Patents, Search of Prior Art and Revelation of Information

Corinne Langinier* and Philippe Marcoul†

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Abstract

The recent deliverance of "business-method-software" patents has been strongly debated. PTO's examiners are accused of granting patents to non deserving-innovations. The failure is mainly due to the lack of prior art. We argue that innovators are liable too, as they do not search for the relevant prior art. We propose a model of bilateral search of information. The innovator can undertake a costly search of prior art, and thus be privately informed of the value of the innovation. Then, he applies for a patent and reveals all or part of the prior art to the PTO. The PTO performs a complementary search to assess the patentability of the innovation. An innovator can conceal some information to increase the probability of being granted a patent. We show that the PTO should not follow the "rule of equal", and commit to an assessment contingent on the quantity of prior art revealed by the innovator. This *ex ante* commitment induces the innovator to search for more prior art.

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*Iowa State University, Ames. langinier@econ.iastate.edu

†Iowa State University, Ames. marcoul@iastate.edu

1 Introduction

In this paper we investigate whether the *ex ante* commitment of the Patent and Trademark Office (PTO) to a certain level of complementary search of prior art information can induce innovators that apply for a patent to search for more prior art information, and to reveal it. Our concern is twofold; first, we focus on how to induce innovators to search for the relevant prior art information, and second, how to incite innovators to reveal it. We show that an innovator who finds a little piece of evidence against the patenting of his innovation will not always provide all the information he has acquired after having searched for it. Nevertheless, there exists a complementarity between the effort of the innovator and the PTO. From a policy implication perspective, we find that the PTO should propose different levels of search of prior art information depending on the amount of prior art that the innovator has revealed while applying for a patent. The more prior art the PTO gets, the more it should try hard to validate this information and thus grant a patent.

The recent surge of "business-method software" patents raises fundamental issues of intellectual property rights (IPR) due to the nature and the novelty of e-commerce. On one hand, some of these e-commerce patents are strongly debated because they seem to protect far too obvious ideas[1]. On the other hand, recent lawsuits indicate that the traditional system is not appropriate for e-commerce innovation[2]. If we accept the legitimacy of these patents, we must understand what could be a better patent system tailored for this specific field, or any new field with the same features.

Among the requirements for an innovation to be patentable it must be novel and non obvious. A patent application must contain references to previous literature and patents upon which the innovation improves or from which it diverges. Thus, innovators must provide information concerning prior art in order to prove that the innovation has not yet been patented or published prior to the time the patent is filed.

A key problem in the e-commerce world is the lack of published papers on software programming. Therefore, it is difficult, but not impossible, to find information concerning existing

[1] Among controversial patents, we can cite the "One-click Purchasing" patent # 5,960,411 owned by Amazon.com. It enables repeat online customers to place orders without re-entering credit-card information or address information. Part of the method covers the way Amazon stores billing information and shipping data. Another contested patent is the patent # 5,794,207: "Buyer-driven sales" held by Priceline.com. It is a method by which a customer could propose a price for a product or service, and the order would be filled if a seller was willing to accept the price.

[2] For instance Amazon.com versus Barnes and Noble. The latter was forced to change its web site.

innovations. And as a matter of fact, most of the e-commerce patents have few prior art (out of 31 software patents, on average only 6 claims have been reported and about half of them have no claim at all[3]). It is thus difficult to judge of the novelty of the innovation. Because of this weak prior information some patents are granted to no deserving innovations. Whose fault is it? The quality of the examination has been extensively criticized: some lawyers and economists claim that the PTO is responsible[4]. Indeed, it seems that the PTO lacks of money and time to perform the necessary search of prior art and then we can blame the PTO of not doing its job and granting patents without enough evidence of their novelty. One can think of a new way of rewarding examiners that are working for the PTO, as it has been suggested by Merges (1999). But the fault is not only on their side. Innovators are liable too. As it is well established that the prior art information is not easily available, innovators have no incentive to make the effort to search for it. This is a classical moral hazard problem. And in fact the research can be done. Any relevant piece of evidence can constitute prior art. It can be a thesis in a French university, a Chinese institution or an Italian institute for instance[5]. So, this is evidence that there is a costly way to find the relevant information for who may want to find it.

Thus, a mechanism must be designed to induce innovators to perform the necessary search of prior art. This new incentive is twofold. On one hand, the innovator will get stronger patent in case of lawsuit, they must be able to defend their patent against an accusation of invalidity. On the other hand, the PTO will examine less patents, and only valuable patents.

We propose the following model. We assume that the value of an innovation that has just been discovered can be good or bad but is unknown to the innovator and the PTO. Furthermore, the innovation can belong to a rich or poor field of prior art. The innovator performs an effort to search for prior art information. He finds a certain amount of prior art (that we restrict to be only three levels: nothing, intermediate or full amount) that may be accompanied by a signal that can be either positive or negative. This signal informs the innovator of the value of the innovation. Aware of this piece of evidence, the innovator reveals some prior art information while applying for a patent. He can reveal all the information he found or just part of it. The PTO observes the announcement of the innovator, updates her beliefs consequently, and then decides to search for extra information to assess the patentability of the innovation. The

[3] Information that can found on the web page of Greg Aharonian, http://www.bustpatents.com/.

[4] See Merges (1999), Beal (1998) and Coppel (2000). They discuss the relevance of the patent protection system in the e-commerce world.

[5] There exists a web site, called Bountyquest.com, on which anybody can post an announcement to find prior art concerning a patent for a reward that goes from $10,000 to $30,000.

search of information of the PTO provides a signal that reveals imperfectly the nature of the innovation. Indeed, the PTO can receive two kinds of signal: a perfect signal that reveals the true value of the innovation, or a random signal. In this last case, with a certain probability the PTO grants a patent (probability that depends on the announced information, and the extra information discovered by the PTO). In this setting, we find that an innovator that receives a negative signal about his innovation has no incentive to always report truthfully when he finds the full amount of prior art. He will thus prefer not to report truthfully all the time (mixed strategies equilibrium). His optimal effort to search for prior art increases with the level of effort undertaken by the PTO when the probability that the innovation belongs to a rich prior art field is high. Thus, we find that a policy in which the PTO, irrespective of the information provided, proposes to make the same effort to search for an extra piece of information is suboptimal. The PTO should commit to different levels of effort depending on the amount of prior art revealed by the innovator: the more prior art is revealed, the higher the effort of the PTO to assess the patentability of the application. This policy permits to increase the effort of the innovator to search for the relevant prior art information. The *ex ante* commitment of the PTO to a higher level of effort when she gets more prior art is in fact inefficient *ex post* as the probability of having a good innovation increases with the amount of prior art revealed.

To the best of our knowledge, in the patent literature very little attention (if not) has been devoted to this problem of search and revelation of prior art information. Patent literature has focused on the importance of patent litigation (Lanjouw and Schankerman (2001)), or settlement in case of patent infringements (Crampes and Langinier (2002)). A great deal of attention has focused on patent rules that affect the value of the patent grant in the context of sequential innovation (Chang (1995), Scotchmer (1996), O'Donoghue (1998), Schankerman and Scotchmer (2001)).

In the context of principal-agent model, Levitt and Snyder (1997) put the emphasis on the role of the transmission of information from agent to principal. They show that contracts must sometimes reward agents for announcing bad news. Thus, as we find in our model, the principal commits *ex ante* to an inefficient *ex post* outcome.

The paper is organized as follows. In section 2 we present the model, the search of prior art strategy for the innovator and the search of complementary information and granting strategy for the Patent and Trademark Office. Section 3 is devoted to the detailed presentation of the technologies of examination of a patent and research of prior art. In section 4 we show the non existence of equilibrium in pure strategies and we show that there exists only equilibrium in mixed strategies in which the innovator does not always reveal the information obtained. In section 5, we derive the optimal level of effort to search for prior art information of the innovation, according to the revelation strategies. We derive policies implications in section 6.

2 The Model

We consider a model with two players: an innovator, that has just made an innovation, and the Patent and Trademark Office (PTO) that will judge of the deliverance of a patent. At the outset, neither the innovator nor the PTO know the social value of the innovation. We assume that it can be good or bad, and thus will conduct to a "good" or "bad" patent if it is patented. Nevertheless, both the innovator and the PTO have prior distribution on the value of the innovation: they believe with probability p that the innovation is good.

Having done the original work, the innovator is nonetheless aware of the inventive degree of his innovation. The innovation can be new or, on the contrary, can be an improvement of another innovation, but can still be patented. This, in turn, determines the field of prior art to which the innovation belongs. It can belong to a field where there is very few prior art, that we call a poor prior art field or, conversely, to a rich prior art field. In order to get more information concerning the value of the innovation, the innovator must undertake an effort to search for relevant prior art. Thus the prior art that the innovator finds depends, on one hand, on his effort of research and, on the other hand, on the field to which the innovation belongs. This information will be helpful for the innovator to update his beliefs concerning the value of the innovation. The observation of the prior art will bring bad or good news. For instance, if the innovator makes enough effort to search for prior art if he believes that he has a good innovation that belongs to a rich field, he will find all the relevant prior art that represents only positive information (good news). On the contrary, if he does very little effort to search for prior art, and believes that it is a bad innovation and there is only few prior art, he will not find any prior art, and thus will not be able to infer any information (positive or negative). All the contingencies will be detailed in the presentation of the different steps of the model.

The innovator then files a patent application that contains his announced amount of prior art. The PTO cannot observe the effort of the innovator nor what he actually found, she can just observe the announced amount of prior art and update in consequence her beliefs. Based on her new beliefs, and on the announced amount of prior art, she can decide to search for more information.

Let us detail each of the components of the model.

2.1 Search of Prior Art

At the beginning of the game an innovation has been made, with probability p the innovation will conduct to a good patent, while with the complementary probability $(1-p)$ it will conduct to a bad patent. The value of the patent is unobservable by both parties.

First, the innovator determines a level of effort to search for a certain amount of prior art that can only take three values $x \in \{0, x_i, \overline{x}\}$. This effort generates a probability of finding a certain amount of prior art $e \in [0,1]$ and has a desutility $k(e) = e^2/(1-e)$. It will undeniably have an impact on the amount of prior art that the innovator will find as will the field in which the innovation belongs to. Let γ be the probability that the innovation belongs to a rich prior art field, and $(1-\gamma)$ the probability to belong to a poor prior art field. Neither the innovator's effort nor the found prior art can be observed by the PTO. Only the innovator is aware of the prior art information he found that can be the maximum amount $(\overline{x})$ that we normalize to be 1, the minimum amount (0) or an intermediate level of prior art (x_i). We now explain under what circumstances the innovator will find one of these levels of prior art. First, the innovator will find the maximum amount of prior art $\overline{x} = 1$ with probability e if the prior art actually exists (i.e. in a rich prior art field). However, he can get only positive information if the innovation is good, or positive and negative information in the case of a bad innovation. To simplify, we assume that the amount of prior art is the same whatever the value of the innovation but, in the case of a bad innovation, the innovator gets an extra (ε) amount of information that is negative. This negative extra amount of information is enough to state that the value of the patent is bad[6]. The innovator will be aware of the value of his innovation after having found the prior art. Second, the innovator will find no prior art with probability $(1-e)$ in the case of a poor prior art field. In this case he cannot get positive nor negative information. Finally, the innovator will find the intermediate level x_i in several cases. Either he generates a probability of finding prior art e if the innovation belongs to a poor field of prior art, or if he generates a probability $(1-e)$ and the innovation belongs to a rich field. In both cases he will get a positive information if the innovation is good, and a positive and negative information if the innovation is bad.

After having searched for more prior art, in most of the cases, the innovator learns the value of his innovation, and then decides to apply for a patent. If he does not find a piece of evidence against patenting, but on the contrary only positive information, he will for sure apply for a patent. On the other hand, if he finds a piece of evidence against patenting (that may invalidate

[6] For instance the innovator can find a little piece of evidence that should invalidate his patent if he is sued in court.

it in court), he may decide not to apply for a patent, or at least not all the time. We could assume indeed, that an innovator that receives a negative signal decides to apply for a patent with a certain probability. This assumption does not alter nor improve our qualitative results, so to simplify we just assume that whatever his type, the innovator applies for a patent.

2.2 Search of Complementary Information and Granting a Patent

Once he has discovered more prior art, the innovator files a patent application with his announced level of prior art $\widetilde{x}$. The PTO observes the level of prior art and updates her beliefs. She then makes the following decision: - If the announced level of prior art is 0, she does not grant a patent claiming that there is not enough information to judge of the novelty of the innovation. - Otherwise, if she observes x_i or $\overline{x} = 1$ she decides to search for complementary information. After having observed $\widetilde{x} = 1$ (resp. $\widetilde{x} = x_i$), she makes an effort[7] $E_1 \in [0,1]$ (resp. $E_{x_i} \in [0,1]$) to search for complementary information at the cost K_1 (resp. K_{x_i}). Therefore she receives two types of signals: either a good signal (with probability $\widetilde{x}E_{\widetilde{x}}$ where $E_{\widetilde{x}} = \{E_1, E_{x_i}\}$) that the innovation is good (or bad) or a random signal (with probability $1 - \widetilde{x}E_{\widetilde{x}}$). If she receives a random signal she will grant the patent with the updated probability of having a good innovation and will not grant the patent with the complementary probability.

To summarize, the timing is the following:

1. The innovator makes two decisions: - the effort to search for prior art - and, once he has learned the value of the innovation, he files a patent application with an announced prior art ($\widetilde{x}$).

2. Then the PTO observes the innovator's announcement, and decides to search for additional information or not. Eventually the PTO decides whether or not to grant a patent.

Insert Timing over Here

3 The Technologies of Examination and Research of Prior Art

We now examine in details the structure of the PTO's technology of examination of patent, and then the innovator's technology of research of prior art.

[7] The effort of the PTO is dichotomic: she can either choose to make an effort, or not to make it after having observed $\widetilde{x}$. We have explored the case of continuous efforts that cost $\frac{a}{2}E_{\widetilde{x}}{}^2$ instead of a fixed cost $K_{\widetilde{x}}$. As it gives a richer model, it also gives non tractable equations that complicated the whole model.

3.1 The Examination Technology of the PTO

The PTO will decide whether or not to search for complementary information and eventually grant a patent based on the received announcement.

If the innovator files a patent application with no prior art at all ($\widetilde{x} = 0$) then the PTO does not deliver a patent arguing that there is not enough information, whatever her beliefs.

If the innovator files a patent with the intermediate or the maximum amount of prior art ($\widetilde{x} = x_i, \overline{x}$), then the PTO will first update her beliefs and then will eventually search for more information. The updated beliefs of the PTO depend crucially on the anticipated behavior of the innovator. There are several possible combinations: - either the innovator always reveals the truth - or only the innovator that receives a positive signal reveals the truth, - or only the innovator that receives a negative signal reveals the truth, - or none of them reveal the truth, - or eventually the innovator randomizes his revelation of the truth.

For the time being let just define a non-specified updated beliefs that will take different values depending on the anticipations of the PTO. After having received a signal $\widetilde{x}$, the PTO is able to update her beliefs, and thus now believes that the innovation is good with probability $\mu_{\widetilde{x}} = \Pr(Good/\widetilde{x})$ and no longer p. The expected payoff of the PTO is thus

$$\mu_{\widetilde{x}} \left[\widetilde{x}E_{\widetilde{x}}\overline{W}_G + (1 - \widetilde{x}E_{\widetilde{x}}) \left(\mu_{\widetilde{x}}\overline{W}_G + (1 - \mu_{\widetilde{x}})\overline{W}_R\right)\right] + (1 - \mu_{\widetilde{x}}) \left[(1 - \widetilde{x}E_{\widetilde{x}})\mu_{\widetilde{x}}\underline{W}_G\right] - K_{\widetilde{x}}$$

where $\overline{W}_G$ represents the social value of the innovation in case the PTO grants a patent on a good innovation, $\overline{W}_R$ the social value of the innovation if the PTO refuses a patent on a good innovation, $\underline{W}_G$ the social value of the innovation if the PTO grants a patent on a bad innovation. Once the PTO has decided to search for more information, if she receives a perfect signal (with probability $\widetilde{x}E_{\widetilde{x}}$) as she already believes that the innovation is good with probability $\mu_{\widetilde{x}}$, she will grant a patent. If she only receives a random signal, she will grant a patent with probability $\mu_{\widetilde{x}}(1 - \widetilde{x}E_{\widetilde{x}})$ and will not grant it with probability $(1 - \mu_{\widetilde{x}})(1 - \widetilde{x}E_{\widetilde{x}})$. When the innovation is good, its social value is $\overline{W}_G$, and when it is bad, it is $\underline{W}_G$ from granting a patent on a bad innovation. We assume that refusing a patent on a bad innovation generates a social value null ($\underline{W}_R = 0$).

We naturally assume that granting a patent on a good innovation is always better than refusing a patent on a good innovation that in turn is better than granting a patent on a bad innovation. Formally we assume that $\overline{W}_G > \overline{W}_R > 0 > \underline{W}_G$. Furthermore we assume that the social value of granting a patent on a good innovation is very high and that the following inequality must be satisfied $\Delta W = \overline{W}_G - \overline{W}_R - \underline{W}_G > 4\frac{K_i}{x_i E_i}$.

To simplify we have assumed that the efforts of the PTO are exogenous, and furthermore we assume that $E_1 > x_i E_{x_i}$. If these efforts were to be the same, this inequality is satisfied as $x_i < 1$. If $E_1 > E_{x_i}$ the inequality can be deduced directly. And last, if $E_1 < E_{x_i}$, the effort after observing 1 has to be not too small to respect the assumption made above.

3.2 The Technology of Research of Prior Art of the Innovator

The innovator makes two decisions: first he determines the level of effort he wants to make to search for prior art, and second he announces the prior art information he eventually got after searching for it.

We first present the announcement of the innovator once he has found some prior art and has received a signal about the value of the innovation. Then we determine the optimal level of effort that the innovator makes to find the appropriate level of prior art.

3.2.1 announcement of the Innovator

We first determine what must be the optimal announcement of the innovator. Once the innovator has made a certain level of effort, he will find some prior art information, that can be $0, x_i$ or $\overline{x}$, and a signal about the value of the innovation. In the case of a good innovation, in a rich prior art field, with probability e the innovator will discover $x = 1$ and a positive signal, and with probability $(1 - e)$ he will discover $x = x_i$ with also a positive signal. In the case of a good innovation in a poor prior art field, with probability e the innovator will discover $x = x_i$ with a positive signal, and with probability $(1 - e)$ he will discover $x = 0$ and thus he is enable to infer the value of the innovation. Conversely, if the innovation is bad, the innovator will get the same structure of information depending on the prior art field but he will get a negative signal, and will thus know that there exists a little piece of information that is not in favor of patenting.

As the innovator cannot report more than he has actually found, if he gets no information $(x = 0)$ he will report nothing $\widetilde{x} = 0$.

If the innovator receives an intermediate level of information $(x = x_i)$, he can announce $\widetilde{x} = 0$ or $\widetilde{x} = x_i$. There is no gain to announce nothing since the PTO does not deliver a patent if the patent application has no prior art information. Thus whatever the value of the innovation, he should always report truthfully, $\widetilde{x} = x_i$.

If the innovator gets all the information $(x = 1)$ he can announce either $\widetilde{x} = 0$, $\widetilde{x} = x_i$ or $\widetilde{x} = 1$. We first compare the gain from reporting something and the gain from not reporting anything. As we have already mentioned, the innovator is always better off by revealing some prior art rather than nothing. So, announcing $\widetilde{x} = 0$ is a strictly dominated strategy that will never been chosen by the innovator. Second, we have to determine under what circumstances the innovator will prefer to report truthfully. An innovator that gets only positive signal on the top of the prior art will get an expected gain from reporting truthfully of

$$E_1\overline{V} + (1 - E_1)\mu_1\overline{V}$$

where $\overline{V}$ represents the profit of the innovator if he patents a good innovation and E_1 the level of effort undertaken by the PTO when she observes $\widetilde{x} = 1$. If he reports $\widetilde{x} = x_i$ instead, he will get

$$x_i E_{x_i}\overline{V} + (1 - x_i E_{x_i})\mu_{x_i}\overline{V}$$

where E_{x_i} is the level of effort of the PTO when she observes $\widetilde{x} = x_i$. The innovator compares both expected payoffs to decide whether or not to report truthfully. He will then report truthfully as long as

$$E_1 + (1 - E_1)\mu_1 > x_i E_{x_i} + (1 - x_i E_{x_i})\mu_{x_i} \tag{1}$$

If the innovator receives a negative signal on the top of the prior art he found, he will get an expected gain from reporting truthfully

$$(1 - E_1)\mu_1\underline{V}$$

where $\underline{V}$ represents the profit of the innovator if he patents a bad innovation. If he reports $\widetilde{x} = x_i$ his payoff is

$$(1 - x_i E_{x_i})\mu_{x_i}\underline{V}$$

Thus, the innovator that gets bad information will report truthfully if

$$(1 - E_1)\mu_1 > (1 - x_i E_{x_i})\mu_{x_i} \tag{2}$$

Equations (1) and (2) represent the conditions under which both types report truthfully.

3.2.2 Search of Prior Art

The innovator determines first the level of effort to make in order to find the relevant prior art. As we solve the game by the end, we will characterize the expected profit of the innovator after having determined the optimal level of effort of the PTO and the optimal amount of prior art that the innovator will decide to reveal.

4 Optimal Report of Prior Art

In this section we characterize the optimal report of the innovator once he has discovered prior art information accompanied of a signal. We derive the Subgame Perfect equilibrium.

4.1 Is it optimal to Report Truthfully?

Imagine first that both types report truthfully. We can thus update the beliefs of the PTO,

$$\begin{aligned}
\Pr(Good/\widetilde{x} &= 1) = \frac{p\gamma e}{p\gamma e + (1-p)\gamma e} = p \\
\Pr(Good/\widetilde{x} &= x_i) = \frac{p(1-\gamma)e + p(1-e)\gamma}{p(1-\gamma)e + p(1-e)\gamma + (1-p)((1-\gamma)e + (1-e)\gamma)} = p \\
\Pr(Good/\widetilde{x} &= 0) = p
\end{aligned}$$

The PTO must choose whether to make the appropriate level of effort $E_{\widetilde{x}}$ when she observes the intermediate level of prior art ($\widetilde{x} = x_i$) or the maximum level of prior art ($\widetilde{x} = 1$). If she observes the maximum level of prior art, the payoff of the PTO if she makes the effort E_1 is

$$p\left[E_1\overline{W}_G + (1-E_1)\left(p\overline{W}_G + (1-p)\overline{W}_R\right)\right] + (1-p)\left[(1-E_1)p\underline{W}_G\right] - K_1$$

and

$$p\left[p\overline{W}_G + (1-p)\overline{W}_R\right] + (1-p)p\underline{W}_G$$

if she does not make any effort. Thus the PTO will search for more information after observing $\widetilde{x} = 1$ as long as

$$E_1 p(1-p)\Delta W - K_1 > 0 \tag{3}$$

where $\Delta W = \left[\overline{W}_G - \overline{W}_R - \underline{W}_G\right]$. The inequality (3) is satisfied for values of p such that $p \in [p_1', p_1'']$. The determination of these cutoff values is defined in the appendix.

If she observes the intermediate level of prior art, the payoff of the PTO is

$$p\left[x_i E_{x_i}\overline{W}_G + (1-x_i E_{x_i})\left(p\overline{W}_G + (1-p)\overline{W}_R\right)\right] + (1-p)\left[(1-x_i E_{x_i})p\underline{W}_G\right] - K_{x_i}$$

if she makes the effort E_{x_i} and

$$p\left[p\overline{W}_G + (1-p)\overline{W}_R\right] + (1-p)p\underline{W}_G$$

if she does not make any effort. Thus, after observing $\widetilde{x} = x_i$, the PTO will search for more information if

$$x_i E_{x_i} p(1-p)\Delta W - K_{x_i} > 0 \tag{4}$$

This inequality is satisfied for values of p such that $p \in [p'_i, p''_i]$. (see appendix for detail). the comparison of the different cutoff values leads to the following inequality $0 < p'_1 < p'_i < \frac{1}{2} < p''_i < p''_1 < 1$ as$\frac{E_1}{K_1} > \frac{x_i E_{x_i}}{K_{x_i}}$ is satisfied.

We summarize this first set of results:

Result 1 *The effort of the PTO depends on her prior beliefs.*

- *For values of p that belong to the interval $[p'_i, p''_i]$, the PTO always makes the effort E_1 after observing $\widetilde{x} = 1$ and E_{x_i} after observing $\widetilde{x} = x_i$.*
- *For values of p that belong to the interval $[p'_1, p'_i[\cup]p''_i, p''_1]$ the PTO makes the effort E_1 after observing $\widetilde{x} = 1$ but no effort after observing $\widetilde{x} = x_i$.*
- *For values of p that belong to the interval $[0, p'_1[\cup]p''_1, 1]$, the PTO makes no effort at all.*

The intuition of this first result is the following. The bigger the uncertainty the more likely the PTO will try to get more information whether she observes the intermediate or the full amount of prior art. When the uncertainty is very small, for values of p very close to 0 or to 1, the PTO strongly believes that the innovation is either bad or good, and it is thus too costly to search for more information. The first terms of equations (3) and (4) becomes very small compared to the second term, the associated cost. For intermediate values of uncertainty, it is too costly for the PTO to search for more information after observing the intermediate level of prior art. This is due to the fact that, in case of observing intermediate level of prior art, the technology of finding more relevant information is less precise than in the case of observing full prior art. Indeed, the PTO will only find relevant evidence with probability $x_i E_{x_i}$ against E_1 after observing the full prior art.

We now describe the best revealing strategy of the innovator. The innovator will report truthfully or not depending on the PTO's decision of making an effort or not.

For values of p that belong to the interval $[p'_i, p''_i]$, inequality (1) becomes $E_1 + (1-E_1)p > x_i E_i + (1-x_i E_i)p$ that is satisfied if $E_1 > x_i E_i$ and inequality (2) becomes $(1-E_1)p > (1-x_i E_i)p$ that can only be satisfied if $E_1 < x_i E_i$. These two inequalities cannot hold simultaneously.

For values of p that belong to the interval $[p'_1, p'_i[\cup]p''_i, p''_1]$, inequality (1) becomes $E_1 + (1 - E_1)p > p$ that is always satisfied and inequality (2) becomes $(1-E_1)p > p$ that is never satisfied. Consequently, the innovator who gets a negative signal has no incentive to report truthfully.

For values of p that belong to the interval $[0, p'_1[\cup]p''_1, 1.]$, inequality (1) becomes $p \geq p$ and inequality (2) becomes $p \geq p$. If we assume that when the innovator is indifferent between reporting the truth or lying he always reports truthfully, then in this case reporting truthfully is the best decision of the innovator.

Result 2 *There exists no equilibrium in which both types report truthfully for $p \in [p'_1, p''_1]$.*

4.2 Is It Optimal to Always Lie?

We have shown that there exits no equilibrium in pure strategies in which both types report truthfully when the *ex ante* uncertainty is the bigger. We now determine whether there exits equilibrium in which one type is better off by always lying.

Consider first that the PTO believes that the good type always reports truthfully, and that the bad type always lies. In this case the updates beliefs of the PTO becomes

$$\Pr(Good/\widetilde{x} = 1) = \frac{p\gamma e}{p\gamma e + (1-p)\gamma e} = 1$$

$$\begin{aligned}\Pr(Good/\widetilde{x} &= x_i) = \frac{p(1-\gamma)e + p(1-e)\gamma}{p(1-\gamma)e + p(1-e)\gamma + (1-p)((1-e)\gamma + e\gamma + (1-\gamma)e)} \\ &= \frac{p(e+\gamma-2e\gamma)}{\gamma + e - 2e\gamma + (1-p)e\gamma} = \mu_{x_i}\end{aligned}$$

$$\Pr(Good/\widetilde{x} = 0) = p$$

If the PTO observes the maximum level of prior art, she prefers not to make any effort to search for more prior art as there is not anymore uncertainty concerning who reveals the full amount of prior art. Indeed, inequality (3) cannot hold. If she observes the intermediate level of effort, she makes the effort E_{x_i} if

$$x_i E_{x_i} \mu_{x_i} (1 - \mu_{x_i}) \Delta W - K_{x_i} > 0$$

This inequality holds for values of p that belong to the interval $[q'_i, q''_i]$. (see appendix)

Thus, for values of $p \in [q'_i, q''_i]$, the innovator who receives a positive signal reports truthfully as inequality (1) is always satisfied ($(1 - x_i E_i)(1 - \mu_{x_i}) > 0$). Furthermore, the innovator that receives a negative signal is better off by telling the truth as well as (2) is also satisfied. Indeed, it is easy to check that $1-(1-x_iE_i)\mu_{x_i}>0$ is always true. Thus an innovator that gets a negative signal has no incentive to lie about it. Thus we cannot have an equilibrium in this case.

The innovator that receives a good signal about his innovation has no incentive to deviate from revealing the truth as the PTO infers that only good types reveal the truth. Nevertheless, an innovator that receives a bad signal is expected to always lies and does not report his full prior art. Thus, the innovator may decide to report truthfully in order to fool the PTO that believes that only good types report the high level of prior art. The innovator that gets a negative signal can get a patent per chance, when the PTO is not sure about the signal she receives. Thus the innovator will only get an expected profit. If now he decides to report all the prior art he found, he will get a patent for sure, as the PTO believes that he is a good type that reports truthfully. This is not an equilibrium.

Imagine now that the PTO believes that the innovator that receives a good signal always lies, and that the innovator that receives a bad signal reports truthfully. Using the same argument, we can show that the PTO will not make any effort when she observes the high level of prior art, but she will not give a patent either. When she observes the intermediate level of effort, she will make a positive effort. A good type will not deviate whereas a bad type will deviate as he can get a patent from fooling the PTO. This is not an equilibrium.

The PTO can believe that none of the types will report truthfully. By the same token we show that this cannot be an equilibrium.

Result 3 *There exists no equilibrium in pure strategies when the ex ante uncertainty is the biggest.*

4.3 Is It Optimal to Not Always Report Truthfully?

When the uncertainty about the value of the innovation is at its biggest level (i.e. when the *ex ante* belief is around 1/2), innovators have no incentive to report truthfully, as they have no incentive to always lie. Thus it can be an equilibrium to report sometimes truthfully. In this section we explore the possibility of having an equilibrium in mixed strategies.

We consider a Perfect Bayesian Equilibrium in which only the innovator that receives a negative signal with the maximum amount of prior art decides to randomize his report decision.

With probability θ he reports truthfully and with the complementary probability he reports the intermediate amount of prior art. An innovator that gets the full amount of prior art with a positive signal reports truthfully.

When the PTO observes the maximum amount of prior art, it can come from an innovator that receives a positive signal or an innovator that receives a negative signal. When she receives an intermediate level of prior art, it cannot come from an innovator that receives a positive signal with the full prior art. But it can come from other types of innovators: an innovator that receives an intermediate level of prior art and a positive or negative signal, or an innovator that receives the maximum amount of prior art and a negative signal, and does not report truthfully.

In this case the updates beliefs of the PTO becomes

$$\begin{aligned}
\mu_1(\theta) &= \Pr(Good/\widetilde{x} = 1) = \frac{p}{p + \theta(1-p)} \\
\mu_{x_i}(\theta) &= \Pr(Good/\widetilde{x} = x_i) = \frac{p(1-\gamma)e + p(1-\gamma)e}{(1-\gamma)e + (1-e)\gamma + (1-p)(1-\theta)e\gamma} \\
\Pr(Good/\widetilde{x} &= 0) = p
\end{aligned}$$

An innovator that receives a negative signal and the full amount of prior art must be indifferent between revealing the full amount and getting $\mu_1(1-E_1)\underline{V}$ and revealing the intermediate level of prior art and getting $\mu_i(1 - x_i E_{x_i})\underline{V}$. Thus, there exists a θ that must satisfy

$$\mu_1(\theta)(1 - E_1) = \mu_{x_i}(\theta)(1 - x_i E_{x_i}). \tag{5}$$

After observing $\widetilde{x} = 1$ (resp. $\widetilde{x} = x_i$) the PTO chooses E_1 (resp. E_{x_i}) according to

$$E_1\mu_1(\theta)(1 - \mu_1(\theta))\Delta W - K_1 > 0 \tag{6}$$

and

$$x_i E_{x_i}\mu_{x_i}(\theta)(1 - \mu_{x_i}(\theta))\Delta W - K_{x_i} > 0 \tag{7}$$

These three equations (5), (6) and (7) must be satisfied simultaneously to get an equilibrium.

When the PTO decides to make the appropriate level of effort depending on the prior art that has been revealed by the innovator, we find that there exists a unique probability θ^* that satisfies equation (5) (see appendix).

Result 4 *For $p \in [r_i', r_i'']$, there exists a unique probability θ^* that satisfies (5):*

$$\theta^* = \frac{(1-E_1)(e(1-\gamma) + \gamma(1-e) + (1-p)\gamma e) - p(1 - xE_i)(e(1-\gamma) + \gamma(1-e))}{(1-p)\left((1 - xE_i)(e(1-\gamma) + \gamma(1-e)) + (1-E_1)\gamma e\right)}$$

- *θ^* decreases with p, and increases with x_i* and *γ.*
- *It is decreasing with E_1 and increasing with E_{x_i}.*
- *It is increasing with e.*

Proof. see appendix ■

When the prior beliefs about the value of the innovation are not too small nor too big, the level of the effort of the PTO as she observes the full amount of prior art is decreasing with the optimal probability θ^*. The intuition is the following. As the PTO increases her effort to search for more information when she is checking a patent application with the full amount of prior art, the screening process will be more accurate. Indeed, the PTO will grant less patent to bad innovation. Thus, an innovator that receives a negative signal and the full amount of prior art will prefer not to report truthfully, and thus the probability of reporting truthfully decreases.

In the same vein, the effort of the PTO when she observes an intermediate level of prior art is increasing with θ^*, or in other words decreasing with $(1-\theta^*)$. as the PTO monitors more carefully the application with the intermediate level of prior art, the innovator that finds all the prior art and a negative signal will prefer to report truthfully.

We can thus posit the following result:

Result 5 *There exists a semi-separating equilibrium in mixed strategies in which the innovator who receives a negative signal with the maximum amount of prior art randomizes his revelation.*

Proof. see appendix ■

5 Search of Prior Art for the Innovator

The very first decision of the innovator is to choose the level of effort to make in order to find the relevant prior art and a signal. The maximization program of the innovator is

$$\underset{e}{Max}\Pi(e) = \{ep\overline{V}[\gamma(E_1+(1-E_1)\mu_1(\theta^*))+(1-\gamma)(x_iE_{x_i}+(1-x_iE_{x_i})\mu_{x_i}(\theta^*))]$$
$$+(1-e)\overline{V}p\gamma[x_iE_{x_i}+(1-x_iE_{x_i})\mu_{x_i}(\theta^*)]$$
$$+e(1-p)\underline{V}[\gamma(1-E_1)\mu_1(\theta^*)\theta^*+\gamma(1-x_iE_{x_i})\mu_{x_i}(\theta^*)(1-\theta^*)+(1-\gamma)(1-x_iE_{x_i})\mu_{x_i}(\theta^*)]$$
$$+(1-e)(1-p)\underline{V}[\gamma(1-x_iE_{x_i})\mu_{x_i}(\theta^*)]$$
$$-\frac{e^2}{1-e}\}$$

where $\mu_1(\theta^*)$ and $\mu_{x_i}(\theta^*)$ are the updated believes of the PTO and θ^* the optimal probability with which the innovator randomizes his decision to reveal all the information[8]. The optimal solution is

$$e^* = \frac{2(p(1-p)(\overline{V}-\underline{V})(\gamma E_1+(1-2\gamma)xE_i)+(1-\gamma)p((1-p)\underline{V}+p\overline{V})+1)-2\sqrt{(1-p)p(\overline{V}-\underline{V})(\gamma E_1+(1-2\gamma)xE_i)+(1-\gamma)p((1-p)\underline{V}+p\overline{V})+1}}{2\left((\overline{V}-\underline{V})(1-p)p(E_1\gamma+xE_i(1-2\gamma))+p(1-\gamma)((1-p)\underline{V}+p\overline{V})+1\right)}$$

The innovator has an incentive the make a high effort if he is in a poor prior art field. Indeed, the PTO does not deliver a patent to an innovator that does not provide at least the intermediate level of prior art.

This solution depends of the levels of effort undertaken by the PTO if she observes the intermediate level of effort or the full amount.

Result 6 *There exists a unique optimal effort of the innovator* $e^* \in [0,1]$. It *is increasing with* E_1 *and decreasing with* E_{x_i} *if* $\gamma > 1/2$.

Once the innovator anticipates correctly what will be the strategies of the PTO, he can choose his level of effort that maximizes his *ex ante* profit function. When the innovator is in a poor prior art field, an increase of the PTO's effort induces the innovator to intensify his prior art research efforts. Indeed, his chances of getting a patent increases with his effort [9].

6 Policy Implications

The effort of the innovator has two opposite effects on the PTO's expected profit. On one hand, as e increases θ increases and it induces more bad innovators to report the full amount of prior art. The objective of the PTO is to reduce the errors (i.e. granting patent to non deserving innovation), she will put more effort (increase E_1). On the other hand, an increase in e reduces $(1-\theta)$ which means that less innovators who found the full amount of prior art will lie. If the PTO believes that the innovation belongs to a rich field of prior art ($\gamma > 1/2$) she will reduce her effort E_{x_i}. Indeed, as the PTO believes that it is a good field of research, the more the innovator increases his effort, the more likely he will get the full amount of prior art, and thus the less likely he will report the intermediate level of prior art. On the contrary, if the PTO

[8] See figure 2 for a graphic representation of the *ex ante* profit of the innovator for some specific values of the parameters.

[9] The probability of being granted a patent is $p(patent) = q_1\mu_1 + q_i\mu_i = p(\gamma + e(1-\gamma))$ where q_1 and q_i represent the probabilities of having a report of the full amount or intermediate amount of prior art as defined in the next section.

believes that the innovation belongs to a poor field of research, the more the innovator increases his effort, the more likely he will report the intermediate level. Thus the PTO is better off by increasing her effort.

We determine the *ex ante* payoff of the PTO

$$\begin{aligned}V_{PTO}(e) &= q_1(\mu_1(E_1\overline{W}_G + (1-E_1)(\mu_1\overline{W}_G + (1-\mu_1)\overline{W}_R)) + (1-\mu_1)(1-E_1)\mu_1\underline{W}_G - K_1) \\ &\quad + q_i(\mu_{x_i}(x_iE_{x_i}\overline{W}_G + (1-x_iE_{x_i})(\mu_{x_i}\overline{W}_G + (1-\mu_{x_i})\overline{W}_R)) + (1-\mu_{x_i})(1-x_iE_{x_i})\mu_{x_i}\underline{W}_G - K_{x_i})\end{aligned}$$

where q_1 (resp. q_i) represents the probability of having a report of the full amount of prior art (resp. the intermediate amount of prior art). These probabilities are defined as follows

$$\begin{aligned}q_1 &= (p + \theta(1-p))e\gamma \\ q_i &= (1-\gamma)e + (1-e)\gamma + (1-p)(1-\theta)e\gamma\end{aligned}$$

We show that the *ex ante* profit of the PTO is strictly increasing with the effort of the innovator. Thus the PTO would always rather prefer the innovator the make the maximum effort. Indeed, he will then be able to find the relevant prior art and to reveal it.

There is nevertheless a negative effect on the PTO's payoff. The higher the effort of the innovator, the higher the probability of reporting the full amount of prior art if he has a negative information. It will then become more difficult for the PTO to be able to identify effortless the nature of the innovation when the innovator reports all the prior art.

Result 7 *A policy in which the PTO proposes the same level of efforts $E_1 = E_{x_i}$ is suboptimal. The PTO should propose $E_1 > E_{x_i}$ in order to induce the innovator to put forth effort if $\gamma > 1/2$.*

The *ex ante* profit of the PTO increases as E_1 increases. This is due to result 6 and the fact that the *ex ante* profit of the PTO is increasing with the effort of the innovator.

If $x_iE_{x_i} < E_1 < E_{x_i}$, the PTO makes more effort to search for complementary prior art information after having observed an intermediate level of prior art. It is actually what we can expect to happen in reality: the PTO's examiners try to search for evidence that will give indication on whether the innovation must be patented or not. In other words, the PTO examiners do the innovators' job.

If the PTO decides to transfer some of her effort from E_{x_i} to E_1 up to the point where $E_1 > E_{x_i}$, according to result 6 it will induce the innovator to search for more prior art. The intuition is the following. If the PTO announces *ex ante* that her level of research will be higher if she observes a fair amount of prior art, and lower if she observes less prior art it will have two effects on the innovator. On one hand, if the innovator is in a rich field of prior art and he expects to have a good innovation, he has a strong incentive to find the more prior art he can in order to reveal it later and thus to be seen by the PTO as a good innovator. Thus innovators that expect to have a good innovation have incentive to put forth effort. Innovators that expect to have a bad innovation after finding the maximum amount of prior art will decide not to report truthfully more often, as the screening in the intermediate prior art level report is less intensive. On the other hand, if the innovation belongs to a poor field of research, the innovator has also a strong incentive to look for the maximum amount of prior art he can find to get a chance to have his patent examined.

In fact there are two effects that work in opposite direction. On one hand, if the PTO increases *ex ante* her effort E_1, this will have a positive direct effect on the effort of the innovator, and a negative indirect effect through θ^*. Indeed, as E_1 increases, θ^* decreases, and this will lead the innovator to reduce his effort. Overall, the first positive effect is bigger than the second, negative effect.

The *ex ante* commitment to $E_1 > E_{x_i}$ is in fact inefficient *ex post*, as the probability of having a good innovation after having received the full amount of prior art is bigger than the probability after having received the intermediate level. Thus, the PTO commits to a level of effort that is *ex post* inefficient.

Appendix

Is it Optimal to report truthfully?

After observing $\widetilde{x} = 1$, the PTO makes an effort E_1 that must satisfied (3). This inequality $E_1 p(1-p)\Delta W - K_1 > 0$ is verified if $p \in [p_1', p_1'']$ where $p_1' = \frac{1}{2E_1\Delta}\left(E_1\Delta - \sqrt{\left(E_1^2(\Delta W)^2 - 4E_1\Delta W K_1\right)}\right)$ and $p_1'' = \frac{1}{2E_1\Delta W}\left(E_1\Delta W + \sqrt{\left(E_1^2(\Delta W)^2 - 4E_1\Delta W K_1\right)}\right)$. It is easy to check that $0 < p_1' < p_1'' < 1$.

After observing $\widetilde{x} = x_i$, the PTO makes an effort E_{x_i} if the inequality (4) is satisfied. It is verified if $p \in [p_i', p_i'']$ where $p_i' = \frac{1}{2xE_i\Delta W}\left(xE_i\Delta W - \sqrt{\left(x^2E_i^2(\Delta W)^2 - 4xE_i\Delta W K_i\right)}\right)$, and $p_i'' = \frac{1}{2xE_i\Delta W}\left(xE_i\Delta W + \sqrt{\left(x^2E_i^2(\Delta W)^2 - 4xE_i\Delta W K_i\right)}\right)$ and we can also easily check that $0 < p_i' < p_i'' < 1$.

Furthermore, $p_1' < p_i' < p_i'' < p_1''$ if the following condition holds:

$$\frac{E_1}{K_1} > \frac{x_i E_{x_i}}{K_{x_i}}. \tag{A\#2}$$

We thus can conclude that $0 < p_1' < p_i' < \frac{1}{2} < p_i'' < p_1'' < 1$.

We also need to check that the PTO makes the effort that corresponds to the revealed prior art information she observes. In other words, if she observes $\widetilde{x} = 1$ she must make E_1 and not E_{x_i}. We thus check that

$$(E_1 - x_i E_{x_i})p(1-p)\Delta W - (K_1 - K_{x_i}) > 0$$

This inequality is satisfied for $p \in [\underline{p}', \underline{p}'']$ where

$$\begin{aligned}\underline{p}' &= \frac{1}{2\Delta W(E_1 - x_iE_{x_i})}(\Delta W(E_1 - x_iE_{x_i}) \\ &\quad + \sqrt{((\Delta W)^2(E_1 - x_iE_{x_i})^2 - 4\Delta W(E_1 - x_iE_{x_i})(K_1 - K_{x_i})))}\end{aligned}$$

and

$$\begin{aligned}\underline{p}'' &= \frac{1}{2\Delta W(E_1 - x_iE_{x_i})}(\Delta W(E_1 - x_iE_{x_i}) \\ &\quad - \sqrt{((\Delta W)^2(E_1 - x_iE_{x_i})^2 - 4\Delta W(E_1 - x_iE_{x_i})(K_1 - K_{x_i})))}\end{aligned}$$

We check that $\underline{p}' < p_1'$ and $\underline{p}'' > p_1''$ when the inequality (A#2) holds.

Is it Optimal to always lie?

By the same token, we determine the cutoff values for p when the bad type lies whereas the good type reports truthfully.

The inequality $x_iE_{x_i}\mu_{x_i}(1-\mu_{x_i})\Delta W - K_{x_i} > 0$ is satisfied for values of p that belong to the interval $[q_i', q_i'']$ where

$$\begin{aligned} q_i' &= \frac{1}{2\left(\Delta W x E_i(\gamma(1-e)+e(1-\gamma))(\gamma(1-e)+e)+K_i\gamma^2e^2\right)}(-\Delta W x E_i(\gamma(1-e)+e(1-\gamma))-2K_i\gamma e \\ &\quad -(\gamma(1-e)+e)\sqrt{(\Delta W x E_i(\gamma(1-e)+e(1-\gamma)))^2-4\Delta W x E_i K_i(\gamma(1-e)+e(1-\gamma))^2}) \end{aligned}$$

and

$$\begin{aligned} q_i'' &= \frac{1}{2\left(\Delta W x E_i(\gamma(1-e)+e(1-\gamma))(\gamma(1-e)+e)+K_i\gamma^2e^2\right)}(-\Delta W x E_i(\gamma(1-e)+e(1-\gamma))-2K_i\gamma e \\ &\quad +(\gamma(1-e)+e)\sqrt{(\Delta W x E_i(\gamma(1-e)+e(1-\gamma)))^2-4\Delta W x E_i K_i(\gamma(1-e)+e(1-\gamma))^2}). \end{aligned}$$

Subgame Perfect Bayesian Equilibrium

There exists a semi-separating equilibrium in which the innovator that finds the whole amount of prior art and a negative signal decides to randomize his revelation decision.

Equations (5), (6) and (7) must be satisfied.

From the first equation, $\mu_1(\theta)(1-E_1) = \mu_{x_i}(\theta)(1-x_iE_{x_i})$, we derive an unique $\theta^* \in [0,1]$,

$$\theta^* = \frac{(1-E_1)(e(1-\gamma)+\gamma(1-e)+(1-p)\gamma e) - p(1-xE_i)(e(1-\gamma)+\gamma(1-e))}{(1-p)\left((1-xE_i)(e(1-\gamma)+\gamma(1-e))+(1-E_1)\gamma e\right)}$$

$\theta^* > 0$ if $p < \overline{p} = \dfrac{(1-E_1)((1-\gamma)e+\gamma)}{(1-xE_i)((1-\gamma)e+(1-e)\gamma)+(1-E_1)\gamma e}$ that we will check later.

We then determine the values of p such that $E_1\mu_1(\theta)(1-\mu_1(\theta))\Delta W - K_1 > 0$. We find that the inequality is satisfied if $p \in [r_1', r_1'']$ where

$r_1' = \frac{1}{2\Delta W E_1}\left(\Delta W E_1 - \sqrt{\left(\Delta W^2E_1^2 - 4\Delta W E_1 K_1\right)}\right)\frac{(1-E_1)(\gamma(1-e)+e)}{(1-xE_i)((1-e)\gamma+(1-\gamma)e)+(1-E_1)\gamma e}$ and

$r_1'' = \frac{1}{2\Delta W E_1}\left(\Delta W E_1 + \sqrt{\left(\Delta W^2E_1^2 - 4\Delta W E_1 K_1\right)}\right)\frac{(1-E_1)(\gamma(1-e)+e)}{(1-xE_i)((1-e)\gamma+(1-\gamma)e)+(1-E_1)\gamma e}$.

And the last inequality, $x_iE_{x_i}\mu_{x_i}(\theta)(1-\mu_{x_i}(\theta))\Delta W - K_{x_i} > 0$ is satisfied if $p \in [r_i', r_i'']$ where

$r_i' = \frac{1}{2xE_i\Delta W}\left(xE_i\Delta W - \sqrt{\left(x^2E_i^2\Delta W^2 - 4xE_i\Delta K_i\right)}\right)(1-xE_i)\frac{(\gamma(1-e)+e)}{(1-xE_i)((1-e)\gamma+(1-\gamma)e)+(1-E_1)\gamma e}$

and

$r_i'' = \frac{1}{2xE_i\Delta W}\left(xE_i\Delta W + \sqrt{\left(x^2E_i^2\Delta W^2 - 4xE_i\Delta K_i\right)}\right)(1-xE_i)\frac{(\gamma(1-e)+e)}{(1-xE_i)((1-e)\gamma+(1-\gamma)e)+(1-E_1)\gamma e}$.

We check that $0 < r_1' < r_i' < \frac{1}{2} < r_i'' < r_1'' < 1$ if (A#2) is satisfied.

Thus for $p \in [r'_i, r''_i]$, inequalities (6) and (7) are verified. Furthermore we check that $\overline{p} > r''_i$ and thus there exists an unique θ^* that belongs to $[0, 1]$.

For $p \in [r'_1, r'_i[\cup]r''_i, r''_1]$, the inequality (6) is satisfied, whereas (7) is not. In other words, the PTO makes an effort E_1 after observing the maximum prior art, but no effort at all after observing the intermediate level. Equation (5) becomes

$$\mu_1(\theta)(1 - E_1) = \mu_{x_i}(\theta)$$

and we derive

$$\theta_1^* = \frac{(1 - E_1)(e(1-\gamma) + \gamma(1-e) + (1-p)\gamma e) - p(e(1-\gamma) + \gamma(1-e))}{(1-p)(e(1-\gamma) + \gamma(1-e)) + (1-E_1)\gamma e(1-p)}$$

that belongs to $[0, 1]$ if $p < \overline{p_1} = \dfrac{((1-\gamma)y + \gamma)(1-E_1)}{(1-\gamma)y + (1-y)\gamma + (1-E_1)\gamma y}$. We check that $\overline{p_1} > r''_1$ and thus $\theta_1^* \in [0, 1]$.

For $p \in [0, r'_1[\cup]r''_1, 1]$, none of the inequalities (6) and (7) are satisfied. Thus the PTO will not make any effort and the innovator is indifferent if

$$\mu_1(\theta) = \mu_{x_i}(\theta)$$

which is only satisfied for $\theta = 1$. This is actually the equilibrium in pure strategies that we have define earlier.

Search of Prior Art for the Innovator

We cannot define a simple explicit solution for the function of profit of the innovator $\Pi(e)$. We can check that $\Pi(0) > 0$ and $\Pi(1) = -\infty$. Furthermore $\left.\frac{\partial \Pi(e)}{\partial e}\right|_{e=0} > 0$. By computing the derivative using a computer program, we find that there exist two values of e that satisfy the FOC. Thus there exists an unique $e^* \in [0, 1]$ such that $\frac{d\Pi(e)}{de} = 0$.

Let $F(e, E_1, E_{x_i})$ be the first order condition of the maximization program, i.e. $F(e, E_1, E_{x_i}) = \frac{d\Pi(e)}{de} = 0$. We can determine the total derivative of this new function: $\partial F = \frac{\partial F}{\partial e} de + \frac{\partial F}{\partial E} dE = 0$ where $E = E_1, E_{x_i}$. From this equality we can easily determine the sign of $\frac{de}{dE} = -\frac{\partial F}{\partial E} / \frac{\partial F}{\partial e}$. Locally, $\frac{\partial F}{\partial e} < 0$. We thus have to determine $\frac{\partial F}{\partial E}$. Using the computer program we find that $\frac{\partial F}{\partial E_1} > 0$ and $\frac{\partial F}{\partial E_{x_i}} > 0$ if $\gamma < 1/2$ and < 0 otherwise. Thus $\frac{de}{dE_1} > 0$ and $\frac{de}{dE_{x_i}} > 0$ if $\gamma < 1/2$ and < 0 if $\gamma < 1/2$.

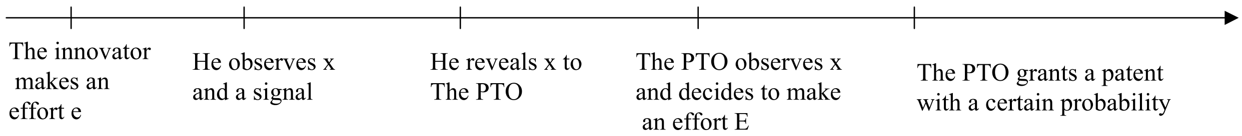
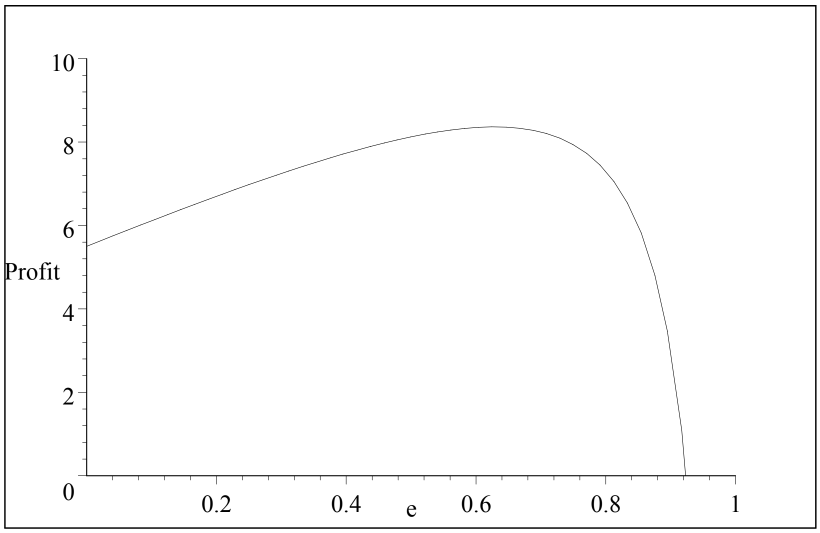


Figure 1: Timing of the Game



Optimal Level of Effort for the innovator for $\gamma = 0.5$, $\overline{V} = 30$, $E_1 = 0.5$, $E_i = 0.4$, $x = 0.5$, $\underline{V} = 10$, $p = 0.5$