Career concerns and choosing whether to compete[*]

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Abstract

In many situations, agents have to decide to compete or not, against others in order to signal themselves. The behavior of researchers, open source developers or start ups illustrate this fact. The purpose of the paper is to examine, in a imperfect signaling framework, the investment strategies of agents who try to manipulate the beliefs of the market. Our results suggest that in the case of homogeneous projects, the decision of agents is not relevant for the market since they will play according to pooling strategies. On the other hand, in the case of cost-differentiated projects, there will be some scope for separating equilibria. Their features are that a low type agent will always invest on the "easy" project whereas a high type, when acting as a follower, will always differentiate himself. We further investigate the willingness of agents to postpone their investment decision and the social optimality of their decision in the presence of reputational concerns.

1 Introduction

In many economic situations, agents get involved on different projects and, hence, have to compete against others in order to signal themselves to future employers. One naive economic intuition would predict that "highly skilled agents" are willing to compete against other players in order to show that they have a relatively higher ability than the others and, hence, get a larger reward. On the other hand "dumb"

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agents should dislike competition since they have high chances to be defeated and, hence, to get a small wage. This paper shows that this intuition is sometimes wrong.

This strategic choice between competing and not competing exists for academic researchers when they decide to invest their time and e¤ort in a particular …eld. It also exists for software developers who invest in Open Source Software (OSS) projects[1] as it is analyzed in Lerner-Tirole [2000] or Crémer, Peyrache and Tirole [2001]. Both papers argue that OSS programmers freely contribute to the development of various softwares in order to show their ability to the rest of the community or potential employers in commercial …rms. Computer skills of most developers in the hacker community has indeed being acquired as a "hobby"and, in the absence of diploma, constitute a non observable information by potential employers. Developers, in order to signal their ability, will then strategically invest on one (or various) OSS project(s). Their decision will be based on their own ability but also on the project's characteristics and their expectations concerning the behavior of others in the OSS community.

In such environments, each individual imposes externalities on the others. In the …rst place, each player should dislike any competition since it can only decrease his chances to be successful. But there is a second dimension to the externalities. The number of players on each project will in‡uence the way the market revise its beliefs about their ability. Competing is a risky strategy but it may signal a relatively higher ability in case of success. The success of a researcher in a very competitive research area will, for example, reveal that the agent is relatively smarter than the others. Symmetrically, failing on an isolated …eld may be very informative on the inability of this agent. At the same time, as it is well known from the literature on adverse selection, if the probability of the low type to succeed when there are no competitors is low, then he may be willing to "hide" himself behind the high type by mimicking his action. The observation that two agents invested on the same project could then constitute a negative signal.

Avoiding competition may then provide a more positive signal. It could indeed signal to future employers that the agent has the ability to tackle some innovative projects. The …rst agent who invests on a given project may then, depending on his ability, try to attract or not followers on the same project through the di¤erent signals sent to the latter. This strong interaction suggests potentially complex strategic behavior by agents who anticipate such externality.

As in Holmström [1982], I develop a model of learning where agents manipulate the assessment of future employers regarding their ability. As in Scharfstein and Stein [1990], the model will also be of sequential entry. A leader will choose between two potentially heterogeneous projects. A follower, who observes this …rst move, will

[1] De…ne the OSS

decide between entering a contest, i.e. compete against the leader, or investing alone on a project. I will consider environments where future employers do not directly[2] observe the quality o¤ered by each contestant and, hence, cannot implement input-based rewards. Moreover the sequentiality in the choice of the agents will not be observed by future employers either. In such setting the labor market will use two pieces of evidence 1) success or failure 2) the number of agents on the project.

I show that although rewards present some "Milgrom [81]" type monotonicity properties with respect to the fact of being successful on its own project, it is not necessarily monotonic with respect to the number of agents involved on the project. Contrary to the intuition one can have, competing might not be more valuable, even for a high type junior, than investing alone. Indeed, in a framework of heterogeneous projects, if separating equilibria exist, the "smart" agent does not "follow the herd" whereas the "dumb one" does. In this setting both pieces of evidence are used by the labor market in order to update its beliefs about the ability of the di¤erent players. These features are robust to the fact that the type of some agents, the "senior" agents, might be common knowledge to everyone.

On the other hand, in the case of homogeneous projects, only the …rst piece of evidence is used by the labor market: agents' decision to compete or not does not constitute a relevant information at equilibrium, only the outcome of the game is. Indeed, both agents (the leader and the follower), independently of the presence of a senior, play according to pooling strategies.

I further investigate the willingness of the di¤erent agents to act as leader or follower by showing that, under some conditions, agents may be willing to postpone their entry decision in order not to have to deter entry by the follower. Finally I show that, even in the presence of reputation e¤ects, these equilibria can be socially optimal.

Related literature

This paper is related to three …elds of the economic theory: signaling games, contests and herding behavior.

Signaling games

- In most signaling models, although the choice of which message to send has strong externality e¤ects[3], it never alters the signal sent by the other senders. Moreover, there is no competition in the signaling game itself in the sense that

[2] Or, at least, not at a zero cost. This is the justi…cation used in the literature to reward agents based on the outcome of a "contest". What matters there is the ranking of agent not the identi…cation of the "distance" between two contribution.

[3] either through the updated beliefs by the receiver as in Spence [1973] or, when competition on a …nal market is considered, through the inverse demand function.

if both agents send the same signal, they will be considered of the same type. Finally, these signals are perfectly observable by the receiver, and the sender perfectly controls the ‡ow of information he transmits. Crawford and Sobel [1982] develop a model where the Sender voluntarily partition the signal space by "only reporting which element of the partition his observation lies in", but the agent still perfectly monitors the amount of information obtained by the other party. My approach of imperfect informative equilibria will then be di¤erent from theirs since the signal received by future employers wil depend on the action taken by the other players.

Literature on tournament

- Most of the literature on contests, as Lazear and Rosen [1981], Nalebu¤ and Stiglitz [1983] or Rosen [1986] focuses on the role of competitive compensation scheme and e¢ciency in a hidden action framework. The incentives of agents to enter a contest and eventually to select a given tournaments among many, is generally not analyzed.[4] Moreover, contrary to this literature, I will not based my analysis on exogenously given reward but rather determine it endogenously.

Literature on herding

- If the spirit of herding models, such as Scharfstein and Stein [1990], is close from mine, there are some substantial di¤erences. Models of herding have the common feature with those of signaling that there is no externality in the ‡ow of information transmitted to the market since there is no competition in the signaling game itself.[5] In other word, experts perfectly control the amount of information they transmit. Moreover, in this model, there will not be any "sharing the blame e¤ects" since, in case of competition, there will be, at most, a single winner.

Outline

The paper is organized as follows. Section 1 lays out the model in a hidden knowledge framework. Section 2 examines the case of two juniors investing on projects. Considering identical projects will allow me to focus on the strategies of the second player since the …rst move is totally uninformative. I stress the inexistence of any separating equilibrium strategies for follower. I will later reintroduce the strategic choice of the …rst agent by analyzing the case of heterogenous projects. In section 3,

[4] Lazear and Rosen [1981] analyzes the incentives of agents to self sort between two di¤erent leagues. But their analysis is conducted with exogenous ranking payo¤s where agents do not choose based on signaling devices.

[5] Two experts who send the same signal are considered to be of the same type

the impact of the presence of a senior on the strategy of the junior will be analyzed. I will show that the equilibria have some common features with those derived in the previous section. Some concluding remarks will appear in section 4.

2 The model

This section sketches the main elements of the structural model. Consider an economy with two agents, A_i; $i = 1;2$, who will sequentially invest on two projects, T_k; where $k = 1;2$ at a type independent cost C^k. The first agent to invest will be referred as the leader and the second one as the follower. Differences among agents are parametrized by a single hidden characteristics, his/her ability θ, which can take two values $\underline{\theta} < \overline{\theta}$: We will call "dumb" an agent of type $\underline{\theta}$ and "smart" an agent of type $\overline{\theta}$. The prior probability that an agent is smart is $\lambda \in (0;1)$. The probability distribution of θ and its support are common knowledge across the different players.

Structure of probabilities

Let us define by $P^i(S/\theta_i;\theta_{-i};n^k)$ the probability of success of agent i when he is of type θ_i and invest on task T_k; agent $-i$ is of type θ_{-i} and n^k agents are involved on task T_k where $n^k = 1;2$. We will assume that when he chooses a task k, a high type agent will always provide output whose quality is acceptable to the market. In other word, a high type agent will always be successful when investing alone. On the other hand, we will assume that a low type agent can only provide a work of acceptable quality with probability p.[6] Formally, we have:

$$P^i(S/\theta_i;\theta_{-i};1) = \begin{cases} 1 & \text{if } \theta_i = \overline{\theta},\ \forall\theta_{-i} \\ p & \text{if } \theta_i = \underline{\theta},\ \forall\theta_{-i} \end{cases}$$

When agents invest on the same task, the probabilities of success are defined as follows:

$$P^i(S/\theta_i;\theta_{-i};2) = \begin{cases} 1 & \text{if } \theta_i > \theta_{-i} \\ 0 & \text{if } \theta_i < \theta_{-i} \\ \frac{1}{2} & \text{if } \theta_i = \theta_{-i} = \overline{\theta} \\ \frac{p}{2} & \text{if } \theta_i = \theta_{-i} = \underline{\theta} \end{cases}$$

[6] One can interpret this approach as follows: when choosing project k, the quality proposed by agent i is modeled as a random variable: $q^k(\theta_i) = \theta_i + \varepsilon^k$ where ε^k, project-specific shocks, have a differentiable probability distribution function $F(.)$, with density $f(.)$, on the support $]-\overline{\theta};\infty[$: Shocks are uncorrelated among different projects. Considering that the minimum quality standard on each project will be set to zero, this is equivalent to assume that any agent $\overline{\theta}$ will at least achieve the quality standard with probability 1 whereas an agent of type $\underline{\theta}$ will do so with a probability $p = 1 - F(-\underline{\theta}) < 1$

Note that when investing on the same task, both low type agents will (or not) obtain a quality above the minimum standard at the same type. This is in line with the common shock interpretation given in the footnote.

A crucial assumption is that worker can't invest on two tasks or play repeatedly so the productivity risk is non diversifiable.

Market's information structure

We will consider that the observation of quality offered by each agent is difficult, or very costly, for the potential employers. This typically characterizes the organization of the Open Source Software industry where a developer will transmit the line of code he wrote to, say, Linus Torvald, the leader of the Linux project, who will then decide to incorporate it, or not, to the core. The potential employers clearly identify the agents who succeeded in their investment, their contributions have been accepted, but observing the lines of code is costly. This feature will be modeled as follows: agents will only report their work to an intermediary who, by perfectly observing the quality offered by each agents, will determine the winner of the game. Thus the quality of the contributions will not be an argument of the wage schedule offered to the agent.

We will further make the following assumptions that we will discuss in the conclusion:

Assumption 1 The market is unable to observe the order in which workers invest on projects.

Assumption 2 The market does not observe the difficulty of the task.

The information structure of the market will then be limited to the observation of

1) the outcome O_i of agent i $(i = 1; 2)$, i.e. success (S_i) or failure (F_i),

2) the number n^k of active people on task k at the end of the game

and the market will, using Bayes' rule, updates his beliefs on the type of the agent i when investing on task k based on the triplet $(O_i; O_{-i}; n^k)$:

The timing: a game of sequential entry

To summarize, we are describing a game where:

- At t=0, nature determines randomly the types of the agents. Agents may or may not learn their own type.
- At t=1, a coin is flipped to determine which agent will be the leader. The leader commits to a choice of one task.
- At t=2, the follower observes the choice of the first one and decides to compete or not to compete.

- At t=3, agents simultaneously invest on tasks and report the quality of their work to the intermediary.
- At t=4, the outcome of the game is determined.
- At t=5, market observes the outcome and the number of active workers on each task. It updates its beliefs based on its information structure.

Preferences

We will now specify agents' objective functions. Players are not rewarded for the task they complete but the wage they will be o¤ered depends directly on the outcome of their investments and their choice decisions. Good performance today will enhance the labor market's perception of one's ability and hence improve one's future earnings. Following Holmström and Ricart i Costa [1986], Scharftein and Stein [1990] we assume that competition leads juniors's spot market wages to be set to the expected value of their abilities.

Each agent will select their signal in order to maximize the di¤erence between the o¤ered wage and the signaling costs. Moreover, the opportunity utility is normalized to zero[7] so the only way they can expect a strictly positive payo¤ is by investing on one of the tasks. Let us call $R^i(O_i; O_{¡i}; n^k)$ the expected reward function of an agent of type μ_i when investing on task k and, considering that agents are risk neutral, $U^k(\mu_i)$ his expected utility. We have

$$\begin{aligned} R^i(O_i; O_{¡i}; n^k) &= \overline{\mu}.P(\overline{\mu}/O_i; O_{¡i}; n^k) + \underline{\mu}.P(\underline{\mu}/O_i; O_{¡i}; n^k) \\ &= \underline{\mu} + 4\mu.P(\overline{\mu}/O_i; O_{¡i}; n^k) \end{aligned}$$

and

$$U^k(\mu_i) = R^i(O_i; O_{¡i}; n^k) ¡ C^k$$

where $4\mu = \overline{\mu} ¡ \underline{\mu}$:

Since the expected return is an increasing function of the revised assessment of the probability that the agent is smart, he will invest in order to generate high values of $P(\overline{\mu}/O_i; O_{¡i}; n^k)$: The reputation concerns may then be in con‡ict with the socially optimal allocation of resources as we will see later.

Strategies and equilibrium

A strategy for each agent is a choice of a task as a function of their information set. The strategy of the leader is simply a mapping from his type set to the set of tasks. On the other hand, a strategy for the follower is a mapping from his type set and the action of the leader to the set of tasks.

We will consider two di¤erent equilibria:

[7] which can be interpreted as if they could not be hired without investing on one of the project

- the competitive case where both agents will invest on the same task. In this setting, there will be at most a single winner, the one of higher type.

- the non-competitive case where agents invest on two di¤erent tasks. In this setting, both agents can be successful on their own task.

Because of their tractability and economic appeal, we will limit our analysis to pure strategy (sequential) equilibria. In the second part, we will even narrow the approach by concentrating on pure strategy separating equilibria.

Preliminary remarks

Before analyzing the di¤erent equilibria of the game, we will start by stating the following Lemma which will be useful for analyzing signaling equilibria in the di¤erent frameworks we will consider.

Lemma 1 (Monotonicity)

The reward function has the following properties

(i) $R(S;F;n^k) \geq R(F;F;n^k)$ for $n^k = 1;2$: and for all possible beliefs by the market.

(ii) $R(S;F;n^k) \geq R(F;S;n^k)$ for $n^k = 1;2$ and for all possible beliefs by the market.

Some elements about the way we computed the expected reward for each possible beliefs by the market are given in the appendix. The sense of this lemma is that the observation of success is more favorable in the sense of Milgrom [81], than observing a failure given that the other agent failed. Stated di¤erently, the posterior probability of a agent's type given observation of success in a given task dominates the posterior probability distribution given observation of failure in the sense of …rst order stochastic dominance. The second point is, somehow, de…ned in the same vein.

3 Equilibria with reputational concerns among juniors

In this section we will call "Junior" an agent whose type is private information and who tries to in‡uence the assessment of the market regarding their ability. We will consider here that both agents will be "Juniors". We will show that heterogeneity among the tasks is necessary for the allocation choice of agents to be a relevant information for the market. Indeed, in the case of identical tasks, the follower will play according to pooling strategies. Further more, when separation equilibria exist, it always involves di¤erentiation by a smart agent when he acts as a follower.

3.1 Signaling in the case of identical tasks

In this section, the fixed costs of investing on each task will, for convenience, be normalized to zero ($C^1 = C^2 = 0$) whatever the type of the agents. We will focus on the decision rule of the follower since the choice of the leader -it does not transfer any information to the second worker- is irrelevant. The revised ability assessment will then be a function of the conjecture about the action of the follower by the market. In the next section we will study the implications of introducing heterogeneity on the task side.

3.1.1 Benchmark

I will start the analysis by considering, as a benchmark, the extreme case where agents do not know their type. As an illustration the reader may think about a OSS programmer without any experience or a PhD students who are not aware of their true ability.

Proposition 1 When he does not know his type:

(i) the follower is indifferent between competing and not competing. His ex post payoff is identical to the reward he would get had the market used its prior beliefs.

(ii) Agents are indifferent between playing first or playing second.

Proof. When agents do not know their type, they do not base their choice on any private information. Thus the observation of competition, or not, does not constitute a signal that restricts the type space of the two agents. The expected reward conditional on the number of agents on each project in given in the following table.

No competition	Competition
$R(S;1) = \underline{\theta} + \Delta\theta \cdot \frac{\lambda}{\lambda + (1-\lambda)p}$	$R(S;F;2) = \underline{\theta} + \Delta\theta \frac{\lambda(2-\lambda)}{\lambda(2-\lambda) + (1-\lambda)^2 p}$
$R(F;1) = \underline{\theta}$	$R(F;S;2) = \underline{\theta} + \Delta\theta \frac{\lambda^2}{\lambda(2-\lambda) + (1-\lambda)^2 p}$

Note that when agents invest on two different projects, their reward is independent of the outcome of the other. On the other hand, when they compete, the revision of beliefs takes into account the potential ability of the competitor.

We can now compute their respective utility when they decide to compete or not to.

$$\begin{aligned}
U_2^{NC}(.) &= \lambda \cdot R(S;1) + (1-\lambda)[p \cdot R(S;1) + (1-p) \cdot R(F;1)] \\
&= \underline{\theta} + \lambda \cdot \Delta\theta \\
U_2^{C}(.) &= \lambda[(1-\frac{\lambda}{2}) \cdot R(S;F;2) + \frac{\lambda}{2} \cdot R(F;S;2)] + (1-\lambda)[(\lambda + \frac{1-\lambda}{2} p) \cdot R(F;S;2) \\
&\quad + \frac{1-\lambda}{2} p \cdot R(S;F;2) + (1-\lambda)(1-p) \cdot R(F;F;2)] \\
&= \underline{\theta} + \lambda \cdot \Delta\theta
\end{aligned}$$

Then, whatever the probability $q \in [0;1]$ that the follower will decide to compete, the expected utility of the first player if:

$$
\begin{aligned}
U_1(.) &= (1 - q)\, U_1^{NC}(.) + q U_1^{C}(.) \\
&= \underline{\mu} + \lambda.\Delta\mu
\end{aligned}
$$

And so Agents are indifferent to the order of the play. ■

The stochastic process described by the utility of the agents is then a Martingale with respect to the choice of the agents. The explanation for this result stems from the fact that, since agents do not know their type, they compute their expected reward based on all possible outcomes of the game. No extra information being obtained during the process, a difference between the ex post and the ex ante reward would shed light on a non-optimal revision of beliefs by the market.

3.1.2 Signaling equilibria

We now turn our attention to the situation where agents know their types. In this subsection, I will show that, as in the case where agents do not know their type, the actions of individuals are not a relevant information for the market, only the outcome of the game will be.

Proposition 2 In the case of identical tasks, when agents know their types:

(i) the only two equilibria are two pooling sequential equilibria where both types decide either to compete or not to compete. These equilibria can be supported by the following out of equilibrium belief: "Any deviator is considered to be a $\underline{\mu}$ agent".

(ii) For any $(\lambda; p)$; none of the 2 pooling equilibria Pareto-dominates the other.

(iii) Agents are indifferent between playing first or playing second

Proof. the proof is provided in the appendix. ■

The inexistence of separating equilibria stems from the fact that independently of the values of the different parameters $(\lambda; p)$, the expected reward when following $\overline{\mu}$ type's strategy is higher than the one when following $\underline{\mu}$ type's strategy and that the two functions never cross. Therefore, it is always better to play according to $\overline{\mu}$ type's strategy.

I also show that for all values of $p > p^*$ (where. $p^* = \frac{\lambda^2}{(1-\lambda)^2+\lambda}$), the high type would be better-off under the "competing pooling equilibrium" whereas the low type would prefer the "non competing pooling equilibrium". For $p < p^*$; the conclusion is reversed: the high type would be better-off under the "non-competing pooling equilibrium" whereas the low type would prefer the "competing pooling equilibrium". It's fairly intuitive that for low values of p, type $\overline{\mu}$ has high chances of being the single winner when investing alone and, then, avoids the possibility of having to compete with another high type. Conversely, for low values of p, type $\underline{\theta}$, having high chances of failing alone (and then be identified as dumb with probability 1) would like to hide himself behind the smart agent.

Finally part (iii) of the proposition follows directly from (i). Indeed, the fact that agents do not have any incentive to postpone their decision and acts as followers stems from the fact that whatever the order of move, both types of agents pool their decision.

Notice that all groups do not loose from the existence of signaling. In both pooling equilibria, agents of high type are better off when they have this opportunity to signal themselves. Indeed, one can easily check that, for all values of $(\lambda, p) \in (0,1)^2$:

$$\begin{aligned} U_2^C(\bar{\theta}) &> \underline{\theta} + \lambda.\Delta\theta \\ U_2^{NC}(\bar{\theta}) &> \underline{\theta} + \lambda.\Delta\theta \end{aligned}$$

On the other hand,

$$\begin{aligned} \underline{\theta} + \lambda.\Delta\theta &> U_2^{NC}(\underline{\theta}) \\ \underline{\theta} + \lambda.\Delta\theta &> U_2^{C}(\underline{\theta}) \end{aligned}$$

meaning that agents of low type are worse off investing in this signaling activity than they would do had the market used its prior beliefs.

Having established that continuation equilibria with type-contingent decisions by the follower does not exist in the case of homogeneous tasks, I will now turn to the case of heterogeneous tasks

3.2 Signaling in the case of differentiated tasks

In the previous section, I have shown that in the case of identical tasks, the actions taken by the different individual were not relevant for the market since both agents play according to the same pooling strategies. The way the market revise its beliefs is clearly not the same if agents are competing or not competing, but the choice to compete, or not, itself themselves does not reveal any information. Assuming that the follower always plays the "non competing equilibrium", this game would be equivalent to a game where there would be no externality among agents.

Now, let us introduce some heterogeneity on the task side. Agents have to bear a cost normalized to zero when choosing the "easy" task (T_E) and a cost C when choosing the "hard" task (T_H). Formally, $C^E = 0$ and $C^H = C$. I will further assume that $\underline{\theta} - C \geq 0$, and so agents are always better off investing on one of the tasks rather than choosing their outside opportunity.

3.2.1 Benchmark

Let us start, as a benchmark, with the simplest case where none of the agents know his true ability. The choice of the leader does not reveal any information neither to the market nor to the follower.

Proposition 3 In the case of heterogenous tasks, when agents do not know their type

(i) the only equilibrium is pooling where all agents play T_E

(ii) agents are indi¤erent between playing …rst or second

Proof. Using the results of proposition 1, we have shown that the agent is indi¤erent between competing or not competing when he bears the same cost of investment. His payo¤ was invariably $µ + ¸ . 4 µ$:

In the di¤erentiated task case, follower will always choose to invest on task T_E whatever the choice of leader since he will bear a smaller cost. The reasoning concerning the leader is exactly symmetric. From this last remark follows (ii). ■

As in the identical tasks setting where agents didn't know their type, both workers are indi¤erent between having to signal themselves or let the market use its prior beliefs. This was the martingale argument. But agents are not indi¤erent between competing and not competing since investing on T_H is now a costly choice. Moreover, since the equilibrium is that both agents pool on T_E, they will still be indi¤erent to the order of their move.

3.2.2 Signaling equilibria

Consider now the case where both agents have private information. When they know their type, the choice of the …rst agent will transmit some information to the second one, and the follower will, using Bayes' rule, update his beliefs on the type of the leader. Both agents will then play strategically.

Notation 1 Let us denote the strategy of the leader by $¾_1(µ_L)$ and the strategy of the follower by $¾_2(µ_F; a_L)$, where a_L is the action taken by the leader.

There is a trivial pooling equilibria on T_E for high values of C: $¾_1(µ_L) = T_E$ and $¾_2(µ_F; T_E) = T_E$: Therefore we will investigate only separating equilibrium strategies.[8]

[8] We have 2^6 potential equilibria in the game just described. We will not consider pooling equilibria nor situation where an agent of type µ could respond di¤erently to the same signal. The potential candidates for equilibrium are then reduced to 36.

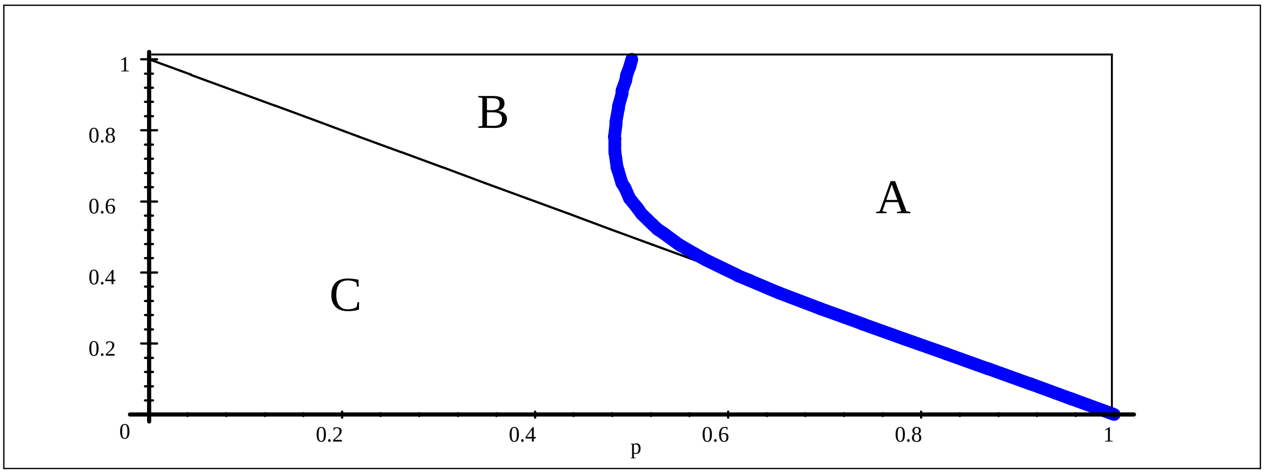

Figure 1:

Proposition 4 *Consider the case of heterogenous tasks, when agents know their type. For all equilibria where, at least one of the agents has a separating strategy, we have:*

(i) The low type agent, independently of his order of move, will always play T_E*, formally:* $\sigma_1(\underline{\theta}) = \sigma_2(\underline{\theta}; T_k) = T_E \ \forall T_k$

(ii) The high type agent, when he acts as a follower, will always differentiate, formally: $\sigma_2(\overline{\theta}; T_k) = T_{-k}$

Proof. The proof is rather long and is available from the author. The methodology is the following: we successively consider all potential separating equilibria; we determine the new assessment of the ability of the agent by future employers in all these situations; finally we analyze if, given these beliefs, any agents have an incentive to deviate. ■

Depending on the values of the parameters $(\lambda; p; C)$, there might be at most either 0 (region A), 1 (region B) or 2 (region C) separating equilibrium strategies (see figure 1). Figure 1 constitutes a graphical illustration of the relevant range of $(\lambda; p)$ for which separating equilibria may be defined.[9]

The only potential equilibrium in region B is equilibrium 1defined by

$$\begin{cases} \sigma_1(\theta) = T_E \ \forall\theta \\ \sigma_2(\underline{\theta}; T_E) = T_E; \sigma_2(\overline{\theta}; T_E) = T_H \end{cases}$$

i.e. the follower does not compete when he is smart and compete when he is dumb.

On the other hand, the two potential equilibria in region C are equilibrium 1 and equilibrium 2 that we define by

[9] If C is defined in the appropriate range.

$$\begin{cases} \sigma_1(\underline{\theta}) = T_E ; \sigma_1(\overline{\theta}) = T_H \\ \sigma_2(\underline{\theta}; T_E) = T_E ; \sigma_2(\underline{\theta}; T_H) = T_E ; \sigma_2(\overline{\theta}; T_E) = T_H ; \sigma_2(\overline{\theta}; T_H) = T_E \end{cases}$$

i.e., the follower plays T_E if he is dumb and always differentiate when he is smart.

A remarkable and common feature of these separating equilibria is that, contrary to a naive intuition, smart agents do not "follow the herd". If future employers anticipate that smart agents compete, the observation of competition would constitute a positive signal. Dumb agents would then have an incentive to hide themselves and to mimic the action of the smart agent. As a consequence, we have:

Corollary 1 The reward function is not necessarily monotonic with the number of agents involved on the task. Formally, for some equilibria, $R(O_i; O_{-i}; 2) \geq R(O_i; O_{-i}; 1)$ and for others $R(O_i; O_{-i}; 2) \leq R(O_i; O_{-i}; 1)$.

The multiplicity of equilibria is driven by the fact that the choice of project itself does not directly affect the expected reward of agents but constitute a way to transmit information to future employers and manipulate their beliefs. Given the formulation of the model, I cannot predict the exact choice made by each agent but can analyze how much information is revealed in equilibrium. In the case of identical tasks, the binary outcome $(S; F)$ was the only relevant information used by the market at equilibrium since the follower was, in all cases, using a pooling strategy. The information structure of the market is, here, richer[10] and the revised assessment becomes a function of the outcome of the game and the number of agents involved in the task.[11]

In addition to the multiplicity of equilibria, potential inefficiency in the choice of agents is another consequence of the reputation effects of the model. Let us denote by V the social value associated to each task. The socially optimal allocation of resources in an economy with no reputation considerations will have both agents choosing the easy task if they are both dumb and if $pV > p^2 2V + 2p(1 - p)V - C$, which is equivalent to $\frac{C}{V} > p$. The smart follower, will always be assigned to the other project if $pV + V - C > V$, i.e. if $\frac{C}{V} < p$. Finally, as long as $\frac{C}{V} < 1$, it is always optimal for both players to choose two different projects when they are both smart.

[10] This is true despite the fact that future employers do not observe the difficulty of each project.

[11] In the setting of equilibrium 1, observing two agents competing reveals that there exists at least one agent who is $\underline{\theta}$. Respectively, observing two agents not competing reveals that there exists at least one agent who is $\overline{\theta}$. Moreover, a failure of one of the agents in a "non competing" framework signals to the market that his competitor is surely of high type. Similarly, equilibrium beliefs in equilibrium 2 are such that the observation of two agents competing reveals that both agents are of low type.

Hence, the equilibria in the presence of reputational concerns as defined above can be in conflict with the optimal allocation of resources.

Note finally that agents may be better off postponing their investment decision in order to observe the choice of the other player. Conversely, by playing first, they may try to deter the follower to compete by sending an appropriate signal. This drives the following proposition:

Proposition 5 (i) In equilibrium 1, both type of agents of agents would be better off playing first

(ii) In Equilibrium 2:

a) a low type agent is indifferent to the order of the move

b) a high type agent would be better off playing second

Proof. It is a matter of routine once considering that $\frac{C}{4\theta} > \lambda.[\frac{2-(1-\lambda)p}{2\lambda} - \frac{2+(1-\lambda)p}{2(\lambda+(1-\lambda)p)}].$ ■

As an illustration, let us try to consider the behavior of a high type agent when playing according to equilibrium 2. He is always weakly better off by postponing his decision investment. He will indeed invest alone in both cases but, playing as a follower, allows him not to have to deter entry by playing T_H. Hence, if the first player is smart, which occurs with probability λ, a smart follower have the opportunity to invest on T_E. If the first player is dumb, which occurs with probability $1-\lambda$, a smart follower will then be indifferent to his order of move.

4 Equilibria in the presence of a senior

We examine now the case where "seniors", agents whose ability is common knowledge to all players, are active on the market. "Seniors" and "juniors" have different objective function. Since seniors' type is common knowledge, their investment decision does not change the assessment of the market regarding their ability and they will be rewarded a bonus B every time they will be successful on a task. Thus, "juniors" will be the only agent trying to influence the assessment of the market regarding his/her ability. As an illustration, one can go back to the example of the open source software community: juniors invest in order to be employed by a software company, seniors are already employed and invest in order to get stock options. Note that if a senior is active on the market, his ability will be part of the information structure of the junior.

I will argue that the basic features of the equilibria I obtained in the precedent section are not modified by the presence of a senior. I will indeed show that heterogeneity among the tasks is again necessary for the allocation choice of agents to be a relevant information for the market. Moreover, if a separating equilibrium for juniors exist, it is such that "dumb juniors" play T_E and "smart juniors" di¤erentiate when acting as followers.

Proposition 6 In the case of identical tasks,

(i) A senior never competes when acts as a follower

(ii) The only equilibria for a junior are two perfect pooling equilibria where he decides either to compete or not to compete. None of the equilibria pareto-dominates the other.

Proof. It is straightforward to show that there is no separating equilibria in this game with homogeneous projects since there would always be a type who would have an incentive to deviate and act as if he was a smart player.

Let us now consider pooling equilibria. One can, once more, easily show that, by setting out-of-equilibrium beliefs such that all deviator is considered as being dumb, we get two pooling equilibria where either both agents compete or not compete. ■

The explanation for the …rst part of this result is as follows. A senior agent does not invest in order to manipulate the beliefs the market has regarding his/her ability but rather in order to be successful with the highest probability. Since the cost of investing on both tasks is the same, a senior will clearly choose to avoid competition when he acts as a follower.

The explanation for the second part of the result is identical to the case where both agents were juniors. If we conjecture that a separating equilibrium exist, then one can show that, independently of the values of the di¤erent parameters $(¸; p)$, the expected reward when following $\overline{\mu}$ type's strategy is higher than the one when following $\underline{\mu}$ type's strategy and the two functions never cross. Therefore, it is always better to play according to $\overline{\mu}$ type's strategy.

Nevertheless, the non-existence of separating strategies, for at least one of the two agents, is not robust to the introduction of heterogeneous tasks. In this setting, we have the following results[12]:

Proposition 7 Consider the case where tasks are heterogeneous and there is a senior agent. For certain values of the parameters, there is a unique equilibria where juniors play according to separating strategies. We have:

(i) The "senior-agent", independantly of his ability, and the low type "junior-agent" play $T_{E:}$

[12] There are two other equilibria. Obviously, a situation where all agents invest on T_E is an equilibrium strategy for high values of C: The other equilibrium is such that, for appropriate values of the parameters, seniors invest either T_H and juniors, whatever their types, always invest on T_E whatever the order of move. This second equilibrium has exactly the same features concerning the behavior of juniors as is stated in the proposition .

(ii) The high type “junior-agent”, when playing as a follower, di¤erentiate.

Proof. The proof is provided in the appendix. ■

The features of the separating equilibrium is very close from the ones with two junior agents. Indeed a low type junior will always invest on the low-cost task and the high type junior, when acts as a follower, will always di¤erentiate. Moreover, juniors and seniors are always indi¤erent with respect to the order of move.

Notice that the presence of a senior increases the informativeness of the equilibrium and permits a more acurate revision of beliefs by future employers. Since the type of “seniors” is common knowledge, the outcome of the game when agents compete is clearly more informative than with two “juniors”. Moreover, in the presence of a senior, since the market perfectly knows what would be the decision taken by the senior in all possible subgames, the …rst play becomes informative both for the follower and future employers.

5 Conclusion

The purpose of this paper has been to analyze the impact of career concerns on the sequential strategic choice of competing on (potentially heterogenous) tasks by di¤erent players. Our approach imposed discipline by assuming that “junior players” only value the reputation e¤ect of their investment whereas “senior agents” based their decision on the direct return they expected. The action of the leader constituted a signal interpreted by the follower before he chose to compete or not on the same task. The outcome of the game and the number of competitors on each project were used by the labor market to revise its assessment on the ability of each players. I have shown that the decision to compete by the follower was, when considering separating equilibria, not valuable. For appropriate values of the parameters, a smart player has indeed an incentive to di¤erentiate himself by investing on a di¤erent project. The presence of a senior, whose ability is common knowledge to all, do not change the nature of the results.

A Lemma

We will first prove a lemma that we will use in the appendix. It allows us to prove that a certain function (the difference in utility obtained by two different actions) is positive for all parameters lying in some set.

Lemma 2 Let Ω be a bounded subset of a n-dimensional space and let $H(\Omega)$ be the boundary of Ω. Let $f(.) : X \rightarrow \Re$ be a continuously differentiable function where X is some open set containing Ω. Furthermore, we will call $\bar{x}$ a stationary point of $f(.)$ if $\frac{\partial f(\bar{x})}{\partial x_i} = 0$ for all i.

If $f(x) \geq 0$ for all $x \in H(\Omega)$ and there is no stationary point $\bar{x}$ of f in Ω such that $f(\bar{x}) < 0$, then $f(x) \geq 0$ for all $x \in \Omega$.

Proof. We will proceed by contradiction. Suppose that the lemma is false. Then there would exist at least one $\tilde{x}$ in the interior of Ω such that $f(\tilde{x}) < 0$. Since the minimum is not a boundary point and f is continuously differentiable, then the minimum must be a stationary point. But, by assumption, the values of f at all stationary point are nonnegative. This contradiction proves the lemma. ■

B Some elements about Lemma 1

The expected reward can be calculated conditionally on the realization of the outcome and the number of agents active on a task:

$$\begin{aligned} R(O_i; O_{-i}; n^k) &= \bar{\mu}.P(\bar{\mu}/O_i; O_{-i}; n^k) + \underline{\mu}.P(\underline{\mu}/O_i; O_{-i}; n^k) \\ &= \underline{\mu} + \Delta\mu.P(\bar{\mu}/O_i; O_{-i}; n^k) \end{aligned}$$

Table 1 present the conditional probabilities, for the different market's beliefs we will handle in the appendix[13],that the agent is of high ability.

Let us define the potential beliefs of the market:

- Situation 1: the choices of the agents does not transmit any information. This is the case , for example, when agents play pooling strategies

[13] The reader has to keep in mind that this description is not exhaustive. Since a full proof is beyond the scope of this paper, we will focus on beliefs corresponding to equilibria (and not all potential equilibria) of the different games we will analyze.

- Situation 2: When agents are competing, then the market anticipate that the follower is of type $\bar{\mu}$. When agents are not competing, then the market anticipate that the follower is a $\underline{\mu}$ player.

- Situation 3: When agents are competing, then the market anticipate that the follower is $\underline{\mu}$. When agents are not competing, then the market anticipate that the follower is $\bar{\mu}$.

- Situation 4: When agents are competing, then the market anticipate that both agents are $\underline{\mu}$. When agents are not competing, then the market anticipate that agents are of opposite types or both $\bar{\mu}$.

	(F,S,1)	(S,F,1)	(S,S,1)	(S,F,2)	(F,S,2)
Situation 1	0	$\frac{\lambda}{\lambda+(1-\lambda)p}$	$\frac{\lambda}{\lambda+(1-\lambda)p}$	$\frac{\lambda[2-\lambda]}{\lambda[2-\lambda]+(1-\lambda)^2p}$	$\frac{\lambda^2}{\lambda[2-\lambda]+(1-\lambda)^2p}$
Situation 2	0	$\frac{\lambda}{\lambda+2(1-\lambda)p}$	$\frac{\lambda}{2[\lambda+(1-\lambda)p]}$	1	λ
Situation 3	0	1	$\frac{1}{2}+\frac{\lambda}{2[\lambda+(1-\lambda)p]}$	$\frac{\lambda}{\lambda+(1-\lambda)p}$	0
Situation 4	0	1	$\frac{\lambda+(1-\lambda)p}{\lambda+2(1-\lambda)p}$	0	0

Let us detail, as an illustration, the way we compute $P(\bar{\mu}/F,S,2)$ in situation 2. Using Bayes rule:

$$P(\bar{\mu}/F,S,2)=\frac{P(F,S,2/\bar{\mu}).P(\bar{\mu})}{P(F,S,2/\bar{\mu}).P(\bar{\mu})+P(F,S,2/\underline{\mu}).P(\underline{\mu})}$$

where,

$$\begin{aligned} P(F,S,2/\bar{\mu}) &= \frac{1}{2}\left[(1-\lambda).0+\lambda.\frac{1}{2}\right]+\frac{1}{2}\left[(1-\lambda).0+\lambda.\frac{1}{2}\right] \\ &= \frac{\lambda}{2} \end{aligned}$$

since the agent play first with probability $\frac{1}{2}$ and, considering that the agent is $\bar{\mu}$, there is no way he can be defeated by an agent $\underline{\mu}$.

In the same manner,

$$P(F,S,2/\underline{\mu})=\frac{1}{2}[(1-\lambda).0+\lambda]=\frac{\lambda}{2}$$

since, considering that the agent is $\underline{\mu}$ imply that he played first (otherwise both agents wouldn't be competing) and his opponent is $\bar{\mu}$ (since there exist at least one agent $\bar{\mu}$).

All probabilities in table 1 are computed using the same reasoning.

C proof of proposition 3

We will only show that the pooling strategies are equilibrium strategies. The proof that every time the market potential belief are that $\underline{\theta}$ competes and $\overline{\theta}$ does not (resp. $\overline{\theta}$ competes and $\underline{\theta}$ does not), $\underline{\theta}$ will indeed have an incentive to mimic the strategy of $\overline{\theta}$ is available from the author.

C.0.3 "Pooling on Competition" as an equilibrium

The observation by the market that both agents are on the same task does not reveal any information concerning their type.

- The reward function are then the following:

$R(S;F;2) = \underline{\theta} + 4\theta\frac{\lambda(2-\lambda)}{\lambda(2-\lambda)+(1-\lambda)^2 p}$
$R(F;S;2) = \underline{\theta} + 4\theta\frac{\lambda^2}{\lambda(2-\lambda)+(1-\lambda)^2 p}$

We can now compute their respective expected utility when follower decide to compete:

$$
\begin{aligned}
U_2^C(\overline{\theta}) &= [(1-\frac{\lambda}{2})\cdot R(S;F;2) + \frac{\lambda}{2}\cdot R(F;S;2)] \\
&= \underline{\theta} + \lambda\cdot 4\theta\cdot[\frac{1+(1-\lambda)^2}{\lambda(2-\lambda)+(1-\lambda)^2 p}] \\
U_2^C(\underline{\theta}) &= \lambda\cdot R(F;S;2)] + (1-\lambda)[\frac{p}{2}\cdot R(F;S;2) + \frac{p}{2}\cdot R(S;F;2) + (1-p)\cdot R(F;F;2)] \\
&= \underline{\theta} + \lambda\cdot 4\theta\cdot[\frac{\lambda^2+(1-\lambda)p}{\lambda(2-\lambda)+(1-\lambda)^2 p}]
\end{aligned}
$$

- Let us assume now when it is observed that the two agents invested on two different tasks, the out-of-equilibrium beliefs of the market are that the deviator is a low type with probability 1. The reward function are then the following:

$R(S;F;1) = \underline{\theta} + 4\theta\frac{\lambda}{\lambda+2(1-\lambda)p}$
$R(S;S;1) = \underline{\theta} + 4\theta\frac{\lambda}{2(\lambda+(1-\lambda)p)}$
$R(F;.;1) = \underline{\theta}$

The expected utilities can then be computed as follows:

$$
\begin{aligned}
U_2^{NC}(\underline{\theta}) = {} & \lambda[pR(S;S;1) + (1-p)R(F;S;1)] + \\
& (1-\lambda)[p^2R(S;S;1) + p(1-p)R(S;F;1) + p(1-p)R(F;S;1) + (1-p)^2R(F;F;1)]
\end{aligned}
$$

$$= \underline{\theta} + \lambda(\Delta\theta)\left[p\frac{2-\lambda}{2(\lambda+2(1-\lambda)p)}\right]$$

$$U_2^{NC}(\bar{\theta}) = (\lambda+(1-\lambda)p)R(S;S;1)+(1-\lambda)(1-p)R(S;F;1)$$

$$= \underline{\theta} + \lambda(\Delta\theta)\left[\frac{2-\lambda}{2(\lambda+2(1-\lambda)p)}\right]$$

- Let us show, using Lemma 1, that for all values of $(\lambda;p) \in (0;1)^2$, agent $\underline{\theta}$ has no interest to move to the NC situation.

If we denote the relative advantage of competing of $\underline{\theta}$ by:

$$\Psi^C(\underline{\theta}) = U_2^C(\underline{\theta}) - U_2^{NC}(\underline{\theta})$$

$$= \lambda.\Delta\theta\frac{\lambda p\left[-5\lambda^2+6\lambda-2\right]+2p(1-\lambda)^2(2+\lambda)}{2\left[\lambda(2-\lambda)+(1-\lambda)^2p\right](\lambda+2(1-\lambda)p)}$$

For all values of $(\lambda;p) \in \Omega$ the sign of $\Psi^C(\underline{\theta})$ is the same as the one of

$$L(\lambda;p) = \lambda p\left[-5\lambda^2+6\lambda-2\right]+2p(1-\lambda)^2(2+\lambda)$$

The set Ω is bounded and $L(\lambda;p)$ is certainly continuously differentiable. We can then apply Lemma 1.

$$\frac{\partial L(\lambda;p)}{\partial p} = 0 \quad , \quad p^s(\lambda) = \lambda\frac{5\lambda^2-6\lambda+2}{2(1-\lambda)^2(2+\lambda)}$$

One can then check that for all $\lambda \in (0;1)$; $L(\lambda;p^s(\lambda)) \neq 0$

Hence there is no feasible $(\lambda;p)$ such that $L(\lambda;p)$ admits a stationary point.

Moreover, $L(\lambda;p)$ is positive on the boundary of Ω :

$$\begin{aligned}
L(0;p) &= 3p^2 > 0 \\
L(1;p) &= 2-p > 0 \\
L(\lambda;0) &= 2\lambda^3 > 0 \\
L(\lambda;1) &= (2-\lambda)\left[1-2\lambda(1-\lambda)\right] > 0
\end{aligned}$$

We can then conclude that $L(\lambda;p)$ is positive in Ω and that it's optimal for $\underline{\theta}$ to compete.

In the same manner, we could show that it's optimal for $\bar{\theta}$ to compete.

Both incentive compatible (IC) are then satisfied:

$$\begin{aligned}
U_2^C(\bar{\theta}) &> U_2^{NC}(\bar{\theta}) \\
U_2^C(\underline{\theta}) &> U_2^{NC}(\underline{\theta})
\end{aligned}$$

- In the same manner, it can be shown after computing the appropriate reward function, that the out-of-equilibrium belief: "the deviator is a high type with probability 1", can't sustain the pooling equilibrium just defined.

C.0.4 "Pooling on No Competition" as an equilibrium

- The reward function by the market will be the following:

$R(S;F;1) = \underline{\theta} + \Delta\theta\frac{\lambda}{\lambda+(1-\lambda)p}$
$R(S;S;1) = \underline{\theta} + \Delta\theta\frac{\lambda}{\lambda+(1-\lambda)p}$
$R(F;.;1) = \underline{\theta}$

and the expected utilities:

$$\begin{aligned} U_2^{NC}(\underline{\theta}) &= \underline{\theta} + \lambda.\Delta\theta.[\frac{p}{\lambda+(1-\lambda)p}] \\ U_2^{NC}(\overline{\theta}) &= \underline{\theta} + \lambda.\Delta\theta.[\frac{1}{\lambda+(1-\lambda)p}] \end{aligned}$$

- Let us assume now when it is observed that the two agents are competing on the same task, the out-of-equilibrium beliefs of the market are that the deviator is a low type with probability 1. The reward function are then the following:

$R(S;F;2) = \underline{\theta} + \Delta\theta\frac{\lambda}{\lambda+(1-\lambda)p}$
$R(F;S;2) = \underline{\theta}$

and the expected utilities:

$$\begin{aligned} U_2^{C}(\overline{\theta}) &= \underline{\theta} + \lambda.\Delta\theta.[\frac{2-\lambda}{2(\lambda+(1-\lambda)p)}] \\ U_2^{C}(\underline{\theta}) &= \underline{\theta} + \lambda.\Delta\theta.[\frac{(1-\lambda)p}{2(\lambda+(1-\lambda)p)}] \end{aligned}$$

- Using Lemma 1, one can check that for all values of $(\lambda;p) \in (0;1)^2$, none of the agent has an interest to move to the C situation.

Both incentive compatible (IC) are then satisfied:

$$\begin{aligned} U_2^{NC}(\overline{\theta}) &> U_2^{C}(\overline{\theta}) \\ U_2^{NC}(\underline{\theta}) &> U_2^{C}(\underline{\theta}) \end{aligned}$$

- In the same manner, it can be shown after computing the appropriate reward function, that the out-of-equilibrium belief: "the deviator is a high type with probability 1", can't sustain the pooling equilibrium just defined.

Finally, it's a routine matter to show that:

- $U(C/\overline{\theta}) > U(NC/\overline{\theta}) \iff p > \frac{\lambda^2}{(1-\lambda)^2+\lambda}$
- $U(C/\underline{\theta}) > U(NC/\underline{\theta}) \iff p < \frac{\lambda^2}{(1-\lambda)^2+\lambda}$.

D Proof of proposition 7

We will start by stating the following lemmas. The proofs for the first two are trivial

Lemma 3 A senior agent, when acts as a follower, will always play E when he observes a first move on H.

Lemma 4 A senior agent, when acts as a leader, will always play E when he anticipates that a follower of high type will play H.

Lemma 5 If a senior plays first and plays H and if the equilibrium strategy of a high type follower is to differentiate, then the low type follower will mimic this strategy.

Proof. Suppose it was not true. By competing, the low type agent would be considered as a low type and would incur the highest cost. But this would contradicts the fact that this is an equilibrium strategy. ■

Lemma 6 There is no equilibrium where the senior plays T_E and the labor market believes that the smart agent compete.

Proof. Suppose such an equilibrium exists. By investing alone, the dumb agent will incur the cost C and will be identified as being dumb with probability one. His payoff is then the lowest he could get. This contradicts the fact that it could be an equilibrium. ■

Lemma 7 (Consistency).

If an agent of type μ_i has an incentive to play k $(k = E; H)$ when he observes a first move on l $(l = k$ or $l \neq k)$ then he also has an interest to play k when he anticipates that the follower will play l.

Proof. Let $H_2^i(k/l)$ the return of agent i when, as a follower, he plays k and the other plays l: If playing k is an equilibrium strategy we have that

$$H_2^i(k/l) > H_2^i(k'/l) \text{ where } k' \neq k$$

Suppose now that it's an equilibrium strategy for player i to play k' when acts as a leader and anticipates that the follower will play l. We then have:

$$H_1^i(k'/l) > H_1^i(k/l)$$

Since the market does not observe the order of move on the agent, we have

$$H_2^i(k/l) = H_1^i(k/l)$$

and the two inequalities are incompatible. ■

Using the 4 previous Lemmas, we get the following claim:

Claim 1 A senior of high type will play E whether he moves …rst or second.

Proof. From lemma 4, a senior, when acts as a leader, could play H if and only if the continuation equilibrium is such that a high type follower always play E whatever the …rst move of the senior. Then from lemma 5, the continuation equilibrium is such that a low type follower also plays E when a …rst move on H has been observed.

Since we decided to focus on situations where junior agents would not play according to the same strategies, we will not consider this case. Then, the only relevant equilibria are the ones where the senior plays E when acts as a leader. By consistency and using lemma 7, a senior agent will play E whatever his order of move. ■

Claim 2 For appropriate values of the parameters, there is a separating equilibrium where a junior agent will play H when he is smart and play E when he is dumb.

Proof. Suppose that the separating equilibrium is de…ned in the opposite way. Then a dumb agent would be identi…ed with certainty and would incur the highest cost. This contradicts the fact that it is an equilibrium strategy. On the other hand, for appropriate values of C a smart junior may have incentives to perfectly signal himself by incurring the high cost. C has to be high enough such that the dumb agent does not have any incentive to mimic the strategy of the smart one but also low enough such that the latter has an incentive to invest of the hard task. Formally, we must have: $4\mu > C > p4\mu$.

Using lemma 7, this is true whatever the order of move of the junior. ■

Claims 1 and 2 fully prove the proposition.

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