5, the features just described are not particular to the present examples, but generic to political equilibria.

In this section we have worked out an example that anticipates the most relevant results of the paper. We move next to the formal analysis of existence and characterization of equilibria.

4 Purely Pragmatic Voting

Let us start by assuming that voters care only about the implemented policy. One could think of this restricted version of the general model as a comparative statics exercise: we recreate the standard political competition model with certainty, except for the implemented policy function that substitutes the winner-takes-all assumption.[16] The advantage of comparative statics is that we are able to isolate the relevance of the altered assumption in explaining final results. Thus, by maintaining the other basic assumptions (including certainty about voting behavior), we can understand the effects of political compromise (or winner-takes-all) on explaining parties' and voters' behavior. The exercise becomes even more justifiable when we discover that perturbing the winner-takes-all assumption produces diametrical results. Full convergence of platforms turns into full divergence. And all citizens decide to vote,[17] in contrast to the winner-takes-all case where no citizen has an incentive to vote because both parties announce the same platform and, therefore, nobody is pivotal.

For the rest of this section we assume that voting behavior is fully driven by its pragmatic component.

Assumption 2 (A2) *For any* τ, $\eta(t;\tau) = 0$ *for all* $t \in T$.

Assumption 2 is equivalent to

$$v(s; t_L, t_R, \nu; \tau) = w_\tau(s; t_L, t_R, \nu).$$

The main conclusions of this section are found in Theorem 4.1, that says that the unique political equilibrium can be only of two possible types: either one party manages to implement its ideal policy, or, more interestingly, parties radicalize and announce extreme platforms.

We will follow the standard procedure to prove the existence of a Nash equilibrium: first, we construct the best-response correspondences, then we look for a fixed point.

Lemma 4.1 describes the voting behavior of the electorate (everybody to the left(right) of the implemented policy votes for the Left(Right) party) and shows that the consistent vote correspondence is a well defined and continuous function.

Lemma 4.1 *Let A1 and A2 hold. Then:*

(i) Given $t_L, t_R \in T$ *and* $\nu \in [0,1]$*, everybody votes.*
Moreover, for $t_L < t_R$*, everybody to the left of* $\hat{t}(t_L, t_R, \nu)$ *votes for L while everybody to the right of* $\hat{t}(t_L, t_R, \nu)$ *votes for R. That is,* $V_L(t_L, t_R, \nu) = \left[\underline{t}, \hat{t}(t_L, t_R, \nu)\right)$*, and* $V_R(t_L, t_R, \nu) = \left(\hat{t}(t_L, t_R, \nu), \overline{t}, \right]$*.*

(ii) $\chi(t_L, t_R)$ *is non-empty and single-valued for all* $(t_L, t_R) \in T^2$*.*

[16]Uncertainty is deliberately excluded from the analysis. We elaborate on this subject in the concluding remarks.

[17]To be precise, a subset of the electorate of measure one prefers to vote, while the zero-measure set of voters with ideal policy equal to the implemented policy are indifferent between voting and abstaining.

(iii) χ *is continuous, except possibly at* $t_L = t_R$.

Proof.
(i) That everybody votes follows directly from the definition of w_τ. Because, for all (t_L, t_R, ν) and for all τ, $w(L; t_L, t_R, \nu, \tau) = -w(R; t_L, t_R, \nu, \tau)$ and $w(A; t_L, t_R, \nu, \tau) = 0$, then either voting for L or voting for R is at least as preferred as abstaining. Now, let $t_L < t_R$ and $t^0 = \hat{t}(t_L, t_R, \nu)$. From (2.1), (2.2) and (2.3):

$$w(L; t_L, t_R, \nu, \tau) = \frac{\partial u}{\partial t}\left(t^0; \tau\right) \cdot g'(\nu) \cdot (t_L - t_R).$$

Thus, $w(L; t_L, t_R, \nu, \tau) > 0$ if and only if $\frac{\partial u}{\partial t}(t^0; \tau) < 0$. By single-peakedness, $\frac{\partial u}{\partial t}(t^0; \tau) < 0$ if and only if $\tau < t^0$. Hence, everybody to the left of t^0 votes for L while everybody to the right of t^0 votes for R.

(ii) From *(i)*, $V_L(t_L, t_R, \nu) = F(t^0)$ and $V_L(t_L, t_R, \nu) + V_R(t_L, t_R, \nu) = 1$. Thus, we only need to show that there exists a unique solution to the following equation in ν:

$$\begin{array}{lll} & \nu = F(g(\nu) t_L + (1 - g(\nu)) t_R) & \text{if } t_L \leq t_R, \\ \text{or} & \nu = 1 - F(g(\nu) t_L + (1 - g(\nu)) t_R) & \text{if } t_L > t_R. \end{array}$$

Consider 3 cases:
<u>Case 1</u>: $t_L < t_R$. Define $\Psi_1(\nu) = F(g(\nu) t_L + (1 - g(\nu)) t_R) - \nu$. Note that $g \in C^1$ (A1), and $F \in C^1$. Then $\Psi_1 \in C^1$, and

$$\Psi_1'(\nu) = f\left(\hat{t}(t_L, t_R, \nu)\right) \cdot g'(\nu) \cdot (t_L - t_R) - 1 < 0,$$

since $f \geq 0$, $g' \geq 0$, and $(t_L - t_R) < 0$. Thus, Ψ_1 is a continuous, strictly decreasing function. Because $\Psi_1(0) = F(t_R) > 0$ and $\Psi_1(1) = F(t_L) - 1 < 0$, there exists a unique ν in $(0, 1)$ that solves the equation $\Psi_1(\nu) = 0$.
<u>Case 2</u>: $t_L > t_R$. Define $\Psi_2(\nu) = 1 - F(g(\nu) t_L + (1 - g(\nu)) t_R) - \nu$. By symmetry with Case 1, there exists a unique solution ν in $(0, 1)$ to the equation $\Psi_2(\nu) = 0$.
<u>Case 3</u>: $t_L = t_R = t$. For any $t \in T$, $\hat{t}(t, t, \nu) = t$. Thus $\nu = F(t)$ is the unique solution we are looking for.

(iii). Let $t_L < t_R$. We know from *(ii)* that $\Psi_1'(\nu) \neq 0$. By the Implicit Function Theorem there exists an open ball $B_\epsilon\left(\check{t}\right)$ of radius ϵ and center $\check{t} = \left(\check{t}_L, \check{t}_R\right)$, and a differentiable function $\psi_{\check{t}} : B_\epsilon\left(\check{t}\right) \to \mathbb{R}$, such that if $(t_L, t_R) \in B_\epsilon\left(\check{t}\right)$, then

$$\Psi_1(\psi_{\check{t}}(t_L, t_R); t_L, t_R) = 0. \tag{4.1}$$

Because, from *(ii)*, the solution to "$\Psi(\nu, t_L, t_R) = 0$" is unique, $\chi(t_L, t_R)$ is a singleton and, thus, can be viewed as a function. Then, (4.1) implies that $\chi(t_L, t_R) = \psi_{\check{t}}(t_L, t_R)$ for all $(t_L, t_R) \in B_\epsilon\left(\check{t}\right)$, i.e., $\psi_{\check{t}}$ is the restriction of χ to $B_\epsilon\left(\check{t}\right)$. Thus, χ is continuous on $B_\epsilon\left(\check{t}\right)$ and, in particular, at $\check{t}$. Since this is true for all $\left(\check{t}_L, \check{t}_R\right)$ with

$\check{t}_L < \check{t}_R$, we conclude that χ is continuous on $\{(t_L, t_R) \in T^2 : t_L < t_R\}$. A similar argument applies for $t_L > t_R$. ■

Because χ is a function, we can project the political game on the policy space. Thus we are able to formulate the implemented policy and the utility function of parties in their *reduced* form. Let $\tilde{t} : T^2 \to T$ be defined by

$$\tilde{t}\,(t_L, t_R) = \hat{t}\,(t_L, t_R, \chi\,(t_L, t_R))\,. \tag{4.2}$$

And, for $J = L, R$, let $\tilde{\pi}_J : T \times T \to \mathbb{R}$ be defined by

$$\tilde{\pi}_J\,(t_L, t_R) = \pi_J\left(\tilde{t}\,(t_L, t_R)\right). \tag{4.3}$$

It follows (Lemma 4.2) that the implemented policy function and the utility of parties are continuous on $T \times T$.

Lemma 4.2 *The functions $\tilde{t}$ and $\tilde{\pi}_J$ are continuous on T^2.*

Proof. Recall that $\tilde{t}\,(t_L, t_R) = g\,(\chi\,(t_L, t_R))\,t_L + (1 - g\,(\chi\,(t_L, t_R)))\,t_R$. By A1, g is a continuous function and, by Lemma 4.1(iii), χ is continuous for $t_L \neq t_R$. Thus, $\tilde{t}$ is continuous for $t_L \neq t_R$.
Let $\bar{t} \in T$, and $\left(t_L^k, t_R^k\right) \to (\bar{t}, \bar{t})$ as $k \to \infty$. It is easy to see that $\tilde{t}\left(t_L^k, t_R^k\right) \to \hat{t}\,(\bar{t}, \bar{t})$. Observe that, $\lim_{k\to\infty} \chi\left(t_L^k, t_R^k\right) = F\,(\bar{t})$, and $\tilde{t}\left(t_L^k, t_R^k\right) = t_R^k + g\left(\chi\left(t_L^k, t_R^k\right)\right)\left(t_L^k - t_R^k\right)$. Then, $\lim_{k\to\infty} \tilde{t}\left(t_L^k, t_R^k\right) = \bar{t} = \tilde{t}\,(\bar{t}, \bar{t})$. It follows that $\tilde{t}$ is continuous for $t_L = t_R$ as well. Hence, $\tilde{t}$ is continuous on T^2.

Finally, since $\tilde{t}$ and π_J are continuous, we have that $\tilde{\pi}_J\,(t_L, t_R) = \pi_J\left(\tilde{t}\,(t_L, t_R)\right)$ is also a continuous function on T^2. ■

Define the **best-response correspondence** of party $J = L, R$, $BR_J : T \rightrightarrows T$, that assigns to each alternative the set of utility maximizers. That is,

$$BR_L\,(t_R) = \arg\max\{\tilde{\pi}_L\,(t, t_R) \;\mid\; t \in T\}$$
$$BR_R\,(t_L) = \arg\max\{\tilde{\pi}_R\,(t_L, t) \;\mid\; t \in T\}\,.$$

The following lemma shows that the best response for each party is to radicalize, unless it can ensure the implementation of its ideal policy.

Lemma 4.3 *Let A1 and A2 hold.*

(i) The best-response correspondence of party J, BR_J, is single-valued and continuous, $J = L, R$.

(ii) Given $t_R \geq \tau_L$, either L can implement its ideal policy, or its best response is $\underline{t}$. That is, either there exists t_L^o such that $\tilde{t}\,(t_L^o, t_R) = \tau_L$, or $BR_L\,(t_R) = \{\underline{t}\}$.

(iii) Given $t_L \leq \tau_R$, either R can implement its ideal policy, or its best response is $\bar{t}$. That is, either there exists t_R^o such that $\tilde{t}\,(t_L, t_R^o) = \tau_R$, or $BR_R\,(t_L) = \left\{\bar{t}\right\}$.

Proof. *(i)* We only prove the lemma for party L. The proof for party R is symmetric.

1. Claim: If $t', t'' \in BR_L(t_R)$, then $\tilde{t}(t', t_R) = \tilde{t}(t'', t_R)$. Suppose not, say $\tilde{t}(t', t_R) < \tilde{t}(t'', t_R)$. By the strict quasi-concavity of π_L: $\tilde{t}(t', t_R) < \tau_L < \tilde{t}(t'', t_R)$. But $\tilde{t}$ is a continuous function (Lemma 4.2). Thus, there exists a t''' between t' and t'' such that $\tilde{t}(t''', t_R) = \tau_L$, i.e. $\tilde{\pi}_L(t''', t_R) = \pi_L(\tau_L) > \tilde{\pi}_L(t', t_R)$, contradicting the fact that t' is a best response.

2. It is easy to see that $BR_L(\tau_L) = \{\tau_L\}$, since $\tilde{t}(t, \tau_L) \neq \tau_L$ for all $t \neq \tau_L$.

3. Let $t_R > \tau_L$ and assume that t' and t'' are best responses for party L to t_R, with $t' \neq t''$. Assume, without loss of generality, that $t' < t''$. Then $t'' < t_R$. Otherwise party L would be better-off by choosing $t_R : \tau_L < \tilde{t}(t_R, t_R) = t_R < \tilde{t}(t'', t_R)$.
   We compute the derivative of $\tilde{t}$ w.r.t. t_L as
   $$\frac{\partial \tilde{t}}{\partial t_L}(t_L, t_R) = -g'(\chi(t_L, t_R)) \frac{\partial \chi(t_L, t_R)}{\partial t_L} \cdot (t_R - t_L) + g(\chi(t_L, t_R)),$$
   where χ is implicitly defined as the unique solution of $F(g(\nu) t_L + (1 - g(\nu)) t_R) - \nu = 0$ (see the proof of Lemma 4.1(*ii*)). Then, using the fact that
   $$\frac{\partial \chi}{\partial t_L}(t_L, t_R) = \frac{f(\tilde{t})\, g(\chi(t_L, t_R))}{f(\tilde{t})\, g'(\chi(t_L, t_R))(t_R - t_L) + 1},$$
   we can conclude that
   $$\frac{\partial \tilde{t}}{\partial t_L}(t_L, t_R) = \frac{g(\chi(t_L, t_R))}{f(\tilde{t}(t_L, t_R))\, g'(\chi(t_L, t_R))(t_R - t_L) + 1} > 0.$$
   But this implies that $\tilde{t}(t', t_R) < \tilde{t}(t'', t_R)$, a contradiction with step 1.

4. A similar argument applies to $t_R < \tau_L$. Thus, the best-response correspondence is single-valued.

5. Continuity is a direct implication of the Maximum Theorem.

(ii). If t_L^o exists, then it is a global maximizer of $\tilde{\pi}_L(t, t_R)$ and clearly the best choice for party L. If there is no $t \in T$ such that $\tilde{t}(t, t_R) = \tau_L$, then it follows from the continuity of $\tilde{t}$ that $\tilde{t}(t, t_R) > \tau_L$ for all $t \in T$ (recall that $\tilde{t}(t_R, t_R) = t_R > \tau_L$). Because $\tilde{t}$ is monotone increasing in t_L (see step 3 above) and π_L is single-peaked, it follows that $\tau_L < \tilde{t}(\underline{t}, t_R) < \tilde{t}(t, t_R)$ for all $t > \underline{t}$. Thus, $BR_L(t_R) = \underline{t}$.
(iii). The proof is a symmetric replica of *(ii).* ■

It follows from the previous lemmas that there is a unique political equilibrium which can take only two possible forms: either one party manages to implement its ideal policy, or, more interestingly, parties radicalize and announce extreme platforms.

Theorem 4.1 *Let A1 and A2 hold. Then:*

(i) a political equilibrium exists;

(ii) the equilibrium is unique; and

(iii) either platforms are polarized at equilibrium (i.e., $(t_L^*, t_R^*) = \left(\underline{t}, \bar{t}\right)$), or one of the parties can implement its ideal policy (i.e. $\tilde{t}\,(t_L^*, t_R^*) = \tau_J$, for some $J = L, R$.)

Proof.
(i) Existence. Let $BR : T^2 \to T^2$ defined by $BR\,(t_L, t_R) = (BR_L\,(t_R)\,, BR_R\,(t_L))$. From Lemma 4.3, BR is a continuous function. Thus, by Brouwer's fixed point theorem, there exists $(t_L^*, t_R^*) \in T^2$ such that $BR\,(t_L^*, t_R^*) = (t_L^*, t_R^*)$. It follows that (t_L^*, t_R^*) is an equilibrium.

(iii) Let (t_L^*, t_R^*) be an equilibrium and suppose that $(t_L^*, t_R^*) \neq (\underline{t}, \bar{t})$. If $t_L^* \neq \underline{t}$, then $\tilde{t}\,(t_L^*, t_R^*) = \tau_L$ (Lemma 4.3). If $t_L^* = \underline{t}$, then $t_R^* \neq \bar{t}$ and $\tilde{t}\,(t_L^*, t_R^*) = \tau_R$ (Lemma 4.3, once more).

(ii) Uniqueness. Suppose that (t_L^1, t_R^1) and (t_L^2, t_R^2) are two different equilibria. From *(iii)*, at least one of them is not $(\underline{t}, \bar{t})$ and it implements the ideal policy of one party. Without loss of generality, take $(t_L^1, t_R^1) \neq (\underline{t}, \bar{t})$, and suppose that $\tilde{t}\,(t_L^1, t_R^1) = \tau_L$ (nothing would change had we taken τ_R instead).

Because τ_L is implemented, $BR_R\,(t_L^1) = \bar{t}$ (Lemma 4.3). That is, $(t_L^1, \bar{t})$ is an equilibrium and, therefore, $t_L^1 = BR_L\,(\bar{t})$. It follows that $(t_L^2, t_R^2) \neq (\underline{t}, \bar{t})$ as well, since BR_L is a function and $(t_L^2, t_R^2) \neq (t_L^1, \bar{t})$. Thus it must be that $\tilde{t}\,(t_L^2, t_R^2) = \tau_R$ and $t_L^2 = \underline{t}$, by a similar reasoning as above.

Therefore, we have shown that $\tilde{t}\,(t_L^1, t_R^1) = \tau_L$, $\tilde{t}\,(t_L^2, t_R^2) = \tau_R$, $t_R^1 = \bar{t}$ and $t_L^2 = \underline{t}$.

From Lemma 4.1, $\chi\,(t_L^1, t_R^1) = F\,(\tau_L)$ and $\chi\,(t_L^2, t_R^2) = F\,(\tau_R)$. Then, using the definition of $\tilde{t}$,

$$\begin{aligned}\tau_L &= \tilde{t}\left(t_L^1, \bar{t}\right) = g\,(F\,(\tau_L))\,t_L^1 + (1 - g\,(F\,(\tau_L)))\,\bar{t}, \text{ and}\\ \tau_R &= \tilde{t}\left(\underline{t}, t_R^2\right) = g\,(F\,(\tau_R))\,\underline{t} + (1 - g\,(F\,(\tau_R)))\,t_R^2\,.\end{aligned}$$

Solving for t_L^1 and t_R^2, we obtain

$$\begin{aligned}t_L^1 &= \frac{\tau_L - (1 - g\,(F\,(\tau_L)))\,\bar{t}}{g\,(F\,(\tau_L))} \geq \underline{t},\\ t_R^2 &= \frac{\tau_R - g\,(F\,(\tau_R))\,\underline{t}}{1 - g\,(F\,(\tau_R))} \leq \bar{t}.\end{aligned}$$

Rearranging terms, we obtain

$$\begin{aligned}\tau_L &\geq g\,(F\,(\tau_L))\,\underline{t} + (1 - g\,(F\,(\tau_L)))\,\bar{t},\\ \tau_R &\leq g\,(F\,(\tau_R))\,\underline{t} + (1 - g\,(F\,(\tau_R)))\,\bar{t}.\end{aligned}$$

But this contradicts the condition $\tau_L < \tau_R$, since g is a non-decreasing function. Therefore, the equilibrium is unique. ■

This section has recreated the standard political competition model with perfect information, substituting the winner-takes-all assumption for a political compromise policy-setting process. The main result (Theorem 4.1) is the disappearance of the convergence tendency of platforms. We observe, on the contrary, the emergence of a divergence tendency that takes parties to radicalize their positions. Therefore, one could argue that platform convergence is not the result of competition between parties, but of gaining nothing from losing.

5 Pragmatic and Ideological Voting

In the previous section we showed that the convergence of parties' platforms, traditionally associated with political competition, is a direct consequence of the winner-takes-all assumption. As soon as we perturb this assumption a little bit and let the opposition intervene (to some extent) into policy-setting, a polarization tendency appears.[18] Recall that an *almost* winner-takes-all implemented policy function (like the one in Figure 1(a)) would be enough to make parties announce radical platforms.

We move next to the study of the full model with voters who care about both platforms and policies. We will see that although a divergence tendency of platforms is still present, parties do not radicalize their positions. The intuition is simple. Voters care about the platform they support, and they will feel alienated by a party that locates too far from their ideal policy. If the electorate concentrates in the middle (i.e. if it holds fairly moderated views), then a party will alienate the core of its constituency by announcing an extreme platform.

Therefore, if one objected that full divergence was as unrealistic as full convergence, we show that the radicalization of parties was a consequence of a simplifying assumption of the particular model studied in the previous section, namely the absence of platform oriented voters.[19] We will see that if we consider a more realistic and more complete model, parties' and voters' behavior resemble closer what we observe.

The inclusion of a non-pragmatic component of voting that depends on the platforms of the parties opens the possibility to abstention. Because abstention notably complicates the analysis, we sacrifice some generality and work within a

[18] This centrifugal force over parties' platforms distinguishes our results from other explanations of platform differentiation, where platforms are subject to a centripetal force but they do not fully converge because of bounded rationality (Kollman et al., 1992), valence issues (Ansolabehere and Snyder, 2000; Aragonés and Palfrey, fc), incumbency advantages –and "valence" issues– (Bernhardt and Ingberman, 1985; Londregan and Romer, 1993), uncertainty and ideology (Wittman, 1977; Calvert, 1985), or internal decision-making (Snyder, 1994).

[19] The divergence tendency found above, instead of disappearing when we consider the complete model, finds a force that works in the opposite direction and diminishes its effect.

simpler framework.[20]

First, we substitute Assumption 1 with the following assumption:

Assumption 3 (A3) $\hat{t}\,(t_L, t_R, \nu) = \nu\ t_L + (1-\nu)\, t_R$, *for all* (t_L, t_R, ν).

That is, we take the implemented policy to be equivalent to proportional representation.[21]

Second, consider a continuum of voters with Euclidean preferences over policies represented by the following utility function:

Assumption 4 (A4) $u\,(t;\tau) = -\frac{1}{2}\,(t-\tau)^2$

Third, let the ideological component of voting be a non-increasing and concave function of the distance between the platform of the party and the "ideology" of the voter:

Assumption 5 (A5) $\eta\,(t;\tau) = \zeta\,(|t-\tau|)$, *with* $\zeta' < 0$ *(decreasing),* $\zeta'' < 0$ *(concave), and* $\zeta(0) = 0$.

Finally, it is convenient for the exposition to adopt the convention $T = [0,1]$.

We want to emphasize that all specifications have been imposed on citizen's preferences. Parties' preferences are only assumed to be single-peaked, and there is no restrictions on the distribution of voters.

A first result (Lemma 5.1) proves what we already observed in the examples of Section 3: the support for each party is an interval around the platform announced by that party. It will follow (Lemma 5.2) that there exists one and only one consistent voting outcome associated to each pair of platforms.

Lemma 5.1 *Let A3, A4, and A5 hold. Given* t_L, t_R, *and* ν, *with* $t_L \neq t_R$, *there exist two intervals*

$$I_\lambda\,(t_L, t_R, \nu) = [\lambda_1\,(t_L, t_R, \nu)\,, \lambda_2\,(t_L, t_R, \nu)]\,, \text{ and}$$
$$I_\rho\,(t_L, t_R, \nu) = [\rho_1\,(t_L, t_R, \nu)\,, \rho_2\,(t_L, t_R, \nu)]$$

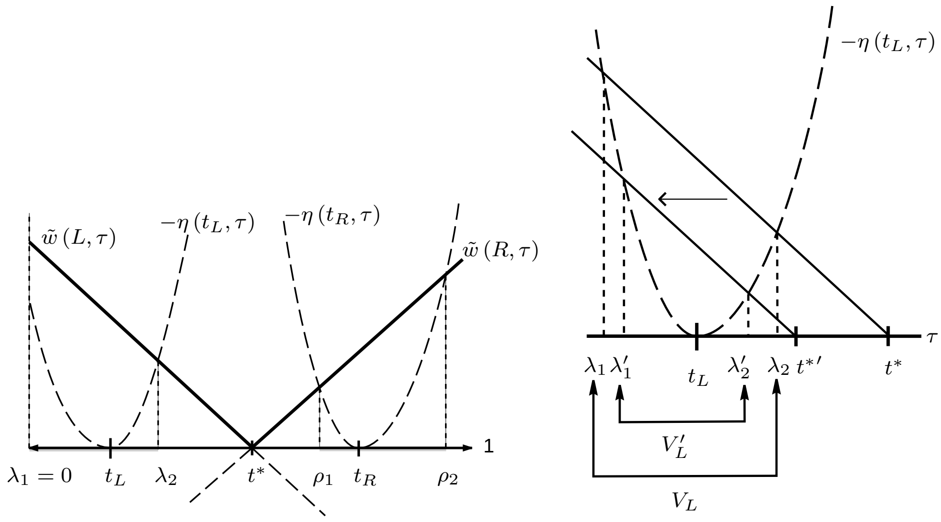
such that if τ *votes for L, then* $\tau \in I_\lambda$, *and if* τ *votes for R, then* $\tau \in I_\rho$. *Therefore,*

$$V_L\,(t_L, t_R, \nu) = \mu\,(I_\lambda\,(t_L, t_R, \nu)) \text{ and}$$
$$V_R\,(t_L, t_R, \nu) = \mu\,(I_\rho\,(t_L, t_R, \nu))\,.$$

Moreover, $V_L\,(t_L, t_R, \nu) + V_R\,(t_L, t_R, \nu) > 0$, V_L *and* V_R *are continuous,* V_L *is non-increasing in* ν, *and* V_R *is non-decreasing in* ν.

[20]We want to emphasize that the main reason for the more restrictive assumptions is to prove the existence of equilibria. Our findings about policy outcome and political agents' behavior do not depend crucially on them. In particular, if an equilibrium existed, the results of Theorem 5.1 and Theorem 5.2 about platforms and policies would still hold.

[21]Ortuño-Ortín (1997) and Gerber and Ortuño-Ortín (1998) restrict their analysis to this type of weight function as well.



(a) Voters concentrate in intervals around the platforms.

(b) As ν increases, the support for L decreases.

Figure 5: Analysis of vote allocation. Voters in the interval $[\lambda_1, \lambda_2]$ vote for L. Voters in the interval $[\rho_1, \rho_2]$ vote for R. The rest of the electorate abstains.

Proof. Intervals and Positive Turnout. (Please refer to Figure 5 for intuition). Take $t_L, t_R \in T$ and $\nu \in [0,1]$. Let $t_L < t_R$ (the proof can be easily replicated for $t_L > t_R$). Write $\tilde{w}(s;\tau) = w(s; t_L, t_R, \nu, \tau)$, $\tilde{\eta}(s;\tau) = \hat{\eta}(s; t_L, t_R, \tau)$, $\tilde{v}(s;\tau) = \tilde{w}(s;\tau) + \tilde{\eta}(s;\tau)$, and $t^* = \hat{t}(t_L, t_R, \nu)$.

From (2.3) and A4, $\tilde{w}(L,\tau) = (t^* - \tau)(t_R - t_L)$ and $\tilde{w}(R,\tau) = -\tilde{w}(L,\tau)$. Because $t_R > t_L$, every $\tau < t^*$ prefers L to R, while every $\tau > t^*$ prefers R to L. Thus, every citizen to the left (right) of t^* either votes for $L(R)$ or abstains.

Observe that $\tilde{w}(L,\tau)$ is an affine and decreasing function of τ. On the other hand, η is concave in τ and $\eta(t_L; t_L) = 0 \geq \eta(t_L; \tau)$ for all τ. Because $\tilde{w}(L, t_L) \geq \eta(t_L, t_L) = 0$ and $\tilde{w}(L, t^*) = 0 \leq \eta(t_L, t^*)$, we can define λ_1 and λ_2 as follows (see Figure 5(a)):

(i) If $\tilde{v}(L,\tau) > 0$ for all $\tau < t_L$, take $\lambda_1 = 0$. Otherwise, let λ_1 be the unique $\tau \leq t_L$ that solves $\tilde{v}(L,\tau) \equiv \tilde{w}(L,\tau) + \eta(t_L,\tau) = 0$. (Uniqueness is obtained from the linearity of $\tilde{w}$ and the concavity of η.)

(ii) Take λ_2 to be the unique $\tau \in [t_L, t^*]$ satisfying $\tilde{v}(L,\tau) = 0$. Observe that $\tilde{v}$ is monotone decreasing in τ over $[t_L, t^*]$ (both $\tilde{w}(L,\tau)$ and $\eta(t_L,\tau)$ are decreasing), and $\tilde{v}(L, t_L) \geq 0 \geq \tilde{v}(L, t^*)$.

It follows that for $\tau \in [\lambda_1, \lambda_2]$, $\tilde{v}(L,\tau) \geq \max\{0, \tilde{v}(R,\tau)\}$. Hence $\tau \in L(t_L, t_R, \nu)$. A symmetric argument obtains ρ_1 as the unique $\tau \leq t_R$ that solves $\tilde{v}(R,\tau) = 0$;

and $\rho_2 = 1$ if $\tilde{v}(R, \tau) > 0$ for all $\tau \geq t_R$, otherwise ρ_2 equals the unique $\tau \geq t_R$ that solves $\tilde{v}(R, \tau) = 0$.

Note that $\lambda_1 = \lambda_2$ only if $t^* = t_L$ (i.e. $\nu = 1$), and $\rho_1 = \rho_2$ only if $t^* = t_R$ (i.e. $\nu = 0$). Therefore, $V_L(t_L, t_R, \nu) + V_R(t_L, t_R, \nu) > 0$.

Continuity and monotonicity. Because party L's voters concentrate in the interval I_λ, we can write

$$V_L(t_L, t_R, \nu) = F(\lambda_2(t_L, t_R, \nu)) - F(\lambda_1(t_L, t_R, \nu)),$$

where λ_2 is implicitly defined as the unique $\tau > t_L$ that solves $v(L; t_L, t_R, \nu, \tau) = 0$. Then, by the Implicit Function Theorem, λ_2 is continuous and

$$\frac{\partial \lambda_2}{\partial \nu}(t_L, t_R, \nu) = -\frac{\frac{\partial v}{\partial \nu}(L; t_L, t_R, \nu, \tau)}{\frac{\partial v}{\partial \tau}(L; t_L, t_R, \nu, \tau)}$$

evaluated at $\tau = \lambda_2$. Because, at $\tau = \lambda_2$,

$$\frac{\partial v}{\partial \nu}(L; t_L, t_R, \nu, \tau) = \frac{\partial w}{\partial \nu}(L; t_L, t_R, \nu, \tau) = (t_R - t_L)\frac{\partial \hat{t}(t_L, t_R, \nu)}{\partial \nu} < 0, \text{and}$$
$$\frac{\partial v}{\partial \tau}(L; t_L, t_R, \nu, \tau) = \frac{\partial w}{\partial \tau}(L; t_L, t_R, \nu, \tau) + \frac{\partial \eta}{\partial \tau}(t_L; \tau) < 0,$$

then $\frac{\partial \lambda_2}{\partial \nu}(t_L, t_R, \nu) < 0$ (see Fig 5(b).) That is, λ_2 decreases as ν increases.

If $\lambda_1 = 0$, it cannot decrease any further. If $\lambda_1 > 0$, then it is continuous and, for $\tau = \lambda_1$,

$$\frac{\partial \lambda_1}{\partial \nu}(t_L, t_R, \nu) = -\frac{\frac{\partial w}{\partial \nu}(L; t_L, t_R, \nu, \tau)}{\frac{\partial w}{\partial \tau}(L; t_L, t_R, \nu, \tau) + \frac{\partial \eta}{\partial \tau}(t_L; \tau)} > 0.$$

Since, first, the numerator is negative: $\frac{\partial w}{\partial \nu} = (t_R - t_L)\frac{\partial \hat{t}}{\partial \nu} < 0$; and, second, the denominator is positive: $\frac{\partial w}{\partial \tau} = -(t_R - t_L) < 0$, $\frac{\partial \eta}{\partial \tau} = -\zeta' > 0$ and $\left|\frac{\partial w}{\partial \tau}\right| < \left|\frac{\partial \eta}{\partial \tau}\right|$ because w is linear in τ, $-\eta$ is convex in τ and $w(t_L, \lambda_1) = \eta(t_L, \lambda_1)$ (see Fig 5(b).)

It follows that
$$\frac{\partial V_L}{\partial \nu} = f(\lambda_2)\frac{\partial \lambda_2}{\partial \nu} - f(\lambda_1)\frac{\partial \lambda_1}{\partial \nu} \leq 0.$$

Similar calculations for V_R obtain that V_R is non-decreasing in ν. ■

Lemma 5.2 *Let A3, A4, and A5 hold. The consistent voting outcome correspondence χ is non-empty, single-valued, and continuous for all $t_L \neq t_R$.*

Proof. Take $t_L \neq t_R$. From Lemma 5.1 V_L and V_R are continuous and $V_L(t_L, t_R, \nu) + V_R(t_L, t_R, \nu) > 0$. Define

$$\Psi(\nu; t_L, t_R) = \frac{V_L(t_L, t_R, \nu)}{V_L(t_L, t_R, \nu) + V_R(t_L, t_R, \nu)} - \nu$$

We know that $\nu \in \chi(t_L, t_R)$ if and only if $\Psi(\nu; t_L, t_R) = 0$ (see (2.6)), that Ψ is continuous and decreasing (Lemma 5.1), that $\Psi(0; t_L, t_R) \geq 0$, and that $\Psi(1; t_L, t_R) \leq 0$. Thus, there exists a unique ν such that $\Psi(\nu; t_L, t_R) = 0$. That is, χ is non-empty and single-valued. Continuity follows directly from the Implicit Function Theorem. ■

The previous lemma shows that there exists a unique voting outcome associated to each pair of differentiated platforms. When both parties propose identical platforms, the implemented policy is unmistakably the common platform, independent of the voting outcome. Therefore, we can write the implemented policy as:

$$\tilde{t}(t_L, t_R) = \begin{cases} \hat{t}(t_L, t_R, \chi(t_L, t_R)) & \text{if } t_L \neq t_R, \\ t & \text{if } t_L = t_R = t. \end{cases}$$

We argued at the end of the previous section that parties proposed extreme platforms because voters were only concerned about pragmatic voting. In that context, everybody voted and therefore, parties did not worry about alienating part of the electorate by becoming too extreme in their platforms. In order to validate our claim we need to show that when a non-pragmatic component of voting is present, parties do not (necessarily) take extreme positions. We start by studying a particular case: a symmetrically distributed electorate.[22]

The following theorem shows that for the symmetric case an equilibrium always exists and, not surprisingly, it is symmetric: parties propose differentiated and symmetric policies, the vote is equally split, and the implemented policy is the median policy, which, in this case, coincides with the mean. It is easy to construct examples where platforms are not extreme. We only need a sufficiently relevant non-pragmatic component of voting and a fairly concentrated distribution (refer to Figure 4(a) in Section 3).

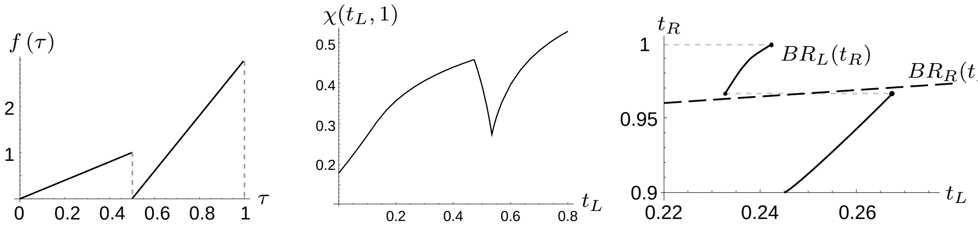
Theorem 5.1 (Symmetry) *Let A3, A4, and A5 hold. Let the electorate be distributed symmetrically around* $\frac{1}{2}$*:* $f(x) = f(1-x)$*. If* $\tau_L \leq \frac{1}{2} \leq \tau_R$*,* $\tau_L \neq \tau_R$*, then:*

(i) there exists a political equilibrium (t_L^*, t_R^*, ν^*)*, with* $t_L^* = \frac{1}{2} - k$*,* $t_R^* = \frac{1}{2} + k$*, for some* $k \in [0, 1/2)$*;*

(ii) the implemented policy is $t^* = \frac{1}{2}$*, and both parties receive the same share of votes,* $\nu^* = \frac{1}{2}$*.*

Proof. Consider the game $\Gamma = (2, T, \Pi_L, \Pi_R)$, where the payoffs of the parties are $\Pi_L(t_L, t_R) = -\tilde{t}(t_L, t_R)$ and $\Pi_R(t_L, t_R) = \tilde{t}(t_L, t_R)$, respectively. This is

[22] One may argue that symmetry facilitates that both parties locate at the extremes. Therefore, finding non-extreme platforms for symmetrically distributed populations strongly supports our claim.



(a) Density function.

(b) χ is not quasi-concave in t_L for $t_R = 1$.

(c) An equilibrium does not exist: the reaction curves do not intersect.

Figure 6: Example of non-existence of equilibrium for very asymmetric distributions. The density function is bi-triangular and $\eta(t;\tau) = -5\,|t-\tau|$.

a two-party zero-sum game. Let

$$\tilde{t}_R^0 = \max_{t_R} \min_{t_L} \tilde{t}\,(t_L, t_R)\,, \text{ and}$$
$$\tilde{t}_L^0 = \min_{t_L} \max_{t_R} \tilde{t}\,(t_L, t_R)\,.$$

Because party R can surely implement at least $\tilde{t}_R^o$ and party L can keep the policy from being more than $\tilde{t}_L^o$, then it cannot be that $\tilde{t}_R^o$ is larger than $\tilde{t}_L^o$, i.e., $\tilde{t}_R^0 \le \tilde{t}_L^0$. On the other hand, a party can always impose a policy $t^* = \frac{1}{2}$ by announcing a symmetric platform to its opponent's (i.e. if R announces t_R, L could announce $1-t_R$, obtain the same fraction of votes than R, and set $\tilde{t}\,(1-t_R, t_R) = \frac{1}{2}(1-t_R) + \frac{1}{2}t_R = \frac{1}{2}$.) It follows that $\tilde{t}_L^0 \le \frac{1}{2}$ and $\tilde{t}_R^0 \ge \frac{1}{2}$. Therefore, $\tilde{t}_L^0 = \tilde{t}_R^0 = \frac{1}{2}$. That is, an equilibrium exits and the implemented policy is $\frac{1}{2}$.

Let (t_L^*, t_R^*) be an equilibrium of Γ. Then $\tilde{t}\,(t_L^*, t) \le \tilde{t}\,(t_L^*, t_R^*) \le \tilde{t}\,(t, t_R^*)$ for all t, i.e., party L cannot lower the implemented policy, neither can party R increase it. Therefore, (t_L^*, t_R^*) is a political equilibrium. Furthermore, because the political game is a strictly competitive game, all equilibria yield the same payoffs for the parties: $t^* = \frac{1}{2}$. ■

The results found under symmetry extend to a more generic case where the electorate is not symmetrically distributed. However, when the distribution is not symmetric one may construct examples where an equilibrium in pure strategies fails to exist[23]: Figure 6 illustrates indeed such an example[24] (details available from the author.) These examples are not easy to find and involve very strong asymmetries. Therefore, in the asymmetric case with an ideological component of voting, existence

[23] Our analysis concentrates in pure strategies equilibria. It is worth noticing however that an equilibrium in mixed strategies always exists (see Fudenberg and Tirole (1991, Theorem 1.3).)

[24] In a previous paper (Llavador, 2000), we showed that an equilibrium may fail to exist also in the classic winner-takes-all model when abstention is an option and the distribution has several peaks.

of equilibria requires some assumptions beyond the ones discussed so far. Ideally, such assumptions would be imposed on the primitives of the model (g, F, u, and η). But, because χ is implicitly derived from a complex computation, this would be an arduous task. We choose instead to postulate the log-concavity of χ, a premise also known in the literature as decreasing *hazard rate*.[25] As Figure 6(b) shows, this condition is violated in our non-existence example.

Theorem 5.2 *Let A3, A4, and A5 hold. Let* χ *be log-concave in* t_L *and let* $1-\chi$ *be log-concave in* t_R*. Then:*

(i) a political equilibrium exists,

(ii) at equilibrium parties propose differentiated but not necessarily extreme policies, and

(iii) the implemented policy is the same in all equilibria.

Proof. *(i).* We follow the steps of the symmetric case. First, we construct a zero-sum game and prove that an equilibrium always exists. Second, we show that if the zero-sum game equilibrium policy locates between the ideal policies of the parties, then it is an equilibrium of the political game. On the other hand, if the equilibrium policy is more extreme than a party's bliss point, there exists a political equilibrium where that party is able to implement its ideal policy.

Consider the two-party, zero-sum game $\Gamma = (T, \Pi_L, \Pi_R)$, with payoff functions $\Pi_L(t_L, t_R) = -\hat{t}(t_L, t_R, \chi(t_L, t_R))$ and $\Pi_R(t_L, t_R) = \hat{t}(t_L, t_R, \chi(t_L, t_R))$, respectively. Use the implemented policy function (A3) to write:

$$\Pi_L(t_L, t_R) = -t_R + \chi(t_L, t_R)(t_R - t_L), \text{ and}$$
$$\Pi_R(t_L, t_R) = t_L + (1 - \chi(t_L, t_R))(t_R - t_L).$$

Therefore, maximizing Π_L (with respect to t_L) and Π_R (with respect to t_R) is equivalent to maximizing

$$\kappa_L(t_L, t_R) \equiv \chi(t_L, t_R)(t_R - t_L) \text{ and}$$
$$\kappa_R(t_L, t_R) \equiv (1 - \chi(t_L, t_R))(t_R - t_L).$$

Next we show that the log-concavity of χ implies the quasi-concavity of κ_L. The function κ_L is quasi-concave in t_L if $\frac{\partial^2 \kappa_L}{\partial t_L^2}(t_L, t_R) \leq 0$ for all t_L such that $\frac{\partial \kappa_L}{\partial t_L}(t_L, t_R) = 0$. Write $\chi_L = \frac{\partial \chi}{\partial t_L}$ and $\chi_{LL} = \frac{\partial^2 \chi}{\partial t_L^2}$. We compute:

$$\frac{\partial \kappa_L}{\partial t_L}(t_L, t_R) = \chi_L(t_L, t_R) \cdot (t_R - t_L) - \chi(t_L, t_R), \text{ and} \quad (5.1)$$
$$\frac{\partial^2 \kappa_L}{\partial t_L^2}(t_L, t_R) = \chi_{LL}(t_L, t_R) \cdot (t_R - t_L) - 2 \cdot \chi_L(t_L, t_R). \quad (5.2)$$

[25] In a similar context, (Roemer, 1997, p.492) also requires the log-concavity of a compound function.

Let $t_L \in T$ such that $\frac{\partial \kappa_L}{\partial t_L} = 0$, then from (5.1) we get $(t_R - t_L) = \frac{\chi(t_L,t_R)}{\chi_L(t_L,t_R)}$ and, substituting into (5.2), we obtain (all functions are evaluated at (t_L, t_R)):

$$\left.\frac{\partial^2 \kappa_L}{\partial t_L^2}\right|_{\frac{\partial \kappa_L}{\partial t_L}=0} = \chi_{LL}|_{\frac{\partial \kappa_L}{\partial t_L}=0} \cdot \frac{\chi}{\chi_L|_{\frac{\partial \kappa_L}{\partial t_L}=0}} - 2 \cdot \chi_L|_{\frac{\partial \kappa_L}{\partial t_L}=0}.$$

Thus,

$$\left.\frac{\partial^2 \kappa_L}{\partial t_L^2}\right|_{\frac{\partial \kappa_L}{\partial t_L}=0} \geq 0 \Leftrightarrow \chi_{LL}|_{\frac{\partial \kappa_L}{\partial t_L}=0} \cdot \chi - 2 \cdot \left(\chi_L|_{\frac{\partial \kappa_L}{\partial t_L}=0}\right)^2 \leq 0, \tag{5.3}$$

since $\chi_L|_{\frac{\partial \kappa_L}{\partial t_L}=0} \equiv \frac{\chi}{t_R - t_L} > 0$.

The log-concavity of χ implies that $\chi_{LL} \cdot \chi - (\chi_L)^2 \leq 0$. It follows from (5.3) that κ_L is quasi-concave. A symmetric argument shows that the log-concavity of $(1 - \chi)$ implies that κ_R is quasi-concave in t_R.

Let $\mathrm{BR}_L(t_R)$ be the set of maximizers of $\Pi_L(t_L, t_R)$ for a given t_R (i.e., the best response of L to t_R). Since κ_L is quasi-concave, BR_L is convex-valued. From the Theorem of the Maximum, BR_L is non-empty and upper hemi-continuous. Similarly, BR_R is also non-empty, convex-valued and upper hemi-continuous. Then, the correspondence $\mathrm{BR} : T^2 \to T^2$:$\mathrm{BR}(t_L, t_R) = (\mathrm{BR}_L(t_R), \mathrm{BR}_R(t_L))$ is non-empty, convex-valued and upper hemi-continuous. By Kakutani's fixed point theorem there exists a $(t_L^*, t_R^*) \in \mathrm{BR}(t_L^*, t_R^*)$. Therefore, (t_L^*, t_R^*) is an equilibrium of Γ.

Let $t^* = \tilde{t}(t_L^*, t_R^*)$. If $\tau_L \leq t^* \leq \tau_R$, then neither party can pull the implemented policy towards its ideal point given the platform announced by the other party. Hence, (t_L^*, t_R^*) is a political equilibrium. Suppose, on the other hand, that $t^* \notin [\tau_L, \tau_R]$. Take, for example, $t^* < \tau_L$. Since Γ is a zero-sum game, it follows that L can secure a policy as to the left as t^*. Because $\tau_L > t^*$, t^* represents a lower payoff for L in the zero-sum game, and party L can definitely secure τ_L. Thus, if $t^* < \tau_L$, there exists a political equilibrium where the implemented policy is τ_L. Similarly, if $t^* > \tau_R$, there exists a political equilibrium with τ_R as the implemented policy.

(iii). Because the political game is a strictly competitive game, the implemented policy is the same in all equilibria.

(ii). Finally, observe that one can never have an equilibrium where both parties propose the same policy. At least one party can always pull the policy a bit closer to its ideal point by differentiating its platform and capturing a small fraction of the vote. As the example in Section 3 shows, platforms may not be extreme at equilibrium. ■

6 Concluding Remarks

The well known and widely used median voter theorem is the standard result in political competition under perfect information about the distribution of the electorate. This elegant theoretical proposition is clearly at odds with the empirical evidence.

In this paper we enrich the theoretical framework to allow for non-trivial policy-setting processes and for sophisticated voters who may abstain. First, we show that platform convergence is a non-robust feature created by the winner-takes-all assumption (Section 4). More importantly, a divergent tendency in platform writing arises when the policy implemented by the winner party is sensitive to the electoral margin of victory. And this is true for any positive degree of sensitivity. Second, we show that, under reasonable assumptions, an equilibrium always exists (Section 5). At equilibrium, voters concentrate around the announced platforms, and abstention occurs among citizens with extreme views as well as with moderate views. High turnout rates obtain even with positive costs of voting. Parties announce differentiated but non-extreme platforms. They may even choose platforms that are more extreme than their ideal policy. Yet the implemented policy tend to be moderated and non-related to the ideal policy of the median voter. Thus our results are consistent with the observation of polarized platforms and moderate policies.

This paper also shows that the whole distribution of the electorate, and not only the (expected) location of a pivotal voter, matters in determining policy outcomes. Consider, for example, two countries identical on everything but on the distribution of their electorate. While country A's electorate follows a concentrated-in-the-middle, unimodal distribution, the electorate of country B is fractured into two radical groups (its density function presents two peaks with the majority of the population located around them). Assume that they share the same median voter, or, if there is uncertainty, that the knowledge about the location of the median voter is the same in both countries. According to our model, parties in country A would move towards the center to avoid alienating the core of their constituency, while parties in country B will radicalize their positions, following the radicalization of the electorate.[26] However, models based on the median voter (with or without uncertainty) would predict the same electoral outcome in both countries.

Thus, summarizing our model, we should observe parties moving towards the 'middle' of the political spectrum when (*i*) the mass of the electorate is concentrated around moderate views, and (*ii*) a party alienates the core of its constituency by radicalizing its platforms. Thus, this is an alternative to the standard claims that parties tend to the middle because of competition, and that platforms converge as the uncertainty level falls (*ceteris paribus*). In this sense, our story is consistent with the observation of a generalized party convergence movement in Western democracies, except for those regions, like Israel or Northern Ireland, where the electorate is divided an polarized.

There remain a number of important theoretical directions to be explored in this class of models. The description of the political process can be enriched by making parties announce not only the platform they stand for, but also how much they will *compromise* if they win the election. Another natural step is extending the results

[26]The implemented policy in country B may or may not be radical, depending on the relative weight of each group and on the "conciliatory" spirit of the winning party (the function g in the model.)

for the case of more than two parties. The analysis of these possibilities is left for future research.

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