An Ex-Ante Examination of the Equity Premium

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Abstract

The equity premium of interest in theoretical models is the extra return investors anticipate when purchasing risky stock instead of riskfree debt. Unfortunately, we do not observe this *ex ante* premium in the data; we only observe the returns that investors actually receive *ex post*, after they purchase the stock and hold it over some period of time during which random economic shocks impact prices. Over the past century US stocks have returned roughly 6% more than riskfree debt, which is higher than warranted by standard economic theory – hence the "equity premium puzzle". In this paper we devise a method to simulate the distribution from which ex post equity premia are drawn, conditional on various assumptions about investors' ex ante equity premium. We calibrate our approach to US data to find that with investors' true ex ante equity premium as low as 2%, the economy could reasonably have produced an ex post equity premium of 6%. We therefore conclude that the 6% ex post equity premium observed in US data may be the result of fortunate market outcomes rather than a true asset pricing puzzle. Results from our simulation-based approach also suggest that only ex ante premia above 2% and below 6% are consistent with dividend yields of the US economy.

Keywords: equity premium puzzle, Monte Carlo simulation

JEL classifications: G12, C13, C15, C22, N22

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Over the past century the average annual return to investing in the US stock market has been roughly 6% higher than the return to investing in riskfree US T-bills. Mehra and Prescott [1985] argue that the US economy's performance has not been sufficiently volatile to warrant such a large premium to holding risky stocks instead of riskless bonds; hence the well known "equity premium puzzle".[1]

The equity premium at issue in economic theory is the premium investors in equilibrium anticipate *ex ante* at the moment they first make the decision to purchase stocks instead of riskfree debt. Conversely, the premium we observe in market data is the return investors actually received *ex post*, after they have held the stock for some time and nature has buffeted the economy with its random shocks. To the extent that the premium investors expected to receive ex ante is much lower than the premium they actually received ex post, there may really be no equity premium puzzle at all. We may simply perceive there to be a puzzle because we are using ex post stock return data as a necessarily inaccurate proxy for investors' ex ante expectations. For example, one could hypothesize that investors may actually demand a premium of only 2% to purchase risky stocks instead of riskless debt; and at the time investors purchased their stocks they may have expected to receive a premium of only 2%. However, subsequent to purchasing their stock the US economy may have been hit by a series of better-than-expected random economic shocks that produced higher excess stock returns than anticipated such that the realized equity premium we observe in the US data is a "lucky" 6% and not the 2% investors expected.

In light of the preceding discussion, to fully examine the equity premium puzzle it seems necessary to examine the ex ante equity premium. We therefore devise in this paper a method to simulate the distribution from which ex post equity premia are drawn, conditional on various values for investors' ex ante equity premium. We calibrate our approach to S&P 500 dividends and US interest rates (not prices or returns) and then conduct statistical tests to

[1] The equity premium literature is large, continuously growing, and much too vast to fully cite here. For recent reviews see Kocherlakota [1996] and Siegel and Thaler [1997].

find that with investors' true ex ante equity premium as low as 2%, the economy could still reasonably produce an ex post premium of 6%. This leads us to conclude that the 6% equity premium observed in US data may be the result of fortunate market outcomes rather than a true asset pricing puzzle. We perform statistical tests using simulation techniques rather than a simple t-test on the estimate of the equity premium from market data because our simulation-based methodology permits more powerful statistical tests and nuanced inference, as will be outlined below.

Although our paper is the first to provide a formal statistical test of the ex ante premium, previous authors have investigated the extent to which ex ante considerations may impact the equity premium. For example, Rietz [1988] investigated the effect that the fear of a serious, but never realized, depression would have on equilibrium asset prices and equity premia. Our work is distinct from his on at least two fronts. First, Rietz studies conditions necessary to obtain an ex ante equity premium as high as 6%; conversely, we develop a method for determining the probability of observing a 6% equity premium ex post even if the ex ante premium is as low as 2%. Second, Rietz assumes the possibility of a catastrophic economic state similar to the Great Depression in order to obtain large equity premia; conversely, we calibrate to post-WWII data during which there are no catastrophic states and still we are able to rationalize ex post equity premia in the range of 6%, even with an ex ante equity premium of only 2%.

Jorion and Goetzmann [1999] take an alternate approach of comparing US stock market performance with stock market experiences in many other countries. They find that, while some markets such as the US and Canada have done very well over the past century, other countries have not been so fortunate; average stock market returns from 1921-1996 in France, Belgium and Italy, for example, are all close to zero, while countries such as Spain, Greece and Romania have experienced negative returns. Jorion and Goetzmann [1999] do not conduct statistical tests, likely because performing statistical tests in the context of the data they have available would present some formidable technical challenges. First, the stock indices they consider are largely contemporaneous, and to the extent that world equity markets are integrated, returns from the various indices are not independent. Statistical tests would have to take into account the panel nature of the data and explicitly model covariances across countries. Second, many countries in the comparison pool are difficult to compare directly to the United States in terms of economic history and underlying data generating processes. (Economies like Egypt and Romania, for example may have equity premia generated from different data generating processes relative to the US.) Since in our paper we simulate many independent economies with the same data generating process as the US over the past 50 years, we avoid both of these issues and are thus able to undertake formal statistical inference.

There are some recent papers that, like our paper, make use of fundamental information in examining the equity premium. However, unlike our study, these studies focus on estimating the *ex post* equity premium, while we consider both the ex post equity premium and the ex ante equity premium. One such paper, by Fama and French [2002], uses historical dividend yields and other fundamental information to calculate estimates of the ex post equity premium which are smaller than previous estimates. Fama and French obtain point estimates of the ex post equity premium ranging from 2.55% (based on dividend growth rate fundamentals) to 4.78% (based on bias-adjusted earnings growth rate fundamentals), however these estimates have large standard errors. For example, for their point estimate of 4.32% based on earnings growth rates not adjusted for bias, a 99% confidence interval stretches from approximately -1% to about 9%. Mehra and Prescott's initially troubling estimate of 6% is easily within this confidence interval and is in fact within one standard deviation of the Fama and French point estimate. Another paper that similarly makes use of fundamental information to form lower estimates of the ex post equity premium is by Claus and Thomas [2001]. The Claus and Thomas study covers a shorter time period relative to the Fama and French study – 14 years versus 50 years – yielding point estimates that are subject to at least as much variability as the Fama and French estimates. Hence, even if fundamental sources of information such as dividends and interest rates are employed in the analysis, conducting inference about the original Mehra and Prescott equity premium puzzle is challenging given the large confidence intervals around the equity premium point estimates from these studies.

In light of the preceding discussion, it is difficult to make concrete statements about the plausible magnitude of the ex ante equity premium. This is because the existing literature has focused on observed measures of the *ex post* equity premium realized in market data instead of the *ex ante* equity premium that forms the foundation for theoretical models. Thus, in our paper we depart from previous studies to focus on the relationship between the ex ante and ex post equity premium. We are able to narrow the range of ex post equity premia that are consistent with a low ex ante equity premium and we are able to obtain new and compelling evidence bounding the range of ex ante equity premia that are consistent with our economy's fundamentals (dividends and interest rates).

The basic methodology used to conduct our simulations, and our key results, are presented in Section 1 below. In Section 2 we describe the exact details of our simulation procedure including the estimation of risk premia used in cash-flow discounting models. In Section 3 we discuss the properties of discounted dividend growth rates and dividend yields that emerge from our simulations. Section 4 concludes.

1 Basic Methodology and Key Results

As noted in the introduction, the equity premium around which economic theory is built is the extra return – i.e., premium – that investors demand to purchase risky stock instead of riskfree debt. As financial economists we do not observe this ex ante premium; we only observe the returns that investors actually receive ex post, after they have purchased the stock and held it over some period of time during which random economic shocks impact prices. Given that the world almost never unfolds exactly as one expects, there is no reason to believe that the stock return we see ex post is exactly the same as the return investors anticipated ex ante. It is therefore difficult to argue that just because we observe a 6% equity premium ex post in the US data, the premium that investors demand ex ante is also 6% and thus a puzzling challenge to economic theory. We therefore ask the question: if investors' true ex ante premium is $X\%$, what is the probability that the US economy would randomly produce an ex post premium of at least 6%? We can then argue whether or not the 6% ex post premium observed in the US data is consistent with various ex ante premium values, X, with which standard economic theory may be more compatible.

The basic methodology we employ is as follows:

(a) Assume a value for the equity premium that investors demand when they first purchase stock (e.g., 2%). This assumed premium, appropriately adjusted for bias as described below, can be added to the riskfree interest rate to determine the discount rate that an investor would rationally apply to a forecasted dividend stream in order to calculate the present-value-price of dividend-paying stock.

(b) Estimate econometric models for the time-series processes driving dividends and interest rates in the US economy, fitted to US dividend and interest rate data. Then use these models to Monte Carlo simulate a variety of potential paths for US dividends and interest rates. The simulated dividend and interest rate paths so produced are of course different in each of these simulated economies because different sequences of random innovations are applied to the common stochastic processes in each case. However, the key drivers of the simulated economies themselves are all still identical to the US economy since all economies share common stochastic processes fitted to US data.

(c) Given the assumed equity premium investors demand ex ante (which is the same for all simulated economies), use a discounted-dividend model to calculate the fundamental stock returns that obtain ex post in each simulated economy, and thus the equity premium that obtains ex post in each simulated economy. In other words, produce equity premia for a distribution of economies that all look identical to the US economy in key respects (i.e., dividend and interest rate processes) – all economies have the same ex ante equity premium, and yet all economies also have different ex post equity premia.

(d) Compare the various ex post equity premia from the various simulated economies with the common equity premium that investors were assumed to have demanded ex ante, asking a variety of questions including whether or not observing an ex post premium of 6% is consistent with the equity premium investors demanded ex ante.

The exact procedures we use to create our simulated economies, to calculate appropriate dividend discount rates given an assumed equity premium, and to conduct various statistical tests, are all explained in Section 2 below. Before discussing any of these details, however, it is instructive to view our key results first.

— Table 1 goes approximately here —

Table 1 reports statistics including the mean ex post equity premia estimates from the simulated economies (in the first column of statistics in Table 1) as well as percentiles of the distribution of ex post equity premia from our simulated economies for ex ante equity premia of 6%, 3%, 2.5% and 2%, respectively. From the top row we see that with an ex ante equity premium of 6% the mean of the simulated economies' equity premium estimates is equal to 6% as it should be. This is one way to confirm that our simulations are producing sensible results – on average the world unfolds ex post as it is assumed it will unfold ex ante. However, the individual simulated economies randomly deviate from this average, depending on how each simulated economy randomly unfolded. The percentiles of the distribution for the case of an equity premium of 6% indicate that a 90% confidence interval (covering from the 5th percentile to the 95th percentile) encloses premia of roughly 2.4% to 8.6%. We also see that 1% of the economies produced ex post premia greater than 9.2% even though the ex ante premium was only 6%. Similarly, 1% of the economies produced ex post premia less than -.1%. The median premium (the 50th percentile) is 6.3%, revealing the slight skew in the empirical distribution of premia.

The remaining rows of Table 1 confirm that our simulated economies produce ex post equity premia with an average equal to the ex ante premium, and that the distribution of equity premia is skewed to the right. Most interestingly, we see from Table 1 that an ex post 6% equity premium is not significantly unusual at the 1% level even with an ex ante equity premium of 2%. Results from Table 1 therefore suggest that the 6% premium we observe in US data may be simply a "lucky" outcome, not a true puzzle to challenge generally accepted economic theory.

— Figure 1 goes approximately here —

Figure 1 plots the frequency distribution of ex post equity premia from thousands of US-like economies, corresponding to the rows of Table 1, with an assumed ex ante equity premium of 6% (Panel A), 3% (Panel B), 2.5% (Panel C), or 2% (Panel D). An equity premium of 3% is the average premium observed across all the countries studied by Jorion and Goetzmann [1999]. An equity premium of 2% is the lowest equity premium we can assume in our simulations while maintaining the ability to produce auxiliary statistics (such as dividend yields and dividend growth-interest rate ratios, as will be discussed below) that are at least broadly consistent with observed US data. These plots provide an informal view of our primary results on the distribution of ex post equity premia given particular ex ante equity premia. As the discussion of Table 1 above indicates, a 6% ex post (estimated) equity premium is not terribly far out in the upper tail of the distribution even when the ex ante (true) equity premium is only 2%. The plots also confirm that the distribution of equity premia estimates is skewed to the right and includes realizations of the ex post equity premium that are negative. Of further interest, these plots show that the distribution changes shape slightly as the premium drops, becoming fatter (larger variance, lower peaked).

We now proceed to describe in detail the manner in which the preceding exercise was conducted, and then to investigate other statistics from our simulated economies, including dividend yields. The consideration of dividend yields will narrow the range of ex ante premia consistent with our economy's observed ex post premium.

2 Simulation Details and Discussion

In this paper we employ numerical simulation procedures to simulate returns and estimate equity premia. Although this particular application of numerical simulation methods is new to the equity premium puzzle literature, there is already a significant literature that uses similar techniques in other asset pricing applications. Indeed, there is body of work that (directly or indirectly) simulates stock prices and dividends, under various assumptions, to investigate price and dividend behavior.[2] However, these studies typically employ restrictions on the dividend and discount rate processes so as to obtain returns from some variant of the Gordon [1962] model and/or some log-linear approximating framework. Rather than employ approximations to solve our price calculations analytically, we instead simulate the dividend growth and discount rate processes directly, and evaluate the expectation through Monte Carlo integration techniques. This approach is computationally burdensome, but it is the only way to evaluate prices, returns and equity premia without approximation error.[3] We also take extra care to calibrate our models to the time series properties of actual data. For example, dividend growth is strongly autocorrelated in the S&P 500 stock market data, counter to the assumption of a log random walk for dividends sometimes employed for tractability in other applications. Thus we model autocorrelation in our simulated dividend growth rates.

2.1 Foundations

The *ex ante* equity premium, which we define as π, is formally defined as the difference between the expected return on risky assets and the expected riskfree rate. (See, for instance, Mehra and Prescott's [1985] Equation 14.) The ex ante equity premium is unobservable, so in practice the *ex post* equity premium is typically estimated using historical equity returns

[2]See, for example, Scott [1985], Kleidon [1986], West [1988a,b], Campbell [1991], Mankiw, Romer and Shapiro [1991], Hodrick [1992], Timmermann [1993,1995], Donaldson and Kamstra [1996] and Campbell and Shiller [1998].

[3]There is still Monte Carlo simulation error, but that is random, unlike most types of approximation error, and it can also be measured explicitly and controlled to be very small, as we have done.

and riskfree rates. Define $\overline{R}$ as the average historical annual return on the S&P 500, $\overline{r}_f$ as the average historical return on US T-bills. Then we can define the ex post equity premium, $\hat{\pi}$, as follows:

$$\hat{\pi} \equiv \overline{R} - \overline{r}_f. \tag{1}$$

To simulate ex post equity premia we simulate $\overline{R}$ and $\overline{r}_f$ for a large number of US-like economies.

The first step in simulating returns is to tie returns to underlying factors that drive asset prices, in particular dividends and discount rates. To do this, we begin by defining P_t as a stock's beginning-of-period-t price, r_t as the rate investors use to discount payments received during period t (i.e., from the beginning of period t to the beginning of period $t+1$), and $\mathcal{E}_t$ as the expectations operator conditional on information available at the beginning of period t. Investor rationality requires that the current market price of a stock, which will pay a dividend D_{t+1} one period from now and then sell for P_{t+1}, satisfy Equation (2):

$$P_t = \mathcal{E}_t \left\{ \frac{P_{t+1} + D_{t+1}}{1 + r_t} \right\}. \tag{2}$$

Then $R_t \equiv \{(P_{t+1} + D_{t+1})/P_t\} - 1$ is the return on stock; i.e., the equity return.

Invoking the standard transversality condition that the expected present value of the stock price P_{t+i} falls to zero as i goes to infinity, and defining the growth rate of dividends during period t as $g_t \equiv (D_{t+1} - D_t)/D_t$, allows us rewrite Equation (2) as:

$$P_t = D_t \mathcal{E}_t \left\{ \sum_{i=1}^{\infty} \left(\Pi_{k=1}^{i} \left[\frac{1 + g_{t+k-1}}{1 + r_{t+k-1}} \right] \right) \right\}. \tag{3}$$

One attractive feature of expressing the present value stock price as in Equation (3) – i.e., in terms of dividend growth rates and discount rates – is that this form highlights the irrelevance of inflation, at least to the extent that expected and actual inflation are the same. Notice that working with nominal growth rates and discount rates, as we do, is equivalent to working with deflated nominal rates (i.e., real rates). That is, $\frac{1+([g_t-I_t]/[1+I_t])}{1+([r_t-I_t]/[1+I_t])} = \frac{(1+g_t)}{(1+r_t)}$, where I_t is inflation. Working with nominal values in the simulations below removes a potential source of measurement error associated with attempts to estimate inflation.

We now execute our strategy of (a) calibrating models to market dividends and interest rates, (b) simulating economies using these calibrated models, (c) calculating returns and equity premia for these economies, and (d) evaluating these equity premia.

2.2 Calibration

The first step in obtaining stock prices from Equation (3) is to estimate time series models for dividend growth and interest rates so that our Monte Carlo simulations will generate dividends and discount rates that share key features with observed dividends and discount rates. The discount rate is defined to be the riskfree interest rate plus a constant[4] premium of $X\%$, where X is chosen to produce a target equity premium as explained in Section 2.3 below. Economic theory admits a wide range of possible processes for the riskfree interest rate, from constant to autoregressive and highly non-linear heteroskedastic forms. The AR(1) model of the logarithm of interest rates, as described in Hull (1993, page 408) will be used here as it fits our data well and restricts nominal interest rates to be positive. Standard specification tests for normality, autocorrelation and ARCH on the error term from an AR(1) model of the logarithm of interest rates do not reject the null of no misspecification. The 1-year T-bill rates on our annual data have mean 0.059 and standard deviation 0.03 over 1952-1998, the

[4] To the extent that the risk premium time-varies, our simulation-based ex ante examination of the equity premium would only be further inclined to yield high ex post *average* equity premia when the ex ante equity premium *process* has a low mean premium. That is, the lower bound on the range of reasonable values for the ex ante equity premium we provide is conservative.

time period we study.[5] The AR(1) coefficient estimate in the regression of log interest rates on lagged log interest rates equals 0.83.

Since dividend growth rates have a minimum value of -100% and no theoretical maximum, a natural choice for their distribution is the log normal. The logarithm of 1 plus the annual dividend growth rate has mean 0.0531 and standard deviation 0.035 for the S&P 500 over 1952-1998. We estimated simple ARMA time series models for the logarithm of 1 plus the dividend growth rate and found the best model by the Bayesian Information Criterion to be an MA(1) model with the MA(1) coefficient equal to 0.60. Standard tests for normality of this error term (and hence conditional log normality of dividend growth rates) do not reject the null of normality, and standard tests for autocorrelation and ARCH fail to reject the null of homoskedasticity and no serial correlation. Finally, the error terms from the MA(1) model of log dividend growth rates and log interest rates are correlated, with a correlation coefficient of 0.21.

Properties of prices and returns produced by Equation (3) depend in important ways on the modeling of the dynamics of the dividend growth and interest rate processes. For instance, the stock price will equal a constant multiple of the dividend level and returns will be very smooth over time if dividend growth and interest rates are set equal to constants plus independent innovations. However, using models that capture the serial dependence of dividend growth rates and interest rates observed in the data, as we do, will typically lead to time-varying price-dividend ratios and variable returns of the sort we observe in the S&P 500 stock market data.

2.3 The Risk Premium Versus the Equity Premium

While the terms equity premium and risk premium are often applied synonymously[6] they do not in general mean the same thing. Recall that the ex ante equity premium, π, is the

[5] The starting year of 1952 was motivated by the U.S. Federal Reserve Board's adoption of a modern monetary policy regime in 1951.

[6] See, for instance, Claus and Thomas [2001] who comment on the equity premium but then calculate the risk premium instead, and Lee, Myers, and Swaminathan [1999] who calculate the equity premium but then use it as a risk premium for discounting future cash flows.

difference between the expected equity return and the expected riskfree rate:

$$\pi \equiv \mathcal{E}\left\{R\right\} - \mathcal{E}\left\{r_f\right\}.$$

The risk premium r_p, however, is the premium added to the riskfree rate, r_f, for the purpose of *discounting future risky cash flows,*; i.e., $r_t \equiv r_{f,t} + r_p$ in Equation (2), as rewritten below:

$$P_t = \mathcal{E}_t\left\{\frac{P_{t+1} + D_{t+1}}{1 + r_{f,t} + r_p}\right\}. \tag{4}$$

Notice that by dividing both sides of Equation (4) by P_t and applying the law of iterated expectations we obtain Equation (5):

$$1 = \mathcal{E}\left\{\frac{1 + R_t}{1 + r_{f,t} + r_p}\right\}. \tag{5}$$

By Jensen's Inequality, the value of r_p that sets the expectation in Equation (5) equal to 1 will not necessarily be the equity premium $\pi \equiv \mathcal{E}\left\{R\right\} - \mathcal{E}\left\{r_f\right\}$ defined above. Only in the special case where the denominator of Equation (5) is not random (e.g., if r_f and r_p are both constant) can we multiply both sides of Equation (5) by $\mathcal{E}\left\{1 + r_f + r_p\right\}$ and derive the equality of r_p and π. Therefore, simply adding $X\%$ to the riskfree rate will not in general yield an equity premium of $X\%$.

In this study, we determine the appropriate risk premium by finding the value of r_p that satisfies the expectation in Equation (4). In our application, this requires adding roughly 20 more basis points to the risk-adjusted discount rate than would have been added had we erroneously used the equity premium in place of the risk premium. In other words, if a 2% equity premium is desired, then we must add a 2.2% risk premium to the riskfree interest rate – a full 10% bias adjustment, which is a relatively important adjustment when one considers the power of compounding in present value calculations. One way to verify that our estimator delivers the correct risk premium for the desired equity premium, and that we have made the equity-premium-to-risk-premium adjustment correctly, is to observe in Table 1 above that our simulations replicate the equity premium assumed in each case. For example, in the 2% ex ante equity premium case we obtain the correct mean of the ex post equity premium of 2%, having added the adjusted risk premium of 2.2% to the riskfree rate in our simulations.

2.4 The Numerical Simulation

We now detail the numerical simulation by which Figure 1 and Table 1 are produced. That is, we detail for the e^{th} economy the formation of the prices (P_t^e), returns (R_t^e), and ex post equity premia ($\hat{\pi}^e$) (where $e = 1, ..., E$ and $t = 1, ..., T$), given dividends, dividend growth rates, risk-free interest rates and the equity premium of the e^{th} economy – D_t^e, g_t^e, and $r_t^e = r_{f,t}^e + \pi$.[7]

In terms of timing and information, recall that P_t^e is the stock's beginning-of-period-t price, r_t^e is the rate used to discount payments received during period t and is known at the beginning of period t, D_t^e is paid at the beginning of period t, g_t^e is defined as $(D_{t+1}^e - D_t^e)/D_t^e$ and is not known at the beginning of period t since it depends on D_{t+1}^e, and $\mathcal{E}_t\left\{\cdot\right\}$ is the conditional expectation operator, with the conditioning information being the set of information available to investors at the beginning of period t. Finally, recall Equation (3), rewritten to correspond to the e^{th} economy:

$$P_t^e = D_t^e \mathcal{E}_t \left\{ \sum_{i=1}^{\infty} \left(\Pi_{k=1}^{i} \left[\frac{1 + g_{t+k-1}^e}{1 + r_{t+k-1}^e} \right] \right) \right\},$$

which can be rewritten as shown in Equation (6),

[7] We set the number of economies, E, at 2000. This is a sufficiently large number of replications to produce results with very small simulation error, as discussed below.

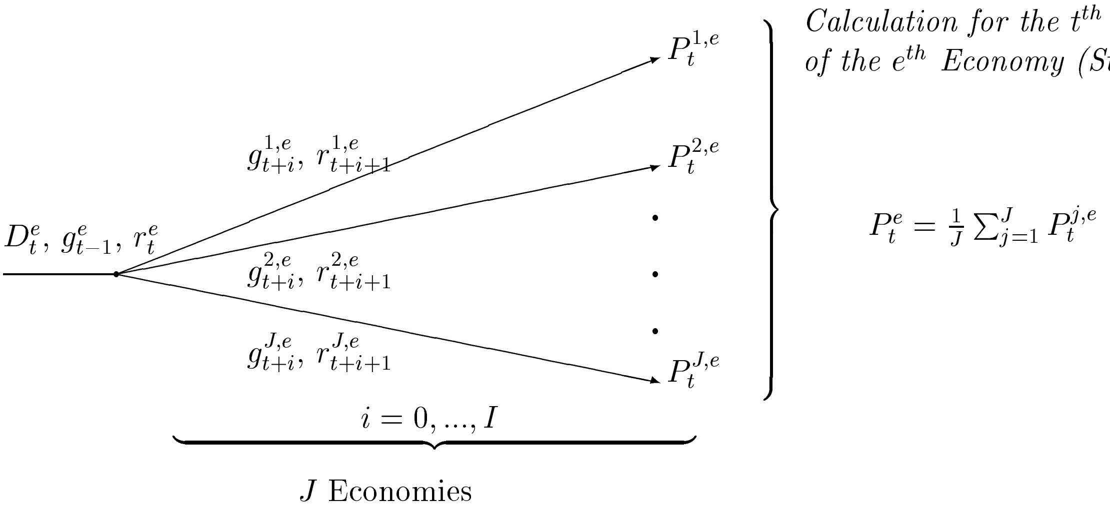
$$P_t^e = D_t^e \sum_{i=1}^{\infty} \mathcal{E}_t \left\{ \Pi_{k=1}^{i} y_{t+k-1}^e \right\}, \tag{6}$$

in which $y_t^e \equiv \frac{1+g_t^e}{1+r_t^e}$ is the discounted dividend growth rate. Returns are constructed as $R_t^e = (P_{t+1}^e + D_{t+1}^e - P_t^e)/P_t^e$, and $\hat{\pi}^e = \overline{R}^e - \overline{r}_f^e$ where $\overline{R}^e = \frac{1}{T}\sum_{t=1}^{T} R_t^e$ and $\overline{r}_f^e = \frac{1}{T}\sum_{t=1}^{T} r_{f,t}^e$.

Based on Equation (3), we generate prices by generating a multitude of possible streams of dividends and discount rates, present-value discounting the dividends with the discount rates, and averaging the results; i.e., by conducting a Monte Carlo integration. Hence we produce prices (P_t^e), returns (R_t^e), and ex post equity premia ($\hat{\pi}^e$) utilizing only dividend growth rates and discount rates. The exact procedure is described below and summarized in Exhibit 1.[8]

Exhibit 1

Diagram of a Simple Market Price Calculation for the t^{th} Observation of the e^{th} Economy (Steps 1 and 2)



Step 1: When forming P_t^e, the most recent fundamental information available to a market trader would be g_{t-1}^e, D_t^e, and r_t^e. The quantities g_{t-1}^e, D_t^e, and r_t^e must therefore be

[8] According to Equation (3), the stream of dividends and discount rates should be infinitely long, however truncating the stream at a sufficiently distant point in time denoted I leads to a very small approximation error. We discuss this point more fully below.

generated directly in our simulations, whereas P_t^e is calculated based on these g, r and D. The objective of Steps 1(a)-(c) outlined below is to produce dividend growth rates and interest rates that replicate the real world dividend growth and interest rate data. That is, the simulated dividend growth rates and interest rates must have the same mean, variance, correlation structure and autocorrelation structure as the observed real world dividend growth rates and interest rates.

Step 1(a): Note that since, as described in Section 2.2, the logarithm of one plus the dividend growth rate is modeled as an MA(1) process, $log(1+g_t^e)$ is a function of only innovations, labelled ϵ_g^e. Note also that since the logarithm of the interest rate is modeled as an AR(1) process, $log(r_{f,t}^e)$ is a function of $log(r_{f,t-1}^e)$ and an innovation labelled ϵ_r^e. Set the initial dividend, D_1^e, equal to the S&P 500's dividend value for 1951 (observed at the end of 1951), and the lagged innovation of the logarithm of the dividend growth rates $\epsilon_{g,0}^e$ to 0. To match the real-world interest rate data, set $log(r_{f,0}^e) = -3.05$ (the mean value of log interest rates required to produce interest rates matching the mean and variance of observed T-bill rates). Then generate two independent standard normal random numbers, $\epsilon_{1,1}^e$ and $\epsilon_{2,1}^e$, and form two correlated random variables, $\epsilon_{r,1}^e = 0.242(0.21\epsilon_{1,1}^e + (1-.21^2)^{.5}\epsilon_{2,1}^e)$ and $\epsilon_{g,1}^e = 0.0305\epsilon_{1,1}^e$. These are the simulated innovations to the interest rate and dividend growth rate processes, formed to have standard deviations of 0.242 and 0.0305 respectively to match the data, and to be correlated with correlation coefficient 0.21 as we find in the S&P 500 return and T-bill rate data. Next, form $log(1+g_1^e) = 0.0531 + 0.60\epsilon_{g,0}^e + \epsilon_{g,1}^e$ and $log(r_{f,1}^e) = -0.18 + 0.94 log(r_{f,0}^e) + \epsilon_{r,1}^e$.[9] Also form $D_2^e = D_1^e(1+g_1^e)$.

Step 1(b): Produce two correlated normal random variables, $\epsilon_{r,2}^e$ and $\epsilon_{g,2}^e$ as in Step 1(a) above, and conditioning on $\epsilon_{g,1}^e$ and $log(r_{f,1}^e)$ from Step 1(a) produce $log(1+g_2^e) = 0.0531 + 0.60\epsilon_{g,1}^e + \epsilon_{g,2}^e$, $log(r_{f,2}^e) = -0.18 + 0.94 log(r_{f,1}^e) + \epsilon_{r,2}^e$ and $D_3^e = D_2^e(1+g_2^e)$.

[9] Notice that the AR(1) parameter for the log interest rate process is estimated to be 0.83 but we have set it to 0.94 in the simulations. It is well known that the coefficient estimate in an AR(1) OLS regression is biased downwards; see for instance Kennedy (1992, page 147). Numeric simulations were employed to determine the appropriate correction for our data, as in Orcutt and Winokur (1969), and this led to the setting of 0.94. The intercept term had to be adjusted as well to reflect this new setting.

Step 1(c): Repeat Step 1(b) to form $log(1 + g_t^e)$, $log(r_{f,t}^e)$ and D_t^e for $t = 3, 4, 5, ..., T$ and for each economy $e = 1, 2, 3, ..., E$. Then calculate the dividend growth rate g_t^e and the discount rate r_t^e (which equals $r_{f,t}^e$ plus the bias-corrected risk premium needed to obtain the desired equity premium).

Step 2: For each time period $t = 1, 2, 3, ..., T$ and economy $e = 1, 2, 3, ..., E$ we next calculate prices, P_t^e. In order to do this we must solve for the expectation of the infinite sum of discounted future dividends conditional on time $t - 1$ information for economy e. That is, we must produce a cross-section of dividends and interest rates that might be observed in periods $t, t + 1, t + 2, ...$ given what is known at period $t - 1$ and use these to solve the expectation of Equation (3). We use the superscript j to index the cross-section of future economies that could possibly evolve from the current state of the economy.

Step 2(a): Set $\epsilon_{g,t-1}^{j,e} = \epsilon_{g,t-1}^{e}$ and $log(r_{f,t-1}^{j,e}) = log(r_{f,t-1}^{e})$ for $j = 1, 2, 3, ..., J$. [10] Generate two independent standard normal random numbers, $\epsilon_{1,t}^{j,e}$ and $\epsilon_{2,t}^{j,e}$ and form two correlated random variables $\epsilon_{r,t}^{j,e} = 0.242(0.21\epsilon_{1,t}^{j,e} + (1 - .21^2)^{.5}\epsilon_{2,t}^{j,e})$ and $\epsilon_{g,t}^{j,e} = 0.0305\epsilon_{1,t}^{j,e}$ for $j = 1, 2, 3, ..., J$.[11] These are the simulated innovations to the interest rate and dividend growth rate processes, respectively. Form $log(1 + g_t^{j,e}) = 0.0531 + 0.60\epsilon_{g,t-1}^{j,e} + \epsilon_{g,t}^{j,e}$ and $log(r_{f,t}^{j,e}) = -0.18 + 0.94log(r_{f,t-1}^{j,e}) + \epsilon_{r,t}^{j,e}$.

Step 2(b): Produce two correlated normal random variables $\epsilon_{r,t+1}^{j,e}$ and $\epsilon_{g,t+1}^{j,e}$ as in Step 2(a) above, and conditioning on $\epsilon_{g,t}^{j,e}$ and $log(r_{f,t}^{j,e})$ from Step 2(a) produce $log(1 + g_{t+1}^{j,e}) = 0.0531 + 0.60\epsilon_{g,t}^{j,e} + \epsilon_{g,t+1}^{j,e}$, and $log(r_{f,t+1}^{j,e}) = -0.18 + 0.94log(r_{f,t}^{j,e}) + \epsilon_{r,t+1}^{j,e}$ for $j = 1, 2, 3, ..., J$.

[10] We choose J to equal 2,000, in order to ensure the simulation error in calculating prices and returns was controlled to be very small. To determine the simulation error, we conducted a simulation of the simulations. Unlike some Monte Carlo experiments (such as those estimating the size of a test statistic under the null) the standard error of the simulation error for most of our estimates (returns, prices, etc.) are themselves analytically intractable, and must be simulated. In order to estimate the standard error of the simulation error in estimating market prices, we estimated a single market price 2,000 times, each time independent of the other, and from this set of prices computed the mean and variance of the price estimate. If the experiment had no simulation error, each of the price estimates would be identical. With the number of cross-sections, J, equal to 2,000 we find that the standard deviation of the simulation error is less than 0.20% of the price, which is sufficiently small as to not be a source of concern for our study.

[11] For our random number generation we made use of a variance reduction technique, stratified sampling. This technique has us drawing pseudo-random numbers ensuring that q% of these draws come from the q^{th} percentile, so that our sampling does not weight any grouping of random draws too heavily.

Step 2(c): Repeat Step 2(b) to form $log(1+g_{t+i}^{j,e})$ and $log(r_{t+i}^{j,e})$ for $i = 2,3,4,...,I$, $j = 1,2,3,...,J$, and economies $e = 1,2,3,...,E$. Solve for the dividend growth rate $g_{t+i}^{j,e}$, the dividends $D_{t+i}^{j,e}$, and the discount rate $r_{t+i}^{j,e}$ (which equals $r_{f,t+i}^{j,e}$ plus the bias-corrected risk premium needed to obtain the desired equity premium) for $i = 0,1,2,...,I$.

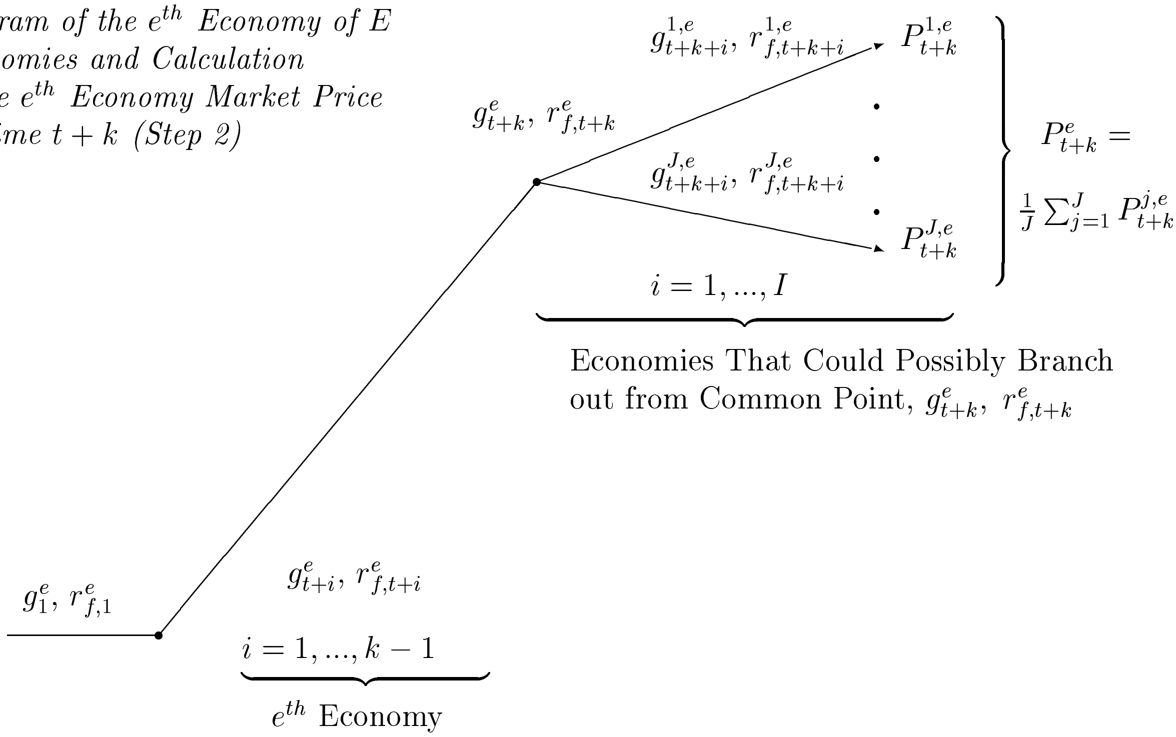
Step 2(d): The present discounted value of each of the individual J streams of dividends is now taken in accordance with Equation (3), with the j^{th} present value price noted as $P_t^{j,e}$. Finally, the price for the e^{th} economy in period t is formed: $P_t^e = \frac{1}{J}\sum_{j=1}^{J} P_t^{j,e}$.

In considering these prices, note that according to Equation (3) the stream of discount and dividend growth rates should be infinitely long, while in our simulations we extend the stream only a finite number of periods, I. Since the ratio of gross dividend growth rates to gross discount rates – i.e., the ys in Equation (6) – are less than unity in steady state, the individual product elements in the infinite sum in Equation (6) – equivalently Equation (3) – eventually converge to zero as I increases. (Indeed, this convergence to zero is exactly what is required for the standard transversality condition that the expected present value of the stock price P_{t+i} falls to zero as i goes to infinity.) We therefore set I large enough in our simulations so that the truncation does not materially effect our results. We find that setting $I = 1000$ years accomplishes this in all cases we studied. That is, the discounted present value of a dividend payment received 1000 years in the future is essentially zero. Also note that the steps above are required to produce P_t^e, D_t^e, g_{t-1}^e, and r_t^e for $e = 1,...,E$ and $t = 1,...,T$; the intermediate terms superscripted with a j are required only to perform the numerical integration that yields P_t^e. This process is illustrated in Exhibit 2. Note that the length of the time series T is chosen to be 47 to imitate the 47 years of annual data we have available from the S&P 500 from 1952 to 1998.

Step 3: After performing Steps 1(a)-(c) and 2(a)-(d) for $t = 1,...T$, rolling out E independent economies for T periods, we construct the market returns for each economy, $R_t^e = (P_{t+1}^e + D_{t+1}^e - P_t^e)/P_t^e$, and the equity premium that agents in the e^{th} economy would observe, $\hat{\pi}^e$, estimated from Equation (1) as the mean difference in market returns and the riskfree rate. These simulated equity premia form the basis for the analysis in Section 1 (including Figure 1 and Table 1 above) of this paper.

Exhibit 2

Diagram of the e^{th} Economy of E Economies and Calculation of the e^{th} Economy Market Price at Time $t + k$ (Step 2)



3 Dividend-Yield-Consistent Equity Premia

In order to assess the reliability of our numerical simulation, it is interesting to consider statistics based on simulated quantities, like the discounted dividend growth rates (the ys in Equation (6) above). We calibrate our simulations to produce interest rates and dividend growth rates consistent with those we have seen in the US economy over the last 50 years. One would therefore anticipate that the discounted dividend growth rates (the ys in Equation (6) above) we produce in our simulations should be consistent with the actual economy's data. Figure 2 and Table 2 below present evidence that the experience of the US economy and the simulated economies is indeed similar.

— Table 2 goes approximately here —

— Figure 2 goes approximately here —

Figure 2 plots the frequency distribution of mean discounted dividend growth rates (y) from our US-like simulated economies, with an assumed ex ante equity premium of 6% (Panel A), 3% (Panel B), 2.5% (Panel C), or 2% (Panel D). In each case the mean y of the actual S&P 500 index (0.94) lies well within the distribution of outcomes from our simulated economies. It falls just to the left of the peak of the distribution of simulated outcomes for the case of an ex ante equity premium of 6%, and increasingly further to the left for ex ante equity premia of 3% or less. Recall that these data, the y, have no price or return data in them; the equity premium only affects y by changing the mean discount rate, so that as the equity premium falls the mean discounted dividend growth rate rises. It is fairly clear from these plots that the mean discounted dividend growth rate observed over recent history in the US is consistent with a broad range of possible ex ante equity premia. Table 2 bears this out, as each moment of discounted dividend growth rates (the mean, standard deviation, skew and kurtosis) lies within the 90% confidence interval of the simulated data for each of the equity premia considered, 6%, 3%, 2.5% and 2%.

It is also interesting to consider statistics based on quantities that are derived from the simulations indirectly, such as the dividend yield, D/P. If our simulations produce dividend yields that are wildly at odds with those of the S&P 500, it would be difficult to defend our simulation-based inferences on the equity premium. (Recall that we do not calibrate our simulations to prices or dividend yields, only to interest rates and dividend growth rates.) Comparing the dividend yields we obtain from the simulated economies to those we witness from the S&P 500 index also provides independent evidence on the likely range of ex ante equity premia that could have produced the US experience of the last 50 years. Indeed this set of fundamental data is used in Fama and French [2002] and Jagannathan, McGrattan, and Scherbina [2001] to argue that the equity premium may be much smaller than 6%.

— Table 3 goes approximately here —

— Figure 3 goes approximately here —

Figure 3 plots the frequency distribution of mean dividend yields from 2,000 US-like economies, with an assumed ex ante equity premium of 6% (Panel A), 3% (Panel B), 2.5% (Panel C), or 2% (Panel D). The first, perhaps most striking thing to notice is that an economy with an ex ante equity premium of 6% would be hard pressed to produce a mean dividend yield as low as the 3.8% we have observed over the last 50 years. With an ex ante equity premium of 3%, or even 2.5%, we see that this observed low dividend yield is more consistent with simulated economies calibrated to the US experience of dividends and interest rates. Finally, the dividend yield of 3.8% for the US is in the extreme *upper* tail of the distribution of dividend yields for the case of an ex ante equity premium of 2%. This suggests that the probable lower end of equity premia our simulations can support is 2%.

Table 3 confirms the visual impression of Figure 3, that a 3.8% dividend yield is below the 1% significance level (see the 1% percentile column) only for the case of the 6% ex ante equity premium, and above the 1% significance level only for the case of the 2% ex ante equity premium. Table 3 also presents additional statistics on the dividend yield for both the S&P 500 index and our 2,000 simulated economies. The second row of each cell of Table 3 presents the standard deviation of the dividend yield. These yields have been very volatile for the S&P 500 index, with a standard deviation of 1.1% annually over the last 50 years. Our simulated economies with an ex ante equity premium of 6% are a little more volatile, with a standard deviation of 1.2% (as shown in the second column). Our simulated economies with equity premia of 3% and lower all have less volatile yields on average but the 90% confidence interval on the standard deviation in each case encloses the 1.1% standard deviation observed in the S&P 500 data. (See the 5% and 95% percentile columns for the standard deviation (σ) row for each equity premium case.) Similarly the higher moments of the dividend yields for the simulated economies match well the actual S&P 500 experience, giving us confidence that our calibrations are successfully reproducing the statistical characteristics of the data, even when considering derived variables like dividend yields, which we did not calibrate our simulations to. All together, these results suggest that an ex ante equity premium as high as 6% is likely not consistent with the broader picture of the S&P 500 index, and that a 2% ex ante equity premium appears to be on the low side of equity premia consistent with the experience on the last 50 years in the US. Note, these are *ex ante* premia. Ex post equity premium estimates of 2% or 6% are well within the likely range of estimates coming from an economy with a true ex ante equity premium of either 6% or 2% – see Table 1 and the columns of percentiles of ex post equity premia estimates generated from economies for various settings of ex ante equity premia.

4 Conclusions

The equity premium of interest in theoretical models is the extra return investors anticipate when purchasing risky stock instead of riskfree debt. Unfortunately, we do not observe this *ex ante* premium in the data; we only observe the returns that investors actually receive *ex post*, after they purchase the stock and hold it over some period of time during which random economic shocks impact prices. US stocks have historically returned roughly 6% more than riskfree debt, which is higher than warranted by standard economic theory – hence the “equity premium puzzle”.

In this paper we have devised a method to simulate the distribution from which ex post equity premia are drawn, conditional on various values for investors’ ex ante equity premium and calibrated to fundamentals of the US economy. Even though ex post estimates provided by recent papers suggest the US equity premium may be falling in recent years, these estimates are imprecise and do not rule out puzzlingly high ex ante equity premia. We have therefore sought to determine whether large ex post equity premium estimates are consistent with a low ex ante equity premium. That is, if investors demand (ex ante) a low equity premium, could we still observe ex post equity premia as high as those which have been deemed puzzling? On the basis of our fundamentals-based analysis, we conclude that the range of ex post equity premium values that have been estimated from historical data are indeed compatible with an ex ante equity premium as low as 2%. Furthermore, results from our simulation-based approach suggest that only ex ante premia above 2% and below 6% are consistent with dividend yields of the US economy; to the best of our knowledge, these are the first fundamentals-based bounds for ex ante equity premia provided in the literature.

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Table 1
Statistics on Ex Post Equity Premium Estimates for the Simulated Market Economies Based on Various Settings of the Ex Ante Equity Premium

Data → / Ex Ante Equity Premium↓	Mean of Ex Post Equity Premia	Percentiles of Ex Post Equity Premia Estimates				
		1%	5%	50%	95%	99%
6%	6.0	-0.1	2.4	6.3	8.6	9.2
3%	3.0	-5.2	-1.5	3.3	6.1	7.1
2.5%	2.5	-5.2	-1.8	2.9	5.6	6.8
2%	2.0	-7.0	-3.0	2.5	5.3	6.2

This table presents means and percentiles of ex post equity premia (reported in percentage form) arising from 2,000 simulated economies. The results reported in each row correspond to simulations in which the ex ante equity premium was set to a value of 6%, 3%, 2.5%, or 2%.

Table 2
Statistics on Discounted Dividend Growth Rates for the S&P 500 Index and Simulated Market Index

Data → Ex Ante Equity Premium↓	Growth Rates	S&P 500	Mean of Simulated Dividend	Percentiles of Simulated Dividend Growth Rates 1%	5%	50%	95%	99%
6%	Mean	0.942	0.943	0.867	0.895	0.947	0.974	0.982
	σ	0.037	0.040	0.026	0.029	0.038	0.059	0.076
	Skewness	0.023	-0.045	-1.055	-0.732	-0.020	0.553	0.792
	Kurtosis	2.631	2.715	1.800	1.985	2.606	3.802	4.572
3%	Mean	0.942	0.969	0.883	0.916	0.974	1.001	1.011
	σ	0.037	0.042	0.027	0.030	0.040	0.064	0.085
	Skewness	0.023	-0.062	-1.123	-0.722	-0.048	0.538	0.846
	Kurtosis	2.631	2.701	1.801	1.960	2.596	3.823	4.625
2.5%	Mean	0.942	0.974	0.892	0.925	0.978	1.007	1.016
	σ	0.037	0.042	0.028	0.030	0.040	0.061	0.078
	Skewness	0.023	-0.059	-1.056	-0.738	-0.047	0.556	0.836
	Kurtosis	2.631	2.691	1.772	1.985	2.581	3.727	4.603
2%	Mean	0.942	0.978	0.889	0.931	0.983	1.011	1.019
	σ	0.037	0.042	0.027	0.030	0.040	0.062	0.083
	Skewness	0.023	-0.049	-1.122	-0.689	-0.036	0.574	0.829
	Kurtosis	2.631	2.719	1.803	1.995	2.578	3.882	4.858

This table presents means and percentiles of the first four moments of discounted dividend growth rates (reported in decimal form) arising from 2,000 simulated economies. The results reported in each set of rows correspond to simulations in which the ex ante equity premium was set to a value of 6%, 3%, 2.5%, or 2%.

Table 3
Statistics on Dividend Yields for
the S&P 500 Index and Simulated Market Index

Data → Ex Ante Equity Premium↓		S&P 500	Mean of Simulated Dividend Yields	Percentiles of Simulated Dividend Yields 1%	5%	50%	95%	99%
6%	Mean	0.038	0.058	0.039	0.043	0.055	0.082	0.098
	σ	0.011	0.012	0.003	0.004	0.011	0.028	0.041
	Skewness	0.843	0.745	-0.316	-0.053	0.694	1.725	2.183
	Kurtosis	3.144	3.216	1.632	1.811	2.790	5.959	8.768
3%	Mean	0.038	0.026	0.015	0.017	0.024	0.041	0.053
	σ	0.011	0.008	0.002	0.002	0.006	0.019	0.031
	Skewness	0.843	0.823	-0.235	0.027	0.783	1.756	2.355
	Kurtosis	3.144	3.368	1.624	1.870	2.936	6.190	9.390
2.5%	Mean	0.038	0.020	0.012	0.013	0.019	0.032	0.042
	σ	0.011	0.006	0.001	0.002	0.005	0.015	0.023
	Skewness	0.843	0.844	-0.191	0.030	0.766	1.842	2.513
	Kurtosis	3.144	3.456	1.618	1.843	2.942	6.795	10.228
2%	Mean	0.038	0.014	0.008	0.009	0.013	0.023	0.033
	σ	0.011	0.005	0.001	0.001	0.004	0.011	0.019
	Skewness	0.843	0.873	-0.192	0.099	0.803	1.907	2.483
	Kurtosis	3.144	3.503	1.629	1.900	2.988	6.993	9.934

This table presents means and percentiles of the first four moments of dividend yields (reported in decimal form) arising from 2,000 simulated economies. The results reported in each set of rows correspond to simulations in which the ex ante equity premium was set to a value of 6%, 3%, 2.5%, or 2%.

Figure 1: Probability Distribution Function of Simulated Economies' Ex Post Equity Premia

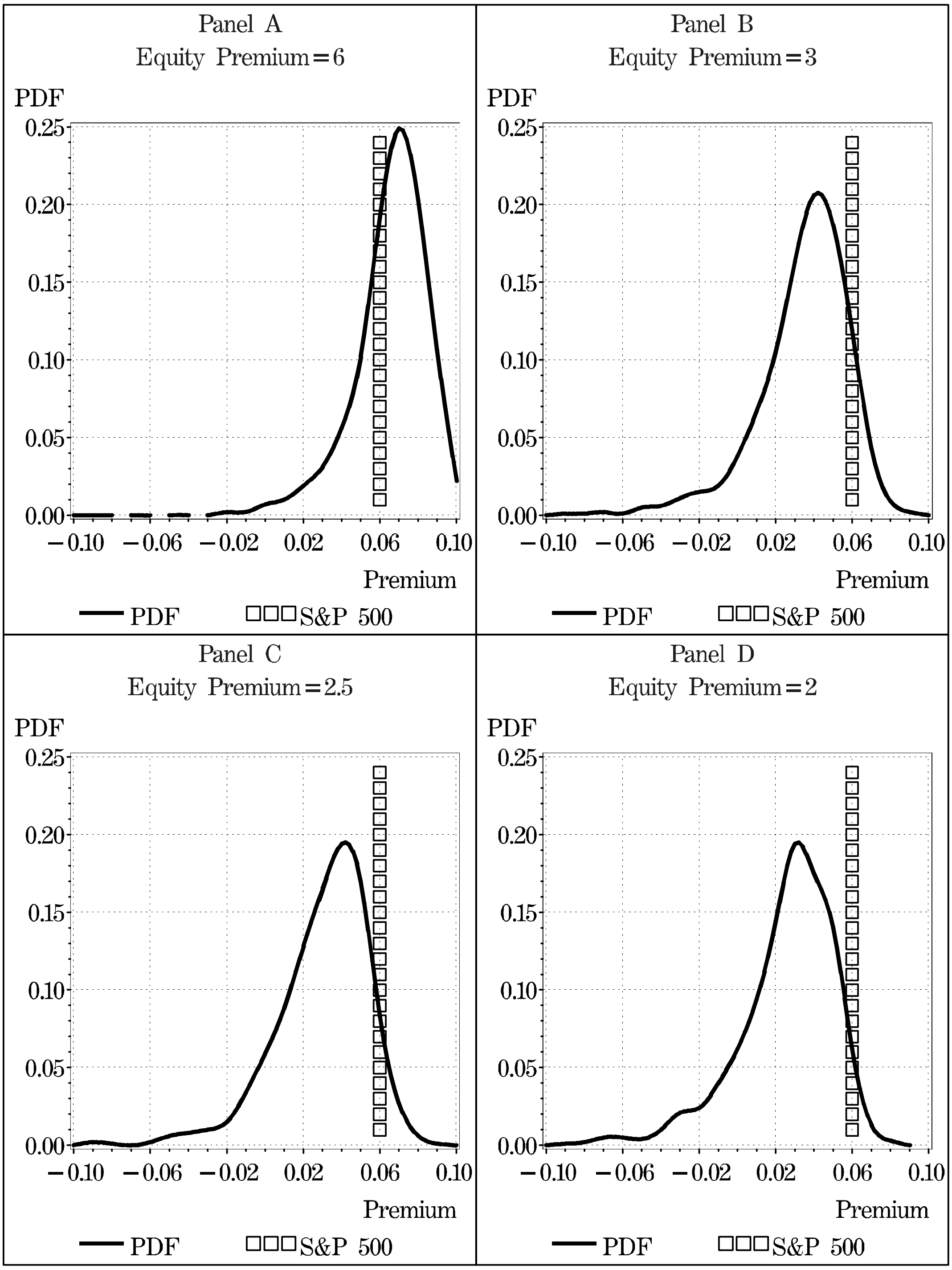

Figure 1: In each plot, the thin line plots the probability distribution function (PDF) of ex post equity premia obtained across 2,000 simulated economies based on an ex ante equity premium of 6% (Panel A), 3% (Panel B), 2.5% (Panel C), or 2% (Panel D). The vertical column of boxes indicates an ex post equity premium of 6% estimated using S&P 500 data. The area to the right of the column is representative of the likelihood of observing an ex post equity premium of 6% if the true ex ante equity premium is 6%, 3%, 2.5% or 2%.

Figure 2: Probability Distribution Function of Simulated Economies' Discounted Dividend Growth Rates

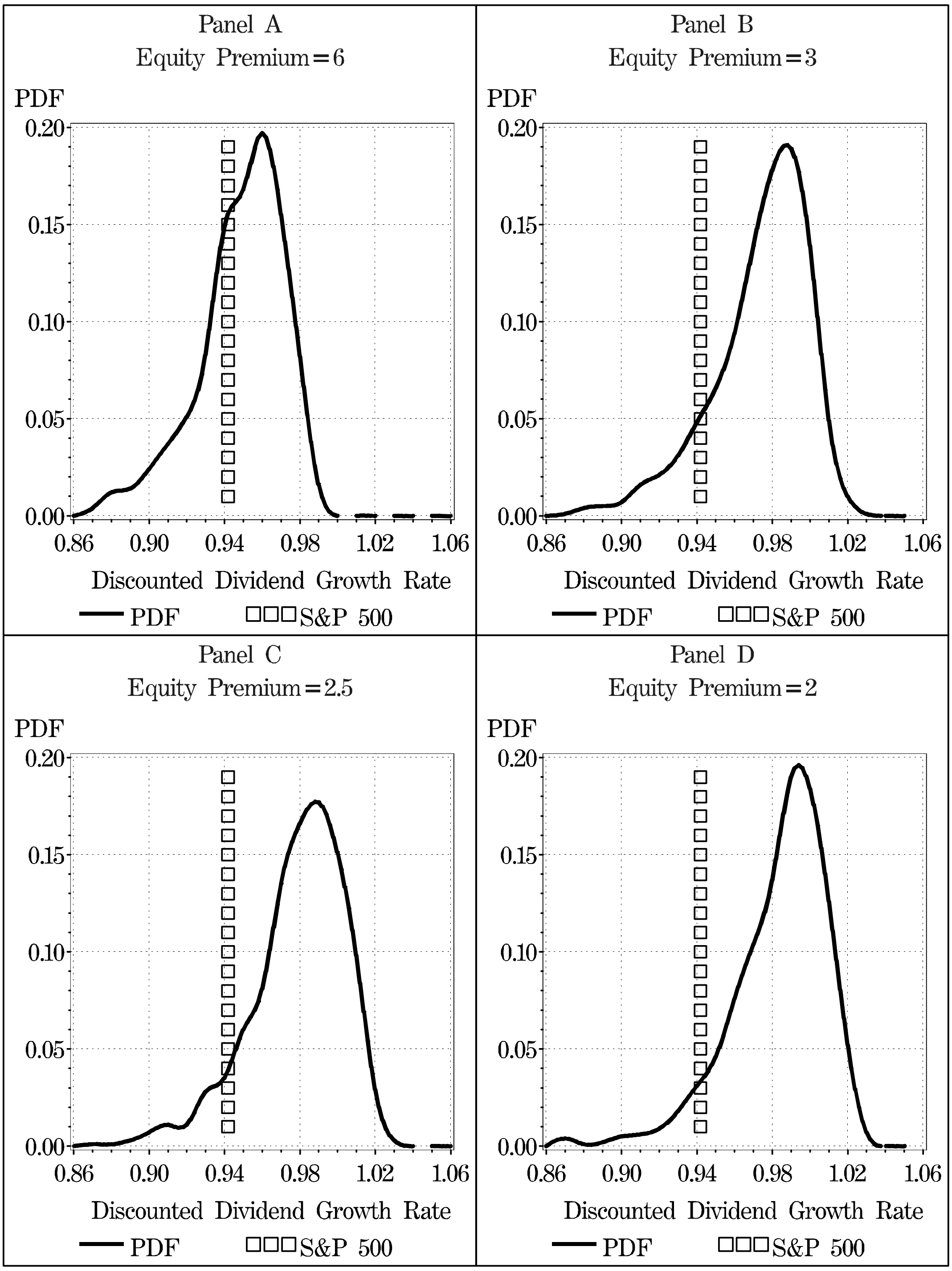

Figure 2: In each panel, the thin line plots the probability distribution function (PDF) of discounted dividend growth rate averages obtained across 2,000 simulated economies based on an ex ante equity premium of 6% (Panel A), 3% (Panel B), 2.5% (Panel C), or 2% (Panel D). The vertical column of boxes indicates an average discounted dividend growth rate of 0.94 estimated using S&P 500 data and an equity premium of 6%. The area to the right of the column is representative of the likelihood of observing of a discounted dividend growth rate average of 0.94 (the S&P 500 average) when the true ex ante equity premium is 6%, 3%, 2.5% or 2%.

Figure 3: Probability Distribution Function of Simulated Economies' Dividend Yields

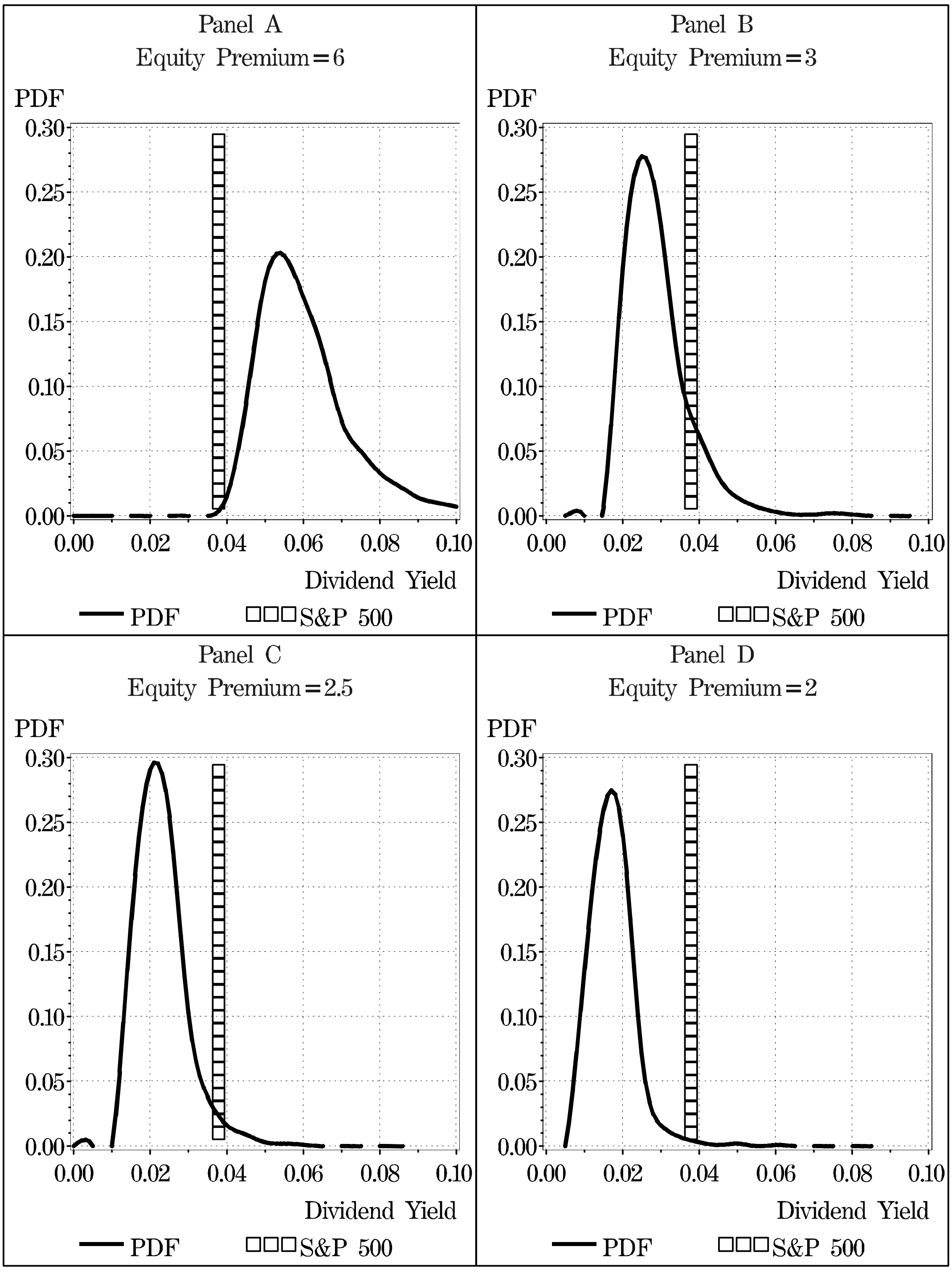

Figure 3: In each panel, the thin line plots the probability distribution function (PDF) of dividend yield averages obtained across 2,000 simulated economies based on an ex ante equity premium of 6% (Panel A), 3% (Panel B), 2.5% (Panel C), or 2% (Panel D). The vertical column of boxes indicates a dividend yield of 3.8% estimated using S&P 500 data. The area to the right of the column is representative of the likelihood of observing of a dividend yield of 3.8% (the S&P 500 average) when the true ex ante equity premium is 6%, 3%, 2.5% or 2%.