Micro Evidence on
Household Energy Consumption

Version of 11 February 2002

Søren Leth-Petersen
AKF
Nyropsgade 37
1602 Copenhagen V - Denmark
Telephone: +45 3311 0300
E-mail: slp@akf.dk

JEL codes: C2, D12, Q41

Keywords: Household energy demand, micro-econometric analysis, nonparametric analysis, separability,

Abstract

The paper contains a micro econometric analysis of household electricity and natural gas demand for a cross section of 2,885 Danish households observed in 1996. The sample includes couples living in single-family dwellings, working full time and having up to two children. The specification of the model is guided by an explorative non- and semiparametric data analysis. Estimates of the final model suggest that electricity consumption depends on the number of children, and depends linearly on total household budget. On the other hand, the consumption of natural gas is found to be independent of the number of children in the household, but to vary with the age of the adults, and to depend linearly on total household budget. Further, demand for natural gas is found to depend on the technical characteristics of the dwelling. Particularly, it is found that the consumption of energy for space heating depends heavily on the building codes in force at the time of the construction of the house.

The dependence between gas demand and demand for electricity is examined in the paper. This is done by testing for separability of demand for gas from demand for electricity, and vice versa. Separability of electricity (gas) from gas (electricity) is tested by estimating demand for electricity (gas) conditional on demand for gas (electricity). Endogeneity of the conditioning variable is allowed for. Exploiting institutional restrictions on the one hand and a minimum of individual time variation, i.e. panel data, on the other, identify the test. The test indicates that demand for electricity is separable from demand for natural gas, and that demand for natural gas is separable from demand for electricity. The result of the test is evidence in favour of single equation modelling of household energy demand.

1. Introduction

Studying household energy demand is important because it constitutes a big part of total energy consumption, approximately 30% of the total final consumption of energy in the Danish economy. The household sector has been submitted to extensive policies designed to reduce energy consumption. As we are all part of the household sector, and because expenditures for energy usually make up an important share of the total household budget[1], it is important to understand what determines household energy consumption in order to understand how policies affect us.

The objective of this study is to estimate demand-relations for natural gas and electricity. When a house is connected to the natural gas grid, gas is used mainly for heating and hot water. Electricity is subsequently used as input to domestic appliances. This is, in part, a result of regulatory measures introduced by the Danish Government.

The household sector in Denmark is submitted to quite extensive regulation on the energy markets. For example, quite restrictive building regulations have been introduced in order to reduce consumption of energy for heating purposes. Building regulations put restrictions on the heat loss of the building shell. Building regulations important for the consumption of energy in single-family houses were introduced in 1977, and tightened further in 1982, and 1985. The first building regulation nearly halved the allowed heat loss of the building. The next left the nominal

---

[1] Energy expenditures make up 6% of the total household budget on average for the whole population, cf. Danmarks Statistik (1998).

heat loss unchanged, but changed the method of calculation of the heat loss. The latter restricted the heat loss allowance even further. Also, other regulative actions have been taken. Huge resources have been allocated into developing collective heating systems by developing district heating, and natural gas distribution grids. The planning of these distribution grids has been made, so that houses located in areas with access to the natural gas grid do not have access to the district heating grid and vice versa. Following the development of these grids, installation of electric heating as the primary heating system in new buildings has been banned since 1988, and from 1994 also in existing buildings in order to promote the use of natural gas and district heating. Supplementary electric heating is not banned, but extensive taxes on electricity have been introduced. In 1996 the share of taxes out of the retail price on electricity was 65%. The purpose of this regulation is that electricity is not used as energy carrier for heating purposes in houses located in areas covered by the district heating or natural gas grid. In 1996, 17% of all single-family houses in Denmark used natural gas, and 35% used district heating as primary energy carrier. In the same year 33% of all single-family houses was furnished with a central heating system based on oil as the primary heating system. For single-family houses using natural gas, district heating or oil, the practice is that only one primary heating system is present in the house. In this way about 85% of all single-family houses in Denmark are furnished with one primary central heating system using only one energy carrier, i.e. either natural gas, district heating or oil. For houses with these types of central heating systems the practice is further that hot tap water is supplied by the central heating system.

The focus of the paper is twofold. First, to get insight into the specification of the cross sectional relationships. Pinning out the specification will be guided by an explorative analysis using non- and semiparametric techniques. Secondly, to obtain knowledge on whether demand for natural gas is separable from demand for electricity and *vice versa*. The extensive planning and regulation of the heating supply would suggest that demand for natural gas for heating is not substituted by demand for electricity except if supplementary electric heating is present. This will be revealed by the separability test. The question of separability between energy carriers also has important implications for modelling household energy demand and for welfare evaluations of, for example, green tax reforms.

The modelling approach in this paper follows a rather extensive number of more specialised energy demand studies, cf. a recent survey by Madlener (1996). These studies are usually single equation studies, based on energy survey data. This type of study often includes an extended set of explanatory variables providing a richer description of the durable stock and other characteristics that are particularly important for the demand for energy. Examples are numerous, e.g. Barnes et al. (1982), Branch (1993), Baker & Blundell (1991), Baker et al. (1989), Dubin & Henson (1988), Dubin et al. (1986), Garbacz (1983, 1985), Green (1987), Green et al. (1986), Haas et al. (1998), Herriges & King (1994), Klein (1988), Morss & Small (1989), Parti & Parti (1980), Poyer & Williams (1993), and Reilly & Shankle (1988). Common to this type of study is that demand for energy is conditioned on goods that have a durable nature, namely the dwelling size, the insulation standard, the heating system and other characteristics of the house, and furthermore the stock of electric appliances, and the family composition. This can be interpreted as a short-run model, because it is reasonable to assume fixed technology only in the shorter run[2]. Demand for an energy carrier will also be modelled conditional on demand for other energy carriers. This approach allows

---

[2] A number of studies stress that the heating and housing technology is potentially simultaneously determined with consumption of energy, i.e. endogenous in the econometric terminology. Some studies have attempted to model this, e.g. Dubin and MacFadden (1984). Others consider the stock of energy-consuming durables as predetermined, and model only purchase of durables in the current period, cf. Halvorsen & Larsen (2001). In the present study the appliance stock and purchase are assumed fixed.

a robust testing procedure for separability of demand for one particular energy carrier from demand for other energy carriers.

The main contribution of the paper, relative to previous papers within this line of literature, lies in offering new insights into the specification of single equation household energy demand relations. For example, the study suggests that the relationship between age and consumption of energy for heating purposes is non-linear. Also, appropriate modelling of effects of building codes on energy consumption for heating purposes appears to be important. Finally, the dependence of demand for natural gas, used mainly for heating purposes, on demand for electricity, used mainly for domestic appliances, is addressed. This has not been done, so far, in the energy demand literature.

The plan of the paper is the following. In section 2 the sample is presented, and in section 3 the data are explored using non- and semiparametric regression analysis. In section 4 the econometric model, and the approach to testing for separability are addressed. In section 5 results are presented, and finally, in section 6 the paper is summarised and concluded.

2. Data

The sample analysed here contains observations of 2,885 households in 1996, and includes information about individual households' consumption of energy, specifically natural gas and electricity. The cross-section is drawn from a larger panel data set covering the period 1990-1997. This characteristic becomes crucial at a later stage for identifying an endogenous parameter. All the households use natural gas for space heating (central heating) and hot tap water. In some cases natural gas is also used for cooking instead of electricity, but this is not observed. As mentioned in the introduction, it is not expected that electricity is used for heating purposes, due to the institutional set up, but solely for domestic appliances. Further, when connected to the natural gas grid, the practice is that no other primary heating system is present in the house. However, it cannot be ruled out that supplementary electric heating exist. It should also be noted that some houses may also have a wood-burning stove, but this information is not recorded in the data set.

The sample is constructed by merging public administrative registers[3] with registers of customers from the electricity utility company, NESA A/S, and the gas utility company, HNG I/S, and thus covers the grid of these companies, i.e. an area surrounding the northern part of Copenhagen. The administrative registers provide information about type, size, and vintage of the house, family composition, age, and information about household income, wealth and tax payments.

The information about income, tax payments and wealth is used to construct a measure of total expenditure for the household. Total household expenditure, for the year we are interested in, is denoted c_t, where lower case t indicates the period of interest. To derive a measure of c_t start out with the following identity describing the total value of the stock of assets at the end of period t

$$\sum_m p_{mt}A_{mt} \equiv \sum_m p_{mt-1}A_{mt-1} + \sum_m i_{mt}A_{mt-1} + \sum_m (p_{mt} - p_{mt-1})A_{mt-1} + e_t - t_t - c_t \qquad (2.1.1)$$

---

[3] Rasmussen (1997) for a more complete description of this part of the sample.

where p_{mt} is the price of asset m at the end of period t and A_{mt} is the stock of asset m at the end of period t, and $\sum_m p_{mt}A_{mt}$ is the value of the total stock at the end of period t. This equal the value of the total stock at the end of the previous period , $\sum_m p_{mt-1}A_{mt-1}$, plus returns to this asset, $\sum_m i_{mt}A_{mt-1}$, where i_{mt} is the return, ie. interests or dividends, to asset m, plus capital gains, $\sum_m (p_{mt} - p_{mt-1})A_{mt-1}$, plus earnings, e_t, less taxes, t_t, and what is allocated for consumption, c_t.

Rearranging (2.1.1) yields

$$c_t \equiv \left(e_t + \sum_m i_{mt}A_{mt-1} - t_t\right) - \left(\sum_m p_{mt}A_{mt} - \sum_m p_{mt-1}A_{mt-1} - \sum_m (p_{mt} - p_{mt-1})A_{mt-1}\right) \qquad (2.1.2)$$

where the first set of brackets yields total disposable income, the second set of brackets yields the change in value of the total stock of assets less capital gains.

For practical implementation of (2.1.2) the stock of assets are divided in to two types of assets, housing assets and non-housing assets. For these two stocks of assets the change in the total value less capital gains is calculated. However, only the capital gain relating to the housing assets is taken in to account. This is because capital gains can not be identified for other types of assets.

The sample is delimited to include couples, both full-time employed[4], living in a single-family house, and having at most two children. No old-age pensioners or students are included in the sample. This rather tight delimitation is taken in order to work with a homogenous sample that prevents matters from being far too complicated when pinning out the specification and subsequently testing for separability.

The data set is constructed by considering the customers of only one energy supplier for each energy type in one year. Therefore, no price variation is present to estimate price responses.

3. Exploring the data

In the demand literature a number of recent studies have undertaken a preliminary data analysis applying non-parametric techniques in order to obtain additional information about some of the relationships present in the data, in particular the Engel-curve relationship. Examples of such studies are Banks et al. (1997), Blundell et al. (1998). This approach offers valuable insight to the specification of the final parametric model to be estimated.

The nonparametric technique applied is kernel regressions. In section 3.1, a description of the kernel regression technique will be presented. Emphasis will be purely on aspects relevant for application. After this, in section 3.2, results of the nonparametric analysis will be presented.

---

[4] Consumption may be traded off with leisure as emphasised by Browning and Meghir (1991). This trade off is not modelled here in order to focus on separability from other energy carriers. Further, the modelling of labour supply decisions would have been hampered by only a few observations being present in other labour supply categories.

3.1 Kernel regression

Nonparametric regression techniques provide a valuable tool for exploring relationships in data by not imposing any restrictions on the functional form of a relationship between two or more variables. In this way nonparametric regression analysis can be used to determine the function g(.) in (3.1.1)

$$y = g(x) + u \tag{3.1.1}$$

An estimate of g(.) can be obtained by applying the Nadaraya-Watson Kernel Estimator, cf. Härdle (1990). In the univariate case the estimator is

$$\hat{g}_b(x_k) = \frac{\sum_h y_h K_b(x_h - x_k)}{n\,\hat{f}(x_k)} \tag{3.1.2}$$

where lower case h indicates the observation number out of n observations, $K_b = b^{-1}k(x/b)$, and k(x) is some kernel function with bandwidth b evaluated at x_k, and $\hat{f}$ is an estimate of the kernel density given by (3.1.3)

$$\hat{f}(x_k) = \frac{1}{n}\sum_h K_b(x_h - x_k) \tag{3.1.3}$$

A number of choices for the kernel function exist, and the choice is not critical, cf. Härdle (1990). In all applications here, the gaussian kernel $k(x) = \left(1/\sqrt{2\pi}\right)\exp\left(-x^2/2\right)$ is applied.

The Nadaraya-Watson kernel regression estimator is calculating a local conditional mean, E(y|x), in an interval b centred at some point x_k. The value of this local conditional mean is, of course, heavily dependent on the size of the interval b. Choosing the size of the interval, called the bandwidth, is crucial in the application of the nonparametric regression techniques. An optimal choice of bandwidth exists in which the mean squared error (MSE) is minimised. The MSE can be approximated by

$$MSE\left[\hat{g}_b(x)\right] = E\left[\left(\hat{g}_b(x) - g(x)\right)^2\right] \approx \frac{1}{nb}V(x) + b^4 B^2(x) \tag{3.1.4}$$

where V is the variance component and B is the bias component. Minimising (3.1.4) yields an optimal bandwidth

$$b^{opt} = \left(\frac{V(x)}{4B^2(x)}\right)^{\frac{1}{5}} n^{-1/5} \tag{3.1.5}$$

Choosing b implies making a trade-off between precision and bias. The contribution to the MSE from V decreases as b increases, and the contribution from B increases as b increases.

Applying cross validation techniques is a way of providing a sample analogue to (3.1.4). A number of cross-validation functions exist. In the present context the generalised cross-validation criterion is applied. This minimises

$$CV(b) = \sum_{k=1}^{n} \left(y_k - \hat{g}_b(x_k)\right)^2 \Xi\left(W_{bk}(x_k)\right) w(x_k)$$

(3.1.6)

Where $\Xi(W_{bk}(x_k)) = (1 - W_{bk}(x_k))^{-2}$ is the Generalised Cross-Validation function, with $W_{bk}(x_k) = K(0) \Big/ \sum_{h=1}^{n} K\left(\frac{x_h - x_k}{h}\right)$, and w(x$_k$) is a weight function. The Ξ-function penalises bandwidths that are too small, and w(x$_k$) is introduced to reduce boundary effects.

To facilitate inference from the estimated nonparametric regression curve confidence bands can be placed around the regression curve. The asymptotic distribution of the kernel estimator is normal, and the mean of the distribution is a function of the bias that is a complicated function of the derivatives of the kernel regression estimate and the derivatives of the distribution of the explanatory variable. However, approximate point-wise confidence bands are suggested by Härdle (1990) who does not take into account the bias of the kernel regression estimate. These confidence intervals are given by

$$Conf(x_k) = \hat{g}_b(x_k) \pm \frac{c_\alpha c_K^{1/2} \hat{\sigma}(x_k)}{\left(nb\hat{f}(x_k)\right)^{1/2}}$$

where $\hat{\sigma}^2(x_k) = n^{-1} \sum_{h=1}^{n} \left(K_b(x_h - x_k) / \hat{f}(x_k)\right)\left(y_k - \hat{g}_b(x_k)\right)$, and $c_K = \int K^2(u)du$, where $u = (x_h - x_k)/b$, and c$_K$≈0.282 for the gaussian kernel, and c_α is the 1- α quantile of the normal distribution.

In this paper nonparametric regression analysis is applied in order to guide the specification of a parametric model. The nonparametric techniques can be used to supply an alternative to test against a parametric null model. One way of doing this is to nonparametrically regress the errors of the parametric model on explanatory variables in order to check if the errors of the parametric model are centred at zero for all levels of the explanatory variables. Placing confidence bands around the nonparametric regression curve can provide an indication of whether the regression curve is in fact centred at zero.

Another approach is to test somewhat more formally the parametric null model against the nonparametric alternative. Zheng (1996) suggests one such test statistic.

$$T^{ZHENG} = \frac{\sum_{k=1}^{n} \sum_{\substack{h=1 \\ h \neq k}}^{n} K\left(\frac{x_h - x_k}{b}\right) \hat{u}_h \hat{u}_k}{\left\{ \sum_{k=1}^{n} \sum_{\substack{h=1 \\ h \neq k}}^{n} 2K^2\left(\frac{x_h - x_k}{b}\right) \hat{u}_h \hat{u}_k \right\}} \quad \sim \quad N(0,1) \qquad (3.1.7)$$

where û is the error from the parametric model.

Here the formulas for the univariate kernel regression estimator are presented. In principle it is straightforward to expand to more regressors. The application of kernel regression techniques is computationally rather heavy, though, and the number of points at which to evaluate the conditional mean is given by p^d, where p is the number of gridpoints per explanatory variable and d is the number of variables. Therefore, only a very limited number of explanatory variables can be included in practice. This has obvious limitations since covariates certainly exist. This is to some extent addressed by first choosing a fairly homogenous sample and second, by stratifying based on some of the expected covariates. Another problem relates to potential stochastic regressors. The kernel regression assumes fixed regressors. For example in the case of regressing consumption of natural gas on total expenditures, total expenditures may be endogenous, since expenditures on natural gas are contained in total household expenditures. This can cause bias in the estimated function.

Blundell & Duncan (1998) suggest that the problem of endogeneous regressors can be adressed by applying a semiparametric instrumental variable technique, where Robinsons (1988) semiparametric estimator is applied in an augmented regression approach. If x is endogenous in (3.1.1), so that $\mathrm{E}[\mathrm{u}|\mathrm{x}] \neq 0$ and $\mathrm{E}[\mathrm{y}|\mathrm{x}] \neq \mathrm{g}(\mathrm{x})$, then the estimator will be inconsistent. However, if a variable z exists where $\mathrm{x} = \pi \mathrm{z} + \upsilon$ and $\mathrm{E}[\upsilon|\mathrm{z}] = 0$ then, the following augmented regression model holds instead of (3.1.1)

$$y = g(x) + \upsilon\rho + \varepsilon \tag{3.1.8}$$

where $\mathrm{E}[\varepsilon|\mathrm{x}] = 0$. Take expectations of (3.1.8) conditional on x, and subtract it

$$y - E[y|x] = (\upsilon - E[\upsilon|x])\rho + \varepsilon \tag{3.1.9}$$

Replace $\mathrm{E}[\mathrm{y}|\mathrm{x}]$ and $\mathrm{E}[\upsilon|\mathrm{x}]$ by their nonparametric estimators, $\hat{\mathrm{m}}_{\mathrm{b}}^{\mathrm{y}}(x)$ and $\hat{\mathrm{m}}_{\mathrm{b}}^{\upsilon}(x)$, and obtain an estimate of ρ by OLS. To do this, replace υ by the residual obtained in a regression of x on z, $\hat{\upsilon} = \mathrm{x} - \mathrm{z}\hat{\pi}$. Then a consistent estimator for g(x) is

$$\hat{g}_b(x) = \hat{m}_b^y(x) - \hat{m}_b^\upsilon(x)\hat{\rho} \tag{3.1.10}$$

In the next section these non- and semiparametric regression techniques are applied to explore the data. It is the intention that this analysis will guide the specification of the parametric model. The tools for making inference will also be applied when the final parametric model has been estimated in order to check for misspecification.

3.2 Applying kernel regressions to energy demand data

Three variables in the data set can be considered continuous and thereby appropriate for non-parametric regression analysis. These are total expenditures, size of the house and age of oldest person in the household. All non- and semiparametric regressions and test statistics are evaluated at generalised cross-validation level, cf. (3.1.6), unless something else is specifically stated.

Figure 3.2.1 presents kernel regressions of consumption of natural gas explained by total household expenditure. Total household expenditure is expected to be endogenous, and is instrumented by household gross income. The estimate is obtained by applying (3.1.10).

[figure 3.2.1. See end of paper]

Figure 3.2.1. gives clear indications that the consumption of both natural gas and electricity is linearly related to total expenditure.

[figure 3.2.2. See end of paper]

Conditional on the number of children linearity is still quite apparent. Figure 3.2.2, illustrates semi-parametric regressions of consumption of natural gas and electricity on total expenditures conditional on the number of children present in the household. Again, total expenditure is instrumented by household gross income, and the estimates are obtained by applying (3.1.10). figure 3.2.2. indicates that the functional form of the relation is not influenced by the number of children.

[figure 3.2.3 See end of paper]

The relation between consumption of energy and the size of the house is illustrated in figure 3.2.3. The nonparametric estimate is plotted together with a parametric quadratic polynomial approximation. The figure indicates that there appears to be a slight curvature in the relation between the size of the house and the consumption of energy for both gas and electricity. The curvature is not very pronounced, though. Testing for linearity using the Zheng-statistic, linearity is not rejected in either case: T^{ZHENG}(gas)=-0,43, T^{ZHENG}(el)=-0,01.

Conditional on the number of children present in the household the picture remains unchanged. The nonparametric regression of consumption of energy on the size of the house conditioned on the number of children is represented in figure 3.2.4.

[figure 3.2.4. See end of paper]

The figure indicates that the functional form is not affected by the presence of children in either the gas relation nor the electricity relation. Further, the level appears not to be influenced in the gas relation whereas some indication is given that a level effect is present in the electricity relation. A dip in consumption of electricity is indicated for lower range of house sizes for families with two children. This is a boundary effect in the nonparametric regression and does not reflect a true shift in the relation.

The structure between consumption of energy and the age of the oldest person in the household is explored in figure 3.2.5.

[figure 3.2.5. See end of paper]

In figure 3.2.5 the nonparametric regressions of consumption of gas and electricity on age of the oldest person in the household are illustrated together with a quadratic polynomial parametric approximation. The top window indicates that the relation between consumption of natural gas and age may be nonlinear. The pattern is not quite clear, though, and a linear approximation cannot be rejected by the zheng test, T^{ZHENG}(gas)=0.45. The relation between consumption of electricity and age appears to have a distinct inverted-U shape, and the Zheng-test for linearity is clearly rejected, T^{ZHENG}(el)=2.62. This shape is more apparent than real though. This becomes evident when the relation is pictured conditional on the number of children. This is done in figure 3.2.6

[figure 3.2.6. See end of paper]

In figure 3.2.6 the nonparametric regressions of consumption of gas and electricity on age conditional on the number of children present in the household are graphed. The graphs indicate that the number of children does not seem to influence the shape of the gas-age relation. In the electricity-age relation the number of children, however, appears to influence the shape of the electricity-age relationship quite strongly. The figure gives indication that the inverted U-shape observed in figure 3.2.5 disappears when conditioning on the number of children, and age effects may not be present at all when conditioning on the number of children.

The explorative analysis has provided an insight into some of the structures present in the data. It appears that total expenditure influences consumption of natural gas and electricity linearly. Some curvature is present in the relationship between the consumption of gas and electricity and the size of the house, but it is not very pronounced. The presence of children in the household is found to have no influence on the shape of the relation, but children may have an influence on the level of consumption of electricity. Age is suggested to influence consumption of natural gas in a, possibly, nonlinear fashion. No evidence was found, though, that consumption of electricity is influenced by age. Analysing the data by univariate kernel regressions has, as already mentioned, its obvious limitations. Therefore, the analysis should only be considered suggestive as to the specification of a more complete parametric model. This is the objective of the next section.

4. Parametric Model, Testing and Estimation

The purpose of this section is to estimate a well-specified parametric model of demand for electricity and gas for households based on the cross section of households, and to test if the demand for natural gas is separable from the demand for electricity and vice versa. Finding the right specification is important because it is obviously desirable to obtain the best description of the data, but also because it facilitates proper testing, for example testing of separability. Testing for separability is interesting for at least two reasons. Firstly, because it provides information about whether the demand for one energy carrier can be modelled without taking into account the demand for other energy carriers. In single equation household energy-demand studies, separability is most often assumed implicitly. Exceptions are Baker & Blundell (1991) and Baker et al. (1989), though. In their studies of British households they assume that demand for electricity and gas is not separable. No study, however, undertakes a formal test for separability. Secondly, testing for separability is interesting because it is possible to check if consumers are substituting towards other types of energy following an increase in the price of, say, natural gas.

Section 4.1 lays out the structure of demand for electricity and gas respectively. Section 4.2 will address the issue of testing for separability. Issues relating to estimation will be dealt with in section 4.3.

4.1 The Structure of Demand

The interest of the paper is in modelling the demand for energy carriers in the household. This is done by modelling demand for electricity and gas respectively as single equations. Households divide their total budget into different types of consumption. For the present purpose we allocate the budget into *three* types of demand: demand for natural gas, x_{gas}, demand for electricity, x_{el}, and demand for all other goods[5], x_{oth}, and denote the corresponding prices of these goods p_{gas}, p_{el}, p_{oth}.

---

[5] 'Other goods' includes food, services, clothing, transport, etc.

Further, let the function $g^{ij}(\bullet)$ be the demand function for good i, the good of interest, conditional on the demand for good j, the conditioning good. Then the conditional demand for good i is given by

$$x_i = g^{ij}\left(p_i, p_{oth}, c_{-j}, x_j, a\right) \qquad \begin{array}{l} \mathrm{i, j = el, gas} \\ \mathrm{i \neq j} \end{array} \qquad (4.1.1)$$

where the subscript identifying household units is suppressed for convenience, and where $c=p_ix_i + p_jx_j + p_{oth}x_{oth}$ is total expenditure of the household on all types of goods, and $c_{-j}=p_ix_i + p_{oth}x_{oth}$, is total expenditure less expenditures on the conditioning good j, cf. Pollak (1969). Also, the demand for good x_i is conditional on a vector, a, containing demographic variables, e.g. age, and family composition.

The conditional demand function is used instead of the unconditional demand function because it is a more convenient vehicle for exploring the structure of consumer preferences and the relations among goods. Applying the conditional demand function has at least two advantages for researchers modelling energy demand. First, the conditional demand function allows modelling demand for an energy carrier while not being occupied with modelling preferences for other goods, including other energy carriers and durables. The possible influence of other goods is taken into account simply by including the quantity consumed. Secondly, it offers a very easy way of testing separability.

4.2 Testing for Separability

If the good of interest is (weakly) separable from the conditioning good then the conditional demand function is given by

$$x_i = g^{ij}\left(p_i, p_{oth}, c_{-j}, a\right) \qquad \begin{array}{l} \mathrm{i,j = el, gas} \\ \mathrm{i \neq j} \end{array} \qquad (4.2.1)$$

This result is due to Pollak (1971). Comparing equation (4.2.1) with (4.1.1) suggests a simple test for separability of the good of interest from the conditioning good. The test amounts to testing the significance of the quantity of the conditioning good in the equation for the good of interest. The test is suggested by Browning and Meghir (1991).

In the present context it is possible to test for separability of demand for electricity (natural gas) from demand for natural gas (electricity) by inserting the quantity of natural gas (electricity) into the equation for electricity (natural gas)[6].

The testing procedure has a number of advantages over a procedure based on the unconditional demand function. First, it does not require that preferences for the conditioning good are correctly specified, as do separability tests based on the unconditional demand function. Secondly, it does not require modelling of the budget constraint of the conditioning good. This is of particular value when dealing with labour supply and durables.

---

[6] Note, it is relevant also to test for separability between demand for energy and demand for non-energy goods, for example food. This is, however, not possible in the present context since other goods are defined residually as a composite good. To test for separability between energy and other goods one needs more detailed information about the consumption of other goods. This is a subject for future research.

4.3 Estimation

Returning to issues related to estimation and the practical implementation of the testing procedure, some of the conditioning goods and other explanatory variables may themselves be stochastic, and some may be unobserved.

Firstly, note that no price variation is present in the data. The price terms in (4.1.1) thus vanish in the estimating relation. The estimating relation including the conditioning variable can now be written

$$x_i = f_i\left(c_{-j}, x_j, a \middle| \beta_i\right) + u_i \qquad \begin{array}{l} \mathrm{i, j = el, gas} \\ \mathrm{i \neq j} \end{array} \qquad (4.3.1)$$

where β is the vector of parameters to be estimated and u is the error term.

A second complication for estimation is that the quantity of the conditioning good may be endogenous. That is, x_{el}, may be endogenous in the equation for x_{gas}, and vice versa. The potential endogeneity problem arises because of unobserved components that are potentially important for the demand for energy. Demand for energy, as opposed to many other types of goods, is characterised as input into a durable that produces services that are the actual object of demand. In this sense it is the portfolio of energy-consuming durables, and the use of them in the household that generates the demand structure. First, consider the error term in (4.3.1) of the electricity equation, u_{EL}. This can be divided into three components

$$u_{EL} = D_{EL} + \mu + \varepsilon_{EL} \qquad (4.3.2)$$

D_{EL} is a vector of unobserved electricity consuming durables, μ is a household idiosyncratic term containing information about household behaviour relevant to the consumption of electricity, and ε_{EL} is random independent error term. The importance of these components for the separability test is outlined below.

The durable vector, D_{EL}, contains information about the stock of electric domestic appliances. This includes durables such as lighting, television sets, computers, washing machine, dishwasher etc., and possibly also includes an electric kitchen stove, and possibly heating durables. All houses in the sample are furnished with a primary heating system based on natural gas. For some reason supplementary electric heating units may also be present in the house, for example in the form of portable electric heating fans.

A similar form for the error term of the gas equation exists.

$$u_{GAS} = D_{GAS} + \lambda + \varepsilon_{GAS} \tag{4.3.3}$$

where D_{GAS} is a vector of gas-consuming durables, λ is a household idiosyncratic term containing information about household behaviour relevant to the consumption of natural gas, and ε_{GAS} is random independent error term. The durable vector, D_{GAS}, contains information about the unobserved characteristics relating to the efficiency of the heating system, and components of the building shell that are of importance to the consumption of energy for heating purposes. Also, D_{GAS} contains information about the possible existence of a gas-based kitchen stove.

Thereby, some types of durables, namely a kitchen stove and some heating equipment, can be contained in either, or both, D_{GAS} and D_{EL}. These elements may influence estimates of the conditioning variable in different ways.

If the kitchen stove is electrically based then this will imply a correspondingly lower level of gas consumption. On the other hand, if the kitchen stove is gas based then this will imply a correspondingly lower level of electricity consumption. Expressed compactly, $E[x_j\ D_i] < 0$, for i, j=el, gas.

All the households in the sample are equipped with a primary heating system that is natural gas based. Some households may, however, for different reasons have some electric heating, so that electric heating substitutes part of the natural gas based heating. Consider two households that are identical except that one household provides all heating by natural gas, and the other provides part of the heating by electricity. In this case, the household with supplementary electric heating will appear with a lower level of gas consumption than the household with only gas-based heating. Expressed compactly, $E[x_{GAS}\ D_{EL}] < 0$. Thus, in the electricity equation, the presence of electric heating durables will cause negative correlation between the error term and the conditioning variable, x_{GAS}.

Not only the unobserved durable vector may have implications for the estimate of the parameter of conditioning variable. Also the role of the unobserved behaviour of the household, captured by μ and λ, may have important implications for the parameter estimate of the conditioning variable. For example, endogeneity can arise because of people having different unobserved preferences for staying home as an alternative to going out. In this example, people having preferences for staying in will need to warm the house and will be likely to use the stock of domestic appliances more. In this way the idiosyncratic term of the electricity equation, μ, will be positively correlated with the idiosyncratic term in the gas equation, λ. Expressed compactly, $E[\mu \ \lambda] > 0$. Other types of household idiosyncrasies can exist, though, and in general it is not possible to infer anything about the sign of the correlation associated with this term.

Summarising, the presence of two kinds of unobserved durables, electrically based heating units and gas/electrically based kitchen stove, can cause negative correlation between error term and the conditioning variable. If electrically based heating durables are present (and used) then this will generate a negative correlation between the error term and x_{GAS} in the electricity equation. Further, it is not observed whether the kitchen stove is gas or electricity based. This can cause a negative correlation between the error term and the conditioning variable in both the gas and the electricity equation. Furthermore, differences in in-house unobserved household idiosyncrasies can cause correlation between the error term and the conditioning variable in both equations.

In order to identify the parameter of the conditioning variable, other variables that are orthogonal to the unobserved components, but still correlated with the conditional variable, must be chosen. These are called instrumental variables. A natural instrumental variable for the conditioning goods would be the market price, since this is surely exogenous. Unfortunately, as already mentioned, the cross-section analysed here does not offer any price variation. Other candidates may be present, though.

A candidate as instrument for x_{GAS} in the electricity equation is the vintage of the house, denote this z_{VINT}. This is potentially a good candidate because building regulations impose massive restrictions on the use of energy for heating purposes while not being concerned with consumption of energy for other purposes. For being a good instrument the following properties must be satisfied

(i) $E[x_{GAS} z_{VINT}] \neq 0$ (ii) $E[\mu\, z_{VINT}] = 0$

(iii) $E[\varepsilon_{EL}\, z_{VINT}] = 0$ (iv) $E[D_{EL}\, z_{VINT}] = 0$

(i) indicates that z_{VINT} must be a good predictor for the consumption of natural gas. This seems a priori plausible, since changing building styles and various building codes have influenced the consumption of energy for heating purposes by putting restrictions on the permitted heat loss of the building shell. (ii) requires the idiosyncratic term to be independent of the vintage of the house. This requirement is potentially critical if a distinct type of energy-savings behaviour exists among the households in the sample, and that these types of households have a higher propensity to live in newer houses than other people. For this to be critical, though, this effect should dominate other household idiosyncrasies. A formal test will clarify if (ii) is a reasonable requirement. (iii) is satisfied by construction. (iv) requires that the vintage of the house is independent of the stock of electricity-consuming durables in the house. This requirement is not trivial. As far as the stock of electricity-consuming durables, that are not producing heating, the assumption is reasonable since building regulations have not imposed any restrictions on durables not providing heating. Institutional restrictions have been imposed on the installation of electrical heating units, though. From 1988 installation of electric heating in new houses has been banned, and from 1994 installation of electric heating has also been banned in existing houses. All houses in the sample are furnished with a primary heating system that is natural gas based, though. Therefore, if electric heating is present in the houses in the sample it must be expected to be present only as supplementary heating units. Supplementary electrical heating is not covered by the ban. The key requirement for the vintage of the house to work as a good instrument for x_{GAS} in the electricity equation is thereby that any presence of supplementary electric heating is unrelated to the age of the house. Following these considerations it is assumed that the vintage of the house does not enter the electricity equation. The appropriateness of this restriction will be tested formally.

No immediate candidate instruments appear for x_{EL}, in the equation for x_{GAS}. However, exploiting that the consumption of electricity and natural gas is also observed in a number of years prior to 1996 (for some of the households) an instrumental variable may be suggested (for these households). To see this, consider a linear version of (4.3.1) where (4.3.3) is inserted and time subscripts are included. For convenience only x_{EL} is included as explanatory variable

$$x_{GASt} = \beta x_{ELt} + (D_{GASt} + \lambda + \varepsilon_{GASt}) \qquad (4.3.4)$$

where t indicates the observation year, 1996, D_{GASt} is a vector of unobserved durable components at time t that is associated with consumption of natural gas. λ is a household idiosyncratic term relevant for the consumption of gas. λ is assumed to be time invariant. Finally, ε_{GASt} is random independent error term entering the gas equation.

The problem that causes potential endogeneity is that $E[x_{EL}\lambda] \neq 0$, i.e. that the household idiosyncratic term is correlated with the consumption of electricity because $E[\lambda\mu] \neq 0$ or $E[D_{EL}\lambda] \neq 0$. Also, potential endogeneity can arise because $E[x_{EL} D_{GAS}] \neq 0$. This can be the case if the kitchen stove is gas based. In this case $E[x_{EL}D_{GAS}]<0$. It is therefore necessary to find a variable that is correlated with x_{ELt} without being correlated with λ and D_{GAS}. One candidate instrument is Δx_{ELt}. For this to work as an instrument for x_{ELt} it is required that

(v) $E[x_{ELt}\Delta x_{ELt}]\neq 0$ (vi) $E[\mu\,\Delta x_{ELt}]=0$

(vii) $E[\varepsilon_{GASt}\,\Delta x_{ELt}]=0$ (viii) $E[D_{GASt}\,\Delta x_{ELt}]=0$

(v) is the requirement that Δx_{ELt} is a predictor of x_{ELt}. (vi)-(viii) are orthogonality conditions. Turning to (vi), the potential correlation caused by $E[\mu\lambda]\neq 0$ is removed by construction since μ is eliminated from the equation for x_{EL} by first-differencing. This leaves $E[\lambda D_{ELt}]=0$ to be satisfied, i.e. that the change in the stock of electricity-consuming durables be independent of the gas specific idiosyncratic term. A formal test will be carried out to check if this is satisfied. For (vii) to be satisfied it is required that $E[\varepsilon_{GASt}\,\Delta\varepsilon_{ELt}]=0$, and $E[\varepsilon_{GASt}\,\Delta D_{ELt}]=0$. These restrictions are satisfied by construction since the ε_{GASt} is an independent error term. For (viii) to be satisfied it is required that $E[D_{GASt}\,\Delta\varepsilon_{ELt}]=0$, and $E[D_{GASt}\,\Delta D_{ELt}]=0$. The former restriction is satisfied by construction since ε_{ELt} is an independent error term from the electricity demand equation. The latter restriction effectively assumes that the acquisition of electrical durables in the present period is uncorrelated with the stock of durables relevant for the gas consumption in the present period. This assumption can be critical. Consider, for example, a household disposing of an electrical stove in period t and replacing it with a gas stove. This implies $\Delta D_{ELt}=-1$ and $D_{GASt}=1$. If this happens to a substantial number of households in the sample then $E[D_{GASt}\,\Delta D_{ELt}]\neq 0$. If, on the other hand, it happens only to very few households in the sample it may not be a problem. The same line of arguments is valid for other types of durables that can produce services using either gas or electricity.

The arguments proposed above carry over to the case where Δx_{ELt-l} (t=1996, l=1,2,…) are applied as instruments, i.e. to the case where lagged values of Δx_{ELt} are also included as instruments for x_{ELt}. In this case, furthermore, $E\left[D_{GASt}\ \Delta D_{ELt-l}\right]=0$ will be more likely to be satisfied. The original data set, from which the sample analysed here is drawn, contains data for consumption of energy dating back to 1990 for some of the households in the sample analysed here. Not all households are, however, observed for the whole period, 1990-1996, and including all lags cut down the sample size unreasonably. On the other side including only Δx_{ELt} as instrument leaves the model just identified, implying that it is not possible to test the validity of the included instrument. Here it is chosen to include $\Delta x_{ELt,}$ Δx_{ELt-1} (t=1996) as instruments for x_{ELt}. This leaves a reasonable number of observations, and leaves one overidentifying restriction. This, in turn, facilitates testing of the overidentifying restriction, i.e. partially testing the validity of the assumptions made above.

Finally, before turning to estimation, note that total expenditure, c, is also likely to be endogenous. As instrumental variable the natural log of household gross income is used.

The vector of parameters, β, of (4.3.1) is estimated by minimising u'Pu, where u is the vector of error terms from (4.3.1) and $P=(Z(Z'Z)^{-1}Z)$ and Z is a matrix of instrumental variables satisfying $E[u|\ Z]=0$. The covariance matrix is estimated using the form suggested by White (1980) that is robust to general forms of heteroscedasticity.

$$V(\hat{\beta}) = (X'PX)^{-1} X'P\hat{\Omega}PX(X'PX)^{-1} \tag{4.3.5}$$

where $\hat{\Omega} = \hat{u}\hat{u}'$ and $\hat{u}$ is the vector of estimated errors.

Further, where relevant, two specification tests are provided. First, a Hausman (1978) test for exogeneity is supplied. The test statistic is given by $(\beta^{IV} - \beta^{0})'[V(\beta^{IV}) - V(\beta^{0})]^{-1}(\beta^{IV} - \beta^{0})$, where β^{IV} are the parameter estimates obtained using the instrumental variables estimator, β^{0} are the estimates obtained assuming exogeneity, and $V(\beta^{IV})$ and $V(\beta^{0})$ are the covariance matrices of the respective estimators. The statistic is chi-squared distributed with degrees of freedom equal to the rank of $[V(\beta^{IV}) - V(\beta^{0})]$ under the null hypothesis that a given set of variables is exogenous.

The second statistic is a Sargan statistic for overidentifying restrictions, cf. Hansen (1982). The Sargan statistic for overidentifying restrictions is given by u'Pu, where u is the vector of errors obtained from instrumental variables estimator and $P=Z(Z'Z)^{-1}Z$. The statistic is chi-squared distributed with $(r_Z - r_X)$ degrees of freedom, where r_Z is the number of instrumental variables and r_X is the number of parameters to be estimated. If the residuals from the demand equation and the instrumental variables have non-zero asymptotic covariances this is taken as evidence that the model is misspecified, i.e. that instruments do not satisfy the requirements for the instrumental variable estimator to be consistent.

5. Results

Two equations have been estimated independently. One equation for the demand for electricity and one equation for the demand for natural gas. Estimation results are presented in table A1.1 and A1.2 in Appendix 1. Table A1.1 contains estimation results for the electricity equation, and table A1.2 contains estimation results for the gas equation. The presentation of the estimation results is divided into two sections. In section 5.1, the specification of the basic model is presented. Then, in section 5.2 the issue of separability in demand for electricity from the demand for gas and *vice versa*, and the problem of endogeneity of the conditioning variable is addressed.

5.1 The Basic Model

In this section the specification and results of the basic model are outlined. The choice of specification builds on the insights of the nonparamtric analysis in section 3. Here it was suggested that the consumption of electricity and gas might be nonlinearly related to the age of the oldest person in the household and the size of the house. On the other hand, the nonparametric analysis pointed out quite clearly that consumption of electricity and gas relates linearly to total expenditures. The nonparametric analysis is limited by only including one explanatory variable, and plays only a suggestive role in the specification of the final parametric model. The preferred specification of the parametric gas demand relation, cf. table A1.2, includes a squared term in the natural logarithm of age of the oldest person in the household while keeping the natural logs of income and square metres linear. For the electricity demand relation the preferred specification includes linear terms in the natural logarithms of income, square metres, and age of oldest member of the household, cf. table A1.1.

Further, in the parametric specification, a set of dummy variables describing the family composition in terms of children is included in both equations. The family composition variables are specified so that all possible combinations of number and age of children are contained within the set of dummy variables. This is done in order to provide the less restricted specification of the importance of children. Finally, two technical dummy variables are included, describing whether the house is two-storeyed and whether it is a non-detached house.

In the gas equation a set of dummy variables indicating the vintage of the house is included. The house vintage is categorised into a set of dummy variables, divided into decades up to 1979 where the first building regulation imposing substantial restrictions on the heat loss of the building shell was put into force. Hereafter, the dummy variables are categorised according to time where the subsequent building regulations were put into force, i.e. in 1983 and 1986. This approach to modelling vintage effects is known as the discontinuous regression approach, cf. Angrist and Krueger (1999), and is implemented to address the non-smooth relationship that is known *a priori* to prevail due to the building codes and changing building styles. First, the electricity demand equation will be commented on, and subsequently the gas equation will be treated.

The preferred specification for the electricity demand relation, cf. table A1.1, includes a linear term in total expenditure, age, and the size of the house. Column 1 contains OLS estimates, where all variables are assumed exogenous. In column 2 estimates are presented where gross household income is included as instrument for total household expenditure and vintage dummies are included as instruments for $\ln(kWh_{GASt})$. The total expenditure elasticity is estimated to 0.28, cf. column 2. This is within the range of what is usually found in studies of household electricity demand, e.g. Baker et al. (1989), and Branch (1993). Consumption of electricity is also increasing linearly with the size of the house. A doubling of the size of the house is roughly associated with a 25% higher level of consumption of electricity. Estimation results indicate that no age effect is present. This is consistent with the indications given by the non-parametric analysis in section 3.

Regressing the errors of the electricity demand relation non-parametrically on total consumption, size of the house, and age provides a check of the validity of chosen specification. These nonparametric regressions are presented in figure 5.1.1, left column, together with approximate point-wise confidence bands. If the model is well-specified with respect to these variables the errors are expected to centre at zero irrespective of the value of the explanatory variable. It is seen that this is indeed the case for all three variables.

Further, in the electricity equation it is seen that children affect the demand substantially. The pattern seems to suggest that the number of children is important to the level of consumption. Families with one child appear to have a level of consumption that is about 14% higher than families without children. Estimates indicate that families with two children are associated with a level of consumption of electricity that is about 20% higher than for families without children. The average contribution per child is thus lower for families with two children than for families with one child. Further, there is some indication that small children do not affect the consumption of electricity as much as older children, although this effect is not statistically significant. Finally, families living in two-storeyed houses or non-detached houses are not associated with a different level of consumption of electricity than families living in single-storeyed detached houses.

Practically all of the studies mentioned in section 1 modelling demand for electricity include a measure of the house size, usually number of rooms, and the number of persons in the household. However, no study, except the present, has modelled the impact of children in detail, while also controlling for age effects, and the importance of the size of the house. This is likely to be important as age and number of children are correlated with the age of the parents, and the number of children is correlated with the size of the house. Only Baker et al. (1989) include a dummy for children aged less than 6, and at the same time controlling for house size and age of reference person by including a dummy for age of the reference person belonging to the age interval 40-65 relative to younger households. The effect on the electricity expenditure share of small children is estimated to be 10%, relative to families without small children, on top of the average contribution a child has by increasing the total number of household members. Estimates from the same study indicate that households where the age of the reference person is 40-65 experience an expenditure share that is about 6% lower than for younger households. Differences in estimates between Baker et al. (1989) and the present study may be caused by differences in modelling approach. However, concerning the study by Baker et al. (1989) it should also be noted that electricity is used also for central heating as opposed to the present study.

Turning to the gas equation, the non-parametric analysis gave the strongest indication for curvature in the age-consumption relationship. Estimates for the gas equation are presented in table A1.2. The first column contains OLS estimates, where all variables are assumed exogenous. In column 2 estimates are presented where gross household income is included as instrument for total household expenditure and $\Delta\ln(kWh_{ELt})$ is included as instrument for $\ln(kWh_{ELt})$. Finally, in column 3 estimates are presented where gross household income is included as instrument for total household expenditure, and $\Delta\ln(kWh_{Elt})$ and $\Delta\ln(kWh_{Elt-1})$ are included as instrument for $\ln(kWh_{ELt})$.

Considering the preferred specification of the gas equation a squared term is included for age of the oldest person in the household whereas only linear terms are included for income and square metres. The linear term in total household expenditure is common to many single equation studies of demand for energy for heating purposes, e.g. Lee and Singh (1994), Garbacz (1985), Klein (1988), Green (1987). The estimated elasticity, cf. column 3, of 0.37 falls in the upper end of the interval of what is usually found in these studies. This may be explained by the fact that many of these studies enter household income instead of total household expenditure or leave total household expenditure exogenous in the demand relation. The linear relation between the size of the house and the level of consumption amounts to an increase in consumption of energy for heating of approximately 50% by doubling the house size. Estimates suggest that consumption is increasing monotonically non-linearly with age. Once having controlled for the size of the house and the age of the oldest person in the household, families with children appear not to be associated with any higher level of consumption of natural gas than families without children.

In figure 5.1.1, right column, nonparametric regression curves of the errors of the gas demand relation on total consumption, size of the house, and age are graphed together with approximate confidence bands. The nonparametric regression curves generally centre at zero. It should be noted, though, that the lower confidence band for total expenditure takes on positive values at the very upper end of the total expenditure range. This indicates that errors may not be centred at zero at this range. The linear specification is maintained, though, because confidence bands are only approximate, and should therefore not be interpreted too rigorous, and because the deviation is only indicated for a very limited part of the sample.

Only one study of demand for gas has contemporaneously included age, children and house size, Baker et al. (1989). Baker et al. (1989) model age effects by including a dummy for age of the reference person being between 40 and 65, relative to reference persons in younger households.

Moreover, they model impacts of children by including a dummy for the presence of children aged less than 6 on top of the average contribution a child has by increasing the total number of household members. Households with reference persons aged 40-65 are estimated to be associated with an expenditure share that is 13% higher than for younger households, while households with small children are found to be associated with an expenditure share that is 14% higher than for households without small children. These results differ from the results obtained in the present study. The differences in modelling approach may explain part of these differences. The contemporaneous and detailed modelling of impacts of children, age effects and influence of the house size is important. This is because age and number of children are correlated with age of the parents, and the number of children is correlated with the size of the house.

The estimated parameters of the technical variables indicate that the consumption of natural gas in non-detached houses is some 4% lower than in detached houses. This is expected, as natural gas is used for space heating, and because this type of houses have relatively less wall area facing outdoor temperature than detached houses. The same considerations should be valid for houses with two storeys. Estimates, however, suggest that there is no effect. This may relate to the second storey being a utilisation of the attic with a lower insulation standard thus neutralising the gain from reducing the surface area facing outdoor temperature. The vintage of the building is of major importance in the demand for natural gas, because of changing building styles, and because of the introduction of building codes. The estimated vintage effect appears to be very robust to the specification of the other parts of the model. Estimated vintage effects are graphed in figure 5.1.2.

[figure 5.1.2 around here. See end of paper]

The significance of the vintage effect in the demand for energy for heating purposes follows what is found in a study of the demand for energy for space heating in apartment blocks, cf. Leth-Petersen & Togeby (2001). The pattern of the vintage effects for single-family houses, cf. figure 5.1.2, indicates that the consumption of energy for heating purposes has been increasingly efficient in houses built after the 1960s. This indicates that energy efficiency improvements were already an issue before the introduction of building codes. However, the figures indicate that gas consumption is significantly lower for buildings built after the introduction of building codes, implying that building codes seem to have had an effect.

5.2 Testing for Separability of Energy Consumption from Consumption of Other Energy

In this section the results of the separability tests are presented. As outlined in section 4.2 testing for separability of electricity from natural gas is done by testing the significance of the quantity of natural gas in the electricity equation and *vice versa*.

Considering the OLS estimate of the quantity of natural gas in the electricity equation, table A1.1 column 1, the parameter estimate of the quantity of natural gas is positive and significant. However, in the instrumental variable regression, table A1.1, column 2, the parameter estimate of natural gas consumption in the electricity equation is insignificant indicating that demand for electricity is indeed separable from demand for natural gas. The quantity of natural gas is instrumented with the vintage dummies. The Hausman test rejects exogeneity, and the Sargan test does not reject the validity of the vintage dummies as instruments. The separability of demand for electricity from demand for natural gas is consistent with the heat planning having been successful, i.e. that the households in the sample do not substitute the use of gas for heating with electricity.

A similar procedure has been applied to the gas equation in order to test if the demand for natural gas is separable from the demand for electricity. Assuming that the demand for electricity is exogenous to the gas demand equation the parameter of electricity demand is estimated significantly positive, cf. table A1.2, column 1, indicating non-separability. The positive sign on the parameter estimate is consistent with in-house behaviour dominating the bias. Consumption of natural gas appears not to be separable from electricity consumption. As mentioned in section 4 the demand for electricity is likely to be endogenous to the demand for natural gas, and this has probably affected the outcome of the test.

Column 2 and 3, table A1.2, present estimates of the gas equation where Δx_{Elt}, column 2, and $\Delta x_{ELt,}$ Δx_{ELt-1}, column 3, are used as instruments for x_{Elt} (t=1996). The estimates are obtained on a subsample of 2,144 observations out of the original sample, applied in estimation of the electricity equation, of 2,885 observations. Considering first the estimate of the parameter of the conditioning variable, where only Δx_{Elt} is introduced as instrument for x_{Elt}, it is seen that the parameter of the conditioning variable, the quantity of electricity consumed, becomes insignificant indicating that demand for electricity is separable from the demand for natural gas. As mentioned in section 3, if many of the households in the sample do change durables from being based on electricity to being based on gas in the observation year Δx_{Elt} may not provide as a good instrument. Therefore, additionally, Δx_{ELt-1}, is introduced as an instrument. This provides one overidentifying restriction, and facilitates testing hereof. Having introduced this additional instrument leaves the parameter estimate of the conditioning variable insignificant like before. The validity of the overidentifying restriction is not rejected by the Sargan test. The Hausman test rejects exogeneity.[7 + 8]

---

[7] The predictive power of the instruments has been investigated by estimating the ”auxillary” regression. Parameter estimates of $\Delta x_{ELt,}$ Δx_{ELt-1} (t=1996) come out with t-values of 23 and 17 respectively, and the overall explanatory power, i.e. R^2, of the regressors in this regression is 0.33. There is, thus, no evidence of weak instruments.

Summarising, the demand for electricity appears to be weakly separable from the demand for natural gas. The analysis also suggests that demand for natural gas is weakly separable from demand for electricity. This means that the allocation of expenditures between electricity and other goods is independent of the demand for natural gas. This corresponds to a situation where expenditure for gas is deducted from the total budget, and the remainder is allocated between electricity and other goods. The other case is similar. The allocation of expenditures between natural gas and other goods is independent of the demand for electricity. The results of the separability tests are based on the assumption that the technology is kept fixed. This means that the results are valid in the short run. In the long run households may adjust the durable stock, for example by acquiring supplementary electrical heating units, undertaking insulation upgrading or by introducing new heating technology.

The results indicate that single equation modelling of household energy demand on cross-section data fares quite well. In the present study this is identified by exploiting institutional restrictions in the form of building codes, on the one hand, and by exploiting a minimum of individual time variation, i.e. panel data, on the other. The analysis suggests that problems of identification in this study relate to unobserved durable stocks, and household idiosyncrasies. This, in turn, indicates that, if durable stocks are observed, then inclusion of durable specific dummies overcomes mutual dependence of demand for different types of energy, not considering unobserved household idiosyncrasies. Energy-consuming durable stocks are often observed in energy and household expenditure surveys. In this way, single equation studies of household energy demand based on this

---

[8] The separability tests are performed in a model where total household expenditure is instrumented with household gross income. The model is estimated on administrative register data, and a measure of total household expenditure is imputed, cf. section 2. In order to check whether the results of the separability tests depend on the imputation of total household expenditure, the tests have also been carried out in a model where household gross income enters directly into the demand relation instead of the imputed measure of total household expenditure. In this reduced form model the conclusion of the test remains unchanged in both the gas and electricity equation.

kind of data and including durable specific dummies are likely not to have been hampered by the implicit separability assumption made in these studies. Examples of this are Branch (1993) and Halvorsen and Larsen (2001).

In order to develop the understanding of separability in demand the effects of an increase in the price of the conditioning good are explored. The treatment follows Pollak (1971). Taking equation (4.1.1) as the starting point the conditional demand function can be written

$$\begin{aligned} x_i &= g^{ij}\left(p_i, p_{oth}, c_{-j}, x_j, a\right) && \mathrm{i, j = el, gas} \\ &= g^{ij}\left(p_i, p_{oth}, \left(c - p_j x_j\right), x_j, a\right) && \mathrm{i \neq j} \end{aligned} \qquad (5.2.1)$$

where c is total expenditure and $c_{-j}=c-p_jx_j$. Differentiating (5.2.1) with respect to p_j yields

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial g^{ij}}{\partial c_{-j}} \frac{\partial c_{-j}}{\partial p_j} + \frac{\partial g^{ij}}{\partial x_j} \frac{\partial x_j}{\partial p_j} \qquad \mathrm{i, j = el, gas} \quad \mathrm{i \neq j} \qquad (5.2.2)$$

In Pollak's terminology the first term is called the *money expenditure effect* and the second term is called the *pure substitution effect*. The substitution effect arises because an increase in the price of the conditioning good implies a change in the demand for the conditioning good, which, in turn, causes reallocation of the non-conditioning goods.[9] The money expenditure effect works by a price

---

[9] Note, that the pure substitution effect dealt with here implies that if a price increase of the conditioning good leads to a decrease in the demand for the conditioning good then the pure substitution effect leaves utility at a lower level. This is in contrast with the Slutsky-Hicks substitution effect that leaves utility unchanged, cf. Pollak (1969).

change of the conditioning good causing the level of expenditures for the non-conditioning goods to change.

The substitution effect $\left(\partial g^{ij}/\partial x_j\right)\left(\partial x_j/\partial p_j\right)$ consists of two terms. The latter term $\left(\partial x_j/\partial p_j\right)$ is the own price effect of the conditioning good. This term is negative if the good is not a Giffen good. The former term $\left(\partial g^{ij}/\partial x_j\right)$ gives the effect on the good of interest of a unit change in the demand for the conditioning good given that the level of expenditures on the non-conditioning goods is kept fixed. Pollak (1969) uses the sign of $\left(\partial g^{ij}/\partial x_j\right)$ to categorise how the conditioning good is related to the non-conditioning good: if $\left(\partial g^{ij}/\partial x_j\right) > 0$ x_i is positively related to x_j, if $\left(\partial g^{ij}/\partial x_j\right) < 0$ x_i is negatively related to x_j, and if $\left(\partial g^{ij}/\partial x_j\right) = 0$ x_i is unrelated to x_j.

In this Pollak (1969)-sense, the demand for natural gas is unrelated to the demand for electricity. Likewise, the demand for electricity is unrelated to the demand for natural gas.

The demand for electricity being unrelated to the demand for natural gas implies that the latter term in (5.2.2), the substitution effect, disappears. This means that a change in the price of natural gas has an effect on the demand for electricity only through the level of expenditures allocated to the demand for electricity and other goods. This implies that if electricity is a normal good, then an increase in the price of natural gas will cause consumption of electricity to decrease if the demand for natural gas is inelastic, as is often found in empirical studies, e.g. Baker et al. (1989).

Similarly, since the demand for natural gas is unrelated to demand for electricity the substitution effect vanishes. A change in the price of electricity thus only results in an expenditure effect. If the demand for electricity is inelastic, as is often found in empirical studies based on micro data, e.g. Branch (1993), and natural gas is a normal good then an increase in the price of electricity will decrease the demand for natural gas. This follows because an increase in the price of electricity leads to an increase in the expenditure on electricity leaving less money to be spent on other goods.

The treatment is based on the assumption that the technology is kept fixed. In this way the results of the tests are relevant particularly in the short run. In the long run it may be expected that households will adjust their stock of technology, e.g. undertake insulation upgrading, adjust the heating technology, etc., following changes in the relative prices of energy.

The results have indicated that the households in the sample using natural gas for space heating experience significant welfare effects in the short run from changes in the relative price of natural gas. This is because they do not substitute the consumption of natural gas with consumption of electricity and do not have any other alternative source of energy supply for substituting natural gas. This is consistent with the intentions of the heat planning. It indicates that increasing substitution possibilities in energy demand for heating purposes is a way of reducing adverse welfare effects of relative price increases on natural gas. Also, results indicate that households experience adverse welfare effects of increases in the relative price of electricity. Electricity is mainly used as input for domestic appliances for other purposes than heating. It is less obvious that it is possible to introduce substitution possibilities for these purposes.

6. Conclusions

The understanding of household energy demand behaviour is an important task since the household segment of the energy market is subject to extensive price fluctuations stemming from the market and because it is submitted to much regulation by politicians, all likely to have significant welfare effects. In this paper a cross-section of households from 1996 has been analysed. Estimates suggest that electricity consumption depends on the number of children, and depends linearly on the natural logarithm of total expenditure, size of the house, and age level. The consumption of natural gas is found to vary nonlinearly with age, and to depend linearly on total expenditure, and the size of the house. Furthermore, demand for natural gas is found to depend on the technical characteristics of the house. Particularly, it is found that the consumption of energy for space heating depends heavily on the vintage of the house reflecting changing building styles and building codes in force at the time of the construction of the house.

Based on the estimated model, separability in demand for natural gas from electricity and *vice versa* was tested using a robust testing procedure. The analysis indicates that demand for electricity is separable from the demand for natural gas. This result is consistent with the extensive planning of the supply of energy for heating purposes in Denmark. The analysis furthermore suggests that the demand for natural gas is separable from demand for electricity. Together, this indicates that market as well as policy induced price changes could have significant welfare effects in the short run since the demand for specific energy carriers is not substituted by demand for other energy carriers.

The study provides evidence in favour of single equation modelling of household energy demand. Applying this approach often implicitly involves assuming separability in demand for one particular type of energy from demand for other types of energy. This assumption is not likely to be satisfied if stocks of energy-consuming durables are unobserved. In the present study exploiting institutional restrictions on energy demand and a minimum of individual household time variation, i.e. panel data, is used to overcome this problem.

References

Angrist, J. D. & Krueger, A. B. (1999); Empirical Strategies in Labor Economics; in O. Ashenfelter & D. Card (eds.), *Handbook of Labour Economics*, vol. 3, chapter 23, Elsevier Science B. V.

Baker, P. & Blundell, R. (1991); The Microeconometric Approach to Modelling Energy Demand: Some Results for UK Households; *Oxford-Review-of-Economic-Policy*; 7(2), Summer, pp. 54-76.

Baker, P., Blundell, R. & Mickelwright, J. (1989); Modelling Household Energy Expenditures using Micro Data; *Economic Journal*, 99, pp. 720-738.

Banks, J. & Blundell, R. & Lebwel, A. (1997); Quadratic Engel Curves and Consumer Demand; *The Review of Economics and Statistics*; Vol. LXXIX, no. 4, pp. 527-539

Barnes, R. & Gillingham, R. & Hagemann, R. (1982); The Short-Run Residential Demand for Natural Gas; *Energy Journal*; 3(1), Jan., pp. 59-72.

Blundell, R. & Duncan, A. (1998); Kernel Regression in empirical Microeconomics; *Journal of Human Resources*; vol XXXIII, no. 1, pp. 62-87

Blundell, R.; Duncan, A.; Pendakur, K. (1998); Semiparametric Estimation and Consumer Demand; *Journal of Applied Econometrics*; 13(5), pages 435-61.

Branch, E. R. (1993); Short Run Income Elasticity of Demand for Residential Electricity Using Consumer Expenditure Survey Data; *The Energy Journal*, vol. 14, no. 4, 1993, pp. 111-121.

Browning, M. and Meghir, C. (1991); The Effects of Male and Female Labour Supply on Commodity Demands; *Econometrica*, 59, no.4, pp. 925-951.

Danmarks Statistik (1998); *Indkomst, forbrug og priser, 1998:11;* Danmarks Statistik, Copenhagen.

Dubin, J.A. & Henson, S. E. (1988); An Engineering/Econometric Analysis of Seasonal Energy Demand and Conservation in the Pacific Northwest; *Journal of Business and Economic Statistics*; 6(1), January, pp. 121-34.

Dubin, J. A. & D. L. MacFadden (1984); An Econometric Analysis of Residential Electric Appliance Holdings and Consumption; *Econometrica*, vol 52, no. 2, pp. 345-362.

Dubin, J.A. & Miedema, A. K. & Chandran, R. V. (1986); Price Effects of Energy-Efficient Technologies: A Study of Residential Demand for Heating and Cooling*; Rand Journal of Economics*; 17(3), Autumn, pp. 310-25.

Garbacz, C. (1983); Electricity Demand and the Elasticity of Intra-marginal Price; *Applied Economics*, 15, pp. 699-701.

Garbacz, C. (1985); Residential Fuel Oil Demand: A Micro-Based National Model; *Applied Economics*, 17.

Green, R. (1987); Regional Variations in US Consumer Response to Price Changes in Home Heating Fuels: The Northeast and the South; *Applied Economics*, 19.

Green, R. D. & Salley, A. G. & Grass, G. & Oesi, A. A. (1986); The Demand for Heating Fuels: A Disaggregated Modeling Approach; Atlantic Economic Journal; 14(4), December, pp. 1-14.

Haas, R. & Biermayr, P. & Zoechling, J. & Auer, H. (1998); Impacts on Electricity Consumption of Household Appliances in Austria: A Comparison of Time Series and Cross Section Analysis; *Energy Policy*, 13, pp. 1031-1040.

Halvorsen, B. & Larsen, B. (2001); The Flexibility of Household Electricity Demand over Time*;* *Resource and Energy Economics*, 23, pp. 1-18.

Hansen, L. P. (1982); Large Sample Properties of Generalized Method of Moments Estimators; *Econometrica*, vol. 50, no. 4.

Hausman, J. (1978); Specification Tests in Econometrics; *Econometrica*, 46, pp. 1251-1272.

Härdle, W. (1990); *Applied Nonparametric Regression*; Econometric Society Monographs, no. 19, Cambridge University Press.

Herriges, J. A. & King, K. K. (1994); Residential Demand for Electricity under Inverted Block Rates: Evidence from a Controlled Experiment*; Journal of Business and Economic Statistics*; 12(4), October, pp. 419-30.

Klein, Y.L. (1988); An Econometric Model of the Joint Production and Consumption of Residential Space Heat; *Southern Economic Journal*, vol 55. no. 2.

Lee, R. S. & N. Singh (1994); Patterns in Residential Gas and Electricity Consumption: An Econometric Analysis; *Journal of Business and Economic Statistics*, vol 12, no. 2.

Leth-Petersen, S. & Togeby, M. (2001); Demand for Space Heating in Apartment Blocks: Measuring Effects of Policy Measures Aiming at Reducing Energy Consumption; *Energy Economics,* Vol. 23, pp. 387-403.

Madlener, R. (1996); Econometric Analysis of Residential Energy Demand: A Survey; *The Journal of Energy Literature*, vol. II, no 2.

Morss, M. F. & Small, J. L. (1989); The Effect of Wood and Coal Stoves on Household Use of Electricity; *Land-Economics*; 65(1), February, pp. 29-37.

Parti, M. & Parti, C. (1980); The Total and Appliance-Specific Conditional Demand for Electricity in the Household Sector; *Bell Journal of Economics*; 11(1), Spr., pp. 309-21.

Pollak, R. A. (1969); Conditional Demand Functions and Consumption Theory; *Quarterly Journal of Economics,* 83(1).

Pollak, R. A. (1971); Conditional Demand Functions and the Implications of Separable Utility; *Southern Economic Journal*; 37(4), April, pp. 423-33.

Poyer, D. A. & M. Williams (1993); Residential Energy Demand: Additional Empirical Evidence by Minority Household Type; *Energy Economics,* Vol. 15(2), pp. 93-100.

Rasmussen, L.E. (1997); *Dokumentation af AKFs forløbsregister*; AKF Forlaget, Copenhagen.

Reilly, J. M. & Shankle, S. A. (1988); Auxiliary Heating in the Residential Sector; *Energy-Economics*; 10(1), January, pp. 29-41.

Robinson, P. M. (1988); Root-N-Consistent Semiparametric Regression; *Econometrica*; 56(4), pp. 931-54.

White, H. (1980); A Heteroscedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroscedasticity; *Econometrcia*; 48(4), May 1980, pp. 817-38.

Zheng, J. X. (1996); A Consistent Test of Functional Form via Nonparametric Estimation Techniques; *Journal of Econometrics*; Vol. 75, pp. 263-289

Table A1.1. Estimates of Electricity Demand Equation

Dependent variable, ln(kwh_{EL})	1. OLS	2. IV/GMM
Constant	5.3399**	3.7779**
	0.4313	0.8327
ln(m^2)	0.2189**	0.2363**
	0.0465	0.0518
ln(total household expenditure) (1)	0.0998**	0.2788**
	0.0195	0.0707
ln(age)	-0.1001	-0.0609
	0.0735	0.0769
1 child, aged 0-6	0.1343**	0.1351**
	0.0577	0.0576
1 child, aged 7-14	0.1520**	0.1357**
	0.0477	0.0482
1 child, aged 15-25	0.1467**	0.1474**
	0.0234	0.0238
2 children, aged 0-6. 0-6	0.0696**	0.0564
	0.0502	0.0502
2 children, aged 0-6. 7-14	0.01936**	0.1830**
	0.0458	0.0461
2 children, aged 7-14.7-14	0.1945**	0.1726**
	0.0389	0.0391
2 children, aged 7-14.15-25	0.2280**	0.2011**
	0.0300	0.0316
2 children, aged 15-25.15-25	0.2214**	0.2122**
	0.0257	0.0263
2 storeys	-0.0222	-0.0132
	0.0587	0.0590
Non-detached house	-0.0104	-0.0127
	0.0272	0.0277
ln(kWh_{GAS})	0.1134**	0.0272 (2)
	0.0297	0.0794
Sargan Statistic	-	0.5959 (3)
Hausman Statistic	-	7.0125 (4)
Number of observations	2,885	2,885

Note: The reference is a couple without children living in a single-storeyed house built in 1960-69.
Standard errors in small numbers below parameter estimates.** indicates significance at 5% level, *indicates significance at 10% level.

(1) Total household expenditures are less the cost of gas consumption. Gross household income is excluded from both equations and used as instrument for total household expenditure.

(2) Vintage variables are excluded from the electricity demand equation, and used as instrumental variables for $\ln(kWh_j)$ in this equation.

(3) The test statistic is chi-squared distributed with 10 degrees of freedom.

(4) The Hausman statistic is calculated only for the variables that are suspected endogenous, i.e. for ln(total household expenditure), and $\ln(kWh_j)$. The test statistic is chi-squared distributed with 2 degrees of freedom.

Table A1.2. Estimates of Gas Demand Equation

	1.	2.	3.
Dependent variable, ln(kwh$_{GAS}$)	OLS	IV/GMM	IV/GMM
Constant	9.8671**	13.4736**	12.8079**
	2.3742	2.8739	.2.7563
ln(m^2)	0.5486**	0.5094**	0.5004**
	0.0362	0.0416	0.0401
ln(total household expenditure) (1)	0.0691**	0.3970**	0.3701**
	0.0152	0.0555	0.0506
ln(age)	-2.1814*	-5.6805**	-5,3198**
	1,2635	1.6025	1.5383
ln(age)^2	0.3078*	0.7773**	0.7300**
	0.1682	0.2135	0.2051
1 child, aged 0-6	-0.0012	0.0151	0.0109
	0.0307	0.0378	0.0362
1 child, aged 7-14	-0.0487	-0.0376	-0.0422
	0.0327	0.0343	0.0337
1 child, aged 15-25	0.0070	0.0272	0.0208
	0.0175	0.0212	0.0201
2 children, aged 0-6. 0-6	0.0092	-0.0015	-0.0024
	0.0338	0.0383	0.0373
2 children, aged 0-6. 7-14	0.0294	0.0368	0.0315
	0.0261	0.0307	0.0295
2 children, aged 7-14.7-14	-0.0281	-0.0225	-0.0295
	0.0244	0.0287	0.0275
2 children, aged 7-14.15-25	-0.0181	-0.0134	-0.0214
	0.0277	0.0331	0.0317
2 children, aged 15-25.15-25	0.0103	0.0270	0.0187
	0.0237	0.0280	0.0266
House vintage, –1899 (2)	0.0751	0.0879	0.0879
	0.0743	0.0728	0.0731
House vintage, 1900-1909 (2)	0.0090	-0.0361	-0.0469
	0.0904	0.0875	0.0861
House vintage, 1910-1919 (2)	0.0975	0.0667	0.0647
	0.0810	0.0907	0.0882
House vintage, 1920-1929 (2)	0.1587**	0.1454**	0.1514**
	0.0329	0.0419	0.0400

(Table 2 continued)

House vintage, 1930-1939 (2)	0.1333**	0.1122**	0.1138**
	0.0263	0.0282	0.0277
House vintage, 1940-1949 (2)	0.0946**	0.0913**	0.0926**
	0.0332	0.0367	0.0361
House vintage, 1950-1959 (2)	0.0771**	0.0760**	0.0769**
	0.0235	0.0268	0.0262
House vintage, 1970-1978 (2)	-0.0590**	-0.0572**	-0.0561**
	0.0147	0.0167	0.0164
House vintage, 1979-1982 (2)	-0.1581**	-0.1826**	-0.1806**
	0.0345	0.0382	0.0369
House vintage, 1983-1985 (2)	-0.2885**	-0.2702**	-0.2733**
	0.0296	0.0333	0.0328
House vintage, 1986-1995 (2)	-0.4071**	-0.4011**	-0.4042**
	0.0255	0.0309	0.0298
2 storeys	0.0287	0.0054	0.0007
	0.0373	0.0401	0.0389
Non-detached house	-0.0381**	-0.0396**	-0.0378**
	0.0162	0.0194	0.0188
ln(kWh$_{EL}$)	0.0685**	-0.0449 (2)	-0.0029 (4)
	0.0144	0.0476	0.0331
Sargan Statistic	-	-	0.2513 (5)
Hausman Statistic	-	37.6810 (3)	39.7576 (3)
Number of observations	2,144	2,144	2,144

Note: The reference is a couple without children living in a single-storeyed house built in 1960-69.
Standard errors in small numbers below parameter estimates.** indicates significance at 5% level, *indicates significance at 10% level.

(1) Total household expenditures are less the cost of electricity consumption. Gross household income is excluded, and used as instrument for total household expenditure in column 2 and 3.
(2) Δx_{ELt} is used as instrumental variable for ln(kWh$_{EL}$) in this equation. This model is just identified.
(3) The Hausman statistic is calculated only for the variables that are suspected endogenous, i.e. for ln(total household expenditure), and ln(kWh$_j$). The test statistic is chi-squared distributed with 2 degrees of freedom.
(4) Δx_{Elt} and Δx_{Elt-1} is used as instrumental variables for ln(kWh$_{EL}$) in this equation. This model has one overidentifying restriction.
(5) The Sargan statistic is chi-squared distributed with 1 degree of freedom.

Figure 3.2.1. Nonparametric regression of consumption of energy on total expenditures for Gas (top panel) and electricity (middle panel), and nonparametric density of total expenditures (bottom panel)

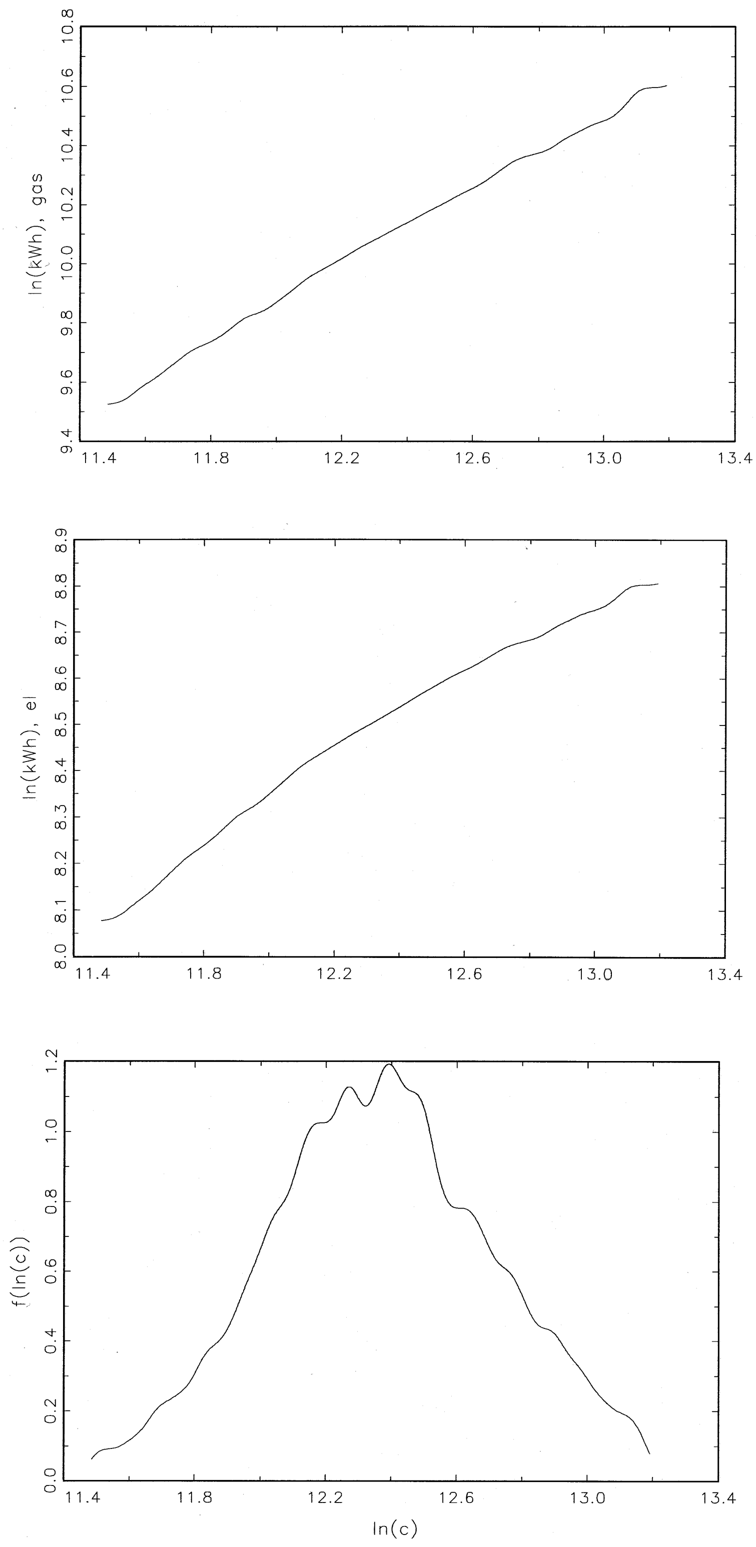


Figure 3.2.2. Nonparametric regression of consumption of energy on total expenditures for Gas (top panel) and electricity (middle panel), and nonparametric density of total expenditures (bottom panel). Solid line: no children, small lines: 1 child, big lines: 2 children.

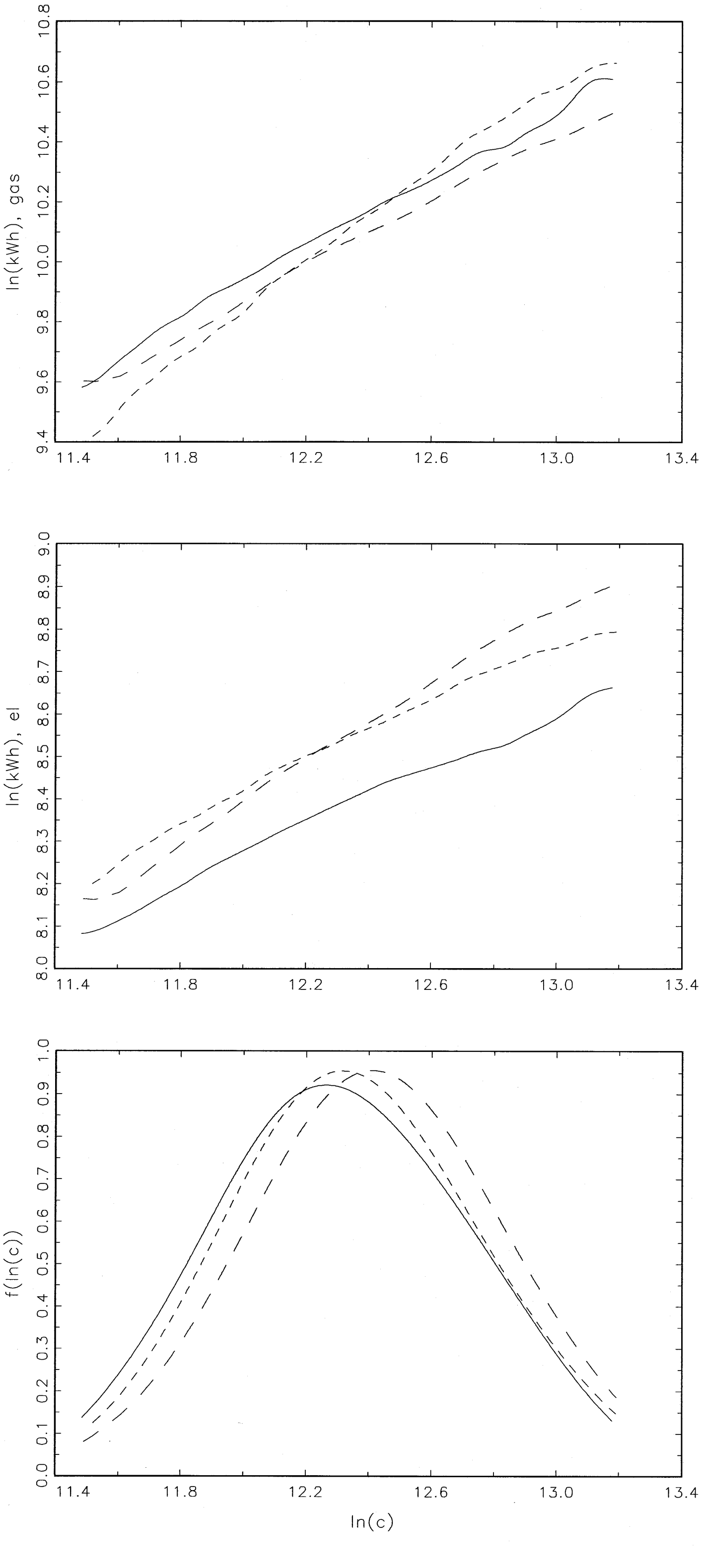


Figure 3.2.3. Nonparametric regression of consumption of energy on size of the house for Gas (top panel) and electricity (middle panel), and nonparametric density of size of the house (bottom panel)

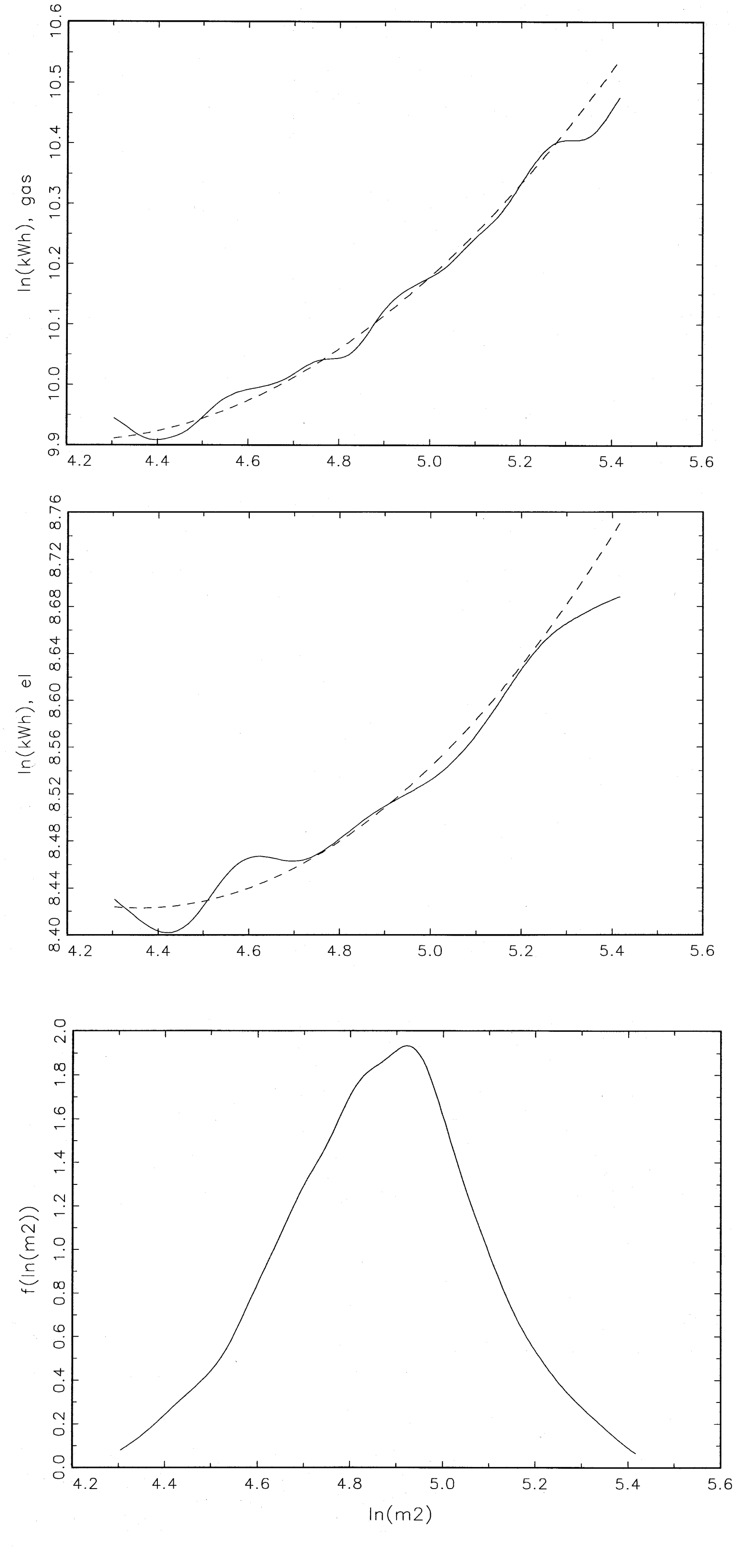


Figure 3.2.4. Nonparametric regression of consumption of energy on size of the house for Gas (top panel) and electricity (middle panel), and nonparametric density of house size (bottom panel). Solid line: no children, small lines: 1 child, big lines: 2 children.

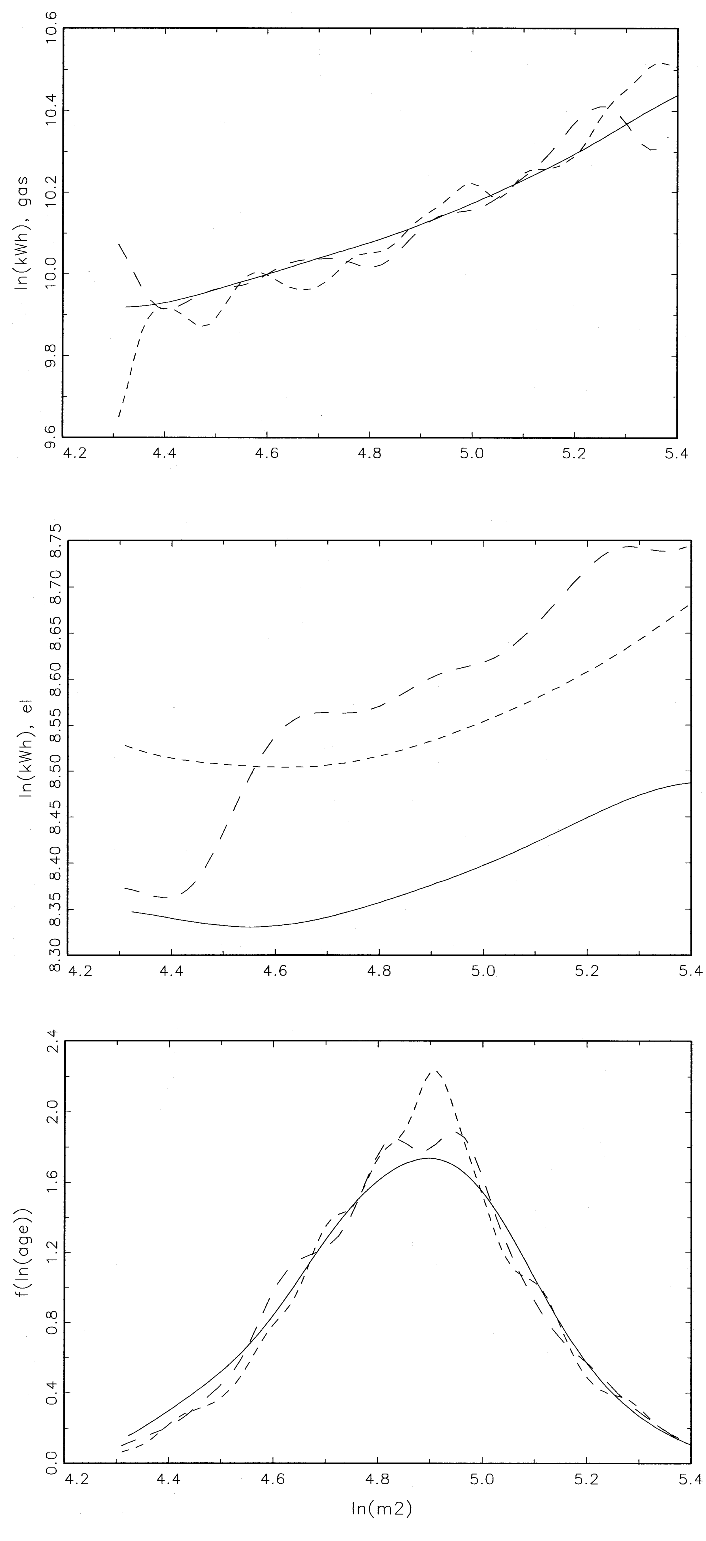


Figure 3.2.5. Nonparametric regression of consumption of energy on age of the oldest person in the household for Gas (top panel) and electricity (middle panel), and nonparametric density of age of the oldest person in the household (bottom panel)

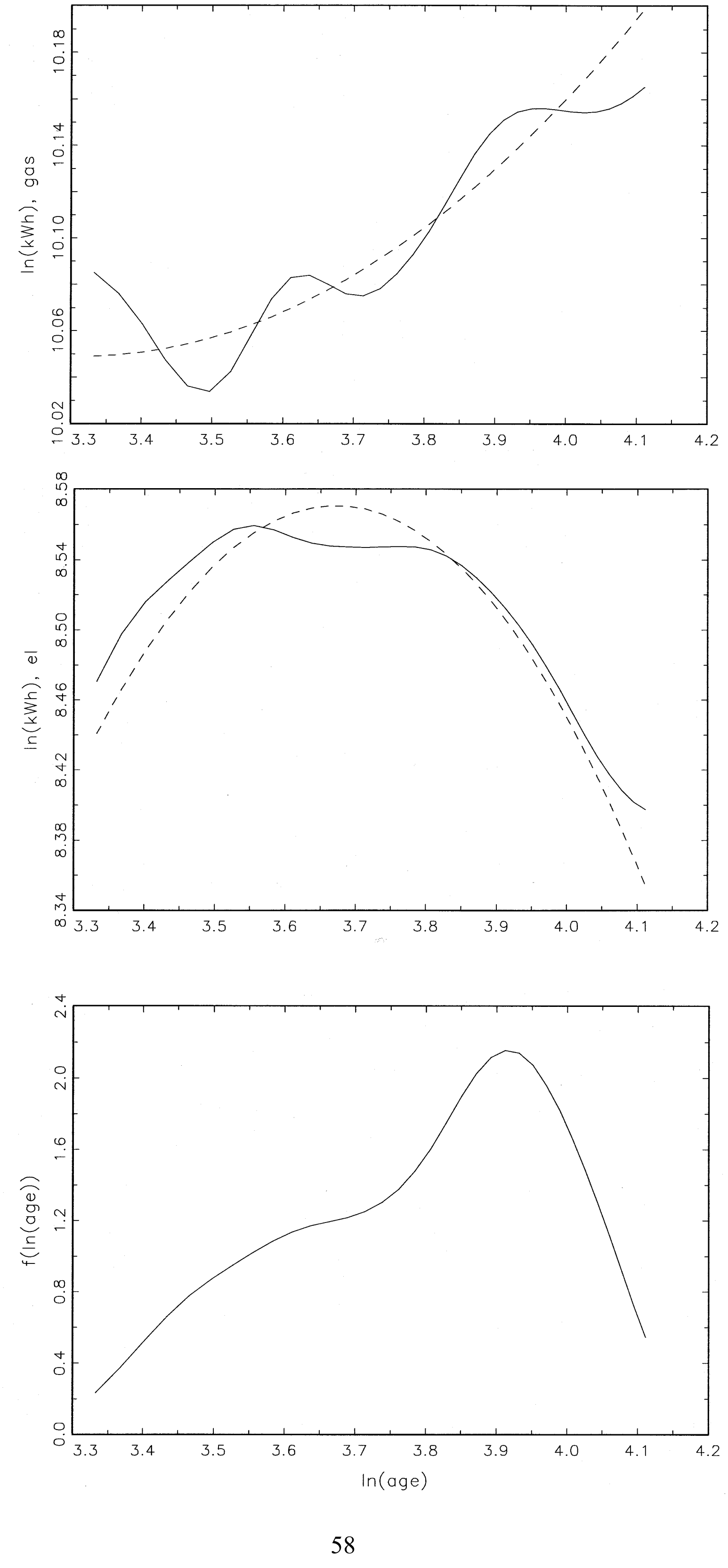
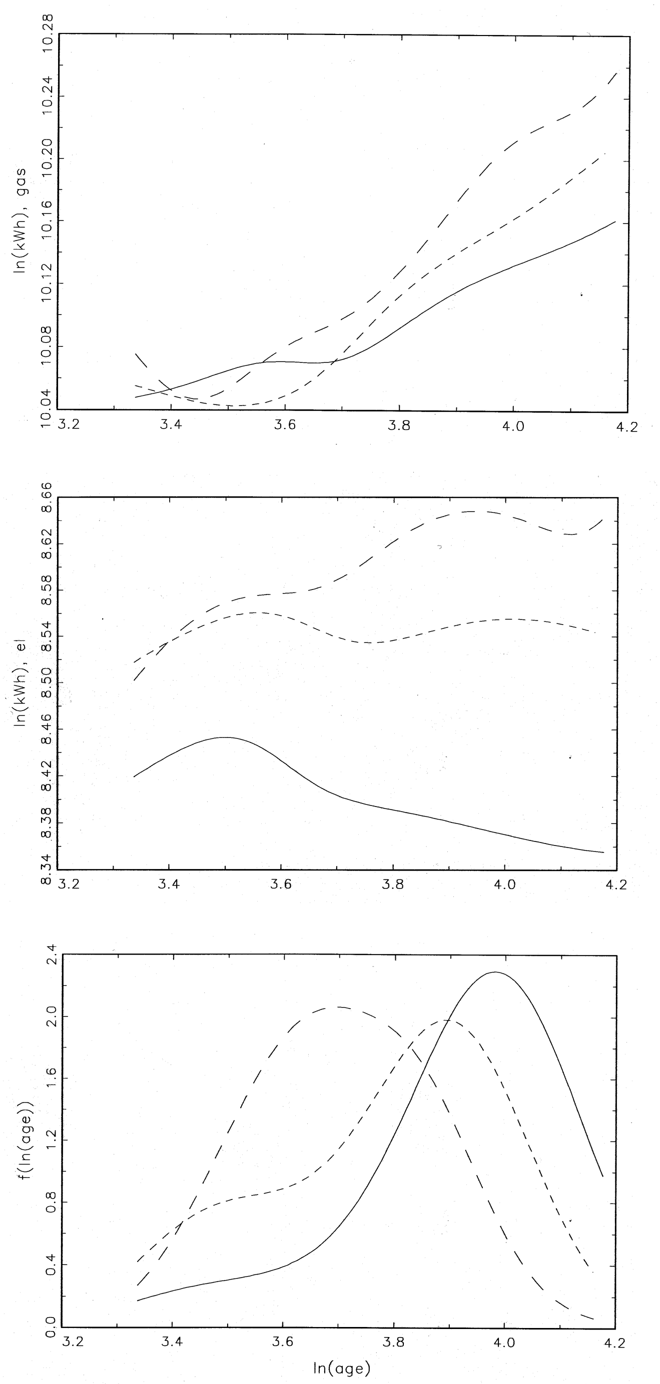
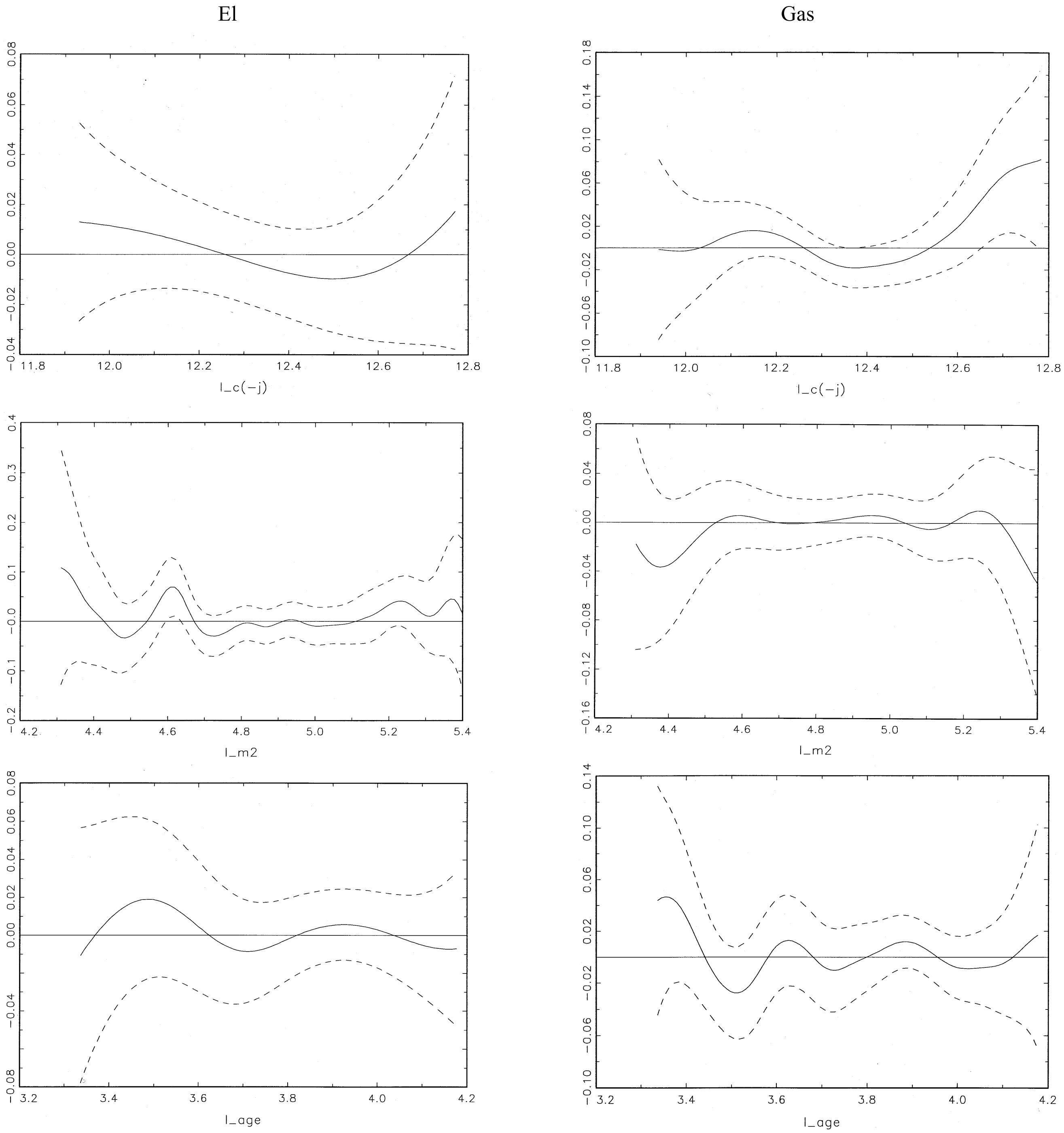


Figure 3.2.6. Nonparametric regression of consumption of energy on age of the oldest person in the household for Gas (top panel) and electricity (middle panel), and nonparametric density of on age of the oldest person in the household (bottom panel). Solid line: no children, small lines: 1 child, big lines: 2 children.



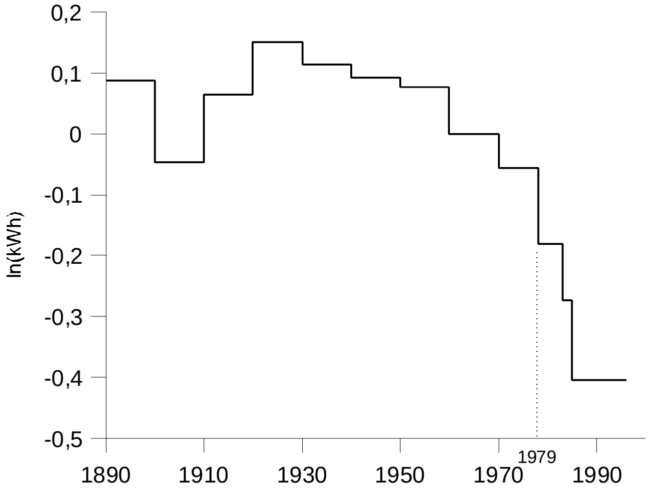
Note: Bandwidths for the nonparametric regresssions presented in this figure are chosen subjectively. This is done because the behaviour of generalised cross validation function is sensitive to areas where the density of observations is low. This is clearly the case for large intervals of the age distributions. For the present purpose, this is only of presentational importance.

Figure 5.1.1. Residual analysis. Nonparametric regressions of errors from estimated parametric model on total expenditures, top row, square metres, middle row, and age, bottom row. Regressions of errors from the electricity equation are located in the left column, and regressions of the errors from the gas equation are located in the right column.



Note: Bandwidths are chosen by generalised cross validation in all regressions except for the bandwidth in the error-age regression of the gas equation, bottom right corner. For this regression the bandwidth is chosen as $\sigma n^{-(1/5)}$.

Figure 5.1.2. Estimated Vintage Effects on Demand for Natural Gas



Note: Variations in the graph should be read as deviations from the level of demand for natural gas in buildings built in the period 1960-1969. Source: Table 1.