Modeling Corner Solutions with Panel Data : Application to Industrial Energy Demand in France.

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Abstract

This paper is providing an initial empirical application of Lee and Pitt's approach to the problem of corner solutions with panel data. This approach deals with corner solutions in a manner consistent with behavioral theory. Furthermore it allows the use of ‡exible form cost functions and general error structure. In this model energy demand, at industrial plant level, is the result of a discrete choice of type of energy to consume and a continuous choice to de…ne the demand level. The econometric model is essentially an endogenous switching regime model which require the evaluation of multivariate probability integrals. We estimate the random e¤ect model by maximum likelihood using a panel of industrial French plants. We verify that estimations predict globally well the model and we simulate the e¤ects of prices variations and a CO_2 tax on energy demand.

JEL Classi…cation: C33, C34, D21, Q41.

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1 Introduction

Econometric energy models are used to evaluate past policy experiences, assess the impact of future policies and forecast energy demand. In addition these models provide information on own- and cross-price elasticities which could shed some light on the e¤ectiveness and impact of di¤erent environmental regulation policies. The consumption of di¤erent types of energy is associated with di¤erent levels of emitted pollution (SO2, CO2,...). Hence, if di¤erent forms of energy are substitute, it may be possible to reduce pollution by taxing the most polluting energies.

Most of research on industrial energy demand makes use of aggregated data. Theses studies usually examine either substitution possibilities between energy forms or substitution between energy (as an aggregate) and other inputs such as labor and capital. Theses studies on aggregate data leads to two particular di¢ culties. First, the aggregation prevents the use of …rm or plant level data directly in models that are designed to be applied at the …rm level. Second, prices used are average prices while the economic theory of cost minimization recognize that the appropriate price to be used as an explanatory variable is the marginal price for energy forms.

Use of disaggregated data allows to remove these two problems but they also raise other modeling issues: The existence of zero expenditure. At least three mechanisms could explain zero expenditure. A person may make no purchase of a particular good simply because the observation period is too short, this is the case in the one or two weeks period of the typical family expenditure enquiry. Alternatively, observed zeros may be purely involuntary, labor supply may be zero because an individual cannot …nd a job, for instance. A third interpretation, which we examine in this paper is that the observed zero is the outcome of a completely free choice. In current prices, the …rm will never purchase the input and is therefore at a corner solution to its utility maximization problem.

When a signi…cant proportion of observations in which expenditure on one or more goods is zero, the econometric model should allow for zero expenditure to occur with positive probability. Usually econometric models assume that expenditures (or cost shares) follow a joint normal distribution and this does not allow for a positive probability of zero expenditure. Standard estimation methods for these models[1] do not take into account zero expenditure and consequently yield inconsistent estimates of parameters. If observations containing zero expenditure are excluded for the purpose of the estimation, this will reduce signi…cantly the sample size and estimators would be biased and inconsistent.

Wales and Woodland (1983) and Lee and Pitt (1986,1987) derived models which o¤er an economic

[1] Such as SUR (Seemingly unrelated Regressions) or maximum likelihood estimator

interpretation of zero expenditure as well as a direct and appropriate method to specify the econometric model.

Wales and Woodland (1983) propose an approach based on Kuhn-Tucker conditions associated to the maximization of a utility function subject to the budget constraint and the non-negativity constraints on goods demands. Zero expenditure are obtained when non-negativity constraints are binding, leading to a corner solution of the conventional utility maximization problem. They apply their method to estimate meat consumption in Australia.

The approach proposed by Lee and Pitt(1986) is based on virtual prices. Their method consists of deriving consumer demand systems from indirect cost or utility functions including popular ‡exible functional forms such as the translog. They de…ne notional demand functions which are de…ned over all of the real line, e¤ective demands are rationed to be non-negative. Notional and observed demands coincide for virtual prices.

The Kuhn-Tucker approach (Wales and Woodland (1983)) and the virtual prices approach (Lee and Pitt(1986)) lead to equivalent regime conditions. The Lee and Pitt's approach has the advantage of allowing the use of ‡exible-form indirect cost function such as the translog.

Lee and Pitt (1987) extend the works of Wales and Woodland (1983) and Lee and Pitt (1986) on consumer demands with binding non-negativity constraints to the problems of estimating the production structure of …rms. They apply the virtual price approach to estimate interfuel substitution between electricity, fuel oil and other fuels from a cross-section of Indonesian …rms in the Weaving and Spinning sector and in the Metal product sector.

Bousquet and Ivaldi (1998) propose a combined and coherent treatment of both the zero expenditures (as in Lee and Pitt(1987)) and missing data (prices)[2]. In this case, price equations are added to the demand system.

BjÁrner & Jensen (2000) focus on substitution between three di¤erent energy inputs, estimations are carried out conditioning on the observed energy pattern. This means that the choice of the type of energy to consume is exogenously de…ned for …rms.

The objective of this paper is to estimate an energy demand system, using the approach of Lee & Pitt (1987) to treat the zero expenditure problem and taking into account the panel form of the data. Panel data control for individual heterogeneity by identifying and measuring e¤ects that are not detectable in pure cross-section or pure time-series data.

We use a panel data sample drawn from a yearly survey on energy consumption conducted by the

[2] This occur because most surveys generally report prices only for the subset of goods purchased.

Service des Statistiques Industrielles (SESSI) of the French Ministry of Industry. The sample contains 324 plants from the Pulp and Paper sector observed over 14 years (1983-1996). The energy survey includes information about expenditures as well as the consumption in physical units for di¤erent form of energy.

The paper is structured as follow. The next section deals with the theoretical basis for two econometric models of producer demand. These models take into account the possibility that expenditures on one or more inputs are zero. Section 3 details the economic model and section 4 presents the econometric model associated; Data used and estimation results are presented in section 5. In section 6, we summarize the main exploitation of our model for practical purposes and policy makers and we conclude in section7.

2 Econometric models of kink solutions

Consider the general case in which the …rst k inputs are not used by the …rm, regime equations are such that:

$$\begin{cases} x_i = 0 & i = 1, ..., k \\ x_i > 0 & i = k+1, ..., n \end{cases} \tag{1}$$

x_i is the …rm demand for input i.

We present in the next subsection the models derived by Wales and Woodland (1983) and Lee and Pitt(1986,1987) which o¤er an economic interpretation of zero expenditure as well as a direct and appropriate method to specify the econometric model.

2.1 Direct approach: Kuhn-Tucker conditions

Wales & Woodland (1983) have considered the problem of estimating consumer demand systems for samples which contain a signi…cant proportion of observations with zero consumption of one or more goods. Their econometric model is derived by maximizing a random direct utility function subject to budget constraints. The Kuhn-Tucker conditions determine the set of non consumed goods. We present here the application of the approach of Wales & Woodland to the estimation of the production structure of …rms.

Consider the …rm's cost minimization problem:

$$\left\{\begin{array}{l} \min_X PX \\ sc\ F(X;q) = 0 \\ \quad X \geq 0 \end{array}\right. \tag{2}$$

where $X = (x_1, \ldots, x_n)'$ is $n \times 1$ vector of inputs and q is the output. The production function F is an increasing function of q and a decreasing function of x_i $(i = 1, \ldots, n)$. Other standard regulatory conditions on F such as differentiability and strict quasi-concavity are assumed. $P = (p_1, \ldots, p_n)$ denote the vector of input prices.

The Lagrangian function is:

$$L = PX - \lambda(0 - F(X;q)) - \gamma X \tag{3}$$

where λ and γ are Lagrange multipliers.

The optimality of X^* is characterized by the following Kuhn-Tucker conditions:

$$\begin{array}{llll} \frac{\partial L}{\partial x_i} = 0 & , & p_i + \lambda \frac{\partial F(X^*;q)}{\partial x_i} - \gamma_i = 0 & (i = 1, 2, \ldots n) \\ \frac{\partial L}{\partial \lambda}\lambda = 0 & , & \lambda F(X^*;q) = 0 & \\ \frac{\partial L}{\partial \gamma_i}\gamma_i = 0 & , & x_i^* \gamma_i = 0 & (i = 1, 2, \ldots n) \\ F(X^*;q) = 0; \ \lambda \geq 0; \ x_i^* \geq 0; \ \gamma_i \geq 0 & & & (i = 1, 2, \ldots n) \end{array} \tag{4}$$

From the regime equation and Kuhn-Tucker conditions it follows that:

- For the consumed inputs $i = k+1, \ldots n$

$$\gamma_i = 0 \quad \text{and} \quad p_i = -\lambda \frac{\partial F(X^*;q)}{\partial x_i} \tag{5}$$

- For the non-consumed inputs $i = 1, 2, \ldots k$

$$\gamma_i \geq 0 \quad \text{and} \quad p_i \geq -\lambda \frac{\partial F(X^*;q)}{\partial x_i} \tag{6}$$

For $i = 1, 2, \ldots k$ if $p_i = -\lambda \frac{\partial F(X^*;q)}{\partial x_i}$ than $\gamma_i = 0$.

as γ_i is the Lagrange multiplier associated to the positivity constraint of the input i.

Define the virtual price for this input i as:

$$\tau_i = -\lambda \frac{\partial F(X^*;q)}{\partial x_i}; \quad i = 1, 2, \ldots k \tag{7}$$

Therefore this regime is characterized by:

$$p_i \geq \tau_i; \quad i = 1, 2, \ldots k \tag{8}$$

Hence, the firm does not use inputs for which the market price is too high (greater than the virtual price)

2.2 Indirect approach: Virtual prices

The approach proposed by Wales & Woodland (1983) derive regime conditions from the optimization problem of the firm. This method rules out the use of more flexible demand specifications for which no explicit specification of the direct cost function needs to be assumed. Lee & Pitt (1986) propose an approach which has the advantage of allowing the use of flexible-form indirect cost function such as the translog. This method consists of deriving the firm demand systems from an indirect cost function. They show how virtual price relationships can take the place of Kuhn-Tucker conditions.

Let $C(P;q)$ the indirect cost function, applying Shephard's lemma, the notional demand equations are:

$$x_i^* = \frac{\partial C(P;q)}{\partial p_i}; \quad i = 1;2;...n \tag{9}$$

These demand equation are deemed notional because they may take negative values since no non-negativity constraint is imposed. The notional demands x_i^* are latent variables which correspond to a vector of nonnegative observed demands x_i as follows. There exist vectors of positive virtual prices $\xi = (\xi_{1;}...\xi_n)$ solution of the demand system:

$$x_i^*(\xi;q) = x_i \quad i = 1;2;...n \tag{10}$$

If demands for the first k inputs are zero, the market prices p are also virtual prices as they exactly support the observed positive demands of inputs $k+1$ to n:

$$\xi = (\xi_{1;}...\xi_k; p_{k+1}; ...; p_n) \tag{11}$$

Comparison of virtual and market prices can help to select among demand regimes, defined as the set of positively consumed goods at the optimum. The regime in which the first k inputs are not consumed is characterized by conditions:

$$p_i > \xi_i; \quad i = 1;2;...k \tag{12}$$

In the switching regime condition (12) virtual prices can be thought as reservation or shadow prices: goods are not consumed unless their reservation price exceeds their market price.

Hence the Kuhn-Tucker approach (Wales and Woodland (1983)) and the virtual prices approach (Lee and Pitt(1986)) lead to equivalent conditions. In this paper we use the Lee and Pitt's approach which has the advantage to allow the use of ‡exible-form indirect cost function such as the translog.

3 Economic model

We consider the production function with four inputs: n_1 types of energy E, n_2 types of labor L, n_3 types of capital K and n_4 types of materials M:

$$Y = f((EN_1 ... EN_{n_1}); (L_1 ... L_{n_2}); (K_1 ... K_{n_3}); (M_1 ... M_{n_4}))$$

This function is weakly separable in the $E; L; K; M$ aggregates if it can be written:

$$Y = f(EN(EN_1 ... EN_{n_1}); L(L_1 ... L_{n_2}); K(K_1 ... K_{n_3}); M(M_1 ... M_{n_4}))$$

Where $EN(EN_1 ... EN_{n_1}); L(L_1 ... L_{n_2}); K(K_1 ... K_{n_3}); M(M_1 ... M_{n_4})$ are aggregate functions and $EN; L; K; M$ are aggregate inputs of energy, labor, capital and materials respectively.

Weak separability means that the marginal rate of substitution between EN_i and EN_j is independent of the quantities of L_l; K_m and M_n demanded $i; j = 1 ... n_1; l = 1; ... n_2; m = 1; ... n_3; n = 1; ... n_4$: For example, cost minimizing or pro…t maximizing choice of energy mix is independent of the capital mix and the level of capital aggregate.

The weakly separable assumption has two signi…cant implications: …rstly, only under this assumption do aggregates exist. Secondly, it implies that …rms' optimal behavior can be modeled as a sequential two-stage procedure. in the …rst stage the optimal amount of aggregate energy demand is determined as a function of its price index and real income. The second stage uses relative prices of fuels to determine the market share of each fuel. In this paper we consider this second stage to determine the demand for energy input components.

Consider the case where a …rm can choose among 3 types of energy: Electricity E, natural gas NG and oils products OP. The production function can be written:

$$Y = f(E; NG; OP; L; K; M)$$

Imposing homothetic weak separability in energy, we can write the production function as

$$Y = f(EN(E; NG; OP); L; K; M) \tag{13}$$

Where EN, the total energy measure, is an appropriately chosen homothetic aggregate function.

Using the theory of duality of cost and production, the cost function corresponding to (13) is also weakly separable and can be written:

$$C = g(P_{EN}(P_E; P_{NG}; P_{OP}); P_L; P_K; P_M; Y)$$

P_{EN} is also an aggregate function. it represents also the price per unit or the cost by unit of energy to the optimizing agent. This cost can be represented by an arbitrary unit cost function. We choose the translog cost form which has the advantage of simplicity and linearity in the logarithm of prices. Another reason for choosing the translog is that it is fairly easy to impose global curvature constraints, this constraint is particularly important in our model as it will be discussed later.

The unit cost of energy is thus described by the function:

$$\ln P_{EN} = \alpha_0 + \sum_{m=1} \alpha_m \ln p_m + \frac{1}{2} \sum_{m=1} \sum_{k=1} \beta_{mk} \ln p_m \ln p_k$$

where index 1 denote natural gas, 2 denote oil product and 3 denote electricity.

The system of energy shares obtained from the translog cost function is defined as:

$$S_m = \frac{p_m X_m}{P_{EN}} = \frac{\partial \ln P_{EN}}{\partial \ln p_m} = \alpha_m + \sum_{k=1} \beta_{mk} \ln p_k \quad m = 1; 2; 3$$

The properties of neo-classical production theory require the following parameters restrictions:

$$\beta_{mk} = \beta_{km}; \quad \forall\, m; k = 1; 2; 3$$

$$\sum_{m=1} \alpha_m = 1 \quad \text{and} \quad \sum_{k=1} \beta_{mk} = 0; \quad \forall\, m = 1; 2; 3$$

4 Econometric model

Our model extends the approach of Lee & Pitt (1986) to the case of panel data. This type of data helps to control for individual heterogeneity. Time series and cross-section studies not controlling for this heterogeneity run the risk of obtaining biased results.

We add disturbances w_{mit} to the system of energy shares described above:

$$S_{mit} = \alpha_m + \sum_{k=1} \beta_{mk} \ln p_{kit} + w_{mit}; \quad m = 1; 2; 3 \tag{14}$$

m denote index for type of energy, i is the plant index and t is the date index.

As is standard in panel data analysis, we decompose the error term as:

$$w_{mit} = \mu_{mi} + \varepsilon_{mit} \quad m = 1, 2, 3 \quad i = 1, ..., N \quad \text{and} \quad t = 1, ..., T \tag{15}$$

where μ_{mi} is the individual firm effect and ε_{mit} is an iid error term for equation m.

Using notations in Schmidt (1990), one can write the system of $M = 3$ equations where the m-th equation is of the form:

$$S_m = \alpha_m + \beta'_m \ln p + w_m; \qquad m = 1, 2, 3 \tag{16}$$

where $S_m = (S_{m11},...,S_{m1T},...,S_{mN1},...,S_{mNT})'$, $\alpha'_m = (\alpha_{m1}, \alpha_{m2}, \alpha_{m3})$, $\beta'_m = (\beta_{m1}, \beta_{m2}, \beta_{m3})$, $\ln p = (\ln p_1, \ln p_2, \ln p_3)'$, $\ln p_m = (\ln p_{m11}, ..., \ln p_{m1N}, \ln p_{mN1}, ... \ln p_{mNT})'$,
$\mu'_m = (\mu_{m1}, ..., \mu_{mN}) \otimes e'_T$ and $\varepsilon_m = (\varepsilon_{m11}, ...\varepsilon_{m1T}, ...\varepsilon_{mN1}, ...\varepsilon_{mNT})$

we suppose that:

$$\mu_{mi} \sim N(0, \sigma^2_{\mu_m}); \quad \varepsilon_{mit} \sim N(0, \sigma^2_{\varepsilon}); \quad E(\mu_{mi}\varepsilon_{mit}) = 0; \quad \forall\, m = 1, 2, 3 \tag{17}$$

with:

$$E(\mu_{mi}\mu_{kj}) = \begin{cases} \sigma^2_{\mu_{mk}} & \text{if} \quad i = j \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, m, k = 1, 2, 3$$

$$E(\varepsilon_{mit}\varepsilon_{kjs}) = \begin{cases} \sigma^2_{\varepsilon_{mk}} & \text{if} \quad i = j \quad \text{and} \quad t = s \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, m, k = 1, 2, 3$$

4.1 The likelihood function

In this study three fuels are identified: electricity E, oil products OP and natural gas NG. To derive the likelihood function for this model, we need to distinguish different regimes. For three-energy models there are seven regimes ($2^3 - 1$) in total. Assuming that it is not feasible to produce without using any kind of energy. In our sample electricity is always employed, hence the choice set is reduced to $2^{3-1} = 4$

cases.

	Energy[3]		
	NG	OP	E
Regime 1	X	X	X
Regime 2	O	X	X
Regime 3	X	O	X
Regime 4	O	O	X

With some constraints on the parameters, we denote $S_m = \alpha_m + \beta'_m \ln p + \mu_m + \varepsilon_m$ the notional share and S^*_m the observed ones for $m = 1; 2; 3$ the regime conditions can be summarized:

Regime	Regime definition	Observed share equations	Regime conditions
XXX	$S^*_1 > 0;\ S^*_2 > 0;$ $1 - S^*_1 - S^*_2 > 0$	$S^*_1 = S_1$ $S^*_2 = S_2$	$0 < S_1$ $0 < S_2$ $0 < S_1 + S_2 < 1$
OXX	$S^*_1 = 0;\ 0 < S^*_2 < 1$	$S^*_1 = 0$ $S^*_2 = S_2 - \frac{\beta_{12}}{\beta_{11}} S_1$	$S_1 \leq 0$ $0 < S_2 - \frac{\beta_{12}}{\beta_{11}} S_1 < 1$
XOX	$0 < S^*_1 < 1;\ S^*_2 = 0$	$S^*_1 = S_1 - \frac{\beta_{12}}{\beta_{22}} S_2$ $S^*_2 = 0$	$0 < S_1 - \frac{\beta_{12}}{\beta_{22}} S_2 < 1$ $S_2 \leq 0$
OOX	$S^*_1 = 0;\ S^*_2 = 0$	$S^*_1 = 0$ $S^*_2 = 0$	$S_1 - \frac{\beta_{12}}{\beta_{22}} S_2 \leq 0$ $S_2 - \frac{\beta_{12}}{\beta_{11}} S_1 \leq 0$

Table 1: Energy regimes presentation

The formal expressions for the likelihood function associated with each regime of demand are detailed in appendix XXX.

4.2 Censored regression models with panel data

For panel data, the presence of individual effects complicates matters significantly. In this case μ_{mi} are unknown parameters and for fixed T the number of parameters μ_{mi} increases with N. This means that μ_{mi} cannot be consistently estimated for a fixed T. However if $T \to \infty$, then the maximum likelihood

[3] X: used energy, O: non-used energy

estimators (MLE) of μ_{mi} and the other parameters are consistent. For the linear regression model when T is fixed, only the other parameters was estimated consistently by first getting rid of μ_{mi} using the Within transformation. This is no longer the case for a non linear model, as demonstrated by Chamberlain (1980).

The two common statistical model specification which are used to analyze pooled cross-section and time-series data are the fixed effects model and the random effects model[4]. Heckman and Macurdy (1980) consider a fixed effects Tobit model to estimate a life-cycle model of female labor supply. They argue that the individual effects have a specific meaning in a life-cycle model and therefore cannot be assumed independent of the explicative variables. Hence, a fixed effects rather than random effects specification is estimated using a two-step iterative method.

There are very few applications of the random effects tobit model with panel data (Maddala 1987b). When the individual specific effects μ_{mi} are random, the error terms are correlated and this results in multiple integrals.

When μ_{1i} and μ_{2i} are independent of exogenous variables and are random sampling from a bivariate distribution G, indexed by a finite number of parameters $\sigma_{\mu_1}; \sigma_{\mu_2}; \rho_{\mu_1\mu_2}$, the marginal likelihood for plant i is:

$$l_i = \int_{\mu_1}\int_{\mu_2} \prod_{t=1}^{T} \prod_{r \in R} \left[(L_r)^{I_{itr}}\right] dG(\mu_1; \mu_2 | \sigma_{\mu_1}; \sigma_{\mu_2}; \rho_{\mu_1\mu_2}) \quad (18)$$

L_r is the likelihood function of regime $r \in R = \{XXX; OXX; XOX; OOX\}$; I_{itr} is such that,

$$I_{itr} = \begin{cases} 1; & \text{if the observation } (i;t) \text{ belongs to regime } r \\ 0; & \text{otherwise.} \end{cases}$$

The sample log-likelihood function is:

$$L = \sum_{i=1}^{N} \log(l_i) \quad (19)$$

4.3 Concavity constraints

Empirically estimated flexible functional forms frequently fail to satisfy the appropriate theoretical curvature conditions. Diewert and Wales (1987) show that one necessary and sufficient condition to global curvature to be satisfied in the case of a translog cost function is that the matrix of parameters $B = [\beta_{ij}]_{i;j=1;2;3}$ should be negative semidefinite.

[4] See Hsiao 1986 and Baltagi 1996

So, in order to impose the concavity restrictions on the translog functional form, we use the following tech,nique due to Wiley, Schmidt and Bramble (1973): we reparametrize the matrix B by replacing it by minus the product of a lower triangular matrix, of the same dimension of B;.times its transpose;

with A a matrix such that: $A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{11}+a_{12} & a_{22} & 0 \end{bmatrix}$

Imposing negative semidefiniteness on the matrix B is equivalent to rewrite as:

$$B = -AA'$$

so:

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} = \begin{bmatrix} -a_{11}^2 & -a_{12}a_{11} & -(a_{11}+a_{12})a_{11} \\ -a_{12}a_{11} & -a_{12}^2 - a_{22}^2 & -a_{12}(a_{11}+a_{12}) - a_{22}^2 \\ -(a_{11}+a_{12})a_{11} & -a_{12}(a_{11}+a_{12}) - a_{22}^2 & -(a_{11}+a_{12})^2 - a_{22}^2 \end{bmatrix}$$

The new parameters satisfy also homogeneity and symmetry constraints imposed. The likelihood function should be rewritten with the new parameters $a_{11}; a_{12}; a_{22}$.

5 Application to industrial energy demand in France

5.1 Data description

The model is applied to study the demand of energy of French plants of Paper and Pulp industry. We use a panel data sample drawn from a yearly survey on energy consumption conducted by the Service des Statistiques Industrielles (SESSI) of the French Ministry of Industry. The sample contains 324 plants from Pulp and Paper sector observed over 14 years ($1983 - 1996$). The energy survey includes information about expenditures as well as the consumption in physical units for different form of energy.

5.1.1 Regimes and plants characteristics

A description of plants characteristics according to there energy regimes is presented in table (2). All plants use electricity, so only four energy regimes are possible for the three types of energy modelled (electricity, natural gas and oil products). We can note that:

- 19% of plants are using the three types of energies, these plants are generally very large (the average number of employees is 211 and the subscribed electricity power is 5726 KW). Those large plants are also energy intensive as they consume about 63% of the overall energy in the sample.

- 65% of plants uses electricity and oil products, this is the most important regime considering the number of plants. These plants are generally not very large (the average number of employees is 115).

- In the same way, 9% of plants use electricity and natural gas. Theses plants are larger than those using electricity and oil products since they use larger number of employees and greater subscribed electricity power.

- Finally a few percent of the plants use only electricity. These are relatively small plants.

5.1.2 Regimes changes

In general plants do not frequently change regime of energy. Figure (??) shows the distribution of the number of transitions between energy regimes over the 14 years of observation. 48% of plants do not change energy regime at all during the observation period. Plants which change energy regime once (24%) switch for a large majority (54%) from the regime $E-OP$ to $E-NG-OP$. 15% and 10% of these plants switch respectively from $E-OP$ to E and $E-NG-OP$ to $E-NG$.

5.1.3 Cost shares evolution

Expenditure shares in electricity is the highest among of the three shares. This share increased over the observation period from 64% to 76%. Oil products and natural gas represent on average respectively 20% and 8% of the total energy cost.

5.1.4 Prices

As explained previously, for each observation, factor prices are only observed conditionally on the realization of strictly positive demand. The most common procedure consists of replacing missing prices by average prices as in Lee and Pitt (1987). But since the observed price distribution is truncated and so observed average prices are lower than the true average, this is not appropriate in our case since: to a zero expenditure we cannot associate a price at which other producers have positive optimal demand.

In our model we substitute the empirical maxima of price distributions for missing prices for each type of energy. This method was originally applied by Flinn et Heckman (1982) to replace non observed wages for non-participants in labor market.

5.2 Estimation of the model

The likelihood function is maximized under the model coherency constraints. The model is non linear in parameters, we use an algorithm which finds values for the parameters using an iterative method. We use Broyen, Fletcher, Goldfab and Shanno (BFGS) method which is called quasi-Newton method. Initial values of parameters are obtained by ISUR (Iterative Seemingly Unrelated Regression) proposed by Zellner.

5.2.1 Estimation results

The loglikelihood function of the model is maximized using MAXLIK routine in GAUSS, to obtain MLE of the parameters $a_{11}; a_{12}; a_{22}; a_1; a_2$, σ_{ε_1}, σ_{ε_2}; $\rho_{\varepsilon_1\varepsilon_2}$; σ_{μ_1}, σ_{μ_2} and $\rho_{\mu_1\mu_2}$. The GAUSS quadrature routine INTQUAD2 is used to evaluate integrals appearing in equation (18). Estimation results are contained in table (3). All parameters are significantly different from zero, except σ_{μ_1} and σ_{μ_2}.

Table (4) displays estimates of the translog cost parameters, obtained under global curvature condition and homogeneity of the cost function. Their standard deviations have been derived by applying results of Serfling (1980) on the moments of functions of normal random variables.

5.2.2 Breusch-Pagan test: Test for random individual effects

In order to test the relevance of estimating the random effect model, we compare our model to a model without random individual effect using the Breusch-Pagan (1980) test. This is a Lagrange-multiplier (LM) test for random effects model. Test hypothesis are:

$$\begin{cases} H_0: & \sigma_\mu = 0 \\ H_1: & \sigma_\mu \neq 0 \end{cases}$$

the test statistic is:

$$LM = \frac{NT}{2(T-1)}\left[1 - \frac{e'(I_N \otimes I_T)e}{e'e}\right]^2$$

where e is maximum likelihood residual vector.

Under the null hypothesis, LM is distributed as chi-squared with one degree of freedom.

Based on the maximum likelihood model residuals, we obtain Lagrange Multiplier (LM) statistics for each energy equation (Electricity, Oil products and Natural gas). These statistics exceed the 5% critical value for chi-squared with one degree of freedom (0.001).

Energy equation	LM statistic
E	0.03035
OP	5.2012
NG	14.9313

The result of the test is to reject the null hypothesis in favor of random effect model. At this point we conclude that the classical regression model with one single constant term is inappropriate for these data.

6 Simulations

One traditional exploitation of econometric production models is to provide cross and own price elasticities in order to study substitution between different inputs. In our model elasticities could not be computed for two reasons:

- When the relative observed share for an energy i is null, we cannot compute elasticities ϵ_{ij} ($\forall j = 1; 2; 3$) since the share appears in the denominator,
- The marginal effect of prices variation on energy relative shares depends upon the regime of energy demand since the observed relative shares expressions differs from one regime to an other.

Our microeconometric approach allows us to distinguish between the qualitative and quantitative effects of price changes in the choice of energy mix. So, substitution effects between energies can be separated in a direct effect on quantities (the distribution of regimes being unchanged), and in an indirect through the probabilities of observing a particular regime.

6.1 Simulation of energy price variation

In order to study energy demand sensitivity to prices changes, we calculate relative cost shares predictions. Suppose we want to predict s periods ahead for the firm i, we first predict the energy regime for this firm i at period s [5] and then calculate cost shares predictions $\hat{S}_{mis}$ associated.

[5] This is done according to energy regime conditions detailed in table (1).

To simulate the e¤ects prices changes on energy demand, we derive from our predicted cost shares the level of energy demand for the three forms of energy considered in our model.

$$\hat{x}_m = \frac{\hat{S}_m \times P_{EN} \times EN}{p_m}; \quad m = E; OP; NG$$

where $\hat{S}_m$ is the predicted cost share for energy m, $\hat{x}_m$ is the predicted demand level for energy m; p_m is the price of energy m, P_{EN} is unit cost of energy and EN is the total quantity of energy used.

The simulation of price variation are obtained by shifting exogenously prices of the three forms of energy considered. Simulation results are presented in …gure (5). Note that the three forms of energy considered are substitute.

- Electricity demand is almost invariant to oil products prices variation.

- Oil products demand is very sensitive to natural gas prices variation

6.2 A carbon tax simulations

Ecological tax reform presents an opportunity to meet the targets for reducing emissions of greenhouse gases set out in the Kyoto Protocol. Carbon and energy taxes have been frequently advocated by economists and international organizations as a policy instrument for reducing carbon dioxide emissions. Increasing number of Western European countries have implemented taxes based on the carbon content of the energy products (Sweden, Norway, The Netherlands, Denmark,...). In France, the government had proposed to reform of the environmental taxation by extending the TGAP to the …eld of energy from 2001. This new energy/CO2 tax on industrial companies was cancelled by the constitutional consulting in December 2000.

Some economic studies[6] evaluate the level of the energy/CO2 tax to implement in France in order to respect it's engagement to Kyoto protocol. They conclude that: "a reasonable value of the ton of carbon within the framework of the plan of …ght against the climate change is between 500 F and 1000 F. But if France chooses to use other policies and instruments to achieve half of it's reduction e¤ort, the level of carbon taxation will be between 500F/tC and 600F/tC".

We simulate en energy/CO2 tax, and study its e¤ects on the three forms of energy demand and on the level of CO2 emissions. Results of these simulations are presented in …gures(6).

We note that demands of natural gas and oil products fail down signi…cantly with the rate of the CO2 tax. Electricity demand increases sharply. The CO2 tax a¤ects less oil products demand, this due

[6] GEMINI-E3 model developed by Alain Bernard and Marc Vielle.

to the fact that there are a lot of taxes on oil products demand in France.

7 Conclusion

This paper has provided an initial empirical application of Lee and Pitt's approach to the problem of corner solutions with panel data. This approach deals with corner solutions in a manner consistent with behavioral theory. It also allows the use of ‡exible form cost functions and general error structure. We present the theoretical outline of the model. We …rst de…ne an indirect translog cost function. Optimal input demands are de…ned by Shephard lemma. Virtual prices concept allow us to characterize zero expenditure. Null demand are considered as the result of endogenous rationing and are explained by price excess. Firms does not consume inputs for which prices on the market exceed the virtual prices. The econometric model is essentially an endogenous switching regime model which require the evaluation of multivariate probability integrals.

We apply Lee and Pitt's approach to estimate energy demand in the pulp and paper sector in France. We use panel data which help us to control for individual heterogeneity. Time series and cross-section studies not controlling for this heterogeneity run the risk of obtaining biased results.

We simulate the e¤ects of prices variations on energy demand. Results of simulations show that

- The three forms of energy considered are substitute
- Electricity demand is almost invariant to oil products price variations
- Oil products demand is very sensitive to natural gas price variations

We also do simulations of a Co2 tax which is a good policy instrument for reducing greenhouse gases emissions to meet the targets set out in the Kyoto Protocol. Simulations shows that:

- Demands of natural gas and oil products fail down signi…cantly with the rate of the CO2 tax. The CO2 tax a¤ects less oil products demand, this due to the fact that there are a lot of taxes on oil products demand in France.
- Electricity demand increases sharply.

This work may provide useful insights for the analysis of future environmental policies impacts (CO2 tax).

Table 2: Descriptive statistics according to energy regime

Variable	E	E-NG	E-OP	E-NG-OP
Observations	343	396	2946	851
Labor	70	167	115	211
Electricity Power (KW)	540	1777	1459	5726
Energy consumption (toe)	355	3069	2209	9563
Electricity consumption (toe)	355	1484	1406	5011
Natural Gas consumption (toe)	0	1585	0	3725
Oil Products consumption (toe)	0	0	803	827
Electricity expenditure (Thousands Euro)	106	269	263	808
Natural Gas expenditure (Thousands Euro)	0	225	0	568
Oil Products expenditure (Thousands Euro)	0	0	148	135
Electricity quantity shares (%)	100	64	72	57
Natural Gas quantity shares (%)	0	36	0	31
Oil Products quantity shares (%)	0	0	28	11
Electricity expenditure shares (%)	100	69	74	62
Natural Gas expenditure shares (%)	0	31	0	28
Oil Products expenditure shares (%)	0	0	26	11
Electricity average price (Euro/toe)	356	310	321	273
Natural Gas average price (Euro/toe)	355	240	360	226
Oil Products average price (Euro/toe)	455	455	303	307

Figure 1: Distribution of the number of energy regime transitions

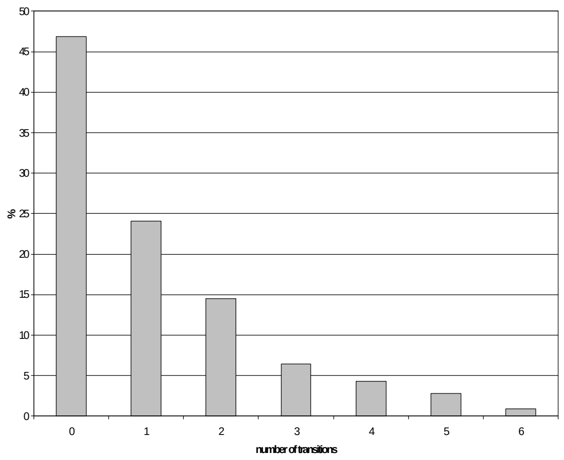


Figure 2: Energy cost shares evolution

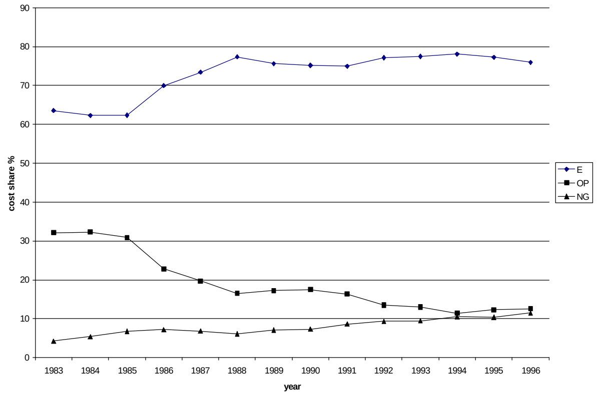


Figure 3: Evolution of the frequency of di¤erents energy regime

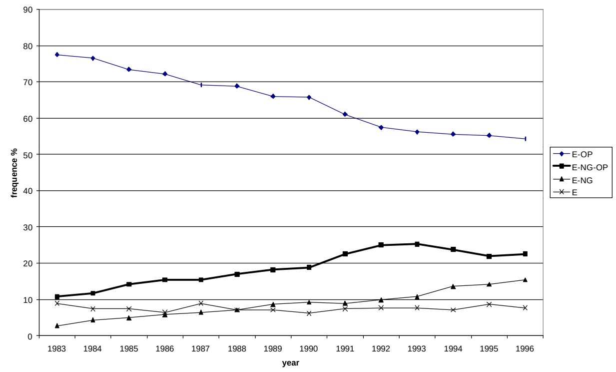


Figure 4: Evolution of energy consumption shares

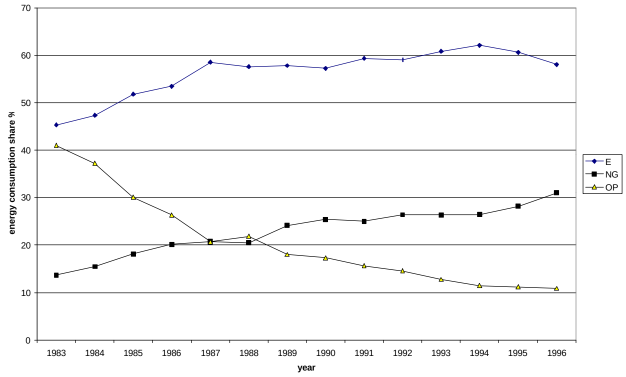


Table 3: Estimates of likelihood function parameters

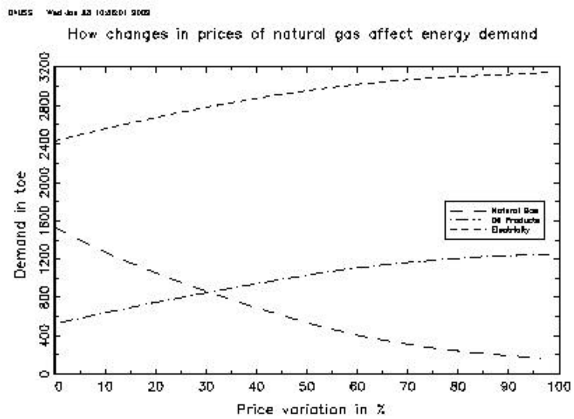
Parameters	Estimate	t-stat
a_{11}	-0.8925	-59.11
a_{12}	0.6472	44.94
a_{22}	0.2442	15.26
a_1	-0.1723	36.20
a_2	0.3924	38.97
ε_1	0.3548	-41.89
ε_2	0.2611	-15.25
$r_{\varepsilon_1\varepsilon_2}$	-0.7080	2.78
μ_1	0.0021	0.16
μ_2	0.0093	0.19
$r_{\mu_1\mu_2}$	-0.2435	-8.70

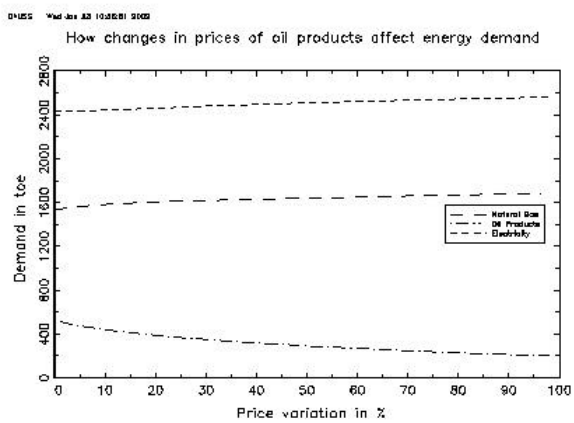
LogL=-1005.345

Table 4: Estimates of parameters of the translog cost function

	1		2		3	
	Natural Gas		Oil Products		Electricity	
Parameters	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
a.	-0.1723	-36.20	0.3924	38.97	0.7799	5.63
b_1.	-0.7965	-12.19	0.5776	13.08	0.2189	4.62
b_2.	0.5776	13.08	-0.4785	-8.11	-0.0991	-2.62
b_3.	0.2189	4.62	-0.0991	-2.62	-0.1198	-5.53
ε.	0.3548	36.04	0.2611	39.07	0.2507	84.46
μ.	0.0021	0.16	0.0093	0.19	0.0095	0.33
$\rho_{\varepsilon..}$	-0.708 (-41.993)			-0.039 (-1.078)		
		-0.677 (-274.54)				
$\rho_{\mu..}$	-0.2435 (-20.22)			-0.975 (-25.15)		
		-0.217 (-32.77)				

Figure 5: Simulations of energy prices variation





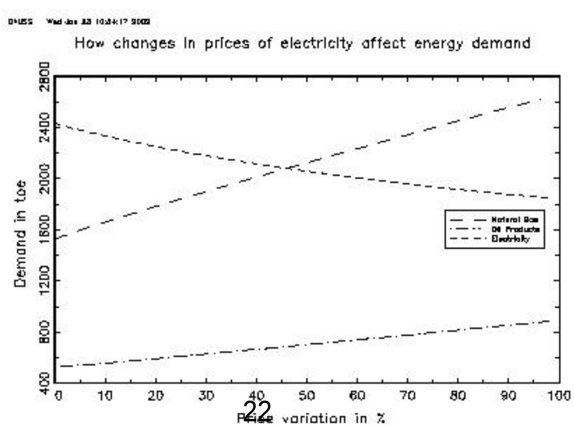
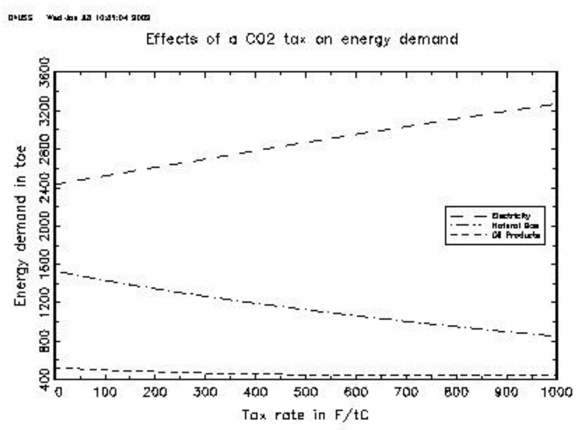
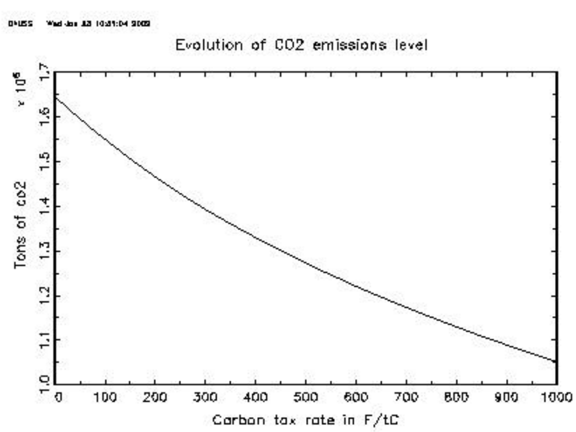


Figure 6: Simulation of a CO2 tax





A Annexes

A.1 Derivation of Estimating Equations and Likelihood function

Denoting $\varphi_m = \alpha_m + \beta'_m \ln p + \mu_m$; $S_m = \varphi_m + \varepsilon_m$ the notional cost share and S^*_m the observed ones for $m = 1, 2, 3$ we present here the derivation of estimating equations and likelihood function for each regime.

A.1.1 Regime 1: XXX

In this case, all type of energy are used, virtual prices are equal to market prices and notional cost shares coincide with observed expenditures. This regime is defined by the system:

$$\begin{cases} S^*_1(p_1, p_2, p_3) = S_1 > 0 \\ S^*_2(p_1, p_2, p_3) = S_2 > 0 \\ 0 < S^*_1(p_1, p_2, p_3) + S^*_2(p_1, p_2, p_3) < 1 \end{cases} \tag{20}$$

The contribution to the likelihood of an observation belonging to this regime is given by:

$$\begin{aligned} L_{XXX} &= f(S^*_1 - \varphi_1, S^*_2 - \varphi_2) \\ &= \frac{1}{\sigma_{\varepsilon_1}\sigma_{\varepsilon_2}} \Phi_2\left(\frac{S^*_1 - \varphi_1}{\sigma_{\varepsilon_1}}, \frac{S^*_2 - \varphi_2}{\sigma_{\varepsilon_2}}; \rho_{\varepsilon_1 \varepsilon_2}\right) \end{aligned} \tag{21}$$

Φ_2 is the bivariate standard normal distribution and $\rho_{\varepsilon_1 \varepsilon_2}$ is the correlation between ε_1 and ε_2.

A.1.2 Regime 2: OXX

In this regime only energy 1 is not used. The observed shares are such that: $S^*_1 = 0, S^*_2 > 0, S^*_3 > 0$.

The virtual price of the first energy η_1 is solution of the equation:

$$S^*_1(\eta_1, p_2, p_3) = S_1 = 0 \tag{22}$$

and is given by:

$$\ln \eta_1 = -\frac{1}{\beta_{11}} S_1 + \ln p_1 \tag{23}$$

The relative share of the second energy is obtained, after substitution for the va:lue of η_1:

$$\begin{aligned} S^*_2(\eta_1, p_2, p_3) &= S_2 = \alpha_2 + \beta_{12} \ln \eta_1 + \beta_{22} \ln p_2 + \mu_2 + \varepsilon_2 \\ &= S_2 - \frac{\beta_{12}}{\beta_{11}} S_1 \end{aligned} \tag{24}$$

This regime conditions are such that:

$$\begin{matrix} \zeta_1 \leqslant p_1 \\ 0 < S_2^*(\eta_1; p_2; p_3) < 1 \end{matrix} \Longrightarrow \begin{matrix} \frac{1}{\beta_{11}} S_1 \geqslant 0 \\ 0 < S_2 + \frac{\beta_{12}}{\beta_{11}} S_1 < 1 \end{matrix} \tag{25}$$

The set $(\varepsilon_1; \varepsilon_2)$ values which satisfy the regime conditions (20) will not overlap with the $(\varepsilon_1; \varepsilon_2)$ values in (25) only if $\beta_{11} < 0$:

With $\beta_{11} < 0$, this regime's conditions are:

$$\begin{cases} \varepsilon_1 \leqslant \varphi_1 \\ 0 < S_2 + \frac{\beta_{12}}{\beta_{11}} S_1 < 1 \end{cases} \tag{26}$$

the likelihood function for this regime is:

$$L_{OXX} = P \left[\begin{matrix} \varepsilon_1 \leqslant -\varphi_1; \\ \varepsilon_2 = S_2^* - \varphi_2 + \frac{\beta_{12}}{\beta_{11}} (\alpha_1 + \beta'_1 \ln p + \mu_1 + \varepsilon_1) \end{matrix} \right]$$

$\Longrightarrow$

$$L_{OXX} = \Phi_1 \left(-\frac{1}{\sqrt{1 - \rho^2_{\varepsilon_1 \omega_1}}} \left(\frac{\varphi_1}{\sigma_{\varepsilon_1}} + \frac{\rho_{\varepsilon_1 \omega_1}}{\sigma_{\omega_1}} \left(S_2^* - \varphi_2 + \frac{\beta_{12}}{\beta_{11}} \varphi_1 \right) \right) \right) \times \frac{1}{\sigma_{\omega_1}} \phi_1 \left(\frac{S_2^* - \varphi_2 + \frac{\beta_{12}}{\beta_{11}} \varphi_1}{\sigma_{\omega_1}} \right) \tag{27}$$

where $\omega_1 = \varepsilon_2 - \frac{\beta_{12}}{\beta_{11}} \varepsilon_1$, σ_{ω_1} with variance ω_1 and $\rho_{\varepsilon_1 \omega_1}$ is the correlation coefficient between ε_1 and ω_1.

A.1.3 Regime 3: XOX

For this regime, calculus could be deduced by symmetry from the regime OXX. The likelihood for this regime is such that:

$$L_{XOX} = \Phi_1 \left(-\frac{1}{\sqrt{1 - \rho^2_{\varepsilon_2 \omega_2}}} \left(\frac{\varphi_2}{\sigma_{\varepsilon_2}} + \frac{\rho_{\varepsilon_2 \omega_2}}{\sigma_{\omega_2}} \left(S_1^* - \varphi_1 + \frac{\beta_{12}}{\beta_{22}} \varphi_2 \right) \right) \right) \times \frac{1}{\sigma_{\omega_2}} \phi_1 \left(\frac{S_1^* - \varphi_1 + \frac{\beta_{12}}{\beta_{22}} \varphi_2}{\sigma_{\omega_2}} \right) \tag{28}$$

A.1.4 Regime 4: OOX

In this regime energy 1 and 2 are not used: $S_1^* = 0; S_2^* = 0$ et $S_3^* > 0$: The virtual prices of good 1 and good 2 satisfy the relations:

$$\begin{cases} \ln \zeta_1 = \ln p_1 - \frac{1}{\beta_{11}\beta_{22} - \beta_{12}^2} [\beta_{22} S_1 - \beta_{12} S_2] \\ \ln \zeta_2 = \ln p_2 - \frac{1}{\beta_{11}\beta_{22} - \beta_{12}^2} [-\beta_{12} S_1 + \beta_{11} S_2)] \end{cases} \tag{29}$$

and the regime conditions are:

$$\left\{\begin{array}{l} \zeta_1 \gtreqless p_1 \\ \zeta_2 \gtreqless p_2 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} \frac{1}{\beta_{11}\beta_{22} - \beta_{12}^2}\left[\beta_{22}S_1 - \beta_{12}S_2\right] \gtrless 0 \\ \frac{1}{\beta_{11}\beta_{22} - \beta_{12}^2}\left[-\beta_{12}S_1 + \beta_{11}S_2)\right] \gtrless 0 \end{array}\right. \tag{30}$$

The $(\varepsilon_1, \varepsilon_2)$ values which satisfy the inequalities will not overlap with those in (20) or (25)only if $\beta_{11}\beta_{22} - \beta_{12}^2 > 0$.

Imposing this inequality, the regime conditions become:

$$\left\{\begin{array}{l} \varepsilon_1 - \frac{\beta_{12}}{\beta_{22}}\varepsilon_2 \gtreqless \frac{\beta_{12}}{\beta_{22}}\varphi_2 - \varphi_1 \\ \varepsilon_2 - \frac{\beta_{12}}{\beta_{11}}\varepsilon_1 \gtreqless \frac{\beta_{12}}{\beta_{11}}\varphi_1 - \varphi_2 \end{array}\right. \tag{31}$$

and the likelihood function is:

$$L_{OOX} = \Phi_2\left(\frac{\frac{\beta_{12}}{\beta_{22}}\varphi_2 - \varphi_1}{\sigma_{\omega_2}}, \frac{\frac{\beta_{12}}{\beta_{11}}\varphi_1 - \varphi_2}{\sigma_{\omega_1}}; \rho_{\omega_1\omega_2}\right) \tag{32}$$

where $\omega_2 = \varepsilon_1 - \frac{\beta_{12}}{\beta_{22}}\varepsilon_2$, with variance σ_{ω_2} and $\rho_{\omega_1\omega_2}$ is the correlation coefficient between $\omega_1$and ω_2.

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