RESOLVING DISTRIBUTIONAL CONFLICTS BETWEEN GENERATIONS

GEIR B. ASHEIM AND BERTIL TUNGODDEN

Abstract. This paper describes a new approach to the problem of resolving distributional conflicts between generations. We impose conditions on the social preferences capturing the following idea: If indifference or preference holds between truncated paths for infinitely many truncating times, then indifference or preference holds also between the untruncated infinite paths. Within this framework, we show (1) how such a condition provides an improved solution to the problem of combining Strong Pareto and impartiality in an intergenerational setting, and (2) how equity conditions well-known from the finite setting can be used to characterize different versions of leximin and utilitarianism.
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1. Introduction

The Suppes-Sen grading principle captures both a concern for equal treatment of generations and the demand for efficiency. And it turns out that this is all that is needed in order to justify sustainable solutions within reasonable technological frameworks.[1]

However, there are two problems with this approach. First, it has been argued that the Suppes-Sen grading principle cannot capture impartiality among generations in a satisfactory manner (Liedekerke and Lauwers [7]). Second, even if we should accept this justification for sustainability, there exists the further problem about how to resolve distributional conflicts between generations going beyond the sustainability question. In the following, we consider both these problems.

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[1]See Asheim, Buchholz, and Tungodden [2].

In Section 3, we look at the problem of intergenerational impartiality. According to Liedekerke and Lauwers [7, p. 163], formal impartiality is ensured by imposing the axiom of Strong Anonymity, which implies indifference to any permutation of utilities of an *infinite* number of generations. As is well-known, this demand cannot be combined with Strong Pareto. Liedekerke and Lauwers suggest to establish a framework where an acceptable trade-off between the demands of impartiality and Strong Pareto can be made. In other words, they suggest that the Suppes-Sen grading principle should be rejected. The Suppes-Sen grading principle is characterized by Strong Pareto and Weak Anonymity (which entails indifference to any permutation of utilities of a *finite* number of generations). However, we argue in the present paper that it is not necessary to give up Strong Pareto in order to satisfy the demands of impartiality in an intergenerational setting beyond Weak Anonymity. We do so by introducing an axiom saying that one infinite utility path should be considered indifferent to another infinite utility path if at any point in time there is a future point in time at which the head of the former utility path is indifferent to the head of the latter. By invoking this axiom, we establish a reasonable formalization of impartiality that does not contradict Strong Pareto and strengthens Weak Anonymity, and hence does not contradict the Suppes-Sen grading principle.

In Section 4, we look at how to resolve distributional conflicts between generations going beyond the sustainability question. Also in this respect, we suggest to introduce a weak condition on the social preferences that makes the infinite intergenerational setting closely related to distributive justice in the finite setting. The condition captures the idea that one infinite utility path should be considered strictly better than another infinite utility path if the head of the former is considered strictly better than the latter at every point in time beyond a certain initial phase. Within this framework, we show how equity conditions well-known from the finite setting can be used in the debate on intergenerational justice. Moreover, we provide characterizations of some of the prominent positions in the literature.

The formal framework and the justification for sustainability based on the Suppes-Sen grading principle is introduced in Section 2, whereas some concluding remarks are provided in Section 5.

2. The framework

There is an infinite number of generations $t = 1, 2, \dots$. The utility level of generation t is given by u_t which should be interpreted as the utility level of a representative member of this generation. Initially (in Section 3 and the first part of Section 4), we assume that the utilities need not be more than ordinally measurable and level comparable, whereas later (in the second part of Section 4) we consider utilities that are also cardinally measurable and unit comparable.

A binary relation R over paths ${}_1\mathbf{u} = (u_1, u_2, \dots)$ starting in period 1 expresses social preferences over different intergenerational utility paths. Any such binary relation R is throughout assumed to be reflexive and transitive on the infinite Cartesian product $\mathbb{R}^\infty$ of the set of real numbers $\mathbb{R}$, where $\infty = |\mathbb{N}|$ and $\mathbb{N}$ is the set of natural numbers. The social preferences R may be complete or incomplete, with I denoting the symmetric part, i.e indifference, and P denoting the asymmetric part, i.e. (strict) preference. For any path ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and any time T, ${}_1\mathbf{u}_T = (u_1, u_2, \dots, u_T)$ denotes the truncation of ${}_1\mathbf{u}$ at T, and ${}_1\tilde{\mathbf{u}}_T$ is a permutation of ${}_1\mathbf{u}_T$ having the property that ${}_1\tilde{\mathbf{u}}_T$ is non-decreasing. Refer to ${}_1\mathbf{u}_T$ as the T-head and ${}_{T+1}\mathbf{u}$ as the T-tail of ${}_1\mathbf{u}$.

In order to define sets of feasible paths we assume that the initial endowment of generation $t \geq 1$ is given by a $n-$dimensional ($n < \infty$) vector of capital stocks k_t. A generation t acts by choosing a utility level u_t and a vector of capital stocks k_{t+1} which is bequeathed to the next generation $t+1$. For every t, the function F_t gives the maximum utility attainable for generation t if k_t is inherited and k_{t+1} is bequeathed; i.e. $u_t \leq F_t(k_t, k_{t+1})$ has to hold for any feasible *utility-bequest pair* (u_t, k_{t+1}) of generation t. Furthermore, it is assumed that the utility level of each generation cannot fall below a certain lower bound $\underline{u}$. This lower bound serves two purposes. First, $\underline{u}$ can be interpreted as the subsistence level of any generation. Moreover, since there are technological limitations on the accumulation of stocks in the course of one period, $F_t(k_t, k_{t+1}) < \underline{u}$ can be used to capture that the bequest k_{t+1} is infeasible given the inheritance k_t. Hence, generation t's utility-bequest pair (u_t, k_{t+1}) is said to be *feasible* at t given k_t if $\underline{u} < u_t \leq F_t(k_t, k_{t+1})$. Hence, the sequence ${}_1\mathbf{F} = (F_1, F_2, ...)$ characterizes the technology of the economy under consideration. Given the technology ${}_1\mathbf{F}$, a utility path ${}_1\mathbf{u} = (u_1, u_2, \dots)$ is feasible at t given k_t if there exists a path

${}_{t+1}\mathbf{k} = (k_{t+1}, k_{t+2}, \dots)$ such that, for all $s \geq t$, generation s's utility bequest pair (u_s, k_{s+1}) is feasible at s given k_s.

A utility path ${}_1\mathbf{v}$ weakly Pareto-dominates another utility path ${}_1\mathbf{u}$ if every generation is weakly better of in ${}_1\mathbf{v}$ than in ${}_1\mathbf{u}$ and some generation is strictly better off. A utility path ${}_1\mathbf{v}$ is said to be efficient if there is no other feasible utility path that weakly Pareto-dominates this path. A feasible utility path ${}_1\mathbf{v}$ is said to be *R-maximal*, if there exists no feasible path ${}_1\mathbf{u}$ such that ${}_1\mathbf{u}\, P\, {}_1\mathbf{v}$. A feasible utility path ${}_1\mathbf{v}$ is said to be *R-optimal*, if ${}_1\mathbf{v}\, R\, {}_1\mathbf{u}$ for any feasible path ${}_1\mathbf{u}$. Any R-optimal path is R-maximal, while the converse need not hold if R is incomplete.

Within this framework, the justification for sustainability in Asheim, Buchholz, and Tungodden [2] rests on one technological assumption and two conditions on the social preferences. First, the following domain restriction is imposed on the technological framework.

Assumption 1 (Immediate Productivity of ${}_1\mathbf{F}$)**.** If ${}_t\mathbf{u} = (u_t, u_{t+1}, \dots)$ is feasible at t given k_t with $u_t > u_{t+1}$, then $(u_{t+1}, u_t, u_{t+2}, \dots)$ is feasible and inefficient at t given k_t.

Secondly, the following two conditions are imposed on the social preferences.

Condition SP (Strong Pareto)**.** For any ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in \mathbb{R}^\infty$, if $v_t \geq u_t$ for all t and $v_s > u_s$ for some s, then ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

Condition WA (Weak Anonymity)**.** For any ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in \mathbb{R}^\infty$, if for some finite permutation π, $v_{\pi(t)} = u_t$ for all t, then ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.[2]

Conditions SP and WA generate the *Suppes-Sen grading principle* R^{S}. The binary relation R^{S} deems two paths to be indifferent if one is obtained from the other through a finite permutation, and one utility path to be preferred to another if a finite permutation of the former weakly Pareto-dominates the other. The Suppes-Sen grading principle R^{S} is a subrelation[3] to the social preferences R if and only if R satisfies SP and WA.

Define sustainability in the following standard way.

[2] A permutation, i.e. a bijective mapping of $\{1, 2, \dots\}$ onto itself, is said to be finite whenever there is a T such that $\pi(t) = t$ for any $t > T$.

[3] Say that R' is a *subrelation* to R'' if (i) ${}_1\mathbf{v}\, I'\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, I''\, {}_1\mathbf{u}$ and (ii) ${}_1\mathbf{v}\, P'\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, P''\, {}_1\mathbf{u}$, with I' and I'' and P' and P'' denoting the symmetric and asymmetric parts of R' and R'', respectively.

Definition 1 (Sustainability)**.** Generation t with inheritance k, is *said to behave in a sustainable manner* if it chooses a feasible utility-bequest pair (u_t, k_{t+1}) so that the constant utility path $(u_t, u_t, \dots)$ is feasible at $t+1$ given k_{t+1}. The utility *path* ${}_1\mathbf{u} = (u_1, u_2, \dots)$ is called *sustainable* given k_1 if there exists ${}_2\mathbf{k} = (k_2, k_3, ...)$ such that every generation behaves in a sustainable manner along $({}_1\mathbf{k}, {}_1\mathbf{u}) = (k_1, (u_1, k_2), (u_2, k_3), ...)$.

We can now state our justification for sustainability.

Proposition 1 (Asheim, Buchholz and Tungodden [2])**.** *If the social preferences R satisfy Strong Pareto and Weak Anonymity, and the technology satisfies immediate productivity, then only sustainable utility paths are R-maximal.*

Proof. See Asheim, Buchholz, and Tungodden [2]. □

3. Intergenerational Impartiality

It has been argued that in order to capture the whole idea of impartiality in an intergenerational setting, we have to deem any two utility paths as equally good if they can be derived from each other by a permutation of the utilities. The problem is that by making a permutation of the utilities of an infinite number of generations, we may end up in a direct conflict with Strong Pareto. By way of illustration, ${}_1\mathbf{u} = (1, 1, 0, 1, 0, 1, \dots)$ can be attained from ${}_1\mathbf{v} = (0, 1, 0, 1, 0, 1, \dots)$ by a permutation where generation 1 gets the utility of generation 2, generation t the utility of generation $t+2$ when t is an even number, and generation $t+2$ the utility of generation t when t is an uneven number. On the basis of this fact, most economists have chosen to adopt a weaker version of impartiality, which only endorses indifference when one of the utility paths can be derived from the other by a *finite* number of permutations. And it is this weak version of impartiality that is captured by Weak Anonymity.

This move, however, does not seem to be entirely satisfactory, because as pointed out by Liedekerke and Lauwers [7, p. 165] (in the present notation): "Weak [Anonymity] only affects T-head of the utility stream and not its T-tail for large enough T. But the T-tail of an infinite utility stream is infinitely larger than its T-head, therefore the [W]eak [Anonymity] condition only guarantees impartiality for a (negligibly) small part of the utility stream".

And Liedekerke and Lauwers suggest that we should adopt a framework that incorporates a trade-off between Weak Anonymity and Strong Pareto.

Weak Anonymity can certainly be criticized along the lines suggested by Liedekerke and Lauwers. However, we will argue that it is not necessary to weaken Strong Pareto in order to capture the idea of impartiality in a stronger manner in an infinite setting. As we will show, a stronger operationalization of impartiality can be attained by combining Weak Anonymity with the following condition.

Condition SIC (Strong Indifference Continuity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}$, then ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.

Strong Indifference Continuity means that if at *any* point in time *into infinity*, there is a future point in time at which the head of a utility path ${}_1\mathbf{v}$ is considered indifferent to the head of a utility path ${}_1\mathbf{u}$, then utility path ${}_1\mathbf{v}$ should be considered indifferent to the utility path ${}_1\mathbf{u}$.

If we combine Strong Indifference Continuity and Weak Anonymity, then we are able to express impartiality in a large number of cases covering permutations of an infinite number of generations. As an illustration, compare ${}_1\mathbf{v} = (0, 1, 0, 1, 0, 1, \dots)$ and ${}_1\mathbf{w} = (1, 0, 1, 0, 1, 0, \dots)$, where ${}_1\mathbf{w}$ can be attained from ${}_1\mathbf{v}$ by a permutation of the utilities of generation 1 and 2, 3 and 4, 5 and 6, and so on. In this case, as argued by Liedekerke and Lauwers [7, p. 162], equal treatment of generations should imply indifference between ${}_1\mathbf{v}$ and ${}_1\mathbf{w}$. This cannot, however, be captured by Weak Anonymity alone. However, if we combine WA and SIC, then we attain the wanted conclusion.

To see this formally, let us introduce the following symmetric relation.

Definition 2 (Strong Infinite Impartiality (SII))**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, the relation ${}_1\mathbf{v}\, R_{\mathrm{S}}^{\mathrm{M}}\, {}_1\mathbf{u}$ holds if, for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$.

Since $R_{\mathrm{S}}^{\mathrm{M}}$ is symmetric, we have that ${}_1\mathbf{u}\, R_{\mathrm{S}}^{\mathrm{M}}\, {}_1\mathbf{v}$, and thus ${}_1\mathbf{v}\, I_{\mathrm{S}}^{\mathrm{M}}\, {}_1\mathbf{u}$, whenever ${}_1\mathbf{v}\, R_{\mathrm{S}}^{\mathrm{M}}\, {}_1\mathbf{u}$. Moreover, the following proposition states that SII is generated by Weak Anonymity and Strong Indifference Continuity.

Proposition 2. *The relation $R_{\mathrm{S}}^{\mathrm{M}}$ satisfies Weak Anonymity and Strong Indifference Continuity, and $R_{\mathrm{S}}^{\mathrm{M}}$ is a subrelation to any R satisfying Weak Anonymity and Strong Indifference Continuity.*

Proof. Trivial. □

Certainly, $R_{\mathrm{S}}^{\mathrm{M}}$ does not cover all permutations involving an infinite number of generations. By way of illustration, this framework does not imply indifference between ${}_1\mathbf{u} = (1, 1, 0, 1, 0, 1, \dots)$ and ${}_1\mathbf{v} = (0, 1, 0, 1, 0, 1, \dots)$. But is that a problem? There is a good reason for preferring ${}_1\mathbf{u}$ to ${}_1\mathbf{v}$, as expressed by SP. Moreover, notice that for any permutation of an infinite number of generations that is not captured by $R_{\mathrm{S}}^{\mathrm{M}}$, we have that beyond a certain point in time, the histories of the two utility paths are *never* considered to be equivalent according to WA. Hence, in these cases, there exists a good reason for not considering the two utility paths equivalent, and thus we will argue that $R_{\mathrm{S}}^{\mathrm{M}}$ captures the claim of impartiality in an appropriate manner in an intergenerational setting. It follows straightforwardly that the justification for sustainability reported in Proposition 1 holds and is not vacuous if SIC is added in addition to SP and WA, because the imposition of SIC strengthens WA and does not contradict SP.

Before we move on to characterizing some prominent distributive rules in an infinite setting, we should like to comment on a slightly weaker version of SIC that has been implicit in much work on intergenerational justice.

Condition WIC (Weak Indifference Continuity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}$, then ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.

WIC states that an infinite utility path should be considered indifferent to another infinite utility path if the head of the former is considered indifferent to the latter *at every point in time* beyond a certain initial phase.

It is straightforward to see that WIC and WA generate the following symmetric relation.

Definition 3 (Weak Infinite Impartiality (WII))**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, the relation ${}_1\mathbf{u}\, R_{\mathrm{W}}^{\mathrm{M}}\, {}_1\mathbf{v}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, ${}_1\tilde{\mathbf{u}}_T = {}_1\tilde{\mathbf{v}}_T$.

Proposition 3. *The relation $R_{\mathrm{W}}^{\mathrm{M}}$ satisfies Weak Anonymity and Weak Indifference Continuity, and $R_{\mathrm{W}}^{\mathrm{M}}$ is a subrelation to any R satisfying Weak Anonymity and Weak Indifference Continuity.*

Proof. Trivial. □

However, if our aim is to extend impartiality to cases involving an infinite number of generations, nothing is gained by combining Weak Anonymity with Weak Indifference Continuity.

Proposition 4. *For any* ${}_1\mathbf{v}$ *and* ${}_1\mathbf{u}$, ${}_1\mathbf{v}\, I^{\mathrm{M}}_{\mathrm{W}}\, {}_1\mathbf{u}$ *if and only if for some* $T \geq 1$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$ *and* ${}_{T+1}\mathbf{v} = {}_{T+1}\mathbf{u}$.

Proof. The if-part of the proposition is trivial, and thus we will only prove the only-if part. Suppose ${}_1\mathbf{v}\, I^{\mathrm{M}}_{\mathrm{W}}\, {}_1\mathbf{u}$. In that case, there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, ${}_1\tilde{\mathbf{u}}_T = {}_1\tilde{\mathbf{v}}_T$ and ${}_1\tilde{\mathbf{u}}_{T+1} = {}_1\tilde{\mathbf{v}}_{T+1}$. Hence, $v_{T+1} = u_{T+1}$, and thus ${}_{\hat{T}+1}\mathbf{v} = {}_{\hat{T}+1}\mathbf{u}$. The result follows. □

Proposition 4 shows that if Weak Anonymity and Weak Indifference Continuity deem two paths indifferent when one is derived from the other through an infinite permutation, then the permutation has no consequence for the levels of utilities that are assigned to generations beyond some finite point in time. Hence, these cases are already covered by Weak Anonymity.

4. Characterizations

Strong Indifference Continuity and Weak Anonymity define a framework that provides both a reasonable account of impartiality and is compatible with the Suppes-Sen grading principle. And within this framework, following Asheim, Buchholz, and Tungodden [2], we can justify sustainability. However, there exists the further question about how to resolve distributional conflicts between generations going beyond the sustainability question, and we now turn to this issue.

From a technical viewpoint, Strong Indifference Continuity establishes a nice link to the standard finite setting of distributive justice. Within the framework of SIC, the comparison of any two infinite utility paths amounts to comparing an infinite number of utility paths each containing a *finite* number of generations, and this is what makes it possible to apply Weak Anonymity in an infinite setting. Similarly, we may apply well-known equity conditions and arguments from the traditional literature on distributive justice more generally if we introduce conditions similar to SIC for the asymmetric part of the social preferences. In this respect, there are two options.

Condition SPC (Strong Preference Continuity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists some $\hat{T} \geq 1$ such

that for all $T \geq \hat{T}$, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, R\, {}_1\mathbf{u}$, and for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$, then ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

Condition WPC (Weak Preference Continuity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$, then ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

There is a problem with SPC, however. It cannot be combined with SIC and the Suppes-Sen grading principle, as the following result shows.

Proposition 5. *There does not exist any R satisfying Weak Anonymity, Strong Pareto, Strong Indifference Continuity, and Strong Preference Continuity.*

Proof. An example suffices for showing that WA, SP, SIC and SPC are incompatible. Consider ${}_1\mathbf{u} = (0, 1, 0, 1, 0, \dots)$ and ${}_1\mathbf{v} = (1, 0, 1, 0, 1, \dots)$. Suppose that R satisfies the conditions of the proposition. By WA and SPC, ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$. However, by WA, SP, and SPC, ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$. The result follows. □

Hence, we will concentrate our attention on Weak Preference Continuity in combination with Strong Indifference Continuity, and in the following illustrate how this framework can be used to characterize the intergenerational version of the Rawlsian leximin principle and the overtaking principle.

Let us now turn to a discussion of the Rawlsian leximin principle. It has been stated as follows in the infinite case (see e.g. Asheim [1, p. 2]), where "WS" indicates that $R^{\mathrm{L}}_{\mathrm{WS}}$ will be shown to correspond to WIC and SPC.

Definition 4 (WS-Leximin)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, the WS-Leximin relation ${}_1\mathbf{v}\, R^{\mathrm{L}}_{\mathrm{WS}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, either ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$ or there is a $s \in \{1, \dots, T\}$ with $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$.

This definition of leximin is not in line with intergenerational impartiality as defined in Section 2, though it satisfies Weak Anonymity. It introduces a questionable property of impatience in the social preferences, which is easily seen by comparing ${}_1\mathbf{u} = (0, 1, 0, 1, 0, 1, \dots)$ and ${}_1\mathbf{v} = (1, 0, 1, 0, 1, 0, \dots)$. In this case, ${}_1\mathbf{v}\, I^{\mathrm{M}}_{\mathrm{S}}\, {}_1\mathbf{u}$, whereas ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{WS}}\, {}_1\mathbf{u}$. Nevertheless, as a standard definition of the Rawlsian leximin principle in an intergenerational context, it is of interest, and we will return to a characterisation of this relation shortly.

In order to have a formulation of leximin in line with $R^{\mathrm{M}}_{\mathrm{S}}$, we need to make the following modification.

Definition 5 (SW-Leximin)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{L}}_{\mathrm{SW}}\, {}_1\mathbf{u}$ holds if, for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$. Moreover, ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{SW}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, there is a $s \in \{1, ..., T\}$ with $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$.

For completeness, we also give a formulation of leximin that is a subrelation to both WS-Leximin and SW-leximin.

Definition 6 (WW-Leximin)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{L}}_{\mathrm{WW}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$. Moreover, ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{WW}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, there is a $s \in \{1, ..., T\}$ with $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$.

Let us start out by characterizing $R^{\mathrm{L}}_{\mathrm{SW}}$. It is well-known that the leximin principle covering finite cases can be characterized by the Suppes-Sen grading principle and the equity condition suggested by Hammond [4, 5].

Condition HE (Hammond Equity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists j, k such that $u_j > v_j > v_k > u_k$ and $u_i = v_i$ for all $i \neq j, k$, then ${}_1\mathbf{v}\, R\, {}_1\mathbf{u}$.

However, in general it is not straightforward to translate this result into the infinite case.[4] However, by combining SIC and WPC, we obtain the following characterization.

Proposition 6. *$R^{\mathrm{L}}_{\mathrm{SW}}$ is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, Strong Indifference Continuity, and Weak Preference Continuity.*

Proof. (If) Assume that R satisfies SP, WA, HE, SIC, and WPC. According to the definition of a subrelation (cf. footnote 3), we have to show that, for any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{L}}_{\mathrm{SW}}\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$, and ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{SW}}\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$. This naturally divides the if-part of the proof into two subparts.

(1) Consider any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$ such that ${}_1\mathbf{v}\, I^{\mathrm{L}}_{\mathrm{SW}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{L}}_{\mathrm{SW}}$, for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$. By WA and SIC it follows that ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.

(2) Consider any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$ such that ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{SW}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{L}}_{\mathrm{SW}}$, there exists a $\hat{T} \geq 1$ such that for any $T \geq \hat{T}$, there is a $s \in \{1, ..., T\}$ with $\tilde{v}_t = \tilde{u}_t$

[4] Lauwers [6] characterizes the maximin relation by a version of Hammond Equity, within a framework where Strong Pareto is relaxed.

for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$. I.e. for any such $T \geq \hat{T}$, the worst off in ${}_1\tilde{\mathbf{u}}_T$ is worse off than the worst off in ${}_1\tilde{\mathbf{v}}_T$. For given $T \geq \hat{T}$, we can now establish a sequence of steps involving conflicts between two generations. The first step involves a conflict between the worst off and the second worst off, the second step a conflict between the worst off and the third worst off, and so on until the conflict between the worst off and the best off is included in the sequence. In each step, let the the worst off generation in ${}_1\tilde{\mathbf{u}}_T$ gain less than $1/T$ of the difference between the utility level this generation attains in ${}_1\tilde{\mathbf{u}}_T$ and ${}_1\tilde{\mathbf{v}}_T$ and the better off generation attain the same as the minimum of what this generation gets in ${}_1\tilde{\mathbf{u}}_T$ and ${}_1\tilde{\mathbf{v}}_T$. Let ${}_1\hat{\mathbf{u}}_T$ be the path established by this sequence. By HE (and SP in cases where the better off generation does not have higher utility in ${}_1\tilde{\mathbf{u}}_T$), each such step is at least a weak improvement according to R. Hence, by transitivity we have that

$$({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\, R\, ({}_1\tilde{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\,.$$

Since R satisfies SP, it follows that

$$({}_1\tilde{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u})\, P\, ({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\,,$$

while WA implies that

$$({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, ({}_1\tilde{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u}) \;\text{ and }\; ({}_1\tilde{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}\,.$$

Hence, by transitivity, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$. Since this holds for all $T \geq \hat{T}$ and R satisfies WPC, it now follows that ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

(Only if) Assume that $R^{\mathrm{L}}_{\mathrm{SW}}$ is a subrelation to R. Then it is trivial to establish that R satisfies SP, WA, and HE. To show that R satisfies SIC, assume that for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}$. Since $R^{\mathrm{L}}_{\mathrm{SW}}$ is a subrelation to R and $R^{\mathrm{L}}_{\mathrm{SW}}$ is complete for comparisons between paths having the same tail, we have that for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I^{\mathrm{L}}_{\mathrm{SW}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{L}}_{\mathrm{SW}}$, this entails that ${}_1\mathbf{v}\, I^{\mathrm{L}}_{\mathrm{SW}}\, {}_1\mathbf{u}$, which in turn implies ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$ since $R^{\mathrm{L}}_{\mathrm{SW}}$ is a subrelation to R. Hence, ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$ holds if for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}$. Thus, we have established that R satisfies SIC. A similar argument shows that R satisfies WPC. □

This result deals with an immediate objection to the Rawlsian leximin position, to wit that the leximin principle is implausible because it assigns *absolute* priority to the interests of the *worst off* generation in cases where it is in conflict with the interest of an *infinite number of future generations*.

Intuitively, this seems a reasonable counterargument. However, the proposition shows that this objection does not approach the problem head on, at least not if we accept the framework of a reflexive and transitive binary relation as the basis of normative evaluation of intergenerational justice. In that case, Proposition 6 tells us that our view on intergenerational justice in general can be determined by considering a particular set of two-generation conflicts. If we agree on assigning absolute priority to the worse off in such a conflict, then we have to assign absolute priority to the worse off in general. Hence, our result provides a defense for the leximin principle in the infinite setting since it seems less difficult to accept the two-generation claim.

Equivalent results can be established for $R^{\mathrm{L}}_{\mathrm{WW}}$ and $R^{\mathrm{L}}_{\mathrm{WS}}$.

Proposition 7. *$R^{\mathrm{L}}_{\mathrm{WW}}$ is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Weak Preference Continuity.*

Proof. By Proposition 4, WA implies that WIC holds. Therefore, the result follows by trivial modification of the parts of the proof of Proposition 6 that involves SIC. □

Proposition 8. *$R^{\mathrm{L}}_{\mathrm{WS}}$ is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Strong Preference Continuity.*

Proof. The result follows by also modifying the parts of the proof of Proposition 6 that involves WPC. □

We must impose an assumption on the technological framework in order to ensure that there exists a maximal path according to $R^{\mathrm{L}}_{\mathrm{SW}}$ and $R^{\mathrm{L}}_{\mathrm{WS}}$ (and thus $R^{\mathrm{L}}_{\mathrm{WW}}$). In this respect, the following domain restriction is of particular interest.

Assumption 2 (Eventual Productivity of ${}_1\mathbf{F}$)**.** For any t and k_t, there exists a feasible and efficient path with constant utility.

Eventual productivity is fulfilled for technologies usually considered in the context of sustainability (see Asheim, Buchholz, and Tungodden [2]). If we take this restriction into account, we can provide a complete justification for an egalitarian approach to intergenerational justice.

Proposition 9. *If the technology satisfies Eventual Productivity, and R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Weak Preference Continuity, then the feasible and efficient path with constant utility is the unique R-optimal path.*

Proof. Eventual productivity guarantees the existence of an efficient utility path ${}_1\mathbf{v}$ with constant utility. Since any alternative feasible path ${}_1\mathbf{u}$ provides at least one generation with lower utility than this constant level, it follows that ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{WW}}\, {}_1\mathbf{u}$. Hence, ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{WW}}\, {}_1\mathbf{u}$ for any other feasible path. It follows from Proposition 7 that $R^{\mathrm{L}}_{\mathrm{WW}}$ is a subrelation to R. Therefore, ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$ for any other feasible path. Hence, ${}_1\mathbf{v}$ is R-optimal, and no other feasible path is R-optimal. □

It follows that the feasible and efficient path with constant utility is preferred to any other feasible path according to any binary relation to which $R^{\mathrm{L}}_{\mathrm{WW}}$ is a subrelation; in particular, this holds for $R^{\mathrm{L}}_{\mathrm{SW}}$ and $R^{\mathrm{L}}_{\mathrm{WS}}$. Hence, the egalitarian path is the unique optimal path also under the two stronger versions of leximin.

The overtaking criterion, introduced by von Weiszäcker [8], represents an important alternative approach to intergenerational justice.[5] As for leximin, there are three versions to consider.

Definition 7 (Catching Up)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, the Catching Up relation ${}_1\mathbf{v}\, R^{\mathrm{U}}_{\mathrm{WS}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t \geq \sum_{t=1}^{T} u_t$.

Definition 8 (Strong Overtaking)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{U}}_{\mathrm{SW}}\, {}_1\mathbf{u}$ holds if, for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $\sum_{t=1}^{T} v_t = \sum_{t=1}^{T} u_t$. Moreover, ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{SW}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t > \sum_{t=1}^{T} u_t$.

Definition 9 (Weak Overtaking)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{U}}_{\mathrm{WW}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t = \sum_{t=1}^{T} u_t$. Moreover, ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{WW}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t > \sum_{t=1}^{T} u_t$.

It turns out that von Weiszäcker [8, p. 85] defines

[5] To analyze "overtaking" and "catching up", we must consider utilities that are also cardinally measurable and unit comparable.

- preference by overtaking (i.e. ${}_1\mathbf{v}$ is preferred to ${}_1\mathbf{u}$ if ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{WW}}\, {}_1\mathbf{u}$, or equivalently, ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{SW}}\, {}_1\mathbf{u}$), and
- optimality by catching up (i.e. ${}_1\mathbf{v}$ is optimal if ${}_1\mathbf{v}\, R^{\mathrm{U}}_{\mathrm{WS}}\, {}_1\mathbf{u}$ for any feasible path ${}_1\mathbf{u}$).

Also in the case of these utilitarian criteria, we may appeal to a set of two-generation conflicts in order to provide characterization.[6]

Condition 2U (2-Person Utilitarianism)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists j, k such that $0 < u_j - v_j \leq v_k - u_k$ and $u_i = v_i$ for all $i \neq j, k$, then ${}_1\mathbf{v}\, R\, {}_1\mathbf{u}$.

By introducing 2-Person Utilitarianism, we overcome an immediate objection to the catching up and overtaking criteria, namely that these criteria allow a large number of smaller gains for many generations to outweigh a greater loss for a single generation. The following results show that this is only a consequence of wanting to follow in two-generation conflicts the interests of the generation that experiences the greater loss.

Proposition 10. *$R^{\mathrm{U}}_{\mathrm{SW}}$ is a subrelation to R if and only if R satisfies Strong Pareto, 2-Person Utilitarianism, Strong Indifference Continuity, and Weak Preference Continuity.*

Proof. (If) Assume that R satisfies SP, 2U, SIC, and WPC. According to the definition of a subrelation (cf. footnote 3), we have to show that, for any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{U}}_{\mathrm{SW}}\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$, and ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{SW}}\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$. This naturally divides the if-part of the proof into two subparts.

(1) Consider any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$ such that ${}_1\mathbf{v}\, I^{\mathrm{U}}_{\mathrm{SW}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{U}}_{\mathrm{SW}}$, for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $\sum_{t=1}^{T} v_t = \sum_{t=1}^{T} u_t$. For this T we can now establish, for each of ${}_1\mathbf{v}_T$ and ${}_1\mathbf{u}_T$, a sequence of steps involving conflicts between two generations. In each step, let the gain for the worse off generation be equal the loss for the better off generation. This procedure leads to two egalitarian T-heads, ${}_1\hat{\mathbf{v}}_T$ and ${}_1\hat{\mathbf{u}}_T$, where the constant utility of the former equals that of the latter. By 2U, each such step is indifferent according to R. Hence, by transitivity, we have that

$$({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, ({}_1\hat{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u}) \;\text{ and }\; ({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}\,.$$

Since ${}_1\hat{\mathbf{v}}_T = {}_1\hat{\mathbf{u}}_T$, it follows by transitivity that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}$. Since R satisfies SIC, we have that ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.

[6] For a discussion of the related result in the finite setting, see d'Aspremont [3].

(2) Consider any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$ such that ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{SW}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{U}}_{\mathrm{SW}}$, there exists a $\hat{T} \geq 1$ such that for any $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t > \sum_{t=1}^{T} u_t$. For given $T \geq \hat{T}$, we can now establish, for each of ${}_1\mathbf{v}_T$ and ${}_1\mathbf{u}_T$, a sequence of steps involving conflicts between two generations. In each step, let the gain for the worse off generation be equal to the loss for the better off generation. This procedure leads to two egalitarian T-heads, ${}_1\hat{\mathbf{v}}_T$ and ${}_1\hat{\mathbf{u}}_T$, where the constant utility of the former is higher than that of the latter. By 2U, each such step is indifferent according to R. Hence, by transitivity, we have that

$$({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, ({}_1\hat{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u}) \;\text{ and }\; ({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}\,.$$

Since R satisfies SP, it follows that

$$({}_1\hat{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u})\, P\, ({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\,.$$

Hence, by transitivity, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$. Since this holds for all $T \geq \hat{T}$ and R satisfies WPC, it now follows that ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

(Only if) Assume that $R^{\mathrm{U}}_{\mathrm{SW}}$ is a subrelation to R. Then it is trivial to establish that R satisfies SP and 2U. Arguments similar to those used in the only-if-part of the proof of Proposition 6 to show that R satisfies SIC when $R^{\mathrm{L}}_{\mathrm{SW}}$ is a subrelation to R, can be used here to establish that R satisfies SIC and WPC. □

Proposition 11. *$R^{\mathrm{U}}_{\mathrm{WW}}$ is a subrelation to R if and only if R satisfies Strong Pareto, 2-Person Utilitarianism, and Weak Preference Continuity.*

Proof. Note that 2U implies that WA holds. By Proposition 4, this in turn implies that WIC holds. Therefore, the result follows by trivial modification of the parts of the proof of Proposition 10 that involves SIC. □

Proposition 12. *$R^{\mathrm{U}}_{\mathrm{WS}}$ is a subrelation to R if and only if R satisfies Strong Pareto, 2-Person Utilitarianism, and Strong Preference Continuity.*

Proof. The result follows by also modifying the parts of the proof of Proposition 10 that involves WPC. □

Since 2U entails WA, any social preferences to which one of these utilitarian criteria is a subrelation, satisfy the Suppes-Sen grading principle.

It is more difficult to establish conditions that guarantee that there exists an optimal (or maximal) path according to the catching up and overtaking criteria, and we leave such a task for another occasion.

5. Conclusion

Liedekerke and Lauwers [7, p. 173] argue that "in an infinite world [S]trong Pareto is an excessive demand. Loosely speaking, it demands from us that, looking upon an infinite crowd, we still see the face of every single [generation] – a difficult task indeed." We believe that this is not so. By introducing a new condition, Strong Indifference Continuity, we have shown how Strong Pareto can be combined with an idea of impartiality that goes beyond Weak Anonymity in an infinite setting. Moreover, within this framework, it is possible to apply interesting equity conditions well-known from the finite setting in order to resolve distributional conflicts between an infinite number of generations. We have applied our analysis to different versions of leximin and utilitarianism, but there are many other possibilities worthy of exploration.

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