Nonparametric Estimation of Supply and Demand Factors with Applications to Labor and Macro Economics*

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Abstract

This paper presents a nonparametric model for estimating sharp bounds on the median of the disturbances in the simultaneous equations model and applies it to estimation of the supply and demand shift variables. A large literature has estimated shift variables in a supply and demand framework to analyze the causes of the fluctuations of the key economic variables. In the existing parametric approaches, the supply and demand shift variables are, however, unidentified either by using only observations of the intersections of the S&D curves or without specifying the S&D functional forms and distributions for the disturbances. In contrast, the model in this paper requires only observations of the intersections of upward-sloping supply and downward-sloping demand curves. It is applied to estimation of the bounds on macro supply and demand shocks since model specification issues have been serious area of conflict in study of the sources of business fluctuations, and estimation of the bounds on labor supply and demand shift variables since an existing research on the causes of wage changes could not identify supply and demand shifters due to the availability of data.

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1 Introduction

This paper presents a nonparametric econometric model for estimating sharp bounds on the median of the disturbances in the simultaneous equations model and applies it to estimation of the supply and demand shift variables in a simple supply and demand framework. The estimation of the causes of the fluctuations of the economic variables is of vital importance in economics. A supply and demand framework has been used for estimating the causes in a large literature. There are, however, several limitations with the existing modeling (parametric) approaches. First, the estimated supply and demand factors depend upon the specification of both functional forms for the structural relationships and distributions for the disturbances. Second, the supply and demand factors are unidentified by using only observations of the intersections of supply and demand curves due to the identification problem. In contrast, the econometric model in this paper requires: (1) upward-sloping supply and downward-sloping demand curves, (2) observing the intersections of the supply and the demand curves, and (3) information of the median of the disturbances. This methodology thus avoids introducing model specification issues into the problem of estimating shift variables and uses only data of the intersections of supply and demand curves to estimate supply and demand shift variables.

The approach is applied to estimation of the bounds on macro-wide supply and demand shocks using panel data on consumption goods (macroeconomics) and labor supply and demand shift variables using panel data on wages and labor inputs by different demographic and skill groups (labor economics). In macroeconomic example, the estimated shocks are subsequently used to investigate relative importance of each shock to business cycle fluctuations. The clarification of the source of business cycle fluctuation is a topic of vital importance in macroeconomics both from theoretical and policy-making perspectives.

Thus many economists have attempted to identify the sources of economic fluctuations. In the existing approaches the data is used to estimate the specified model and then the estimated disturbances are investigated to see the sources. Such existing methodologies require both specifications of functional forms for the structural relationships and the distributions for the disturbances; thus, the estimated sources depend upon the specification of both functional forms and distributions for the disturbances. Specification issues have been a serious area of conflict in macroeconomics, e.g. in Keynesian-versus-neoclassical debates. In contrast, this paper estimates the supply and demand shocks without specifying the functional forms or the distribution of the disturbances.[1]

For labor economics, this methodology is applied to estimating the causes of the change in the wage structure. Katz and Murphy (1992) and Murphy and Welch (1992) introduced a supply-demand framework into this issue.[2] They tried to assess the supply and demand factors causing the differentials of wage changes between demographic and skill groups. However, since the only intersections of the labor supply and demand functions (the equilibrium wages and labor input) are observed, the supply and demand factors causing the wage change could not be estimated. Thus, they "require that observed prices and quantities must be "on the demand curve"" (Katz and Murphy, 1992, p.46,l.40-41). In contrast, this paper uses panel data on wages and labor input to estimate the supply and demand factors causing the wage disparity of between-group.

Introducing additional assumptions into the econometric model, the bounds

[1] The structural vector autoregression methods are widely used for estimation of supply and demand shocks. (e.g., Blanchard and Quah (1989), Shapiro and Watson (1988), Gali (1992) and Bayumi and Eichengreen (1994)). They employed the multivariate time series analysis with the long run restriction induced by economic theoretical models to identify the aggregate demand and supply disturbances. However, none of them used the information on the relationship between shocks and equilibrium in the framework of upward-sloping supply and downward-sloping demand curves to identify the sources, although they use this information to check whether or not the estimated coefficients have plausible signs. In this paper this plausible information is used to estimate shocks.

[2] Katz and Autor (1999) provides a good summary of this issue.

on the shift variables narrow. The narrower estimated bounds may be more useful from the policy-making perspective. Bounds on the shift variables are estimated under a sequence of cumulative assumptions that have often been employed in the existing parametric estimations: (1) distribution of the disturbances is symmetric around zero and (2) a normal distribution with zero mean and unknown variance. The relative importance of the assumptions that were additionally introduced for estimation is investigated by using the estimated shift variables.

Basic ideas of the econometric methodology in this paper is related to Manski (1997) and Manski and Pepper (2000). Manski estimated the bounds on the probability that treatment response functions $h()$ at some covariates s is less than a specified constant c, i.e., $P[h(s) \leq c]$, by observing covariates, realized treatments and realized outcomes for a random sample of individuals. He assumed such prior information that response functions are monotone, semimonotone, or concave-monotone. On the other hand, the methodology in this paper estimates the bounds on the median of the disturbance that generates the distribution of $h()$ inside of the probability in order to investigate the causes of the fluctuations of economic variables. Therefore, his and our purposes and methodologies are different.

This paper is organized as follows. Section 2 discusses the methodology. Section 3 employs this methodology and Panel data on consumption goods and labor supply in the United States to investigate the macro supply and demand shocks and labor supply and demand shift variables. Section 4 concludes the paper.

2 Sharp Bounds on the Median of Disturbances in the Supply- Demand Framework

We study the following model with two endogenous variables and two disturbances and the methodology to estimate the medians of the disturbances.

Model 1:

$$\begin{cases} q = f(p) + u \\ q = g(p) + v \end{cases} \tag{1}$$

where p and q are endogenous variables, and u and v are disturbances. The population is formalized as a measure space (J, Ω, P) of agents, with P a probability measure. Then $P[(u,v),(p',q')]$ gives the distribution of disturbances and realized variables. $f()$ and $g()$ are an increasing and a decreasing function in p, respectively, and thus, the solution (q,p) satisfying equation (1) is unique, given (u,v).

Let us choose one point $(\overline{q}, \overline{p})$ that has associated disturbances: $(0,0)$, among the solutions satisfying equation (1) as a normalization. $NE(\alpha), SE(\alpha), NW(\alpha)$ and $SW(\alpha)$ represent the north east, south east, north west and south west regions of $(\overline{q}+\alpha, \overline{p})$ for some real number α, respectively (See Figure 1).

<<Figure 1>>

Proposition 1 *Suppose Model 1. For any real number* α*,*

(i) $P((q,p) \in SE(\alpha)) \leq P(u \geq \alpha) \leq 1 - P((q,p) \in NW(\alpha))$

(ii) $P((q,p) \in NE(\alpha)) \leq P(v \geq \alpha) \leq 1 - P((q,p) \in SW(\alpha))$

These bounds are sharp.

Proof) See appendix.

$P(u \geq \alpha) = 1 - F_u(\alpha)$ and $P(v \geq \alpha) = 1 - F_v(\alpha)$, where F_u and F_v are the cumulative distribution function of the disturbances u and v, respectively.

Corollary 2 *Suppose Model 1. For any real number* α*,*

(i) $P((q,p) \in NW(\alpha)) \leq F_u(\alpha) \leq 1 - P((q,p) \in SE(\alpha))$

(ii) $P((q,p) \in SW(\alpha)) \leq F_v(\alpha) \leq 1 - P((q,p) \in NE(\alpha))$

These bounds are sharp[3].

Fixing α, the bounds on $F_u(\alpha)$ are estimated by using the inequality (i) in Corollary 2 and the analogous sample probability: $P((q,p) \in NW(\alpha))$ and $P((q,p) \in SE(\alpha))$. Similarly, the bounds on $F_u(\alpha)$ are estimated for any α in the set of real number. Consequently, the bounds on the function, $F_u(\bullet)$, are estimated. Similarly, the bounds on the function, $F_v(\bullet)$, are estimated by using the inequality (ii) and the analogous sample probability: $P((q,p) \in SW(\alpha))$ and $P((q,p) \in NE(\alpha))$, for any real number α.

The goal of our methodology is to estimate the medians of u and v, which are defined as $\overline{u}$ and $\overline{v}$, respectively.

Lemma 3 *Suppose that F_u and F_v are strictly increasing in the neighborhood of $\overline{u}$ and $\overline{v}$, respectively.*

(i) Take

$\alpha_1 = \min\{a_1 | P((q,p) \in NW(a_1)) = 0.5\}, \alpha_2 = \max\{a_2 | P((q,p) \in SE(a_2)) = 0.5\}.$

Then,

$$\alpha_2 \leq \overline{u} \leq \alpha_1 \tag{2}$$

(ii)" Take

$\beta_1 = \min\{b_1 | P((q,p) \in SW(b_1)) = 0.5\}, \beta_2 = \max\{b_2 | P((q,p) \in NE(b_2)) = 0.5\}.$

Then,

$$\beta_2 \leq \overline{v} \leq \beta_1 \tag{3}$$

[3] As an extreme example, for some α the support of $P((q,p))$ may be concentrated in the $SE(\alpha)$ and $NW(\alpha)$ regions. Then, $F_u(\alpha)$ for this α is identified. However, $0 \leq F_v(\alpha) \leq 1$ and therefore, are uninformative on identification of v. On the other hand, for some α the support of $P((q,p))$ may be concentrated in the $NE(\alpha)$ and $SW(\alpha)$ regions. Then, $F_v(\alpha)$ for this α is identified. However, $0 \leq F_u(\alpha) \leq 1$, and therefore, are uninformative on identification of v.

Proof) See Appendix.

Introducing assumptions narrow the estimates of the bounds of the medians of the disturbances. The following two assumptions on the distributions of u and v are considered.

Assumption 1.

The distributions of u and v are symmetric around the medians of u and v, respectively.

Assumption 2.

The distributions of $u-\overline{u}$ and $v-\overline{v}$ are known by an econometrician, where $\overline{u}$ and $\overline{v}$ are medians of u and v, respectively.

Lemma 4 *Suppose that assumption 1 holds and F_u and F_v are strictly increasing.*

(i) Take $\alpha_1(\gamma)=\min a_1(\gamma),\alpha_2(\gamma)=\max a_2(\gamma),\alpha_3(\gamma)=\min a_3(\gamma)$ *and* $\alpha_4(\gamma)=\max a_4(\gamma)$ *satisfying*

$$\begin{array}{l}\gamma=P\left((q,p)\in NW(a_1(\gamma))\right)=1-P\left((q,p)\in SE(a_2(\gamma))\right)\\ =1-P\left((q,p)\in NW(a_3(\gamma))\right)=P\left((q,p)\in SE(a_4(\gamma))\right)\end{array}.$$

Then,

$$\max_{\gamma}\left[\alpha_2(\gamma)+\alpha_4(\gamma)\right]/2\leq\overline{u}\leq\min_{\gamma}\left[\alpha_1(\gamma)+\alpha_3(\gamma)\right]/2.$$

(ii) Take $\beta_1(\gamma)=\min b_1(\gamma),\beta_2(\gamma)=\max b_2(\gamma),\beta_3(\gamma)=\min b_3(\gamma)$ *and* $\beta_4(\gamma)=\max b_4(\gamma)$ *satisfying*

$$\begin{array}{l}\gamma=P\left((q,p)\in SW(b_1(\gamma))\right)=1-P\left((q,p)\in NE(b_2(\gamma))\right)\\ =1-P\left((q,p)\in SW(b_3(\gamma))\right)=P\left((q,p)\in NE(b_4(\gamma))\right)\end{array}$$

Then,

$$\max_{\gamma}\left[\beta_2(\gamma)+\beta_4(\gamma)\right]/2\leq\overline{v}\leq\min_{\gamma}\left[\beta_1(\gamma)+\beta_3(\gamma)\right]/2$$

Proof) See Appendix.

Lemma 5 *Assume assumption 2. Define the distributions of $u-\overline{u}$ and $v-\overline{v}$ as $\widetilde{F_u}$ and $\widetilde{F_v}$, respectively.*

(i)

$$\max_{\alpha}\left\{\alpha - \widetilde{F_u}^{-1}\left[1 - P((q,p)\in SE(\alpha))\right]\right\} \leq \overline{u} \leq \min_{\alpha}\left\{\alpha - \widetilde{F_u}^{-1}\left[P((q,p)\in NW(\alpha))\right]\right\} \tag{4}$$

(ii)

$$\max_{\alpha}\left\{\alpha - \widetilde{F_v}^{-1}\left[1 - P((q,p)\in SW(\alpha))\right]\right\} \leq \overline{v} \leq \min_{\alpha}\left\{\alpha - \widetilde{F_v}^{-1}\left[P((q,p)\in NE(\alpha))\right]\right\} \tag{5}$$

3 Applications

3.1 Macro-wide Supply and Demand Shocks

This section applies our methodology and panel data on consumption expenditures in the United States to estimate the bounds on the supply and demand shocks. We use the US National Income Account's *personal consumption expenditures by type of product* from 1951 to 1997 for the quantity and price indices in the model.

Suppose that (real) quantities and prices of categories of personal consumption goods (or final products) in the United States are determined via the following model, which is the extension of Model 1, equation (1).

Model 2:

$$\begin{cases} \widehat{q_{it}} = f_{it}(\widehat{p_{it}}) + \mu_t + \varepsilon_{it} \\ \widehat{q_{it}} = g_{it}(\widehat{p_{it}}) + \nu_t + \xi_{it} \end{cases} \tag{6}$$

where $\widehat{q_{it}} = q_{it} - \overline{q_{it}}$ and $\widehat{p_{it}} = p_{it} - \overline{p_{it}}$. $(\overline{q_{it}}, \overline{p_{it}})$ is a normalization. The medians of ε_{it} and ξ_{it} for i are zero. $f_{it}(\ )$ and $g_{it}(\ )$ are strictly increasing and decreasing functions, respectively. $f_{it}(0) = g_{it}(0) = 0$. i is the index of the category of personal consumption goods, t is the time index, and q_{it} and p_{it} are, respectively, quantities and prices of categories of personal consumption goods. μ_t and ν_t respectively represent common supply and demand shocks. ε_{it}

and ξ_{it} respectively represent idiosyncratic supply and demand shocks. $\mu_t + \varepsilon_{it}$ and $\nu_t + \xi_{it}$ correspond to u and v in Model 1, respectively, and thus μ_t and ν_t correspond to $\overline{u}$ and $\overline{v}$, respectively.

This model demonstrates that the quantities and prices of personal consumption goods co-move across the categories of goods due to the common supply and demand shocks, μ_t and ν_t, systematically. On the other hand, their independent movement is caused by independent supply and demand shocks, ε_{it} and ξ_{it}. It is crucial that the supply and demand curves, $f_{it}(\ )$ and $g_{it}(\ )$, do not have to be specified and may be different across the categories and time.

We can interpret the model 2 as the model of panel data with fixed effects. Thus, $(\widehat{p_{it}}, \widehat{q_{it}})$ are observations, and (μ_t, ν_t) are unknown parameters ("individual effects") to be estimated. We use Lemmas 3, 4 and 5 to estimate μ_t and ν_t. In Lemmas 4 and 5, we assume that the distributions of ε_{it} and ξ_{it} for i are time-invariant. The procedure of estimation is as follows. First, we fix time t and α. Second, we estimate the bounds on μ_t and ν_t, which correspond to the bounds on $\overline{u}$ and $\overline{v}$, respectively, in Lemmas 3, 4 and 5, by replacing the probabilities indicating the bounds with the corresponding sample frequencies. For example, the estimate of $P\left((\widehat{q_{it}}, \widehat{p_{it}}) \in NW(\alpha)\right) =$ the number of the samples of $(\widehat{q_{it}}, \widehat{p_{it}})$ which locate in the $NW(\alpha)$ region at time t / the total number of the samples of $(\widehat{q_{it}}, \widehat{p_{it}})$ at time t. Similarly are obtained the estimates of the other probabilities, $P((\widehat{q_{it}}, \widehat{p_{it}}) \in SE(\alpha))$, $P((\widehat{q_{it}}, \widehat{p_{it}}) \in SW(\alpha))$ and $P((\widehat{q_{it}}, \widehat{p_{it}}) \in NE(\alpha))$. Third, we take any real number $\alpha\,(\in [-\infty, \infty])$ and repeat the procedure of the first and the second steps for each α. Fourth, we repeat the procedure (the first, second and third steps) for any time t in the sample periods, and then obtain the time series of the estimates of the bounds on μ_t and ν_t.

How to choose the normalization point $(\overline{q_{it}}, \overline{p_{it}})$ depends on economic problems to which this methodology is applied. For example, the following model based on innovations is often investigated in macroeconomics.

Model 3:

$$\left\{ \begin{array}{l} \widehat{q_{it}} = f_{it}\widehat{p_{it}} + \widehat{\mu}_t + \widehat{\varepsilon_{it}} \\ \widehat{q_{it}} = g_{it}\widehat{p_{it}} + \widehat{\nu}_t + \widehat{\xi_{it}} \end{array} \right. \tag{7}$$

where $\widehat{z_{it}} = z_{it} - E(z_{it} \mid I_{t-1})$, ($z = q, p, \mu, \nu, \varepsilon$ and ξ) and I_{t-1} is information available at t-1.

That is, we take $(\overline{q_{it}}, \overline{p_{it}}) = (E(q_{it} \mid I_{t-1}), E(p_{it} \mid I_{t-1}))$ as a normalization. The $\widehat{z_{it}}$s are innovations and considered unexpected changes of the variables and shocks. It should be noted that $E_t(\widehat{z_{it}}) = 0$ $(z = q, p, \mu, \nu, \varepsilon, \xi)$, where $E_t(\widehat{z_{it}})$ is the mean of $\widehat{z_{it}}$ over time.

We apply two types of formations of expectations, $E(x_{it} \mid I_{t-1})$, in order to replace them with data. Both types are often used in macroeconomics.

(A): $E(x_{it} \mid I_{t-1}) = E_t\left(x_{it}\right)$, where E_t is the mean over t.

(B): $E(x_{it} \mid I_{t-1}) = a(L)x_{it}$, where $a(L) = a_1L + a_2L^2 + ... + a_nL^n$.[4]

Figures 2-7 depict the estimated bounds of the common supply and demand shocks by pairing Lemmas (3, 4, 5) with ((A),(B)).

<<Figures 2-7>>

When Lemma 5 is applied for estimation, we assume the distribution of ε_{it} and ξ_{it} a normal distribution where their mean is zero and their variances are σ_ε^2 and σ_ξ^2, satisfying the followings, respectively,

$$Max_\alpha \left\{\alpha - \Phi_\varepsilon^{-1}\left[1 - P((\widehat{q_{it}}, \widehat{p_{it}}) \in SE(\alpha))\right]\right\} \leq Min_\alpha \left\{\alpha - \Phi_\varepsilon^{-1}\left[P((\widehat{q_{it}}, \widehat{p_{it}}) \in NW(\alpha))\right]\right\}$$

and

$$Max_\alpha \left\{\alpha - \Phi_\xi^{-1}\left[1 - P((\widehat{q_{it}}, \widehat{p_{it}}) \in NE(\alpha))\right]\right\} \leq Min_\alpha \left\{\alpha - \Phi_\xi^{-1}\left[P((\widehat{q_{it}}, \widehat{p_{it}}) \in SW(\alpha))\right]\right\},$$

where $\Phi_y\left(x\right) = 1/\sigma_y \phi\left(x/\sigma_z\right)$ $(y = \varepsilon, \xi)$.

[4] If we know $a(L)$, we can identify $\widehat{\mu}_t = \left[1 - a(L)\right]\mu_t$ and $\widehat{\nu}_t = \left[1 - a(L)\right]\nu_t$, that are independent over time. In the empirical results in this section, $a(L) = L$ is applied. Conversely, without knowing $a(L)$, we can identify $\mu_t = \left[1 - a(L)\right]^{-1}\widehat{\mu}_t(= \widehat{\mu}_t + b_1\widehat{\mu_{t-1}} + b_2\widehat{\mu_{t-2}} + ...)$ and $\nu_t = \left[1 - a(L)\right]^{-1}\widehat{\nu}_t(= \widehat{\nu}_t + b_1\widehat{\nu_{t-1}} + b_2\widehat{\nu_{t-2}} + ...)$. In other words, we can identify the accumulated common supply and demand shocks that cause shifts of supply and demand curves in the current term, respectively.

Interpretation of the figures

The National Bureau for Economic Research (NBER) identifies troughs. Troughs occur in the years 1954, 1958, 1961, 1970, 1975, 1980, 1982 and 1991, according to this Institute. The figures suggest which supply or demand shocks are responsible for each recession. The estimated bounds in all figures have some tendency in common. That is,

(1) In the 1954 recession, an unfavorable demand shock was responsible for the recession.

(2) In the 1958 recession, an unfavorable supply shock caused the recession.

(3) In the 1961 recession, an unfavorable demand shock explained the recession.

(4) In the 1970 recession, a supply shock was responsible for the recession.

(5) In the 1974 recession, a supply shock caused the recession.

(6) In the 1980 recession, a supply shock accounted for the recession.

(7) In the 1982 recession, a demand shock was responsible.

(8) In the 1991 recession, a demand shocks explained the recession.

Shapiro and Watson (1988), Gali (1992) and Bayumi and Eichengreen (1994) uses the structural vector autoregression methods to estimate the causes of economic fluctuations. They assume that aggregate demand shocks do not cause the shift of the aggregate supply schedule in the long run for identification. The inferred causes of recessions in this paper is somewhat similar to Shariro and Watson's.

3.2 Labor Supply and Demand Factors

This section applies our methodology and data on wages and labor input in the United States to estimate the bounds on the labor supply and demand shift variables. The procedure is the same as the application to macroeconomic supply and demand shocks.

The data of the index of mean of real weakly wages in 1982 dollar and labor supply in person hours measured in efficiency units from 1963 to 1987 for the United States are the same as those used in Katz and Murphy (1992) and Murphy, Riddell and Romer (1998). The raw data comes from the March Current Population Survey. Following these papers we categorize the data into 64 groups defined by sex, education, and experience. To investigate the factors of the wage differentials by sex and education, we classify 64 groups of the data into four categories: the male high-school equivalents, the male college equivalents, the female high-school equivalents and the female college equivalents. The inflation rate of real wage and the growth rate of labor supply are used as $\widehat{p_{it}}$ and $\widehat{q_{it}}$, respectively (i and t represent groups and time, respectively).

Figures 8-15 show the estimations of the bounds on labor supply and demand shift variables in the four categories. The expectation is assumed the mean over time. The distribution of the disturbance is assumed to have a zero mean in figures 8-11 and to be a normal distribution in figures 12-15.

<<Figures 8-15>>

Interpretation of the figures

Katz and Murphy (1992) found that college wage premia, which is defined as the difference of wages between the high-school and college equivalents, rose in 60s, declined substantially in 70s, increased sharply in 80s and continued to rise modestly in 90s. Blau and Kahn (1999) summarized the estimation results of the male and female wage differentials as follows. The differentials increased in 60s and decreased in 70s and 80s. In the first half of 80s, it decreased more rapidly for high school equivalents than for college equivalents.

Our estimation of the bounds on supply and demand shift variables suggest that:

For the college wage premia for male,

(1) In the latter half of 60s, the demand for college graduates increased

whereas their labor supply decreased, raising their wage. This explains why the wage disparity between educational levels rose in 60s.

(2) In 70s, on the contrary the demand for college graduates declined although their labor supply rose, decreasing the their wage. The behavior of and for college equivalents is important for the wage disparity in 70s.

(3) In 80s, the demand for college graduates increased but their labor supply decreased, raising the their wage. On the other hand, the demand for high-school graduates decreased whereas their labor supply increased, lowering the their wage. This explains why the wage disparity between educational levels rapidly rose in 80s.

For the male and female wage differentials,

(1) In 60s, in the contrary to the factor changes for male, the demand for female decreased and their labor supply increased. Thus the female wage declined whereas the male wage rose. This explains why the wage disparity between sex increased in 60s.

(2) In 70s, both demands for male and female college graduates decreased and both supply of male and female college graduates increased, except for 1975. However, the demand for female high school graduates rose and their supply declined, causing the male and female wage differentials to decrease in 70s.

(3) In 80s, both demands for male and female high school graduates decreased. However the supply of female high school graduates decreased in the first half of 80s whereas the supply of male high school graduates increased. The decrease of demand for both sexes caused both wages to decrease. The different high-school graduate's labor supply behaviors between sexes however caused the female wage to increase and the male wage to decrease, thus lowering the wage disparity between sexes of high-school graduates. This finding accounts for the fact in 80s stated above.

4 Conclusion

This paper presents the methodology to identify the supply and demand shift variables without identifying the full system but by assuming upward-sloping supply and downward-sloping demand curves. Employing this methodology and the US postwar Panel data on consumption goods and labor supply, this paper estimates the supply and demand shift variables to clarify the sources of business cycle fluctuation and the sources of the relative wage changes of between-group. This paper therefore provides another viewpoint tackling identification of shocks and shift variables, that is, fully nonparametric methods.

Since the assumption for identification is less restrictive than the existing parametric approach, the presented methodology estimates the bounds on shift variables but cannot pin them down. Instead, introducing additional assumptions narrows the bounds. However, what assumptions are proper to introduce and in what degree the introduced assumptions narrow the estimates of the bounds are important inquiries from theoretical and policy-making perspectives.

The lag structure needs to be specified for estimation of shocks when shocks and variables are considered to be time-dependent. It is better that the lag structure can be also nonparametrically estimated.

These topics, however, are left for future research.

5 Appendix

Proof of Proposition 1.

Since f is monotone increasing in p, for any real number α,

Thus,

$$\begin{cases} (q,p) \in SE(\alpha) \Rightarrow u > \alpha \\ (q,p) \in NW(\alpha) \Rightarrow u < \alpha. \end{cases}$$

Thus,

$$P\left((q,p)\in SE\left(\alpha\right)\right)\leq P(u>\alpha)\leq 1-P\left((q,p)\in NW\left(\alpha\right)\right)$$

These bounds are sharp, since the empirical evidence and prior information are consistent with the hypothesis $\{(q,p)\in SE\left(\alpha\right)\}$ and the hypothesis $\{(q,p)\in NW\left(\alpha\right)\}$.

Since g is monotone decreasing in p, for any real number α,

$$\begin{cases} (q,p)\in NE\left(\alpha\right)\Rightarrow v>\alpha \\ (q,p)\in SW\left(\alpha\right)\Rightarrow v<\alpha. \end{cases}$$

Thus,

$$P\left((q,p)\in NE\left(\alpha\right)\right)\leq P(v>\alpha)\leq 1-P\left((q,p)\in SW\left(\alpha\right)\right)$$

These bounds are sharp, since the empirical evidence and prior information are consistent with the hypothesis $\{(q,p)\in NE\left(\alpha\right)\}$ and the hypothesis $\{(q,p)\in SW\left(\alpha\right)\}$. Q.E.D.

Proof of Lemma 3.

Corollary 2 and the definition of α_1 imply that

$$P\left((q,p)\in NW\left(\overline{u}\right)\right)\leq F_u(\overline{u})=0.5=P\left((q,p)\in NW\left(\alpha_1\right)\right).$$

Since $P\left((q,p)\in NW\left(\alpha\right)\right)$ is increasing in α and $F_u(\alpha)$ is strictly increasing in α in the neighborhood of $\overline{u}$,

$$\overline{u}\leq\alpha_1.$$

Corollary 2 and the definition of α_2 imply that

$$1-P\left((q,p)\in SE\left(\alpha_2\right)\right)=0.5=F_u(\overline{u})\leq 1-P\left((q,p)\in SE\left(\overline{u}\right)\right).$$

Since $P\left((q,p) \in SE\left(\alpha\right)\right)$ is decreasing in α and $F_u(\alpha)$ is strictly increasing in α in the neighborhood of $\overline{u}$,

$$\alpha_2 \leq \overline{u}.$$

Hence,

$$\alpha_2 \leq \overline{u} \leq \alpha_1.$$

Q.E.D.

Proof of Lemma 4.

Define the γ quantile of F_u as m_γ^u.

Corollary 2 and the definition of α_1 imply that

$$P\left((q,p) \in NW\left(m_\gamma^u\right)\right) \leq F_u(m_\gamma^u) = \gamma = P\left((q,p) \in NW\left(\alpha_1\left(\gamma\right)\right)\right).$$

Since $P\left((q,p) \in NW\left(\alpha\right)\right)$ is increasing in α and $F_u\left(\alpha\right)$ is strictly increasing in α,

$$m_\gamma^u \leq \alpha_1\left(\gamma\right).$$

Corollary 2 and the definition of $\alpha_2\left(\gamma\right)$ imply that

$$1 - P\left((q,p) \in SE\left(\alpha_2\left(\gamma\right)\right)\right) = \gamma = F_u(m_\gamma^u) \leq 1 - P\left((q,p) \in SE\left(m_\gamma^u\right)\right).$$

Since $P\left((q,p) \in SE\left(\alpha\right)\right)$ is decreasing in α and $F_u(\alpha)$ is strictly increasing in α,

$$\alpha_2\left(\gamma\right) \leq m_\gamma^u.$$

Hence,

$$\alpha_2(\gamma) \leq m^u_\gamma \leq \alpha_1(\gamma).$$

Subtracting $\overline{u}$ from both sides,

$$\alpha_2(\gamma) - \overline{u} \leq m^u_\gamma - \overline{u} \leq \alpha_1(\gamma) - \overline{u}. \tag{8}$$

Similarly for the $(1-\gamma)$ quantile of F_u,

$$\begin{aligned} P\left((q,p) \in NW\left(m^u_{1-\gamma}\right)\right) &\leq F_u(m^u_{1-\gamma}) = 1-\gamma = P\left((q,p) \in NW\left(\alpha_3(\gamma)\right)\right). \\ 1 - P\left((q,p) \in SE\left(\alpha_4(\gamma)\right)\right) &= 1-\gamma = F_u(m^u_{1-\gamma}) \leq 1 - P\left((q,p) \in SE\left(m^u_{1-\gamma}\right)\right). \end{aligned}$$

Hence,

$$\alpha_4(\gamma) \leq m^u_{1-\gamma} \leq \alpha_3(\gamma).$$

Thus,

$$\alpha_4(\gamma) - \overline{u} \leq m^u_{1-\gamma} - \overline{u} \leq \alpha_3(\gamma) - \overline{u}. \tag{9}$$

Since the symmetry of the distribution of u around $\overline{u}$ imply $m^u_{1-\gamma} - \overline{u} = -\left(m^u_{1-\gamma} - \overline{u}\right)$, equations (8) and (9) imply that

$$\left(\alpha_2(\gamma) + \alpha_4(\gamma)\right)/2 \leq \overline{u} \leq \left(\alpha_1(\gamma) + \alpha_3(\gamma)\right)/2. \tag{10}$$

Since equation (10) holds for any γ,

$$\max_\gamma \left[\left(\alpha_2(\gamma) + \alpha_4(\gamma)\right)/2\right] \leq \overline{u} \leq \min_\gamma \left[\left(\alpha_1(\gamma) + \alpha_3(\gamma)\right)/2\right].$$

Similarly for $\overline{v}$, since $P\left((q,p) \in Sw(\alpha)\right)$ and $P\left((q,p) \in NE(\alpha)\right)$ are increasing and decreasing in α, respectively,

$$\max_{\gamma} \left[\left(\beta_2 \left(\gamma \right) + \beta_4 \left(\gamma \right) \right) /2 \right] \leq \overline{v} \leq \min_{\gamma} \left[\left(\beta_1 \left(\gamma \right) + \beta_3 \left(\gamma \right) \right) /2 \right].$$

Q.E.D.

Proof of Lemma 5.

Since $F_u(\alpha) = \widetilde{F_u}(\alpha - \overline{u})$, Corollary 2 implies that

$$P\left((q,p) \in NW\left(\alpha \right) \right) \leq \widetilde{F_u}(\alpha - \overline{u}) \leq 1 - P\left((q,p) \in SE\left(\alpha \right) \right).$$

Since $\widetilde{F_u}$ is known, taking inverse of $\widetilde{F_u}$,

$$\widetilde{F_u}^{-1} \left[P\left((q,p) \in NW\left(\alpha \right) \right) \right] \leq \alpha - \overline{u} \leq \widetilde{F_u}^{-1} \left[1 - P\left((q,p) \in SE\left(\alpha \right) \right) \right].$$

I.e,

$$\alpha - \widetilde{F_u}^{-1} \left[1 - P\left((q,p) \in SE\left(\alpha \right) \right) \right] \leq \overline{u} \leq \alpha - \widetilde{F_u}^{-1} \left[P\left((q,p) \in NW\left(\alpha \right) \right) \right].$$

Since it holds for any real number α,

$$\max_{\alpha} \left\{ \alpha - \widetilde{F_u}^{-1} \left[1 - P\left((q,p) \in SE\left(\alpha \right) \right) \right] \right\} \leq \overline{u} \leq \min_{\alpha} \left\{ \alpha - \widetilde{F_u}^{-1} \left[P\left((q,p) \in NW\left(\alpha \right) \right) \right] \right\}.$$

Similarly, since $F_v(\alpha) = \widetilde{F_v}(\alpha - \overline{u})$ and $\widetilde{F_v}$ is known,

$$\max_{\alpha} \left\{ \alpha - \widetilde{F_v}^{-1} \left[1 - P\left((q,p) \in SW\left(\alpha \right) \right) \right] \right\} \leq \overline{v} \leq \min_{\alpha} \left\{ \alpha - \widetilde{F_v}^{-1} \left[P\left((q,p) \in NE\left(\alpha \right) \right) \right] \right\}.$$

Q.E.D.

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Figure 1.

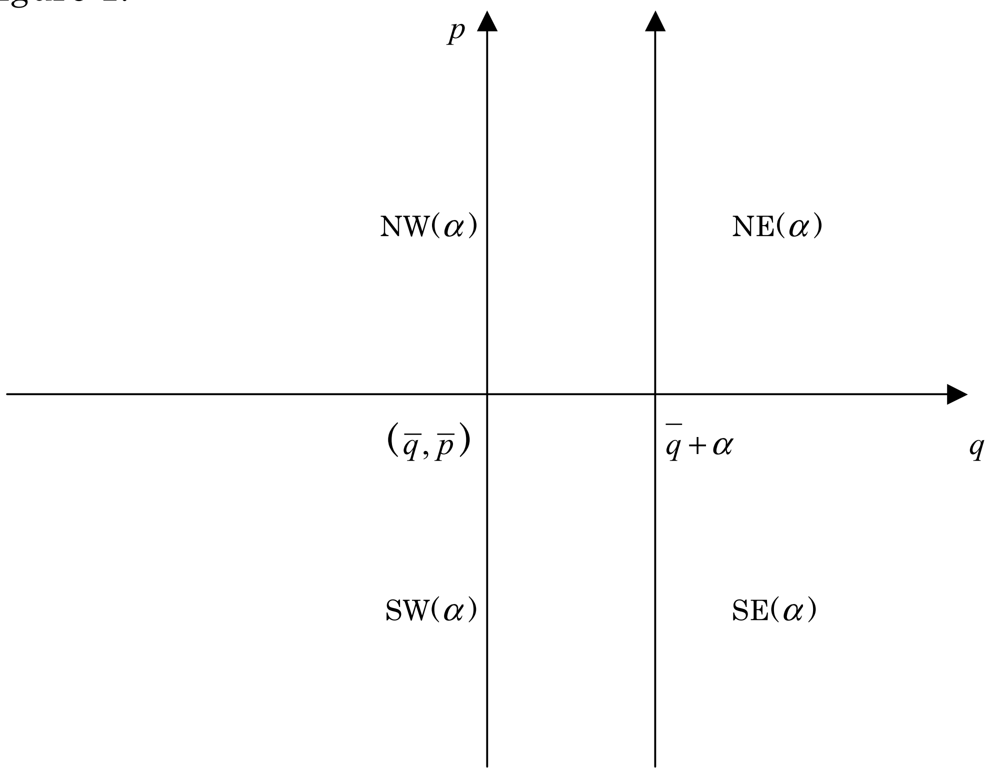
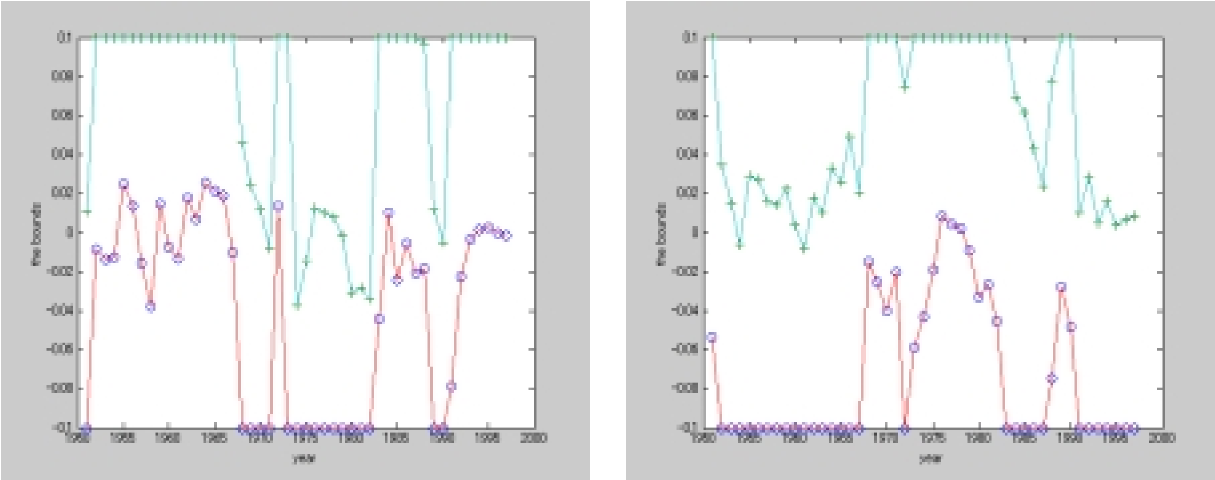


Figure 2. Macro supply and demand shocks

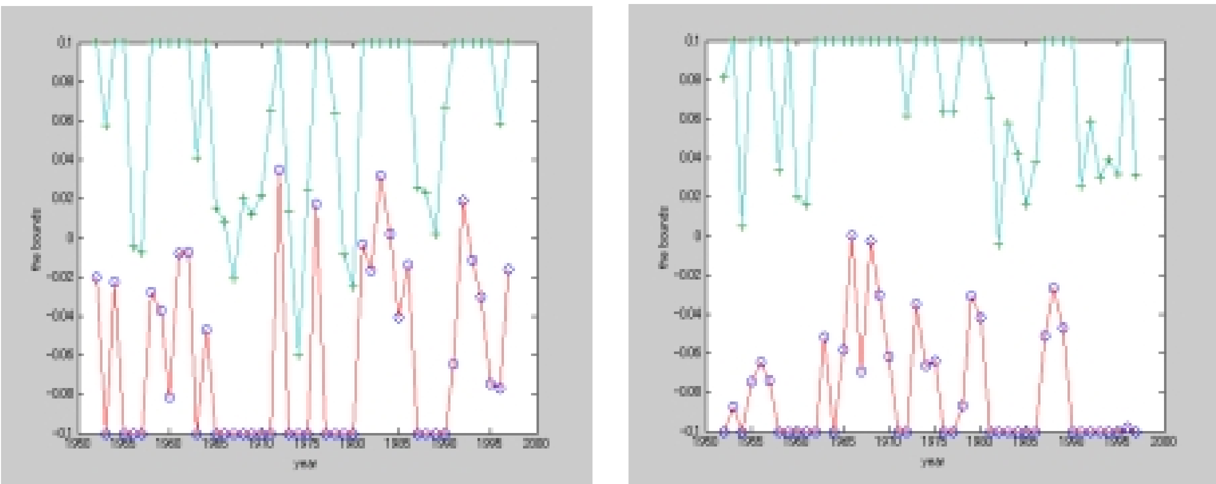
(a) Supply shocks (b) Demand shocks



Note. It is assumed that the median of the disturbances is zero and the expectation is the mean over time.

Figure 3. Macro supply and demand shocks

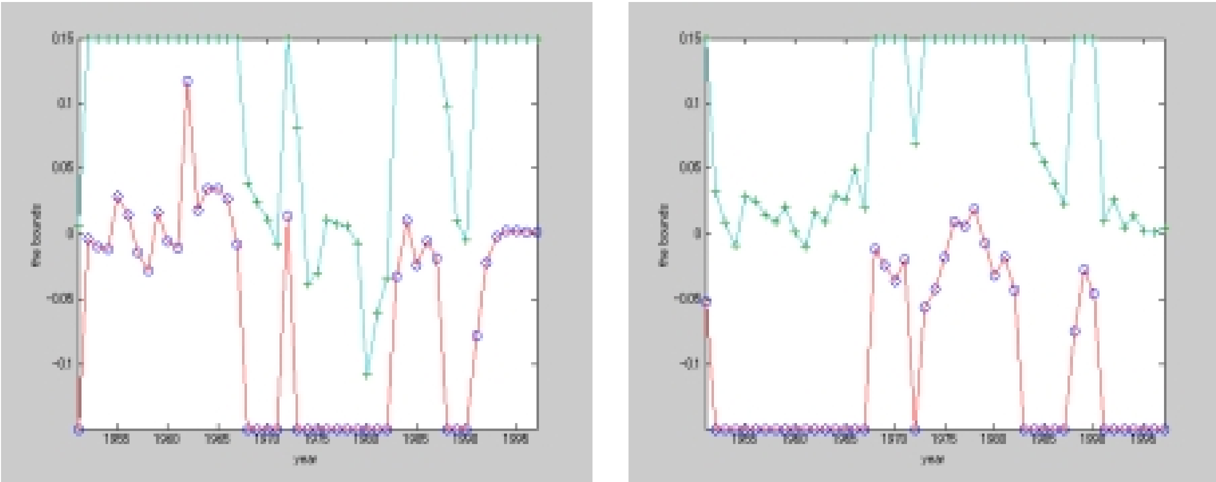
(a) Supply shocks (b) Demand shocks



Note. It is assumed that the median of the disturbances is zero and the expectation is a lagged variable.

Figure 4. Macro supply and demad shocks

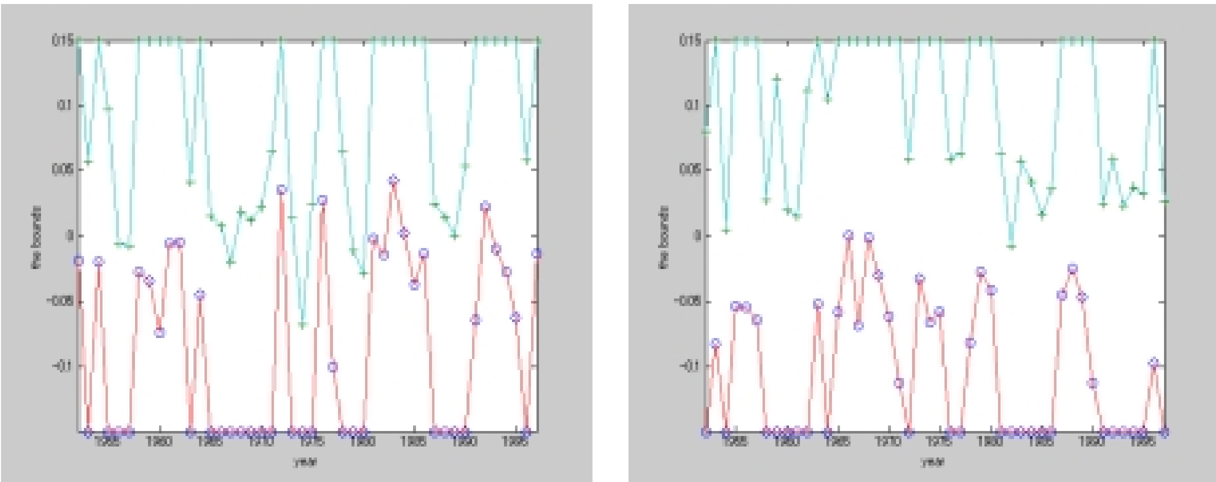
(a) Supply shocks (b) Demand Shocks



Note. It is assumed that the disturbances are symmetric around zero median and the expectation is the mean over time.

Figure 5. Macro supply and demad shocks

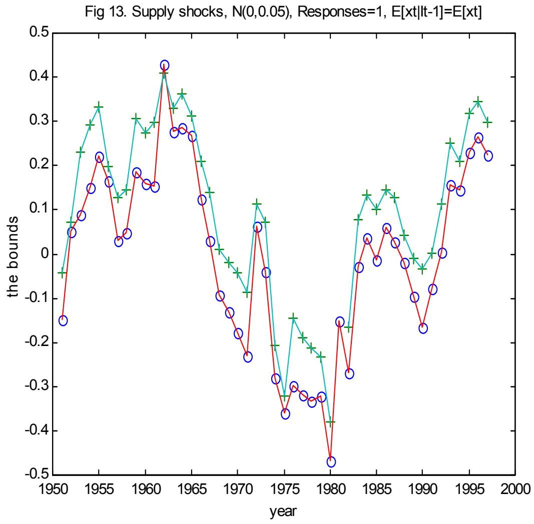
(a) Supply shocks (b) Demand Shocks



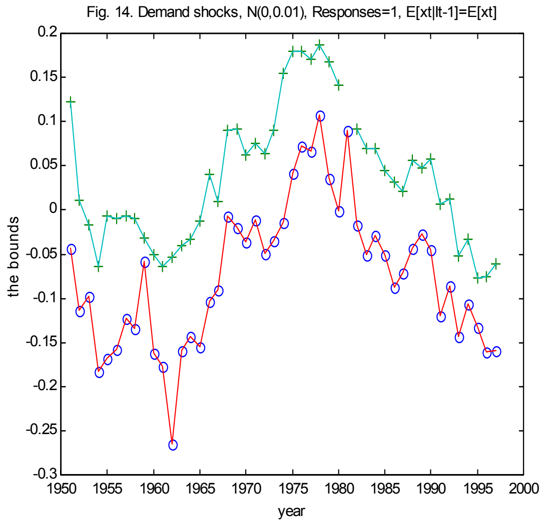
Note. It is assumed that the disturbances are symmetric around zero median and the expectation is a lagged variable.

Figure 6. Macro supply and demand shocks

(a) Supply shocks



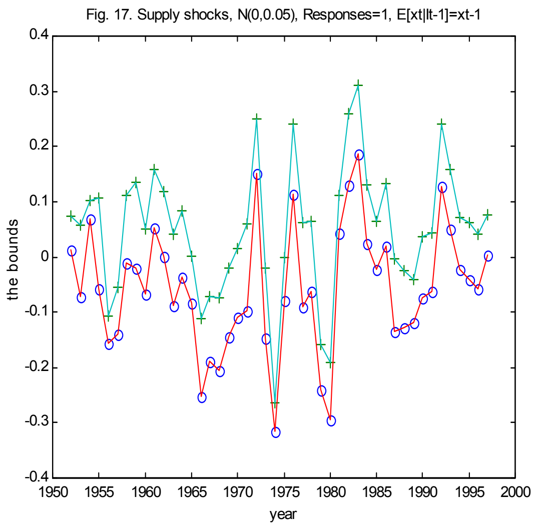
(b) Demand Shocks



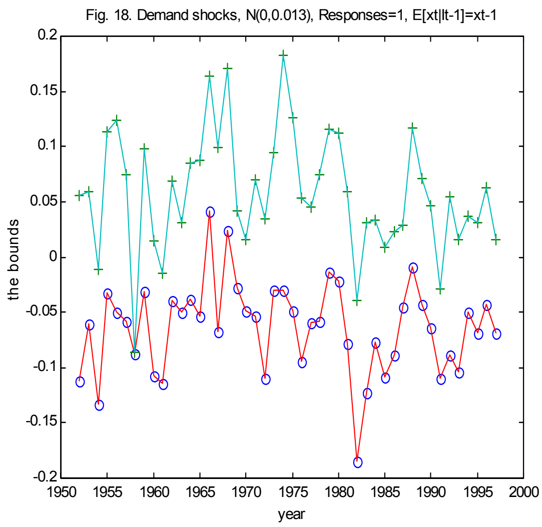
Note. It is assumed that the disturbances follow a normal distribution with zero mean and the expectation is the mean over time.

Figure 7. Macro supply and demand shocks

(a) Supply shocks



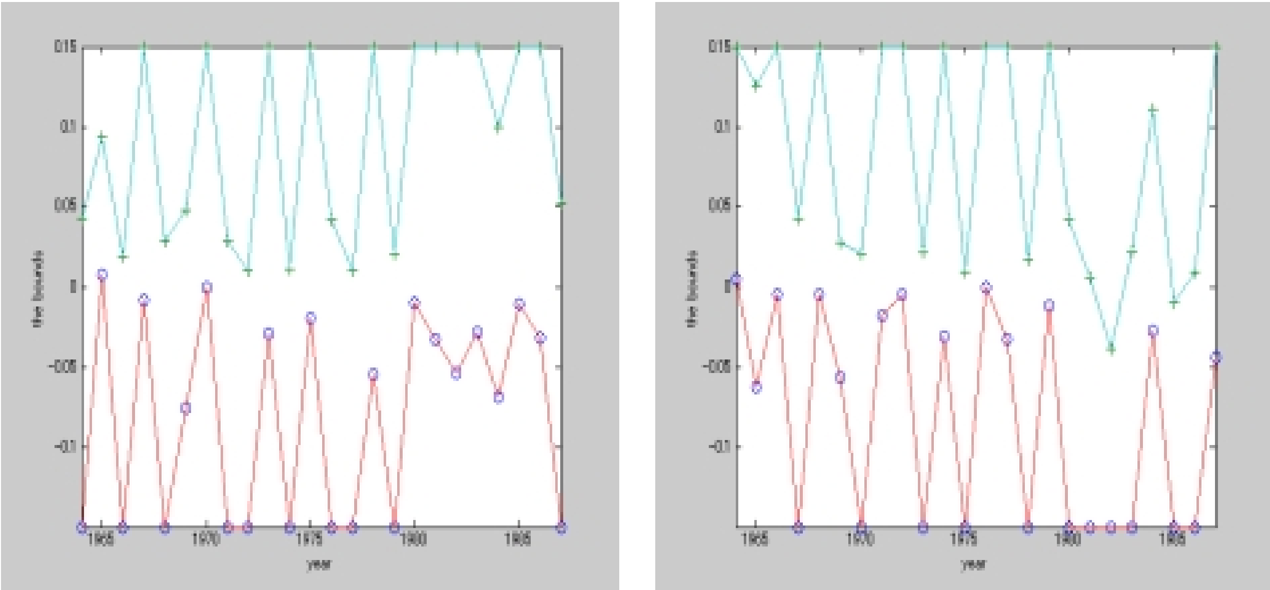
(b) Demand shocks



Note. It is assumed that the disturbances follow a normal distribution with zero mean and the expectation is a lagged variable.

Figure 8. Male High School equivalents

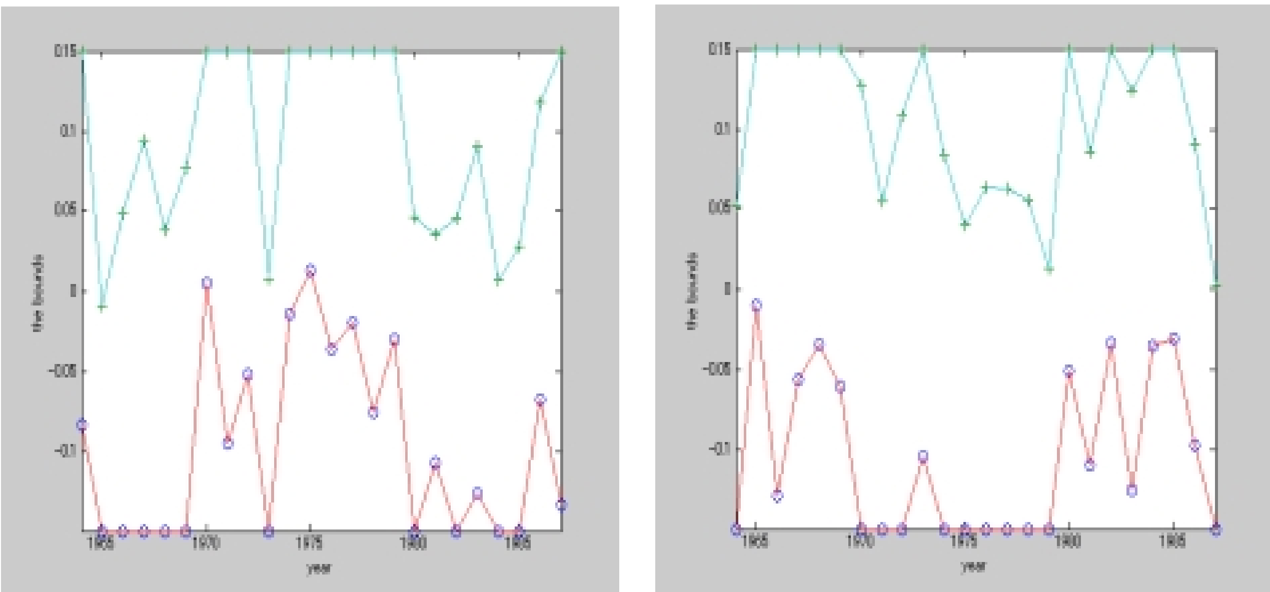
(a) Labor supply shifter (b) Labor demand shifter



Note. It is assumed that the median of the disturbances is zero.

Figure 9. Male College equivalents

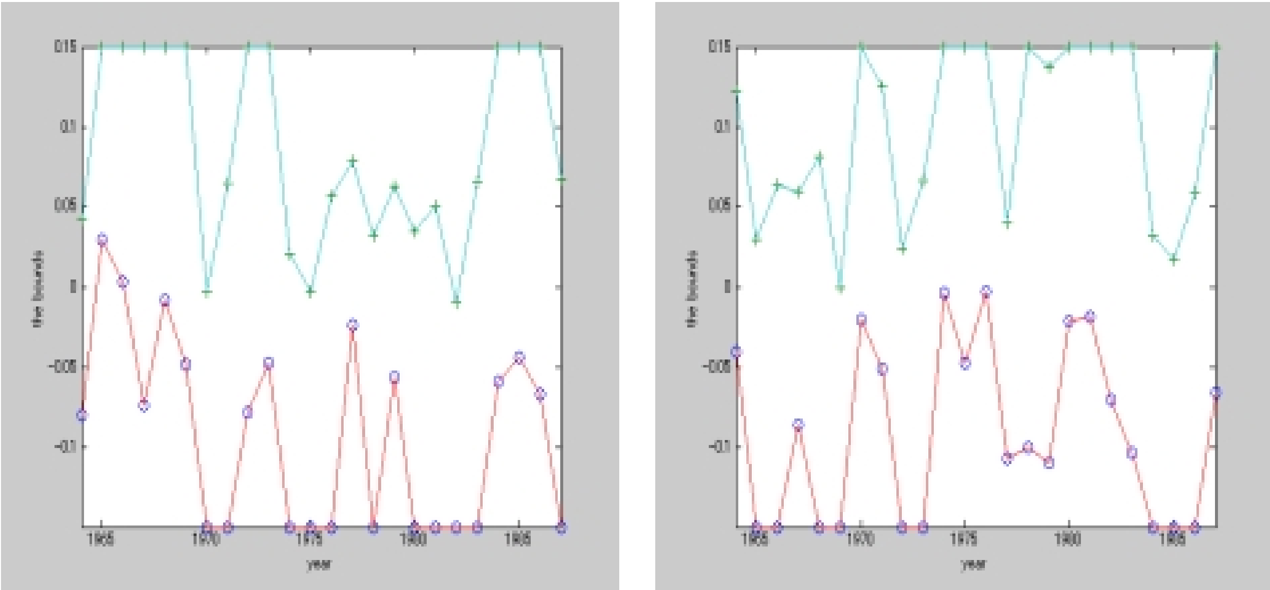
(a) Labor supply shifter (b) Labor demand shifter



Note: See the footnote of figure 8.

Figure 10. Female High School equivalents

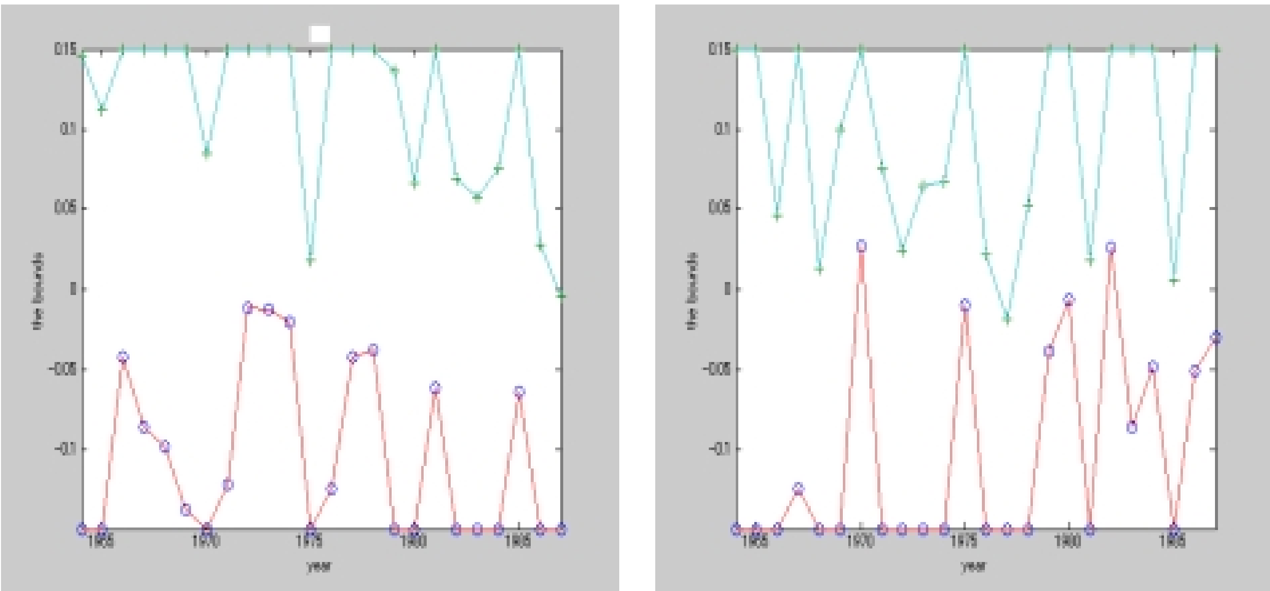
(a) Labor supply shifter (b) Labor demand shifter



Note: See the footnote of figure 8.

Figure 11. Female College equivalents

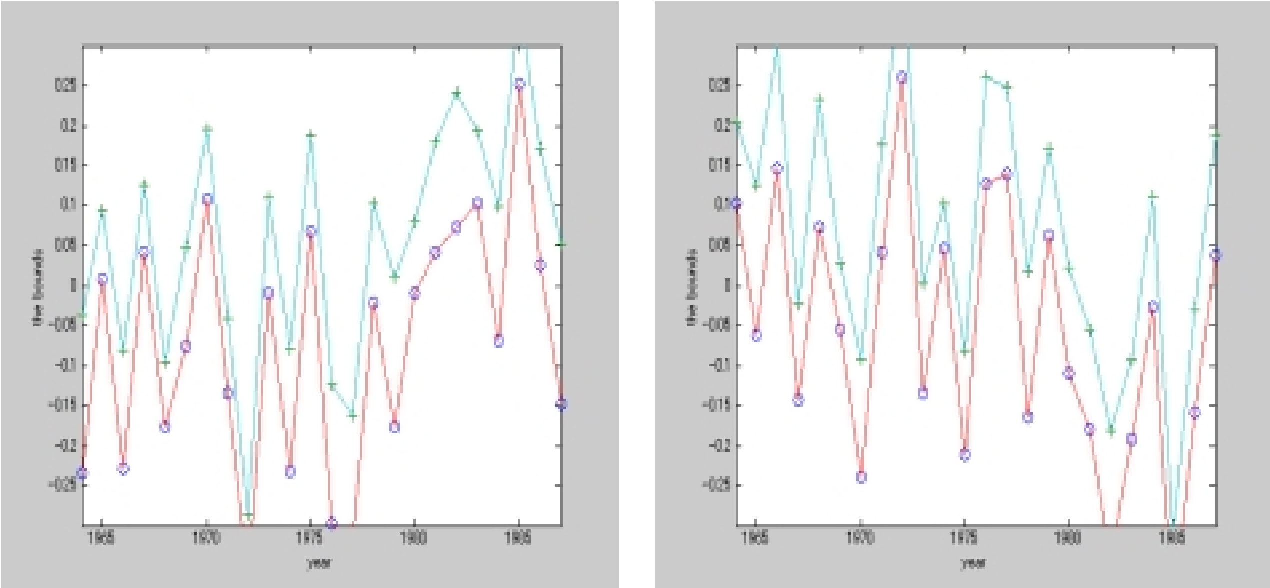
(a) Labor supply shifter (b) Labor demand shifter



Note: See the footnote of figure 8.

Figure 12. Male High School equivalents

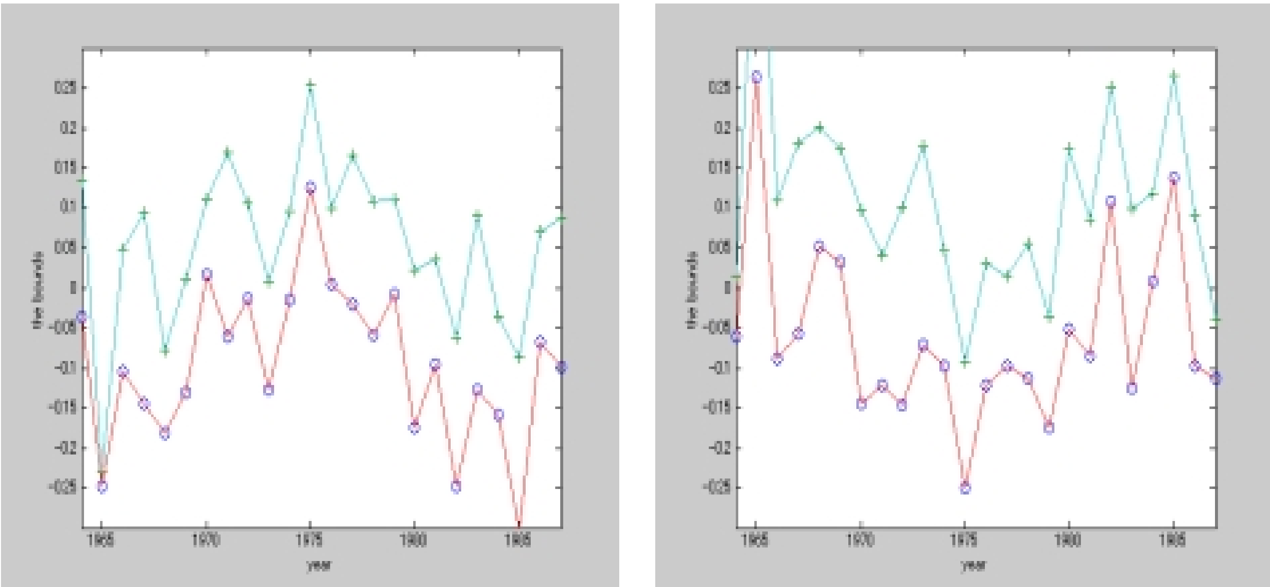
(a) Labor supply shifter (b) Labor demand shifter



Note: It is assumed that the disturbances follow a normal distribution.

Figure 13. Male College equivalents

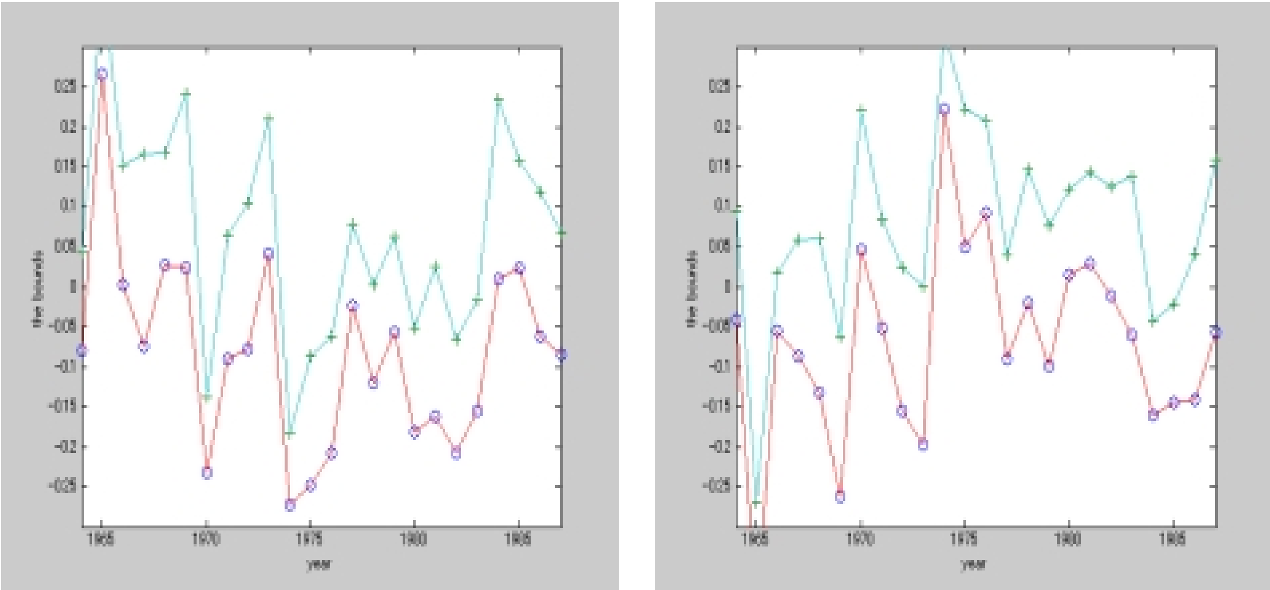
(a) Labor supply shifter (b) Labor demand shifter



Note: See the footnote of figure 12.

Figure 14. Female High School equivalents

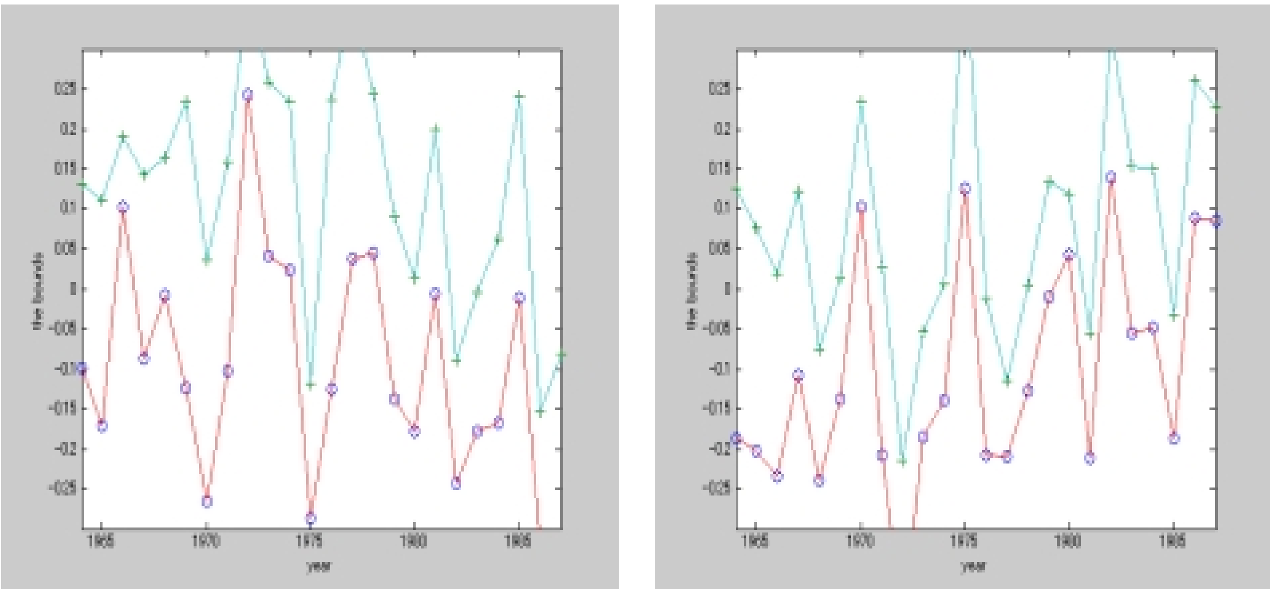
(a) Labor supply shifter (b) Labor demand shifter



Note: See the footnote of figure 12.

Figure 15. Female College equivalents

(a) Labor supply shifter (b) Labor demand shifter



Note: See the footnote of figure 12.