An empirical investigation of two nonlinear models in real exchange rate series

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Abstract

In this paper we examine whether the Markov switching (MS) models and exponential smooth transition autoregressive (ESTAR) nonlinear models can give any additional insights into real exchange rate behaviour during the period 1974-1998. The results from MS models show that there are clearly long swings in the real exchange rate series, which can be characterize as a depreciation regime and an appreciation regime. Furthermore, the regime staying probabilities are relatively high, although the process is eventually mean reverting. The ESTAR model results implies random walk behaviour for small deviations from PPP and fast adjustment for large deviations, which are in line with the MS model.

JEL classification: C22; F31; F41

Keywords: Real exchange rate; Markov switching; ESTAR

1 Introduction

In this paper we examine the nonlinear dynamics of adjustment to the long run PPP. Meese and Rogoff (1983) documented the failure of linear nominal exchange rate models, and some recent studies have provided evidence of the empirical failure of the linear real exchange rate models (see for example Michael *et al.* 1997, Sarantis 1999, Sarno 2000, and Baum *et al.* 2001). The number of both theoretical and empirical extensions of the nonlinear real exchange rate models have been growing in the literature. One possible source for nonlinear dynamics in the real exchange rate series is that market frictions, such as transportation costs and tariffs, permit deviations from the law of one price to grow quite large without precipitating goods arbitrage (O'Connell 1998). A number of researchers have argued that transaction costs are a key explanation for relatively low adjustment speed in PPP (Benninga and Protopapadakis 1988 and Sercu, Uppal and Van Hulle 1995). The theoretial support for transaction costs approach has been provided by Dumas (1992), Williams and Wright (1991), Uppal (1993) and Coleman (1995). They show that in the presence of transaction costs, the adjustment of real exchange rates toward PPP is not necessarily a linear process. Heckscher made the same observation already in 1916, but there has been only a few empirical analyses of this observation.

Dumas (1992) provides a nonlinear model structure for the PPP deviations. In Dumas' one good and two countries model, the deviations from PPP (i.e. the dynamics of the real exchange rate) should follow a nonlinear mean reverting process if we assume spatially separated markets and proportional transaction costs. He also shows that the speed of adjustment towards equilibrium varies nonlinearly and depends on the extent of the deviation from PPP. According to Dumas' model the process is divergent if we are inside the transaction and the deviations will be rapidly extinguished if we are outside the transaction band. The existence of other factors, such as the uncertainty of the shock persistence or sunk costs of the activity of arbitrage, may widen transaction bands (Krugman 1989 and Dixit 1989). Dumas' model implies that deviations from PPP can last very long time, although they do not necessarily follow a random walk (Michael *et al.* 1997). Although the whole real exchange rate process displays mean reversion, the probability of a move away from the parity value is greater than the probability to move towards parity. This asymmetric behaviour suggests the possibility that the real exchange rate can be characterized by long swings. Dumas' theoretical model is one way to explain why the empirical studies for mean reversion in real exchange rates have been largely unsuccessful.

Many authors have shown that the European exchange rate dynamics is typically characterized by periods of relatively calm phases, when the exchange rate arrangements are credible, and by periods of sudden and short lived phases of speculative attacks (Engel and Hakkio 1996, Peria 1999, and Bessec 2000). For example, Bergman and Hanson (1999) suggest that the real exchange rate between the major currencies in the post Bretton Wood period can be described by a stationary, two state Markov switching regime AR(1) model. They find that Markov models outperform the nonstationary real exchange rate models. Bessec (2000) also uses Markov models and finds that the European exchange rates of the ERM members display mean reversion in the credible exchange rate regime, and that they adjust to the PPP during the more volatile period.

Michael *et al*. (1997) apply a smooth transition autoregressive model to analyze long-run PPP for long period data. Their study show empirically that the nonlinear model provides strong evidence of mean reverting behaviour for PPP deviations. An interesting feature in their work is that the estimated model is consistent with Dumas' hypothesis (i.e. real exchange rates behave like random walks for small deviations, but are strongly mean reverting for large deviations). Obstfeld and Taylor (1997) use band threshold autoregressive (B-TAR) model to capture the nonlinear behaviour of the real exchange rates. They assume that nonlinearity of the real exchange rate comes from the transaction costs. In their model framework, the equilibrium value of the real exchange rate can be anywhere in the band and not necessarily in any fixed equilibrium point of PPP. Their model provides evidence of mean reversion for real exchange rate series, which are as low as two months mean reversion in some cases. O'Connell (1998) tests a threshold autoregressive (TAR) model for the post Bretton Woods data and finds that there are no differences between large and small deviations from PPP. He argues that both deviations are equally persistent. Kuo and Mikkola (2000) compare the forecasting performances of a linear AR model and TAR model for US/DEM real exchange rate. They find that the TAR model does not give consistently better forecasts than the linear AR model.

In this paper we extend the recent studies of linear real exchange rate models to the nonlinear models. Our theoretical background is the Dumas' model for the real exchange rate series. First, we apply the methodology of the Markov switching regime models to describe the real exchange rate process in the post Bretton Woods period. The Markov models allow for two states to exist with a series of shifts between the states occurring in a probabilistic fashion. Thus, the shifts occur endogenously rather than being imposed by the researcher. We show that the level of real exchange rate is generated by a stationary first order two-state Markov switching autoregressive model, where the states can be named as a depreciation regime and a appreciation regime. Second, we use smooth transition autoregressive (STAR) type of nonlinear econometric modelling technique. We apply especially exponential STAR (ESTAR) models to exploit nonlinear dependencies in real exchange rates. These models were originally developed by Teräsvirta and Anderson (1992), and their statistical properties and estimation were examined in Granger and Teräsvirta (1993). We show that linear nonstationarity tests may not be able to detect real exchange rate mean reversion property if the true real exchange rate series is a stationary nonlinear process. Also, the speed of adjustment towards long-run real exchange rate level varies more than proportionately with the size of shock.

2 Nonlinear adjustment to PPP

The last ten years have witnessed an explosion of interest among econometricians in the testing, estimation, and spesification of nonlinear models (Potter 1999). The successful modelling of nonlinearities in macroeconomic time series will help us to understand the data generation mechanism behind the series. According to Potter, it is important that at least two conditions are fulfilled in order to properly model the nonlinearities in the macroeconomic time series. Firstly, economic time series must actually contain nonlinearities, and secondly we need a statistical method, which reliably notifies the possible nonlinearities in the data. The first condition is usually easy to fulfill but the second one is much more problematic.

The long run PPP can be written as

$$s_t = \beta_0 + \beta_1 p_t + \beta_2 p_t^* + q_t \qquad (1)$$

where s_t is the log of the nominal exchange rate (denoted as domestic price of foreign currency), p_t and p_t^* are the logs of the domestic and foreign price indices, β_0 is a constant, and q_t is under the null hypothesis a stationary error term representing the real exchange rate series. The strong form PPP imposes the joint restrictions of symmetry ($\beta_1 = \beta_2$) and proportionality ($\beta_1 = -\beta_2 = 1$). These restrictions are not, however, consistent with the empirical data due to for example measurement errors (Taylor 1988 and Cheung and Lai 1993), differential composition of price indices (Patel 1990) and differential productivity shocks (Fisher and Park 1991) as pointed out (Baum *et al.* 2001). However, a necessary condition for the weak form PPP to hold in the long run is that the real exchange rate (q_t) in (1) without any parameter restrictions is stationary. Thus, we have estimated the PPP relationship parameters in (1) by ordinary least squares regression. The estimated parametes are presented in the Appendix (Table A.1). The parameter estimates are in line with the studies of Cheung and Lai (1993) and Baum *et al.* (2001) such that the estimated coefficients are not consistent with the strong form PPP. We conclude that it would be rather unreasonable in our nonlinear model examination to assume the strong form PPP restrictions in model (1) and by estimating the parameters we avoid the imposition of incorrect coefficient restrictions, which could affect to the true mean reversion property of real exchange rate series.

The linear cointegration methodology is a useful way of testing the long-run PPP theory empirically. While cointegration implies mean reverting behaviour in the relationships between analyzed variables, the conventional cointegration methodology assumes a linear process (i.e. the adjustment process to the equilibrium is both continous and has a constant speed). However, the failure of linear cointegration could be a consequence of the nonlinear behaviour of the system (1). In addition, as pointed out by Pippenger and Goering (1993), the possibility of nonlinear behaviour or transaction boundaries in the real exchange rate series brings into question the usefulness of the linear cointegration methodology and the unit root test methodology. They find that the power of conventional tests falls dramatically under a threshold process. In addition, an interesting line of research considers the importance of nonlinearity in the error correction mechanism of the real exchange rate towards its long run equilibrium value. This type of nonlinear error correction mechanism (NECM) can not be detected by any linear nonstationarity tests (Sarno 2000).

Economic theory offers several possible explanations for the presence of nonlinearities in real exchange rates. Already, according to Lucas critique policy shifts may lead to breaks in the trend of the macroeconomic series and thus might generate long swings for the series. Regular random walk models do not take account of any effects of observed policy changes. For example, Kaminsky (1993) shows theoretically that a change from a contractive monetary police to an expansive monetary policy increases the exchange rate depreciation. If we assume that prices are not as flexible as exchange rates the same reasoning applies also to the real exchange rate series. Moreover, heterogeneity of participants in the foreign exchange market is often cited as the major source of nonlinearities in the real exchange rate process (Sarantis 1999). According to Peters (1994) and Guillaume *et al.* (1995) investors' heterogeneity comes from different investment horizons, geographical location, various type of risk profiles and institutional constraints.

2.1 The Markov switching regime model for the real exchange rate series

The Markov switching regime model differs from the models with exogenously imposed breaks in that the timing of the breaks is entirely stochastic. In the Markov models, the inferences are drawn on the base of probabilistic estimates of the most likely state prevailing at each point of series time during the observation period. The major advantage of the Markov switching regime model is the flexibility in modelling time series with respect to regime shifts.

The basic idea in the two-state Markov regime switching model for the real exchange rates is that the real exchange rates can be divided in general to an appreciation or a depreciation regime and a random process governs the switches between the regimes (Klaassen 1999). Engel and Hamilton's (1990) modelled such long swings in the exchange rate data and suggested a Markov switching random walk model with drift for exchange rate series. They found that mean squared errors from the Markov switching regime model were lower than the mean squared errors for the single regime random walk model. Engel and Kim (1999) examine the behaviour of UK/USA real exchange rate under the assumption that the real exchange rate series is integrated of order one. They suggest that the deviation from the permanent component can be modelled as a three state Markov switching regime model. The states can be characterized in their model by low, medium and high variance.

In the Markov regime switching process the probability of being in a particular state is only dependent in which state the process was in the previous period. The regime switching model is originally based on Hamilton's (1989,1990) model for exchange rates. We use the model application of Bergman and Hanson (1999), where the size of the autoregressive parameter is also estimated and do not presume it to be unity as in Hamilton exchange rate models. We use only one autoregression term, because it is generally believed that the short run autocorrelations in exchange rates are small (see for example West and Cho 1995). The restrictions for the observed variables are ergodicity of the regime process and weak stationarity of the real exchange rate variables. We assume also that the probabilities of switching from one regime to the other are constant over time. The model has two elements. The first element is the regime process, which is based on two unobservable regime paths s_1 and s_2 and the second is the mean equation. Within these two regimes, the mean real exchange rate change is μ_1 or μ_2, which are assumed to be constant over time. If the mean regimes are different then persistence of these two regimes leads to the long swings. Thus, the model is consistent with Dumas' theoretical considerations.

In Bergman and Hanson model the real exchange rate (q_t) is generated by the following Markov regime switching model:

$$q_t = \mu_{s_t} + \alpha_{s_t} q_{t-1} + \varepsilon_t, \qquad (2)$$

where ε_t is $N(0, \sigma_{s_t}^2)$ distributed and the initial value q_0 is fixed. In this formulation, the changes in the log of the real exchange rates are normally distributed with mean μ_i and variance σ_i^2 in each of the two possible regime states of the world (i = 1,2). Thus when $s_t = 1$ the change in the real exchange rate is μ_1, and when $s_t = 2$ the change is μ_2. The unobserved random regime state variable s_t is independent of the past q_t

conditional on s_{t-1}. Real depreciation and real appreciation are modelled as switching regimes of the stochastic process generating the growth rate of the real exchange rate series. The regimes are associated with different conditional distributions of the growth rate, and mean is assumed to be positive in first regime and negative in the second regime. We assume that regime path ($s_{t-1}, s_{t-2}, \ldots$) follows a first order Markov process with time homogeneous transition probabilities,

$$P\{s_t = j \mid s_{t-1} = i\} = p_{ij} \ , \qquad (3)$$

for i and j =1,2. This process is completely described by the following constant transition probabilities:

$$\begin{aligned} &P(s_t = 1 \mid s_{t-1} = 1) = p_{11} \\ &P(s_t = 2 \mid s_{t-1} = 1) = p_{21} = 1 - p_{11} \\ &P(s_t = 2 \mid s_{t-1} = 2) = p_{22} \\ &P(s_t = 1 \mid s_{t-1} = 2) = p_{12} = 1 - p_{22} \end{aligned} \qquad (4)$$

Thus p_{ij} is equal to the probability that the Markov chain model moves from state i at time t-1 to state j at time t. The regime path process s_t depends on past realizations of the real exchange rate series q_t and regime path s_t only through previous value of regime path s_{t-1}. It should be noted that we do not assume that the real exchange rate series follow long swing process. The regime persistence can also be assymmetric. The regime means μ_1 and μ_2 should be opposite in signs and the values for both regime staying probabilities p_{11} and p_{22} should be large under the hypothesis of long swing. The unconditional probabilities of the stationary distributions that the process is in each of the regimes are given by

$$P(s_t = 1) = \frac{1 - p_{22}}{2 - p_{11} - p_{22}}, \text{ and} \qquad (5)$$

$$P(s_t = 2) = \frac{1 - p_{11}}{2 - p_{11} - p_{22}}, \qquad (6)$$

see Hamilton (1994) for derivation of this result.

Equations (2) and (3) are important for our Markov switching regime model, because they are related directly to the long swing models. To obtain estimates of the parameters and the transition probabilities governing the Markov chain of the unobserved state, we need an iterative estimation technique. Estimates of parameters for the two most likely regimes are calculated using maximum likelihood estimation techniques. The likelihood is calculated for each possible state, and the probability that a particular state is prevailing is obtained by dividing the likelihood of particular state by the total likelihood for both states. Hamilton (1990) used an expectations maximisation (EM) algorithm, which is useful for the case where all parameters change. The EM algorithm introduced by Dempster *et al.* (1977) is designed for a general class of models where the observed time series depend on some unobservable stochastic variable. For analysing and inferencing the Markov chain states we need to smooth the EM algorithm estimates so that it contains information from the whole sample. After smoothing, the estimate of probabilities is commonly explained so that a state is prevailing when the probability estimate for the particular state is greater than 0.5.

2.2 STAR-models

In the Engle-Granger (1987) and Johansen (1988) linear cointegration methodologies, the speed of adjustment to equilibrium is independent of the magnitude of disequilibrium. One way of modelling nonlinear adjustment involves application of the TAR model (Tong 1993). This would be appropriate for example if there is a threshold level of the absolute deviation from parity beyond which real exchange rate becomes mean reverting. The TAR model might be appropriate in modelling deviations from the law of one price for individual goods for which a given discrete threshold is relevant. We can think for example that the threshold value is the size of transaction costs in arbitraging the good internationally. According to Granger and Teräsvirta (1993) the nonlinear adjustment process can also be characterized in terms of a smooth transition autoregressive (STAR) model. In the STAR framework, the fixed thresholds of a standard threshold autoregressive (TAR) model are replaced with a smooth function, which need to be continuos and non-decreasing (Tong 1993). We think that a smooth transition (STR) model might be more appropriate than TAR model, when we are examining movements of the consumer price indices based real exchange rates, which involves a range of goods with different costs of arbitrage.

Although the STAR models look very much like the Markov switching regime model, there is a crucial difference between these two models. In the threshold model, the regimes are defined by the past values of the time series itself, and in the Markov switching regime models regimes are defined by the exogenous state of Markov chain (Potter 1999). Smooth transition regression models are also more preferable for post Bretton Woods data than for example unit root tests, since the modelling strategy does not require very long time series (Teräsvirta 1994). The STAR models have interesting properties. First of all the STAR models do not assume a sharp switch from one regime to the other like the TAR or the Hamilton's Markov switching regime models. In foreign exchange markets with a large number of investors, each switching at different times due to for example different heterogenous beliefs, a smooth transition seems to be more appropriate. Even if economic agents make only dichotomous decisions, it is unlikely that they change their behaviour simultaneously. Hence, as Teräsvirta (1994) notes, for aggregated processes the change in regime may be smooth rather than discrete. Another feature of the STAR models is that they nest linear regression model, and we can thus use linear Lagrange multiplier (LM) tests for testing the null of linearity before fitting any nonlinear model (Teräsvirta 1994). We can also use LM tests for choosing between the alternative STAR specifications. The need for symmetry in the response to positive and negative deviations from PPP leads to the exponential STAR (ESTAR) model described by Teräsvirta (1994). The ESTAR model can be viewed as a generalization of the double threshold TAR model. It is particularly attractive in the PPP context, as the strength of equilibrating force increases when the degree of absolute disequilibrium grows.

Following Michaet *et al.* (1997), the STAR model for the real exchange rate series (q_t) can be written as

$$q_t = k + \sum_{i=1}^{p} \pi_i q_{t-i} + (k^* + \sum_{i=1}^{p} \pi_i^* q_{t-i}) F(q_{t-d}) + u_t , \qquad (7)$$

where k and k^* are regime constants, q_t is assumed to be stationary and ergodic process, u_t is independently, identically and normally distributed with mean zero and constant variance, d is the delay parameter, and F is a transition function which is bounded by zero and one. In these models, nonlinearities arise through conditioning on lagged real exchange rates. The adjustment takes place in every period, but the speed varies with the extent of the deviation from parity. In contrast with TAR model, the regime of the STAR model changes gradually.

The specification form of the most commonly used transition function is

$$F(q_{t-d}) = 1/\{1 + \exp[-\gamma(q_{t-d} - c^*)]\}, \qquad \gamma > 0. \qquad (8)$$

This is the logistic function and, c^* is a threshold value indicating the half way point between two regimes. The gamma (γ) parameter measures the speed of transition from one regime to the other. It determines the smoothness of the change from one regime to the other. The logistic function (8) is not the most plausible for the modelling of the real exchange rate series, since it is not symmetric for positive and negative deviations from PPP. The need of symmetry leads to exponential STAR (ESTAR) model described by Teräsvirta (1994). The transition function in the ESTAR model is

$$F(q_{t-d}) = \{1 - \exp[-\gamma(q_{t-d} - c^*)^2]\}, \quad \gamma > 0, \qquad (9)$$

The ESTAR model suggests that the two regimes have rather similar dynamics, while the transition period can have different dynamics. The transition function is U-shaped, and symmetric around c^* in the sense that the local dynamics are the same for high and for low values of the real exchange rate series (q_t). It should be noted that for the globally stable model the middle regime does not necessarily have to be locally stable.

In the ESTAR model, the inner regime corresponds to $q_{t-d} = c^*$ when $F = 0$ and thus the model (3.7) becomes a linear AR(p) model

$$q_t = k + \sum_{i=1}^{p} \pi_i \, q_{t-i} + u_t \, . \tag{10}$$

The outer regime corresponds to $q_{t-d} = \pm \infty$ when $F = 1$ and (7) becomes a different type of AR(p) model

$$q_t = k + k^* + \sum_{i=1}^{p} (\pi_i + \pi_i^*) q_{t-i} + u_t \, . \tag{11}$$

We prefer the choice of the ESTAR model, since adjustment to PPP deviations can be excpected to be same for both positive and negative deviations from equilibrium. The ESTAR model can also be viewed as a generalization of a particular form of the two threshold TAR model. A symmetric TAR model can be spesified in which the outer regimes are the same. Such a model is the limiting case of (7) when $\gamma \to \infty$.

The ESTAR model (7) can be reparameterized as follows

$$\Delta q_t = k + \lambda q_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta q_{t-i} + (k^* + \lambda^* q_{t-1} + \sum_{i=1}^{p-1} \phi_i^* \Delta q_{t-i}) F(q_{t-d}) + u_t \, . \tag{12}$$

The most important parameters of (12) are λ and λ^*. Our previous discussion of nonlinearity in the real exchange rates suggests that the larger the deviation from PPP, the stronger the tendency to move back to equilibrium. This implies that while $\lambda \geq 0$ is possible, we must have $\lambda^* < 0$ and $\lambda + \lambda^* < 0$ for the model to be globally stable. That is, for small deviations real exchange rate series may follow a unit root or even explosive behaviour, but for large deviations the process is mean reverting. Thus, for small deviations from equilibrium value, the real exchange rate process may adjust very slowly or not at all, but for large deviations, the process adjusts very quickly towards equilibrium level. The model may be viewed as a nonlinear error correction model in the form of a smooth transition autoregressive process.

In linear error correction model (12) the error correction coefficient λ must be significantly negative for the stable model. The conventional linear cointegration tests of PPP are based on a linear AR(p) model, such as

$$\Delta q_t = k' + \lambda' q_{t-1} + \sum_{i=1}^{p-1} \phi_i' \Delta q_{t-i} + u_t . \tag{13}$$

If the true real exchange rate series is not linear, then the parameter λ' in regression (13) will be between λ and $\lambda + \lambda^*$. Hence the null hypothesis of no cointegration may not be rejected against the stationary alternative, even though the true process is globally stable. Thus, the failure to find linear cointegration relationship does not necessarily invalidate the long run PPP hypothesis.

2.3 Linearity tests for STAR-models

It is reasonable to execute linearity tests if we assume that the real exchange rate series might be nonlinear. In testing linearity, we follow the suggestion by Granger and Teräsvirta (1993) and Teräsvirta (1994), and estimate an auxiliary regression. The first step is to specify the linear part of the model. The linear part of the real exchange rate series (q_t) can be presented as

$$q_t = \beta_0 + \sum_{i=1}^{p} \beta_i q_{t-i} + \varepsilon_t = \beta_0 + \beta' w_t + \varepsilon_t , \tag{14}$$

where $\beta = (\beta_1, \ldots, \beta_p)'$, $w_t = (q_{t-1}, \ldots, q_{t-p})'$ and ε_t is a white noise residual term. The order of the autoregressive part (p) is usually determined by the Akaike information criteria (AIC). In the second step, the null hypothesis of linearity is tested against a smooth transition autoregressive alternative. We use the auxiliary regression

$$q_t = \beta_0 + \beta' w_t + \delta_1' z_t^1 + \delta_2' z_t^2 + \delta_3' z_t^3 + u_t , \tag{15}$$

where $z_t^k = w_t q_{t-d}^k = (q_{t-1}q_{t-d}^k, ..., q_{t-d}^{k+1}, ..., q_{t-p}q_{t-d}^k)'$, and k = 1,2, and 3. The transition variable is chosen from the linearity test H_0: $\delta_1 = \delta_2 = \delta_3 = 0$. The null hypothesis may be tested by LM test (for details see Teräsvirta 1994). It should be noted that when we are using the LM test, we do not need to estimate the model under alternative specification. The order of autoregression is chosen on the basis of serial correlation tests on the residual vectors from alternative autoregressive representations. As Teräsvirta (1994) points out, neglected autocorrelation structure may lead to false rejections of the linearity hypothesis in favor of the presence of nonlinearities. He also warns against the use of automatic selection criteria for choosing the autoregressive lag order without testing for residual autocorrelation. To spesify the value of the delay parameter *d*, the estimation of (15) is carried out for a wide range of values, $1 \leq d \leq D$. In cases where linearity is rejected for more than one value of *d*, *d* is set equal to value which minimize the p-value of the linearity test.

When choosing between LSTAR and ESTAR models for those real exchange rates where linearity is rejected we can use the following sequence of nested tests (Sarantis 1999)

$$H_{01}: \delta_3 = 0 \qquad (16)$$

$$H_{02}: \delta_2 = 0 \mid \delta_3 = 0 \qquad (17)$$

$$H_{03}: \delta_1 = 0 \mid \delta_2 = \delta_3 = 0, \qquad (18)$$

If we reject the hypothesis (3.16), we select the LSTAR model. If we accept (16) but reject (17), we have evidence to choose the ESTAR model spesification. If we accept (16) and (17) but reject (18), we choose the LSTAR model again. However, Granger and Teräsvirta (1993) and Teräsvirta (1994) argue that this sequence of tests application may lead to wrong conclusions, if the higher order terms of the Taylor expansion used in deriving these tests are disregarded. They recommend that we should compute the p-values for all F-tests of (16) to (18) and make the choice of the STAR model on the basis of the lowest p-value.

3 Empirical Analysis

The empirical analysis is based on quarterly observations of spot exchange rates and consumer price indices for the United States, the United Kingdom, Germany, Finland, and Sweden. The data period ranges from the first quarter of 1974 to the last quarter of 1998.

3.1 The Markov switching regime model

We specify a univariate two regime AR(1) Markov switching model for all the nine real exchange rate changes. The Markov switching regime model that we consider is based on Hamilton (1989). We assume that the real exchange rate series is allowed to follow a similar autoregressive process in both regimes. Thus, we think that the data generation process for the real exchange rate series is similar in both regimes, but differs in variances and/or regime staying probabilities. We do not think that this is an unreasonable assumption for the real exchange rate series. We use the Markov switching regime model to test whether the real exchange rate series include long swings. The parameter vector $\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2, p_{11}, p_{22})$ is estimated independently for each series. The difference between our model and the model used in Engel and Hamilton (1990) and in Engel (1994) is that we allow the autoregressive parameter to differ from unity. The parameter vector is estimated by maximum likelihood method and the sample likelihood is a function of the observed values of the changes in the logs of real exchange rates. The maximum likelihood estimation is performed by using the EM algorithm. The EM algorithm is particulary useful for real exchange rate series since all parameters are allowed to switch. The states s_1 and s_2 are unobserved, and we must make inferences about the probability of the state based on the observed data.

Our maximum likelihood estimation results are reported in Table 1. The results show that almost all real exchange rate series are well characterized by an appreciation and a depreciation regime. The estimates show that the states differ not only in means but also in variances. The estimates suggest that the first regime (positive) can be considered as a rising real exchange rate (real depreciation) and the second regime (negative) as a declining real exchange rates (real appreciation). For all country pairs the estimated means are positive in the first regime and negative in the second regime. The test results show that there is more variability in the real exchange rate in the depreciation regime. The probability of staying in one state is large for all country pairs. The point estimates of p_{11} range from 0.553 to 0.913, while the estimates of p_{22} range from 0.809 to 0.937. These estimated probabilities show that if the system is in either state one or two, it is likely to stay in that state.

Table 1. Markov Switching Regime Model estimates for real exchange rates

	UK/US	GE/US	FI/US	SW/US	UK/GE	FI/GE	SW/GE	UK/FI	SW/FI
μ_1	4.626	3.062	2.591	3.455	4.725	3.646	1.789	3.067	0.166
	(1.602)	(1.309)	(0.846)	(1.252)	(2.379)	(0.919)	(0.828)	(1.038)	(0.207)
μ_2	-2.045	-3.281	-3.133	-2.835	-1.207	-1.148	-1.435	-1.337	-0.389
	(0.703)	(0.760)	(0.532)	(0.415)	(0.538)	(0.246)	(0.294)	(0.592)	(0.529)
σ_1^2	21.655	17.673	20.648	24.845	18.560	10.169	18.561	10.113	1.112
	(7.993)	(6.459)	(4.249)	(7.313)	(8.355)	(3.439)	(4.526)	(3.314)	(0.407)
σ_2^2	13.168	10.236	6.394	6.125	10.519	3.517	2.622	9.086	10.223
	(3.405)	(2.861)	(1.708)	(1.791)	(2.103)	(0.634)	(0.671)	(1.931)	(3.458)
p_{11}	0.739	0.847	0.873	0.825	0.553	0.785	0.837	0.766	0.913
	(0.141)	(0.107)	(0.069)	(0.076)	(0.203)	(0.098)	(0.083)	(0.112)	(0.056)
p_{22}	0.878	0.809	0.837	0.853	0.892	0.937	0.887	0.908	0.888
	(0.079)	(0.087)	(0.073)	(0.058)	(0.075)	(0.032)	(0.063)	(0.065)	(0.096)
AR	0.921	0.953	0.939	0.940	0.928	0.932	0.864	0.883	0.917
	(0.039)	(0.031)	(0.035)	(0.033)	(0.039)	(0.036)	(0.048)	(0.045)	(0.039)

Note: Standard errors are in parentheses.

In Table 2 we test first the hypothesis H_0: $\mu_1 = \mu_2$ using Wald test. Under this null hypothesis, the two states differ only by their variances. If the null hypothesis is true the real exchange rates follow a random walk model with heteroskedastic errors. The null hypothesis can be rejected for all the real exchange rate series except the SWE/FIN series at 5 per cent significance level. The failure to model SWE/FIN real exchange rate dynamics by Markov switching approach can be explained by their macroeconomics similarities with respect to each other. Both countries display strong interdependence regarding monetary policy during the past 25 years. When Finland or Sweden changed their exchange rates through devaluations, the other country was also forced to change its' exchange rate policy. Thus, the exchange rate changes had quite similar effects for both countries in the long run.

The test problem under the above null hypothesis is that if one regime governs the real exchange rate series data, the parameters for the second regime are not identified. This makes the asymptotic distribution of the Wald test not χ^2 as Hansen (1992) shows. Garcia (1998) try to solve this problem by deriving the correct asymptotic distribution of the likelihood ratio statistic. We circumvent this problem as in Hamilton (1989) by expecting that all parameters are identified. Indeed, our estimation algorithm (EM) results show in Table 1 that all parameters are identified, and we can circumvent the singularity problem of the information matrix. However, we should note that for the case of FIN/SWE both regimes are insignificant and thus we have no identification problem.

The second hypothesis is that $H_0: p_{11} + p_{22} = 1$. Under this null hypothesis the distribution for the regime path process s_t is independent of s_{t-1}. According to Engel and Hamilton (1989) large values of p_1 and p_2 are characterized by long swings in the series. Table 2 shows that the null hypothesis is strongly rejected for all country pairs at the 5 per cent level. We conclude that movements in the real exchange rate series are well described by long swings and when the real exchange rate series enters the state one or two it stays there for years. The expected duration of the state *i* can be calculated by $1/(1-p_{ii})$ (Hamilton 1989). On average, state one (i.e. real depreciation) lasts from two to eleven periods, and state two (i.e. real appreciation) lasts from five to fifteen periods. Thus, the estimated probabilities show that the stay in the real depreciation regime has been shorter than the stay in the real appreciation regime.

Table 2. The Wald tests of the null hypothesis for stochastic process and independency of the regime path

Series	H$_0$: $\mu_1 = \mu_2$	H$_0$: $p_1 + p_2 = 1$
UK/US	19.962	24.980
GER/USA	32.621	39.024
FIN/USA	32.022	10.091
SWE/USA	23.191	55.513
UK/GER	7.361	4.161
FIN/GER	28.815	54.197
SWE/GER	12.960	79.318
UK/FIN	20.711	23.923
SWE/FIN	0.868	69.169

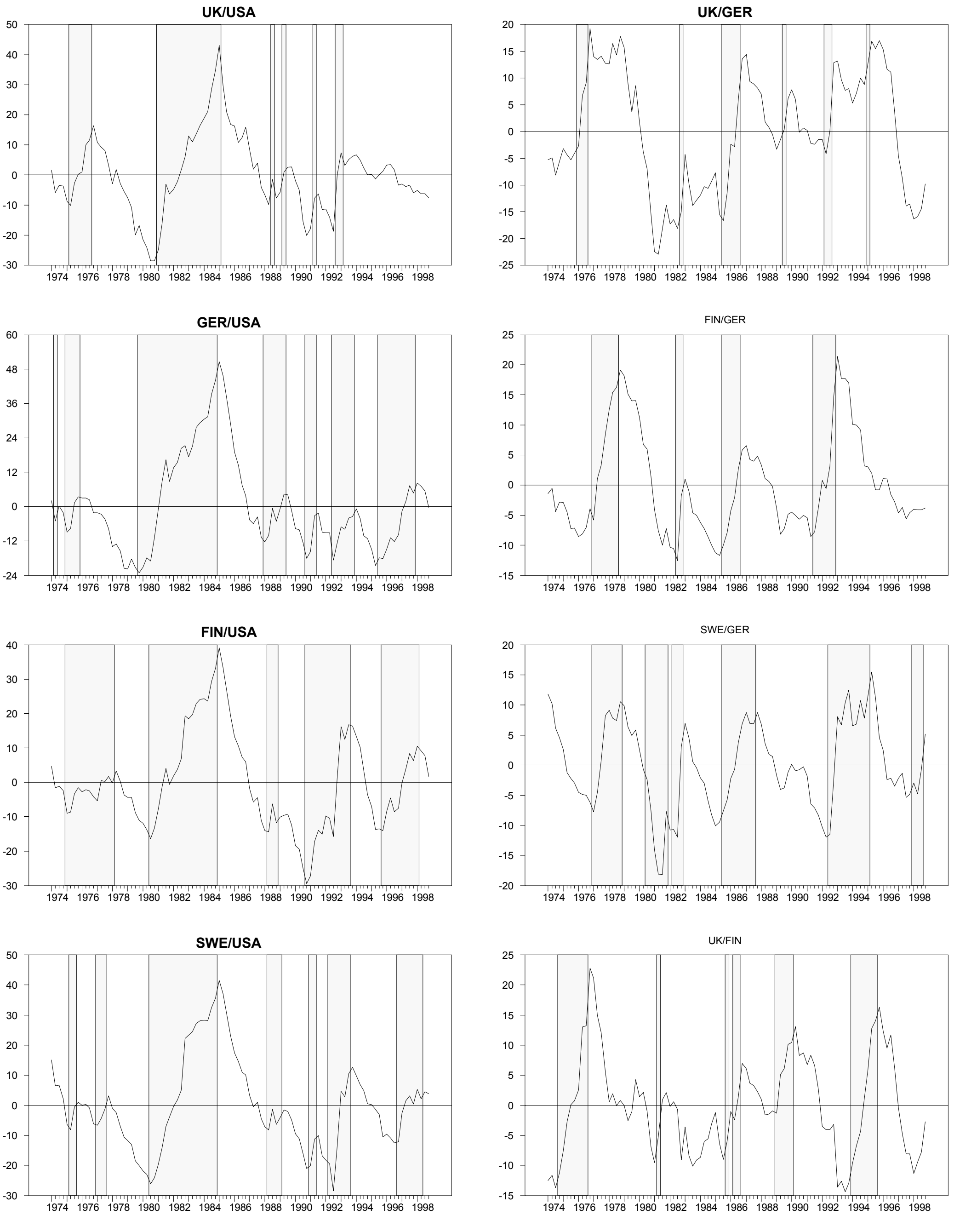
Notes: A Wald statistic for testing H$_0$: $\mu_1 = \mu_2$ is given by

$\frac{(\hat{\mu}_1 - \hat{\mu}_2)^2}{Var(\hat{\mu}_1) + Var(\hat{\mu}_2) - 2Cov(\hat{v}_1, \hat{\mu}_2)} \approx \chi^2(1)$, and a Wald test for H$_0$: $p_1 + p_2 = 1$ is given by

$\frac{[\ (p_{11} - (1 - p_{22})\ ]^2}{Var(\hat{p}_{11}) + Var(\hat{p}_{22}) - 2Cov(\hat{p}_{11}, \hat{p}_{22})} \approx \chi^2(1)$.

Another way to examine persistence of regimes is by inspecting estimated regime probabilities. We use smoothed regime probabilities, which use the complete data set and thereby smoothing the *ex ante* probabilities (i.e. the conditional probability that the process is in a particular regime at time t using only information available at time t-1). Thus, smoothed probabilities give the most informative answer to the question in which regime the variable is at time t most likely. The smoothed probabilities with the real exchange rate series are shown in Figure 1. The shaded areas show the smoothed probabilities when the probability of staying in regime one is above 0.5 representing the real depreciation regime. The real appreciation regime is not shaded. The probabilities show that regime classification is as one might expect. As Figure 1 shows, the smoothed probabilities identify correctly the movements of the real exchange rates. For example, the appreciation of the US dollar against the European currencies in the first half of 1980s and depreciation after that is well captured by the smoothed regime switch probabilities.

Figure 1. Smoothed probabilities of being in regime 1.



The Markov switching regime estimation results are consistent with the Dumas' theoretical model, in which the real exchange rates exhibits long swings. Moreover, the results are in line with the Mussa's (1986) observation, that the real exchange rates are not "nominal exchange rate neutral", since the real exchange rates states appears to shift at points when there are also significant nominal exchange rate events.

3.2 Linear models against ESTAR models

To test for real exchange rate linearity against the ESTAR spesification we use similar artificial regression as Baum *et al.* (2001) for second order, that is

$$q_t = \beta_{00} + \sum_{j=1}^{k} (\beta_{oj} q_{t-j} + \beta_{1j} q_{t-j} q_{t-d} + \beta_{2j} q_{t-j} q_{t-d}^2) + \varepsilon_t , \qquad (19)$$

where q_t is the real exchange rate series. The rejection of the null hypothesis (i.e. linearity) H_0: $\beta_{1j} = \beta_{2j} = 0$ for all j suggesting the use of ESTAR models. In carrying out linearity tests we have considered values for the delay parameter (d) over the range $1 \leq d \leq 16$, and calculated the p-values for the linearity test in each case. The autoregression length parameter (k) for the real exchange rate series is chosen on the basis of the AIC criterion. According to Michael *et al.* (1997) overspecification of the autoregressive model is not as serious a problem as underspesification since autocorrelated errors may affect the linearity tests. With the proper choice of lag length (k), we vary the delay length (d) in order to provide the strongest probability for nonlinearity. If linearity is rejected for more than one value of d, then the estimate of d is chosen by the lowest p-value of the above linearity test.

The linearity test results are shown in Table 3. We use ordinary F-test as an approximation to the LM test since it is generally known to have relatively good size and power properties in finite samples. The test results show that for seven cases out of nine the test classifies the time series as nonlinear ESTAR model at the 5 per cent level of significance. We can therefore proceed to build nonlinear models for these exchange rates. Linearity test is not rejected for country pairs UK/GER and UK/FIN.

Table 3. Linearity tests results for real exchange rate series

Countries	AIC based Lag (k)	Delay parameter (d)	P-value
UK/US	8	3	0.033*
GER/US	2	5	0.031*
FIN/US	4	8	0.021**
SWE/US	8	9	0.031*
UK/GER	2	1	0.255
FIN/GER	5	3	0.00019***
SWE/GER	5	3	0.000009***
UK/FIN	4	9	0.284
SWE/FIN	2	10	0.039*

Notes: The F statistic tests the null hypothesis of linearity against the alternative of nonlinearity; d denotes the delay parameter. Marginal significance levels are calculated using the appropriate F-distribution. The test is the minimum p-value over the interval $1 \leq d \leq 16$. The selection of the maximum lag, k, of the linear AR model is made using the AIC statistic. The *,**,and *** are 5%, 2.5%, and 1% significance levels, respectively.

3.3 The ESTAR spesification models

Assuming that linearity is rejected, we can proceed to estimate the ESTAR models for the real exchange rate series. We consider the possibility that the first differences of the logarithmic real exchange rate series are nonlinear. We assume further that if the real exchange rates are nonlinear, we can adequately characterize nonlinearity by the ESTAR models. We estimate an ESTAR model by nonlinear least squares estimation technique, which provides estimators that are consistent and asymptotically normal. Tong (1993) shows that these conditions also hold for STAR models if series is stationary, ergodic and the error terms are independently and identically distributed. In each case we follow the recommendation of Granger and Teräsvirta (1993) and standardize the transition parameter (γ) by the sample variance of dependent variable and using starting value $\gamma = 1$ for the estimation algorithm.

In Table 4, we present the LR-test results for the following restrictive hypothesis for the model (12)

$$H_0^A : k = k^* = c^* = 0 \tag{20}$$

$$H_0^B : 1 + \lambda = -\lambda^* \text{ and } \phi_j = -\phi_j^*, \text{ given } H_0^A \tag{21}$$

$$H_0^C : \lambda = 0, \text{ given } H_0^A \text{ and } H_0^B. \tag{22}$$

If the restrictions are true, they are imposed to obtain a more parsimonious model. This is an important concern in the estimation of the STAR models (Granger and Teräsvirta 1993). The ESTAR model spesification simplifies considerably if all three restrictions are valid. If we reject one of the restrictions then they are not imposed in the ESTAR models. The first null hypothesis H_0^A implies that both constant terms (k and k^*) are in both regimes zero, and that the equilibrium value of the real exchange rate series (q_t) in the model (12) is also zero. The second hypothesis H_0^B implies that the real exchange rate series (q_t) is in the outer regime (i.e. when F =1) a white noise process. The third hypothesis H_0^C implies that in the middle regime (i.e. when F = 0) the real exchange rate series (q_t) has a unit root.

Results from Table 4 indicate that all three restrictions are relevant for three bivariate real exchange rate series, namely FIN/USA, SWE/GER, and SWE/FIN. For five cases out of seven the first hypothesis (H_0^A) is valid. Only for cases GER/USA and FIN/GER we can not reject the first restriction (19) implying that PPP equilibrium value is not zero. However, the constant terms (k and k^*) may be expected to be zero, because the real exchange rate series is estimated as the residuals of the cointegrating regression. The ESTAR model for these cases is estimated without any restrictions (19), (20), and (21). For two cases, namely UK/USA and SWE/USA real exchange rate series, the H_0^B is not rejected implying that the real exchange rate series is not white noise in the outer regime. However, the stability condition ($\lambda + \lambda^* < 0$) is satisfied at a 1 per cent significance level.

Table 4. The LR test results for the restriction hypothesis

Statistics	UK/USA	GER/USA	FIN/USA	SWE/USA	FIN/GER	SWE/GER	SWE/FIN
LR_1	1.1507	12.25^c	0.002	1.651	6.672^c	0.237	0.007
LR_2	225.1^c		0.000	303.1^c		0.003	0.005
LR_3			0.120			0.359	0.362
λ_1	0.103			0.033			
	(0.201)			(0.072)			
λ_2	-0.282			-0.215			
	(0.197)			(0.080)			
F-test for $\lambda_1+\lambda_2 = 0$	10.520^c			13.146^c			
Concl.	ESTAR	ESTAR with constant	ESTAR	ESTAR	ESTAR with constant	ESTAR	ESTAR

Notes: LR_1, LR_2 and LR_3 are likelihood ratio test statistics corresponding tests of (3.20), (3.21), and (3.22). The stability hypothesis H_0: $\lambda_1+\lambda_2 =0$ is tested using F-test, with degrees of freedom 1 and 99-k, where k is the lag length. (c) denotes significance at 1 % level.

Table 5 presents estimated parameter values and residual diagnostics of ESTAR models for seven country pairs. The ESTAR models are estimated for those real exchange rate series, which nonlinear behaviour was found by the test results in Table 3 at the 5 per cent significance level. The results show that the transition parameter (γ) varies across country pairs. The interpretation of the t-statistics for transition parameter (γ) in Table 5 should be done carefully, since under the null hypothesis of model (12) the real exchange rate series follow a random walk process. Hence, we can not assume that the "t-test" statistics are normal (Taylor and Peel 1998). However, our t-ratio test results in Table 5 are sufficiently larger according to Dickey-Fuller critical values to find significant transition parameters for five cases out of seven. For the case GER/USA we have not significant t-value for the γ parameter according to Dickey-Fuller table, but we have reported the test results because the t-test value differs markedly from those that are insignificant. Thus, only for the cases UK/US and SWE/US we find that the transition parameter can not be distinguished from zero. The residual diagnostic tests are satisfactory for most of the cases. There does not seem to be any autocorrelations or any ARCH-effects in any of the residuals. Only the normality test (Jarque-Bera) show slight non-normality in some cases. The variance ratio (V) between nonlinear (NL) and linear (L) model shows a reduction in the residual variance in all cases for the nonlinear model spesification.

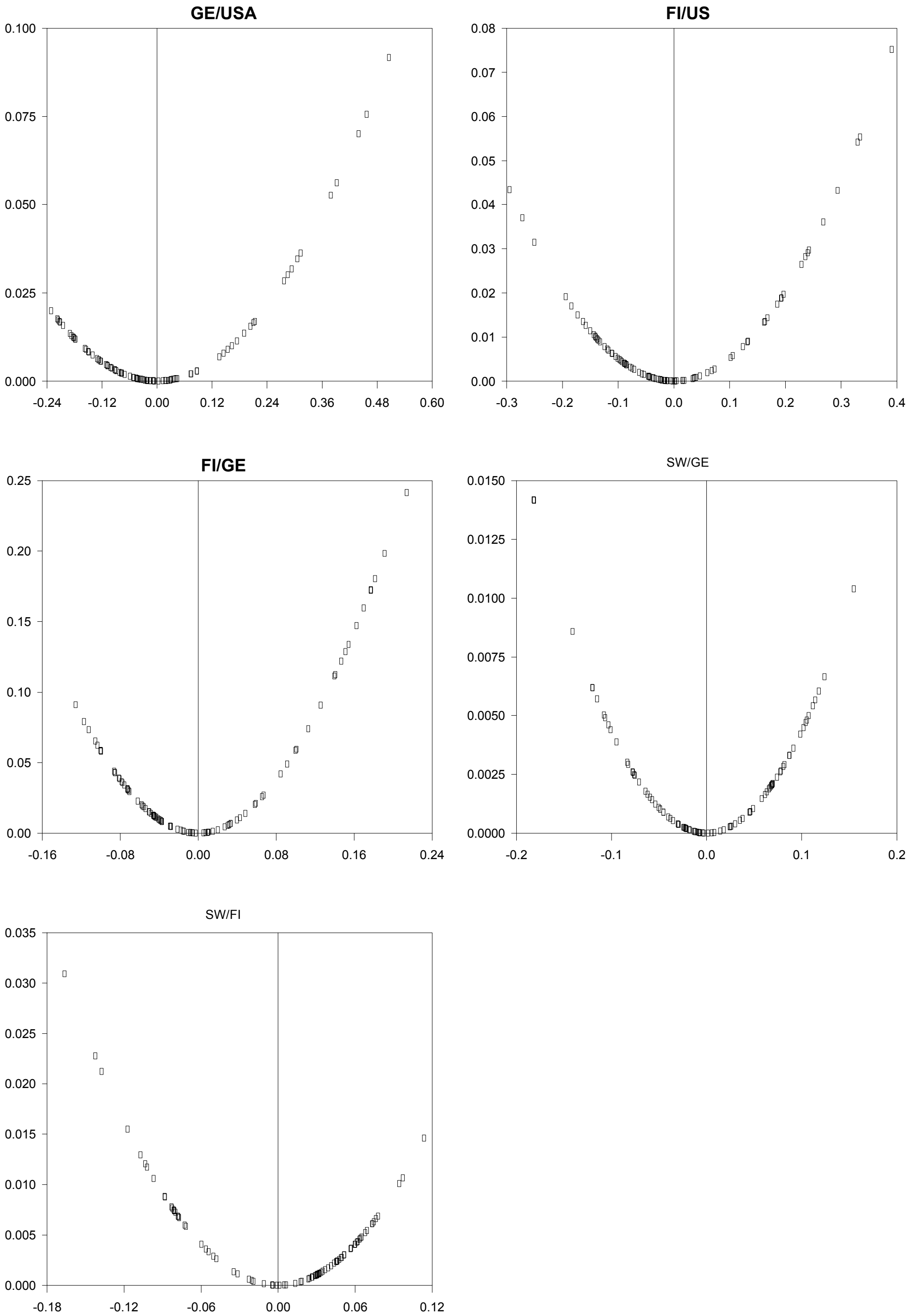
Table 5. Estimates of the ESTAR model for deviations from PPP

	UK/US	GE/US	FI/US	SW/US	FI/GE	SW/GE	SW/FI
d	3	5	8	9	3	3	10
γ	1.3039	0.0097	0.0132	0.344	0.0125	0.0495	0.0176
t-value	(0.542)	(2.291)	(3.371)	(0.447)	(3.254)	(3.617)	(3.039)
ϕ_1	0.126	0.2966	0.269	0.001	0.261	0.254	0.420
s.e.	(0.057)	(0.000)	(0.087)	(0.057)	(0.089)	(0.073)	(0.089)
DW	1.91	1.92	1.94	1.97	1.95	1.77	1.93
Q(1)	0.026	0.011	0.015	0.001	0.034	1.025	0.209
Q(4)	0.648	1.797	0.586	0.385	0.430	1.431	1.177
$ARCH_1$	0.858	0.024	3.746*	0.396	1.522	2.845*	2.659
$ARCH_4$	1.496	0.917	4.584	1.372	3.500	9.055*	6.697
JB	4.095	1.992	8.765*	29.13***	21.62***	5.462*	1.973
SSR	0.0478	0.043	0.049	0.044	0.026	0.032	0.025
V= s^2_{NL}/s^2_L	0.965	0.959	0.964	0.936	0.992	0.969	0.943
c		0.128			0.055		
s.e.		(0.038)			(0.016)		

Notes: Standard errors and t-test values are given in parentheses. Q_1 and Q_4 are the Box-Ljung Q statistics for the absence of serial correlation up to first and fourth order, respectively. $ARCH_1$ and $ARCH_4$ are tests for the ARCH effects up to first and fourth order, respectively. Both Q_n and $ARCH_n$ tests have a χ^2 distribution with n degree of freedom. SSR is the standard error of regression and DW is Durbin-Watson autocorrelation statistics. JB is the Jarque-Bera normality test. (***),(**), and (*) denote significance at 1%, 5%, and 10% levels, respectively.

In Figure 2 we have plotted the significant ESTAR transition functions $F(y_{t-d})$ against lagged CPI based PPP deviations series. In all cases the figures show a reasonable number of observations for both sides of c^*, indicating ESTAR type of nonlinearity. The figures show also much more observations for smaller real exchange rate values, which indicates very slow, albeit significant reversion towards the long run PPP. The amplitudes of transition functions are much smaller than for example in the study of Michael *et al.* (1997). This might reflect the fact that deviations from PPP are much more persistent for post Bretton Woods period than for two century period. Similar conclusions have also drawn Baum *et al.* (2001) in their study. Furthermore, the slow speeds of adjustment are consistent with the difficulties to reject the unit root hypothesis in real exchange rate series.

Figure 2. Estimated transition functions



The ESTAR estimation results are in line with the theoretical adjustment process described in Dumas (1992). In the middle regime, the deviations from the PPP equilibrium value might follow in some cases unit root process, but in the outer regime the deviations from parity are diminished. In all models, where the ESTAR transition function is significant, the real exchange rate series satisfy also the stability condition. The transition parameter estimates show that the speed of adjustment back to PPP is rather slow for post Bretton Woods period.

4 Conclusions

In this paper we use two popular nonlinear models to model the real exchange rate series. The difference between the autoregressive Markov switching regime model and the ESTAR model is that they differ in how they model the movement between regimes. For the Markov model the movements between regimes are unrelated to the past realizations of the process, and the estimate results are modelled unobserved Markov chain process. The ESTAR model moves between regimes depending on the past realizations of the process.

The real exhange rate series are often characterized by a random walk model. This chapter provides empirical evidence that, for a number of OECD countries, the real exchange rate is well characterized by a nonlinear but stationary process. The central question of the Markov switching regime models is whether long swings exist in the real exchange rate series. The results suggest the real exchange rates are reasonably well represented by two state Markov switching AR(1) regime models. The real exchange rate movements can be characterized by long swings of an appreciation and a depreciation regime for the post Bretton Woods period. The results also show that the 1980s can be characterized by a real depreciation regime for the US dollar based real exchange rate series with relatively high persistence, although the process is eventually mean reverting. The 1990s can be characterized as a real appreciation process with high persistence for the US dollar based real exchange rate series.

We evaluate the linear autoregressive model agains the ESTAR alternative using nine different bivariate real exchange rate series. The ESTAR models are applied to the real exchange rate series for which the linearity hypothesis is rejected, and display local instability and global stability. Thus, the real exchange rate can be characterized by a unit root or even explosive process in the neighbourhood of its long run equilibrium, but it adjusts faster when the size of the deviation from real exchange rate equilibrium value increases. The empirical evidence exhibits ESTAR type of nonlinearity for the seven series. The transition parameter estimates for the ESTAR models show that the speed of transition from one regime to the other is quite slow for most of the country pairs. The parameter estimates implies random walk behaviour for small deviations from PPP in some cases, but fast adjustment for large deviations from PPP.

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APPENDIX

Table A3.1. Parameter estimates for the model (3.1)

Series	$\beta' = (s, p^*, p)$
UK/US	(1, -1.76, 2.78)
GER/USA	(1, -3.40, 2.33)
FIN/USA	(1, -2.49, 4.05)
SWE/USA	(1, -1.67, 2.68)
UK/GER	(1, 0.10, 0.88)
FIN/GER	(1, 0.02, 0.97)
SWE/GER	(1, -0.50, 0.48)
UK/FIN	(1, -4.94, 5.91)
SWE/FIN	(1, 4.89, 5.88)