

The Strategic Equivalence of Rent-Seeking, Innovation, and Patent-Race Games*

Michael R. Baye
Indiana University

Heidrun C. Hoppe
Universität Hamburg

October 2001

Abstract

Conditions are identified which guarantee the strategic equivalence of rent-seeking, innovation, and patent-race games. Our results permit one to apply theorems and results intended for rent-seeking games to other games, and vice versa. We conclude with several examples that highlight the practical utility of our results. (**JEL Numbers: D00, L00, D72; Keywords: Contest, Rent Seeking, Innovation, R&D, Patents**).

*Baye: Department of Business Economics, Kelley School of Business, Indiana University, 1309 East Tenth Street, Bloomington, IN 47405-1701, USA, E-Mail: mbaye@indiana.edu. Hoppe: Universität Hamburg, Institut für Allokation und Wettbewerb, Von-Melle-Park 5, D-20146 Hamburg, Germany, E-Mail: HOPPE@hermes1.econ.uni-hamburg.de. We are grateful to Dong Chen for comments on a preliminary draft of this paper.

1 Introduction

Over the past two decades, a number of important papers have independently analyzed games of rent-seeking and innovation; see Nitzan (1994), and Reinganum (1989) for surveys of these two literatures. While it is well-known (cf. Baye, Kovenock, and de Vries, 1996) that many of these games have similar “all-pay” structures in which winners and losers alike forfeit the resources expended to win the prize, a formal analysis of the “duality” between rent-seeking and innovation games is lacking. This paper represents a first attempt to formalize the relationship between rent-seeking contests and innovation games.

The models of rent-seeking considered in this paper include the seminal model of Gordon Tullock, as well as more general contests such as those in Dixit (1987) and Skaperdas (1995). In addition, our analysis covers two different classes of innovation games: patent races, which are typically used to model the competition to be first, and innovation tournaments, which are most frequently used to formalize the competition to be best.¹ The paper considers the classic patent race game as pioneered by Loury (1979) and Dasgupta and Stiglitz (1980), and contributes to the literature on innovation tournaments by offering a model in which R&D efforts determine not only the probability of winning, but also the value of the winner’s prize.²

¹See Taylor (1995) for a discussion of the alternative approaches to the modeling of R&D.

²Much of the existing literature treats the value of the prize as an exogenous parameter; see Taylor (1995) and Fullerton and McAfee (1999) for studies of sponsored research tournaments.

We model R&D as a process where each firm runs parallel experiments (funded through R&D) and chooses to go forward with its “best” project. This view of R&D dates back at least to Nelson (1961), who notes that parallel-path strategies are a standard practice in many industrial laboratories. Our analysis of R&D extends the “parallel-path” monopoly model of Evenson and Kislev (1976) to an oligopolistic setting in which multiple firms engage in R&D.

Based on these general models of rent-seeking and innovation, we identify conditions under which innovation games are strategically equivalent to contests and, in particular, to the Tullock game. These duality results permit one to apply results derived in the contest and rent-seeking literatures to the innovation context, and vice versa. We conclude with a number of examples that highlight some practical applications of our results.

2 Three Classes of Games

This section presents the contests and innovation games considered in this paper. The interested reader may consult the literature and surveys identified in the Introduction for a more extensive discussion of the various games.

2.1 Rent-Seeking Contests

Let x_1, x_2, \dots, x_n denote the effort levels of the n players, let $v(x_1, x_2, \dots, x_n)$ represent the value of the contested prize, and let $p_i(x_1, x_2, \dots, x_n)$ denote the probability that firm i wins the contest. Player i 's expected payoff is

$$\pi_i(x_1, x_2, \dots, x_n) = v(x_1, x_2, \dots, x_n) p_i(x_1, x_2, \dots, x_n) - x_i.$$

In the sequel, we shall let $C(v, p, n)$ denote this class of contests.

Notice that Dixit (1987) corresponds to the special case where v is constant and p is a twice continuously differentiable, symmetric function of players' efforts. The all-pay-auction (cf. Baye, Kovenock and de Vries, 1996) corresponds to the special case where v is constant and p is discontinuous (the player with the greatest x_i wins with probability one). The most celebrated special case is, of course, the Tullock rent-seeking game in which player i 's expected payoff is

$$\pi_i(x_1, x_2, \dots, x_n) = v \frac{x_i^R}{\sum_{j=1}^n x_j^R} - x_i,$$

where v is constant and $R > 0$. We denote the family of Tullock games as $T(v, R, n)$. Equilibria for $T(v, R, n)$ (including the case where $R > 2$) are known; see Baye, Kovenock and de Vries (1994).

2.2 Innovation Tournaments

Consider n firms who compete to enter a new market with an innovation. Suppose the values of ideas for the new product are distributed according to F on $[0, 1]$. The

ideas are borne from innovation activities, which we model as experiments whose outcomes are random in that firms cannot pinpoint ideas that will be valuable. A firm can, however, increase its likelihood of having a valuable idea by increasing its R&D effort, which is represented by increasing the size of the sample it draws from F . The sample size for firm i is determined by the number of scientists that firm i employs, and is denoted by m_i . We assume that the firm that discovers the best idea y wins a patent of value y , while all other firms gain nothing.³

The firms simultaneously choose how many ideas to draw from F . Each draw costs c such that the total cost to firm i of m_i draws is cm_i . We assume that $c < \int_0^1 yf(y) dy$,

³The existing literature on innovation tournaments assumes that the value of the prize is exogenously given (Taylor, 1995; Fullerton and McAfee, 1999). We introduce the endogenous-prize formulation for two reasons. First, we wish to cover the two extreme forms of innovation competition: patent races where R&D accelerates the innovation of a fixed value (see below), and tournaments where R&D increases the value of the innovation but where timing is no issue. Second, in many economic situations, R&D efforts affect the profits that are associated with having the best idea. Consider, for example, a situation where firms can invest in R&D to reduce the cost of producing a new homogenous product. With constant returns to scale and Bertrand price competition in the product market, only the firm with the best innovation will profit from selling the new product, and better ideas will generate higher monopoly profits. Another example is a research tournament in which the firm with the best idea wins an exclusive right for commercializing the idea, such as the tournament recently sponsored by the Federal Communications Commission to develop the best technology for high-definition television (HDTV), where the technology of the winner was chosen as the HDTV standard.

where f denotes the probability density function of y . Thus, R&D costs for a single draw are less than the expected value of the idea associated with it. This is a necessary condition for a firm to invest in one or more ideas.

Let $z_i \equiv \max \{y_{i1}, y_{i2}, \dots, y_{im_i}\}$ denote the best idea of firm i , and $m_{-i} = \sum_{j \neq i} m_j$ denote the number of scientists hired by all firms but i . The value to firm i of hiring m_i scientists when the rivals have hired m_{-i} scientists is

$$\Pr \left(\max_{j \neq i} \{z_j\} \leq z_i \right) z_i,$$

which is a random variable. To find the expectation of this random variable, note that $\Pr (\max_{j \neq i} \{z_j\} \leq z_i) = (F(z_i))^{m_{-i}}$ and $\Pr (\max_k \{y_{ik}\} \leq z_i) = (F(z_i))^{m_i}$, $k = 1, 2, \dots, m_i$. Thus, the expected value to firm i of hiring m_i scientists when the rivals hire m_{-i} scientists is

$$\begin{aligned} v(m_i, m_{-i}) &= \int_0^1 \left(\Pr \left(\max_{j \neq i} \{z_j\} \leq z_i \right) z_i \right) d(F(z_i))^{m_i} \\ &= \int_0^1 (F(z_i))^{m_{-i}} z_i m_i (F(z_i))^{m_i-1} f(z_i) dz_i, \end{aligned}$$

and firm i 's payoff in the innovation tournament is given by

$$\pi_i(m_1, m_2, \dots, m_n) = v(m_i, m_{-i}) - cm_i.$$

Let $I(F, c, n)$ denote this family of innovation games.

2.3 Patent Races

Consider the classic model of a patent race as pioneered by Loury (1979) and Dasgupta and Stiglitz (1980). There are n firms. The firm that innovates first will obtain

a patent of infinitely long life, while all other firms gain nothing. Let v denote the value of the patent. It is assumed that if firm i chooses a lump-sum R&D investment x_i ,⁴ its probability of making a discovery on or before time t is

$$1 - e^{-h(x_i)t}$$

where $h(x_i)$ is the “hazard rate” of firm i , that is, firm i ’s conditional probability of making a discovery between time t and time $t + dt$, given that no innovation has occurred at or before t . The function h is defined on $[0, 1]$ and assumed to be increasing and concave.⁵ Letting r denote the interest rate, firm i ’s payoff in the patent race is given by

$$\pi_i(x_1, x_2, \dots, x_n) = \int_0^\infty h(x_i) v e^{-(\sum_{j=1}^n h(x_j) + r)t} dt - x_i,$$

Let $P(h, v, r, n)$ denote this family of patent races.

3 Results

We first provide sufficient conditions for an innovation tournament $\gamma \in I(F, c, n)$ to be strategically equivalent to a contest $\Gamma \in C(v, p, n)$. To ease the exposition, let $x_{-i} = \sum_{j \neq i} x_j$ denote the aggregate rival effort.

⁴In another class of patent race models, as pioneered by Lee and Wilde (1980), R&D costs are modelled as a flow cost that each firm pays until one firm makes a discovery and wins the patent.

⁵The assumption of concavity is made to ensure uniqueness of the equilibrium.

Theorem 1 For any distribution of ideas (F), an innovation game $\gamma \in I(F, c, n)$ is strategically equivalent to a contest $\Gamma \in C(v, p, n)$ with success function

$$p(x_1, x_2, \dots, x_n) = \frac{x_i}{x_i + x_{-i}},$$

prize function

$$v(x_1, x_2, \dots, x_n) = \frac{1}{c} \left[1 - \int_0^1 (F(z))^{x_i + x_{-i}} dz \right],$$

and where each player's strategy space is $\{0, 1, 2, \dots\}$.

Proof. In an innovation game $\gamma \in I(F, c, n)$, firm i 's expected payoff can be written as

$$\pi_i(m_1, m_2, \dots, m_n) = m_i \int_0^1 (F(z))^{m_i + m_{-i} - 1} z dF(z) - cm_i.$$

Let $v = z$, $u = (F(z))^{m_i + m_{-i}}$, and integrate by parts to obtain:

$$\begin{aligned} \pi_i(m_1, m_2, \dots, m_n) &= \frac{m_i}{m_i + m_{-i}} \int_0^1 v du - cm_i \\ &= \frac{m_i}{m_i + m_{-i}} \left(uv \Big|_0^1 - \int_0^1 u dv \right) - cm_i \\ &= \frac{m_i}{m_i + m_{-i}} \left[(F(z))^{m_i + m_{-i}} z \Big|_0^1 - \int_0^1 (F(z))^{m_i + m_{-i}} dz \right] - cm_i \\ &= \frac{m_i}{m_i + m_{-i}} \left[1 - \int_0^1 (F(z))^{m_i + m_{-i}} dz \right] - cm_i. \end{aligned}$$

■

Corollary 1 The prize function identified in Theorem 1 is increasing in x_i , x_{-i} , and $x_i + x_{-i}$, but at a decreasing rate.

Proof. Using Leibniz's rule,

$$\begin{aligned} \frac{\partial v(x_1, x_2, \dots, x_n)}{\partial x_i} &= \frac{\partial v(x_1, x_2, \dots, x_n)}{\partial x_{-i}} = \frac{\partial v(x_1, x_2, \dots, x_n)}{\partial (x_i + x_{-i})} \\ &= -\frac{1}{c} \int_0^1 F(z)^{x_i + x_{-i}} \ln F(z) dz > 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial^2 v(x_1, x_2, \dots, x_n)}{\partial x_i^2} &= \frac{\partial^2 v(x_1, x_2, \dots, x_n)}{\partial x_{-i}^2} = \frac{\partial^2 v(x_1, x_2, \dots, x_n)}{\partial (x_i + x_{-i})^2} \\ &= -\frac{1}{c} \left(\int_0^1 \frac{\partial}{\partial x_i} \left(F(z)^{x_i + x_{-i}} \ln F(z) \right) dz \right) \\ &= -\frac{1}{c} \left(\int_0^1 F(z)^{x_i + x_{-i}} \ln^2 F(z) dz \right) < 0 \end{aligned}$$

■

Together, Theorem 1 and Corollary 1 reveal that one may view an innovation tournament as a standard contest in which the underlying “prize” is not only an increasing function of each firm’s own R&D effort, but also an increasing function of the rivals’ individual and aggregate efforts.⁶ The innovation tournament thus exhibits positive externalities to R&D investments. The reason is that to win the tournament a firm must surpass the rivals’ R&D efforts. Since higher R&D effort increases the likelihood of making a more valuable discovery, and since the value of the discovery determines the value of the prize, rivals’ R&D efforts have a positive indirect impact on the value of the prize.

⁶Note, however, that since the strategy space in the innovation game is discrete, this exact isomorphism holds only for contests in which the strategy space is discrete.

Example 1 Suppose $F(z) = z$ on $[0, 1]$. Then by Theorem 1, we know that an innovation game $\gamma \in I(F, c, n)$ is strategically equivalent to a contest $\Gamma \in C(v, p, n)$ where

$$p(x_i, x_{-i}) = \frac{x_i}{x_i + x_{-i}}$$

and

$$v(x_i, x_{-i}) = \frac{1}{c} \frac{x_i + x_{-i}}{x_i + x_{-i} + 1}.$$

Our next theorem establishes a relationship between innovation games and Tullock contests.

Theorem 2 There exists an innovation game $\gamma \in I(F, c, n)$ that is strategically equivalent to a Tullock game $\Gamma \in T(v, R, n)$ with $R = 1$ and a discrete strategy space.

Proof. Consider an innovation game in which $c = 1$ and the distribution of ideas depends on R&D effort in the following manner:

$$F^*(y) = (G(y))^{\frac{1}{m_i + m_{-i}}}.$$

As in the proof to Theorem 1, the payoff function for an arbitrary innovation game $\gamma \in I(F, c, n)$ may be written as

$$\pi_i(m_1, m_2, \dots, m_n) = \frac{m_i}{m_i + m_{-i}} \left[1 - \int_0^1 (F(z))^{m_i + m_{-i}} dz \right] - cm_i.$$

Thus, when $c = 1$ and $F = F^*$,

$$\begin{aligned}\pi_i(m_1, m_2, \dots, m_n) &= \frac{m_i}{m_i + m_{-i}} \left[1 - \int_0^1 (F^*(z))^{m_i + m_{-i}} dz \right] - m_i \\ &= \frac{m_i}{m_i + m_{-i}} \left[1 - \int_0^1 G(z) dz \right] - m_i \\ &= \frac{m_i}{m_i + m_{-i}} v - m_i,\end{aligned}$$

where v is a constant. ■

The proof of Theorem 2 reveals that a duality between an innovation tournament and the Tullock game can be established in the presence of diseconomies of density in the total number of R&D experiments. This occurs when the profitability of a single R&D experiment declines with the total number of experiments in the industry. We show that such diseconomies of density can offset the positive externalities in R&D efforts (Corollary 1) so that an innovation tournament can reduce essentially to the Tullock game with $R = 1$.⁷

Next, we turn to patent races. It is easy to see that, in the limit as the interest rate tends to zero, a patent race $\gamma' \in P(h, v, r, n)$ with hazard rate $h(x_i) = x_i^R$, is formally equivalent to a Tullock game $\Gamma' \in T(v, R, n)$. In particular, note that player i 's payoff in a patent race is

$$\pi_i(x_1, x_2, \dots, x_n) = \int_0^\infty h(x_i) v e^{-(\sum_{j=1}^n h(x_j) + r)t} dt - x_i$$

⁷Substantial diseconomies of density appear to be present, for example, in industries where firms use patents simply to “colonize” unexplored areas of technology, such as the computer industry. See *The Economist*, “Patent wars”, April 6, 2000.

$$= v \frac{h(x_i)}{\sum_{j=1}^n h(x_j) + r} - x_i,$$

so in the limit as the interest rate tends to zero, the payoff function is identical to that in a Tullock game for $h(x_i) = x_i^R$.

However, results for the Tullock game cannot be directly applied without imposing further restrictions. The reason is that the patent race game assumes $h(x_i)$ to be an increasing and concave function, defined on $[0, 1]$. For the duality to be exact, one must assume $R \leq 1$ (to obtain concavity). Furthermore, for $R \leq 1$, it is well-known from the literature on Tullock games that there exists a unique symmetric Nash equilibrium in which, for all i :

$$x_i = \frac{(n-1)}{n^2} v R.$$

Hence, imposing an upper bound on v is sufficient to guarantee that $0 \leq h \leq 1$. These observations, coupled with arguments similar to those in the proof of Theorem 2, yield the following duality results.

Theorem 3 *In the limit as r approaches 0, a patent race $\gamma \in P(h, v, r, n)$ with hazard rate*

$$h(x_i) = x_i^R, \text{ for } R \leq 1,$$

is strategically equivalent to a Tullock game $\Gamma \in T(v, R, n)$ in which $R \leq 1$ and $v \leq n^2/(n-1)$. Furthermore, every patent race with hazard rate

$$h(x_i) = x_i$$

is, in the limit, strategically equivalent to an innovation tournament $\Gamma \in I(F, c, n)$

with

$$F(y) = (G(y))^{\frac{1}{m_i + m - i}}. \quad (1)$$

Note that a decrease in the interest rate, r , is equivalent to shrinking the units in which time is measured. As r approaches zero, a patent race converges to its limiting static form. Note further that the cumulative distribution of valuable ideas, as given by (1), exhibits diseconomies of density in the total number of R&D experiments.⁸

4 Applications and Concluding Remarks

We conclude by illustrating the practical utility of the above results. First, consider the work of Chung (1996), who analyzes a contest in which the probability of winning takes the symmetric, logit form as in the Tullock game or the innovation tournament described above, and in which the prize is an increasing function of aggregate efforts. He compares the aggregate equilibrium effort with the socially efficient effort. The latter is defined as the level of effort that maximizes social surplus, $v(X) - x$, where $X \equiv \sum_{j=1}^n x_j$. Chung shows that the contest generates socially wasteful (excessive) effort, and furthermore, that rent dissipation is incomplete. To ensure that the maximization problem has a unique interior solution, Chung makes the following three assumptions:

⁸See Footnote 7 for an example.

(Ai) $v(X)$ is twice continuously differentiable, where $X \equiv \sum_{j=1}^n x_j$.

(Aii) $v(X) \geq 0$, $v' > 0$, $v'' < 0$, and $v'(X) > 1$ as $X \rightarrow 0$.

(Aiii) There exists a finite, positive \bar{X} such that $v(\bar{X}) - \bar{X} = 0$.

From Theorem 1 and Corollary 1 we know that an innovation tournament is isomorphic to a contest in which the prize is increasing in aggregate effort. In order to apply Chung's results in an innovation context, one must simply verify that assumptions (Ai)-(Aiii) are satisfied under an innovation interpretation.

Proposition 1 *An innovation tournament $\gamma \in I(F, c, n)$ satisfies assumptions (Ai), (Aii), and (Aiii).*

Proof. By Corollary 1, assumption (Ai) and the first part of assumption (Aii) are satisfied. We will now establish that $v'(X) > 1$ as $X \rightarrow 0$. Using Leibniz's rule,

$$v'(X) = -\frac{1}{c} \int_0^1 F(z)^X \ln F(z) dz > 0.$$

Hence,

$$\lim_{X \rightarrow 0} v'(X) > 1$$

is equivalent to

$$c < -\int_0^1 \ln F(z) dz. \tag{2}$$

Let $x = z$ and $u = \ln F(x)$. Integrating the right-hand side of (2) by parts yields

$$-\int_0^1 u dx = -x \ln F(x) \Big|_0^1 + \int_0^1 \frac{x F'(x)}{F(x)} dx.$$

Condition (2) is thus satisfied if

$$c < \lim_{x \rightarrow 0} \left(-x \ln F(x) \Big|_0^1 \right) + \int_0^1 \frac{x F'(x)}{F(x)} dx.$$

Using l'Hôpital's rule, we obtain

$$\lim_{x \rightarrow 0} x \ln F(x) = \lim_{x \rightarrow 0} \frac{\frac{1}{F(x)} F'(x)}{\frac{-1}{x^2}} = - \lim_{x \rightarrow 0} \frac{x F'(x)}{F(x) / x}.$$

Note that

$$\lim_{x \rightarrow 0} \frac{F'(x)}{F(x) / x} = 1,$$

so it follows that $\lim_{x \rightarrow 0} x \ln F(x) = 0$. Condition (2) is therefore satisfied if

$$c < \int_0^1 \frac{x F'(x)}{F(x)} dx$$

But note that $c < \int_0^1 x F'(x) dx$ holds in any innovation tournament $\gamma \in I(F, c, n)$, so it must be that $v'(X) > 1$ as $X \rightarrow 0$. Finally, it is straightforward to verify that for sufficiently large X , social surplus is negative at such a solution, i.e. that assumption (Aiii) is satisfied as well. ■

Thus, we may conclude that an innovation tournament in which firms strive to be the best involves over investment in R&D. That is, the positive externalities of R&D effort on the value of the prize (Corollary 1) are outweighed by the negative externalities that each firm imposes on the other firms' probability of winning the tournament. This result, in turn, has implications for R&D and patent policies.

In a similar fashion, one may use the above theorems in conjunction with other results in the contest literature to derive additional policy implications for innovation

games. For instance, when the distribution of ideas satisfies the conditions under which an innovation tournament is strategically equivalent to a Tullock game with $R = 1$ (Theorem 2), or when the interest rate is small so that a patent race is strategically equivalent to the Tullock game with $R = 1$, one can use results from the contest literature to establish that the Nash equilibrium to an innovation game is invariant to sequence of moves (cf. Dixit, 1987; Baye and Shin, 1999) or leads to over investment in R&D (Dixit, 1987; Baik and Shogren, 1992).

Likewise, the above theorems permit one to use results in the literature on innovation and R&D games to shed light on contests and rent-seeking games. For example, Jensen and Showalter (2001) recently examined the impact of the use of leverage to finance R&D on total R&D expenditures in a patent race, and found that debt acts as a commitment to lower R&D investments. Using the isomorphisms identified above, Jensen and Showalter's results have obvious implications for the impact of borrowing on the level of effort in rent-seeking and other contests.

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