Getting the Ball Rolling: Voluntary Contributions to a Long-Term Public Project

by

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Abstract

I consider an incomplete information model of voluntary contributions to a long term public project. While agents can observe the progress of the project and their own costs of contribution, they have incomplete information about the contribution costs of others. I show that the equilibrium pattern of contributions is influenced by the interplay of two opposing incentives: First, agents prefer to free ride on others for contributions. However, second, agents also wish to encourage each other to contribute by increasing their own contributions. Main findings of the paper include: (1) Agents make concessions toward the completion of the project by increasing their contributions as the project moves forward, (2) as additional agents join the group, existing agents increase their contributions in some states and reduce it in others. In particular, agents increase their contributions in the initial stages of the project to increase its value to others and secure their future contributions as a result, (3) despite this nonmonotonicity, agents do strictly benefit from the presence of additional agents, (4) the project progresses too slowly from the social standpoint.

1 Introduction

Voluntary provision of many public goods requires completing a sequence of intermediate stages or subprojects before realizing the full return.[1] Examples include a group of

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[1] My focus is on voluntary contributions. In reality, it is not possible for agents to write enforceable contracts stipulating the pattern of contributions over time– because contributions are not verifiable and

countries undertaking several subprojects toward cleaning the nearby ocean or restoring earth's ozone layer [Murdoch and Sandler (1997)], a group of donors contributing repeatedly toward a public library or a bridge of certain size, a group of lobbyists attending a series of committee meetings to influence the policy decision, and a group of researchers working on several subprojects toward completing a joint grand project. In most cases, although agents can observe the progress of the project over time[2], they may lack important information about certain characteristics of others like their current financial status or costs of contribution.

The objective of this paper is to determine the equilibrium pattern of contributions, and in particular the incentives that induce them when there is incomplete information. For instance, how would equilibrium contributions change as the project progresses and nears completion? What happens as the size of the group increases? Would the project be completed, if at all, at the socially efficient rate?

In general, contributions to a public good suffer from the well-known free-rider problem. While agents benefit from others' contributions at no cost, they bear the full cost of their own, leading them to rely "too much" on others for contributions. When public projects require a sequence of contributions, however, a second effect, countervailing the free-riding incentive, emerges. To see this, suppose there is at first a single agent that is currently indifferent to start contributing to a public project. Now, suppose this agent is informed that another agent could be interested in contributing to this project if it were one step further along or sufficiently "mature". That is, assume that the second agent's current valuation of the project is too small to attract his immediate contribution. If the free-rider effect were the only effect here, then the first agent would continue to remain indifferent

because these agreements cannot be enforced. If such contracts were feasible though, agents could improve upon voluntary contributions outcome. See, for instance, the sizable literature on the mechanism design aspect of public good provision, e.g., Cornelli (1996), d'Aspremont, Cremer and Gerard-Varet(1990), d'Aspremont and Gerard-Varet (1979), Gradstein (1994), Ledyard and Palfrey (1994), Maskin (1999), and Palfrey and Srivastava (1986).

[2] For instance, it is conceivable that countries can readily observe what portion of the ocean has been cleaned, and how many more subprojects are left to finish cleaning the rest. Also, researchers working on a joint project can observe how many important results have been derived and how many more are left to make the work viable.

about making a contribution and starting the project. However, the presence of another agent might in fact break down this indifference and encourage the first agent to start the project in order to attract contributions in the future periods. In a sense, the first agent might invest in the second agent to free ride in the future. Using the terminology of Bolton and Harris (1999)[3], I call the latter effect the "encouragement" effect and examine how it interacts with the free riding incentive throughout the relationship.

The organization and a brief preview of my findings are as follows. In section 2, I present the model, which builds on the discrete public good framework of Palfrey and Rosenthal (1988, 1991). There are N risk-neutral agents who work on a joint project yielding its full return after a sequence of stages are completed. The interaction among agents is modeled as an infinitely repeated Markov game and Markov Perfect Equilibrium (MPE) is the equilibrium concept throughout. The Markov behavior requires that agents base their decisions only on the progress of the project, which is summarized by the current state. In each period, agents simultaneously decide whether or not to contribute upon commonly observing the state of the project and privately observing their own cost of contribution. The costs of contribution come from a distribution which is independently and identically distributed over time and across agents.[4] These costs can be interpreted in many ways depending on the context. For instance, they might reflect countries' financial status fluctuating with the volatile market conditions, or researchers' opportunity costs of working on a project changing from time to time according to outside options. I assume that agents' contributions are perfect substitutes so that as long as one agent contributes, the project moves forward. Otherwise, it stops. No side-payments are permitted and contributions are non-refundable.[5] The complicated dynamic interaction can be simplified by

[3] Although my focus here is on considerably different issues from those addressed by Bolton and Harris (1999), the models share common ground, which I discuss in detail in the analysis.

[4] This assumption, aside from being realistic, rules out the strategic learning among agents in a public good context, which is the topic of Bliss and Nalebuff (1984) and Gradstein (1992), among others. In these papers, agents have private information about their costs of contribution and these costs do not change over time. Thus, as time passes by, agents leak information to others often resulting in inefficient delays in contributions. Although this type of strategic learning is an important element of public good provision, here I abstract from this aspect to better focus on the effects of the progress of the project on agents' contribution pattern.

[5] These assumptions are widely used in the literature. They make most sense when contributions are

employing the dynamic programming arguments. It turns out that agents follow a simple equilibrium cut-off cost strategy in each state to make the contribution decision. This cut-off is endogenously determined reflecting both the free-rider and the encouragement effects. We say that as the cut-off increases, so do agents contributions or efforts.[6]

In Section 2, by restricting attention to the symmetric equilibrium, I show that this game possesses a unique MPE. In equilibrium, agents make concessions toward the completion of the project by increasing their efforts, which highlights the presence of the positive encouragement effect. In general, others' increasing their efforts along with the progress has two opposing effects on one's decision: On the one hand, it makes his effort less pivotal thereby facilitating the free-rider effect. On the other hand, it brings the future returns generated by greater future contributions closer thereby facilitating the encouragement effect. The latter effect mitigates the former and thus in equilibrium each agent raises his effort level as the project moves forward. This also implies that agents' valuations of the project increase at an increasing rate as it progresses.

In Section 3, I consider the comparative statics with respect to the group size and the discount factor. The noteworthy finding here is that equilibrium contributions are nonmonotonic in the group size. As one more agent joins the group, other agents may increase their contributions in some states and reduce it in others. In general, the encouragement effect is stronger in a larger group in the initial stages of the project leading agents to increase their contributions in response to an additional agent– simply because the gain from encouraging others is greater in a larger group. This implies that long-term projects are more likely to take off with a larger group. However, as the project nears completion, the encouragement effect loses strength and the free riding incentive becomes more dominant in a larger group, leading agents to lower their contributions. In fact, it is possible that an additional agent might slow down the progress of the project. Despite this nonmonotonicity, agents do strictly benefit from the presence of an additional agent. This is because the cost saving associated with an additional agent more than offsets the reduced pace. Interestingly, as the group size tends to infinity, the equilibrium contributions approach to the socially optimal level.

The rest of the paper is organized as follows. In Section 4, I analyze the socially

irreversible expenditures or in the form of physical effort, for example.

[6] I use the terms contribution and effort interchangably throughout.

optimal second-best benchmark, where a social planner determines the cut-off strategies for agents prior to observing the costs in each period. This eliminates the inefficiency stemming from the coordination of efforts and, not surprisingly, agents strictly benefit from this coordination. Somewhat surprisingly, I also show that agents contribute too infrequently from the social standpoint and thus the project progresses too slowly. This points to the strength of the free riding incentive in the model, and implies that the encouragement effect cannot fully mitigate the free-rider effect. In Section 5, I extend the model to incorporate complementaries in agents' efforts. I further discuss the presence of an encouragement incentive in other-seemingly unrelated- settings in Section 6, and finally suggest future research directions in Section 7.

Before proceeding, it is useful to further relate and distinguish the present paper from the previous work. In a one-period incomplete information setting, Menezes, Monteiro, and Temimi (2001) and Palfrey and Rosenthal (1988, 1991) consider private provision of a discrete public good. My model can be considered as an extension of their analyses to a dynamic setting in which a sequence of complementary discrete public goods are produced. Palfrey and Rosenthal (1994) extend their one-period setting to an infinitely-repeated game where agents receive the full return in each period if the public good is produced in that period and they confront theoretical findings with experimental results. While my theoretical model is also infinitely-repeated, agents receive the full benefit only when the project is completed.[7] Bagnoli and Lipman (1989) and Palfrey and Rosenthal (1984) show the possibility of efficient provision of a discrete public good in a one-period complete information model.[8] Admati and Perry (1991), Fershtman and Nitzan (1991), and Marx and Matthews (2000) also consider dynamic contribution games with different assumptions. I further relate my results to theirs in the main analysis below.

2 The Model

Consider a dynamic extension of Palfrey and Rosenthal (1988, 1991). There are N agents who work on a long-term joint project. The progress of the project is observed by all

[7] Also see Bac (1996), McMillan (1979), and Pecorino (1999) who consider the possibility of cooperation in infinitely-repeated game settings of public good provision under different assumptions.

[8] To these models, Nitzan and Romano (1990) introduce uncertainty in the production cost and show that the unique equilibrium usually encompasses inefficient provision.

agents. Let T and $r(T)$ denote the state and the corresponding return to each agent in that state, respectively. The project is a non-excludable public good yielding its full return, $v > 0$, after T^H consecutive states or subprojects are completed. Namely,

$$\textit{Assumption 1}: r(T) = \begin{cases} v \ for \ T \geq T^H \\ 0 \ \ forT < T^H \end{cases}$$

Assumption 1 indicates that there are no immediate returns before completing the project. While this assumprion seems more appropriate for some projects like a research paper or a public bridge, other projects might yield intermediate payoffs. By making each subproject *ex-ante* identical, I abstract from the latter complication to better highlight the dynamics in the model . Zero return, however, is pure normalization.

The interaction among agents is modeled as an infinitely repeated Markov game and the MPE is the solution concept.[9] Informally, a MPE consists of strategies for each agent constituting a perfect equilibrium for all payoff relevant histories described by the state T. This concept also has intuitive appeal in my setting as I wish to investigate the effects of the progress of the project on agents' incentives to contribute. In each period, agents observe the state of the project and simultaneously decide whether or not to contribute. Like in Rosenthal and Palfrey's setting, contribution is a 0-1 decision denoted by the indicator variable s. As long as one contributes, the project moves forward. Otherwise, it stops.[10] This type of production function assumes agents' contributions are perfect substitutes and admits no complementarity. Nonetheless, I will show that agents' equilibrium contributions exhibit some complementarity. I discuss a more general production function in Section (5). The cost of contribution or effort comes from a twice continuously differentiable cumulative distribution, $F(c)$, which is *independently and identically distributed over time and across agents.* The support of distribution is $[0, \bar{c}]$ with $\bar{c} > 0$ and $F'(c) = f(c) > 0$. Agents discount the future returns by $\delta \in (0, 1)$. Let $W_i(T, c_i)$ be agent

[9] See, for example, Fudenberg and Tirole (1991) ch. 13 for a review.

[10] This is clearly an approximation. In reality, working on a project, for instance, does not guarantee its immediate progress. A more realistic setting would be where given that someone works on the project, it moves to the next state with some probability. Conversely, it might also be that if no agent contributes, the project might depreciate. These are interesting extensions. However, I conjecture that as long as these transition probabilities remain constant over states, qualitative results in the paper continue to hold.

i's value function when he realizes c_i and the project is in state T. Suppose that $\lambda^*_{-i}(T)$ is the equilibrium probability that at least one agent other than i contributes. Agent i, then, solves the following dynamic program:[11]

$$W_i(T, c_i) = \max_{s_i \in \{0,1\}} \left\{ \begin{array}{c} r(T) + s_i[-c_i + \delta W_i(T+1)] \\ +(1-s_i)\delta[\lambda^*_{-i}(T)W_i(T+1) + (1-\lambda^*_{-i}(T))W_i(T)] \end{array} \right\} \quad (1)$$

where $W_i(T) \equiv E_c[W_i(T, c)]$ and E_c is the expectation operator with respect to c.[12]

According to (1), agent i decides whether or not to contribute to the project upon observing the state of the project and his own current cost. If he contributes, i.e., $s_i = 1$, he bears the full cost of his effort in which case the project moves forward with certainty and he receives the discounted expected continuation value, $\delta W_i(T+1)$. However, if he decides not to, i.e. $s_i = 0$, then he will have to rely on others for contribution. In this instance, he conjectures the expected probability that at least one other agent will contribute, which in turn determines his discounted future expected continuation value, $\delta[\lambda^*_{-i}(T)W_i(T+1) + (1-\lambda^*_{-i}(T))W_i(T)]$. In each decision though, he receives the return, $r(T)$. Define the following cutpoint:

$$x_i(T) \equiv [1 - \lambda^*_{-i}(T)]\delta \Delta W_i(T) \quad (2)$$

where $\Delta W_i(T) \equiv W_i(T+1) - W_i(T)$.

Equation (1) reveals that agent i's equilibrium strategy simply has the following cut-off property:[13]

$$s_i^*(T, c) = \begin{cases} 1, if\ c \leq x_i(T) \\ 0, otherwise \end{cases} \quad (3)$$

[11] Note that agents incur the cost at the time they volunteer in this model. This might create *ex-post* inefficiency in equilibrium as contributions in excess of one are wasted. However, below we will see that agents take this possibility into account when contributing.

[12] Since time enters only through discounting and the cost distribution is stationary over time, I drop the time index throughout the analysis. Also, below I show that the value functions exist and are well-behaved.

[13] Since $c \in [0, \bar{c}]$, I adopt the convention that $x_i(T) = 0$ for $x_i(T) < 0$ and $x_i(T) = \bar{c}$ for $x_i(T) > \bar{c}$.

We say that as $x_i(T)$ increases, so does agent i's contribution or effort.[14][15] According to (2), this effort is monotonic in the likelihood of being the pivotal contributor and in the discounted net future gains. For instance, if agent i conjectures that someone else would take an action with certainty, i.e., $\lambda^*_{-i}(T) = 1$, then he would choose $x_i(T) = 0$ and just free ride on others in state T. Also, for agent i to take an action, he must be expecting a positive net return in the future. The net return, $\Delta W_i(T)$, can be also interpreted as the shadow value of the progress. The complicated dynamic interaction endogenously determines this shadow value and thus the effort levels in each state.

Since agents adopt payoff-relevant strategies only, it is clear from (1) that for $T \geq T^H$, $\Delta W_i(T) = 0$ and thus the unique equilibrium is $x_i(T) = 0$ for all i. That is, no agent would take action once the project is completed. This further implies the boundary condition that for $T \geq T^H$:

$$W_i(T) = \frac{v}{1-\delta} \tag{4}$$

Also, since agents are *ex-ante* symmetric, it seems reasonable to focus on the symmetric equilibrium for $T < T^H$ as well. Suppose $x_i(T) = x(T)$ for all $i = 1, ..., N$ in equilibrium. To determine the MPE, I first define the following function and record its properties:

$$B(x;\delta) \equiv \frac{1}{\delta}\frac{x}{[1-F(x)]^{N-1}} - \int_0^x [1-F(c)]dc \tag{5}$$

Lemma 1. *(a)For $T < T^H$,$\overline{W}_i(T+1) = B(x(T);\delta)$ and $\overline{W}_i(T) = B(x(T);1)$ where $\overline{W}_i(T) = (1-\delta)W_i(T)$.*

(b) $B'(x;\delta) > 0$ for $x \in (0, \overline{c}]$, and $B'(0;\delta) = \frac{1}{\delta} - 1$.

Proof. All proofs are contained in the appendix. ■

[14] This is in *ex-ante* contribution sense. As the cutpoint increases, agent i becomes more likely to make a contribution. In Appendix B, I show that the same qualitative results in the paper would emerge if agents were allowed to make continuous contributions.

[15] Agent i is indifferent about whether or not to contribute if he draws a cost $c = x_i(T)$. Given the cost distribution is continuous, the probability of this event is zero and thus it has no effect on the analysis.

Part (a) of Lemma 1 indicates how the backward induction works to determine the MPE given (4). This is also illustrated in Figure 1. Here, $\overline{W}_i(T)$ represents the average per period return in state T. Armed with this observation, the following proposition characterizes the equilibrium:

Proposition 1. *There exists a unique symmetric MPE with the following properties: For $T < T^H$,*

(i) $x(T) > 0$ for all T.

(ii) $x(T)$ is strictly increasing in T.

(iii) $W_i(T)$ is strictly increasing at an increasing rate in T. That is, $\Delta W_i(T) > 0$ and $\Delta W_i(T) > \Delta W_i(T-1)$.

Part (i) has two implications: First, the project of any lenght takes off with positive probability despite no immediate returns. In equilibrium, agents are willing to take current losses for future returns. Second, the project is finished with some positive probability. In Section 4, I show that the equilibrium rate of completion is too slow from the social standpoint.

The intuition behind part (ii) is more involved. In equilibrium, the cutpoint for agent i given in (2) reduces to:

$$x_i(T) = [1 - F(x(T))]^{N-1} \delta \Delta W_i(T) \tag{6}$$

Everyone else's increasing effort has two opposing effects on one's decision summarized on the RHS of (6): On the one hand, it makes his effort less pivotal and thus facilitates free-rider effect. Formally, as $x(T)$ increases $[1 - F(x(T))]^{N-1}$ becomes smaller. On the other hand, it brings future returns generated by greater future efforts closer and creates an encouragement effect given by $\delta \Delta W_i(T)$. As part (ii) of Proposition 1 indicates, the latter effect mitigates the former and therefore each agent raises his effort as the project moves forward. Put differently, agents view their own efforts as strategic substitutes for others' current efforts and as strategic complements to the future efforts. This is so in equilibrium even though there are no complementaries in the production function and the underlying game structure is not supermodular.

This observation accords with Bolton and Harris (1999). They consider a strategic experimentation setting where agents privately choose whether to invest in a safe or a risky asset in each period. At the end of each period, the choices and payoffs become common knowledge. Thus, the information about the risky asset is a public good that helps all agents make better future decisions.[16] Like the present setting, Bolton and Harris also identify both a free-rider effect and an encouragement effect toward the provision of the informational public good. On the one hand, agents attempt to free-ride on the experiments of others since experimentation entails an opportunity cost. On the other hand, an agent may be encouraged to experiment more to bring forward the time at which the information by others' future experimentation becomes available. Interestingly, this observation runs counter to that of Fershtman and Nitzan (1991). They analyze the steady state provision of a public good in a complete information differential game context with linear Markov strategies. The main finding is that agents free ride not only on the current contributors but also on the future ones. Namely, in Fershtman and Nitzan's setting, agents consider their contributions as strategic substitutes not only for current contributions of others but also for future contributions. Therefore, the Markov perfect equilibrium yields a lower steady state level of public good than does the open-loop equilibrium in which agents commit to a pattern of contributions and not respond to the accumulated amount. As Wirl (1996) indicates, the main source of this finding is not that agents use Markov strategies but they are restricted to use linear strategies. He concludes that if other smooth nonlinear strategies are considered, one will see that there are multiple steady state equilibria, some of which yield a higher level of public good than does the open-loop equilibrium.

Since the present model is stationary, we can easily determine the average waiting time in each state. Note that the equilibrium probability that the project will move from state T to $T+1$ is given by $\lambda^*(T) \equiv 1 - [1 - F(x(T))]^N$. Thus, the average waiting time in state T is $w(T) = \frac{1}{\lambda^*(T)}$. Part (ii) of Proposition 1 implies that this waiting time shrinks as the project moves forward.

The last part of Proposition 1 reveals that the value agents' attach to the project increases at an increasing rate over states. In other words, for $T < T^H$, the shadow value of progress is strictly positive and it increases with the progress.

Proposition 1 also has another implication. In the model, I assume that the progress or state is publicly observable. However, for some projects such as a research paper, it

[16] Note that the belief about the productivity of risky asset is the state in Bolton and Harris' setting, which, unlike mine, is random.

might be privately known only by the current contributors. Proposition 1 then implies that contributors have a strict incentive to inform others about the progress as soon as they contribute–both to avoid duplication of efforts and to secure greater future efforts. Thus, as long as contributions can be costlessly communicated with noncontributors, the results with observable state hold. In the remainder of the paper, I will continue to assume that the state is publicly observed.

3 Comparative Statics

In this section, I explore how the group size and the discount factor affect agents' equilibrium decisions and the valuation of the project.

What happens as the number of agents increases? Since this does not reduce agents' final returns from the project, it seems intuitive that one more agent in the group would cause a reduction in everyone's effort due to the free-rider effect. However, free-riding on someone requires that that agent sufficiently value the project and be willing to put further effort. As part (iii)of Proposition 1 indicates, the value agents attach to a long-term project might be too small in early stages of the project and so are their effort levels. This suggests that perhaps the best strategy for each agent is to first make sure that working on the project is attractive enough to others by increasing his own effort. By doing so, the project moves forward and its value increases. That is, when the value of the project is small, agents might encourage each other by increasing their own effort. When there are more agents in the group, the payoff of this encouragement is larger–simply because one unit increase in one's own effort will encourage more agents and secure higher future efforts. Therefore, it seems reasonable to conjecture that in early stages of the project, the encouragement effect increases with the size of the group. As the project matures however, the encouragement effect should lose its steam and the free-rider should become stronger in a larger group. This is because for each agent, there are more agents to free ride on, which suggests that the larger the group is, the worse the free rider effect is once the project matures. To analyze these issues, I first note the following result:

Lemma 2. *Let k denote the length of the project. Then, $lim_{k\to\infty}x(T^H - k) = 0$.*

Recall that part (ii) of Proposition 1 implies that when there are more steps to finish the project, agents contribute less as the value of the project is initially small. Lemma 2 further complements this finding by indicating that if there are sufficiently large number of steps, agents' contributions converge to 0 and the project virtually does not take off. The backward induction in Figure 1 further illustrates this finding. Now, we are ready to record the following comparative static:

Proposition 2. *For* $N \geq 1$,

(i) $x(T^H - 1; N+1) < x(T^H - 1; N)$

(ii) For sufficiently large k, $x(T^H - k; N+1) > x(T^H - k; N)$

(iii) For $T < T^H$, $\lim_{N\to\infty} x(T; N) = 0$, *and* $\lim_{N\to\infty} \lambda^*(T; N) = 1$

This is perhaps the most striking result of the paper. Proposition 2 implies that agents' efforts are non-monotonic in the group size. To better grasp the intuition behind this nonmonotonicity, consider first the one-shot variation of the model. That is, suppose agents contribute only once and if at least one contributes, they all receive a payoff of $v > 0$. Otherwise, no payoff is received. Let x_0 be the symmetric equilibrium cut-off in this case. It is clear that the unique x_0 solves:

$$\frac{x_0}{[1 - F(x_0)]^{N-1}} = \delta v \tag{7}$$

From (7), it is easy to see that $x_0(N)$ is strictly decreasing in N.[17] Since the free-rider effect is the only effect in this static variation, agents reduce their efforts when one more agent joins the group. This further highlights the presence of the encouragement effect that runs against the free-rider effect in our dynamic setting. The latter effect is more pronounced when the project nears completion as indicated in part (i) of Proposition 2. In fact, in the penultimate state before the completion of the project, the encouragement effect vanishes completely. When there is only one more stage to finish the project, agents do not need to put additional effort to encourage each other given one more contribution will finish the project anyway. However, agents do encourage each other in the early stages of the project when their values attached to the project are small. This encouragement effect is stronger in a larger group, which is indicated in part (ii) of Proposition 2.[18] This also implies that long-term projects are more likely to start with larger groups.

In equilibrium, how much effort agents put in a *given state* is also ambiguous in the

[17] Differentiating both sides of (7) with respect to N, we obtain $\frac{\partial x_0}{\partial N} = \frac{x_0 \log[1-F(x_0)]}{1+\frac{(N-1)x_0 f(x_0)}{1-F(x_0)}} < 0$.

[18] In light of Lemma 2, part (ii) of Proposition 2 might seem puzzling. However, what it really implies is that as $k \to \infty$, $x(T^H - 1; N+1)$ converges to 0 at a slower rate than does $x(T^H - 1; N)$.

group size [see Firgure 3]. Even so, as the group size tends to infinity, each agent expects someone in the group will draw a cost close to 0 and thus chooses a cutpoint close to 0 as a result. Interestingly, this holds in all states as recorded in part (iii). This does not mean, however, the project would never move forward. In contrast, I show in the appendix that the expected equilibrium probability that the project moves from T to $T+1$, i.e. $\lambda^*(T;N)$ converges to 1. The increase in the number of agents more than offsets the reduction in the cutpoint. In fact, in Section 4, I demonstrate that as $N \to \infty$, agents' welfare approaches to the socially efficient level.[19]

I further demonstrate these findings within a numerical example as depicted in Figure 2 and 3. In the example, there are 19 states to complete the project, i.e., $T^H = 19$.[20] The cost of effort is distributed uniformly in $[0,3]$ and the project yields a value $v = 2$ to each agent upon completion. Also, agents discount future returns by $\delta = .9$.

Refer to Figure 2. When the group consists of a single agent, obviously there are no free riding or encouragement incentives. Note that in this case, the agent would finish the project with certainty in the penultimate state, $T = 18$, i.e., $x(18) = \bar{c} = 3$. However, the project virtually does not take off for the states below $T = 6$, i.e., $x(T) \approx 0$ for $T \leq 6$. If there is one more agent in the group, agents encourage each other for the initial 10 states by choosing higher cutpoints than the single-agent case. Thus, the project is more likely to take off with a larger group in these states. However, for the remaining 8 states, the free-rider effect becomes stronger resulting in lower effort levels. Note that one more agent in the team does not mean the project will move at a faster rate in every state. For instance, in the penultimate state, the expected probability that the project will be finished is strictly less than 1 whereas each agent would finish it with certainty if he were alone in that state. As the group size grows further to $N = 3$ (see Figure 3), the critical state below which agents put higher efforts in a larger group goes down. This is because with more agents, the project matures faster and the free-rider effect becomes more dominant starting in an earlier state. The example also shows that agents' effort

[19] This result coincides with Bliss and Nalebuff's who also find in a war of attrition model of public good provision that even though the expected time of supplying the good is ambigious in the group size, it shrinks to 0 as the group size tends to infinity. Hence, they conclude for sufficiently large population the free-rider problem disappears completely.

[20] Figures only show the states 1 through 18 since we already know that $x(T) = 0$ for $T \geq 19$.

levels stay relatively steady in a larger group.

Despite the non-monotonicity in agents' effort levels as a function of the group size, agents do strictly benefit from an additional group member as I record in the following Proposition:

Proposition 3. *For $T < T^H$, $W_i(T, N+1) > W_i(T, N)$.*

Proposition 3 shows that an increase in the number of agents is a Pareto improvement in the sense that each agent has a higher welfare in each state. This seems intuitive for the states in which agents increase their effort levels as this improves the pace of the project resulting in a higher value.[21] However, when the project matures, an additional agent exacerbates the free-rider incentive and might even slow down the progress of the project. Even so, the cost saving associated with one additional agent more than offsets this reduced pace.

Comparative static with respect to the discount factor δ is similar to the one with respect to the progress of the project. By explicitly introducing the dependence on δ, we can re-write (6) as:

$$x_i(T;\delta) = [1 - F(x(T;\delta))]^{N-1}\delta\Delta W_i(T;\delta) \tag{8}$$

An increase in δ has three effects on i's contribution: First, since he now cares more about the future, he is willing to take larger current losses and thus increases his cutpoint. This is the direct effect reflected in the second term on the RHS of (8). Second, given that others contribute more in response to a higher δ, agent i's contribution becomes less pivotal, i.e., $[1 - F(x(T;\delta))]^{N-1}$ is smaller, and thus he tends to free ride and decrease his contribution. Third, others' increasing contributions brings the future returns generated by future contributions closer and thus encourages agent i,reflected in the last term of (8). The following result shows that the two positive effects dominate the negative free-rider effect and thus each agent raises his effort in response to a higher discount factor:

Proposition 4. *For $T < T^H$,*

(i) $x(T;\delta)$ is strictly increasing in δ.

(ii) $W_i(T;\delta)$ is strictly increasing in δ.

(iii) $lim_{\delta \to 1} x(T;\delta) = x_1$ where $B(x_1;1) = v$.

[21] This is clearly seen from Lemma 1 where $B(x;1)$ is stricly increasing in x.

According to Proposition 4, agents strictly benefit from being more patient.[22] This is true despite there are no reputational concerns in my setting, which is in contrast to several other dynamic models of public good provision, e.g., Bac (1996), Marx and Matthews (2000), McMillan (1979) and Pecorino (1999), where agents adopt trigger strategies. In such settings, as the discount factor becomes larger and agents become more patient, the punishment becomes more severe for any deviation, which helps sustain the cooperative outcome. Here, I focus on the Markov behavior which excludes such trigger strategies. Nonetheless, the relationship grows to be more cooperative in response to a higher discount factor. Interestingly, the average waiting time, $w(T)$, decreases with δ. That is, the more impatient agents are, the longer they must wait to benefit from the public good.[23]

The last part of Proposition 4 indicates agents use the same equilibrium cut-off strategy regardless of the state of the project as discounting vanishes. In particular, it is interesting to note that agents would contribute as if they were in the penultimate state before the completion of the project. Given that there are a finite number of states to finish the project, when the cost of waiting is negligible, agents assume the project will ultimately reach the state $T = T^H - 1$ and choose the same cutpoint $x(T) = x_1$ in all other previous states too.[24] Since we already know that there is no encouragement effect in $T = T^H - 1$, we can infer that the encouragement effect vanishes in all other states as $\delta \to 1$.[25] Thus, the free-rider effect is the only effect that governs the relationship in this case. [26] This also implies that positive discounting is a necessary condition to have an encouragement

[22] Other useful and standard interpretations of the discount factor are to view δ as the probability that the group will survive in the next period, and also as the frequency of interaction, where a greater frequency corresponds to a higher δ.

[23] Perhaps, this is why we are usually told that "patience pays off".

[24] Below I show that $x_1$is still less than the socially optimal cut-off.

[25] This result is a reminiscent of similar conclusions in Cabral and Riordan (1994), Lewis and Yildirim (2001), and Spence (1981). These papers consider the strategic effects of a learning curve on dynamic competition or auction settings. The models assume either a finite horizon problem like in Spence's or an infinite horizon with finite number of states to reach the bottom of the learning curve like in the latter papers. Similar to mine here, they conclude that as the discounting vanishes, in any given state, firms behave as if they exhausted all the learning economies already. For instance, Cabral and Riordan's dynamic duopolists price their products at the marginal cost that would be obtained once all the learning economies exhausted.

[26] Applying part (i) of Proposition 2, we see that $x_1(N)$ is strictly decreasing in N. Thus an additional

effect, which is somewhat ironic. One would expect that when δ is large and the future weighs heavily, agents would encourage each other. Instead, they follow a rather static equilibrium strategy which entails only the free-riding incentive. In general, the reason why an agent tries to encourage others when $\delta < 1$ is that this increases the future value of the project that induces others to further their efforts. This allows agents to realize the returns sooner rather than later. However, when the cost of waiting is negligible, agents do not need to encourage others to finish the project sooner. They can simply wait for a more favorable cost realization.

4 Benchmark: Social Optimum

There are two sources of inefficiencies in the noncooperative contribution game I have analyzed: (1) Agents simultaneously choose their cut-off points, i.e., $x_i(T)$, and (2) agents have incomplete information about others' costs of contribution. The first-best would occur if neither of these problems existed. Namely, in a first-best situation, a social planner would maximize the total welfare given that he fully knows agents' current costs of contribution. The optimal strategy in the first-best situation would be to assign the project to the lowest cost agent in each state unless this cost is too high. However, given the informational constraints, this solution is infeasible in the present setting, which relies on voluntary contributions. Therefore, the second-best situation where the social planner coordinates agents' cut-off points before the costs of contribution are realized seems more appropriate as a benchmark.

Suppose a social planner maximizes the total welfare, or equivalently the average welfare per agent in the group by choosing a cut-off point, $x_i^{**}(T)$, for agent i.[27] Let $W^{**}(T)$ be the optimal *average* payoff per agent in state T. It is clear that once the project is completed, the social planner will choose $x_i^{**}(T) = 0$ for $T \geq T^H$ and thus have no agent contribute. This implies the boundary condition: For $T \geq T^H$,

$$W^{**}(T) = \frac{v}{1-\delta}$$

agent causes a reduction in effort by group members, which is exactly what we would expect if the free-rider effect were the only effect. This is in sharp contrast to the finding in Propsoition 2 that for $\delta < 1$, agents' equilibrium efforts are in general non-monotonic in N.

[27] Alternatively, suppose agents are able to write a one-period contract about how much contribution each agent will make before they realize the costs of contribution.

For the intermediate states however, $W^{**}(T)$ satisfies the following dynamic program:

$$W^{**}(T) = \frac{1}{1-\delta} \max_{\{x_i(T)\}} \left\{ -\frac{1}{N} \sum_{i=1}^{N} \int_0^{x_i(T)} cdF(c) + \left[1 - \prod_{i=1}^{N} (1 - F(x_i(T))) \right] \delta \Delta W^{**}(T) \right\} \tag{9}$$

The first-order condition for $x_i(T)$ implies that

$$x_i^{**}(T) = [1 - \lambda_{-i}^{**}(T)] N \delta \Delta W^{**}(T) \tag{10}$$

where again $\lambda_{-i}^{**}(T)$ represents the optimal probability that at least one agent other than i will contribute.[28] Comparing (10) and (2), we see that when determining the cut-off for agent i, the social planner takes the effect of i's contribution on the total welfare rather than just on i's own welfare. Just like before, I will assume that the social planner optimally assigns symmetric cut-off points to all agents. That is, in equilibrium $x_i^{**}(T) = x^{**}(T)$. Now define the following function:

$$B^{**}(x) = \frac{x}{\delta N [1 - F(x)]^{N-1}} - \int_0^x [1 - F(c)] dc + \left(1 - \frac{1}{N} \right) x [1 - F(x)] \tag{11}$$

A variant of Lemma 1 determines how the social planner finds the optimal cut-off points for the group.

Lemma 3. *(a) For* $T < T^H, \overline{W}^{**}(T+1) = B^{**}(x^{**}(T); \delta)$ *and* $\overline{W}^{**}(T) = B^{**}(x^{**}(T); 1)$ *where* $\overline{W}^{**}(T) = (1 - \delta) W^{**}(T)$.

(b) $B^{**\prime}(x; \delta) > 0$ *for* $x \in (0, \overline{c}]$, *and* $B^{**\prime}(0; \delta) = \frac{1}{N}(\frac{1}{\delta} - 1)$.

Part (a) of Lemma 3 indicates that the backward induction for the second-best solution works exactly the same way as the noncooperative case. Not surprisingly, when there is only one agent, the noncooperative solution is also the social optimal. Since the qualitative properties of $B^{**}(x; \delta)$ coincides with that of $B(x; \delta)$, the existence of a unique social optimum easily follows by applying the proof of Proposition 1. Here we are interested in knowing whether the coordination of efforts strictly benefit agents in all states, and more importantly whether the project progresses at the socially optimal rate. The following proposition answers these questions:

Proposition 5. *For* $N > 1$, and $T < T^H$,

(i) $W_i(T) < W^{**}(T)$.

[28] It is easy to see that the maximized function is strictly quasi-concave.

(ii) $x(T) < x^{**}(T)$.

(iii) $\lim_{\delta\to 1} x(T) = x_1 < \lim_{\delta\to 1} x^{**}(T) = x_1^{**}$.

(iv) $\lim_{N\to\infty} W_i(T^H - k) = \lim_{N\to\infty} W^{**}(T^H - k) = \frac{\delta^k v}{1-\delta}$ *for* $k \geq 1$.

According to part (i) of Proposition 5, agents strictly benefit from coordinating their efforts. This is because the equilibrium cutpoints are in the social planner's choice set. Part (ii) implies that agents contribute too infrequently to the project and thus the project progresses too slowly from the social standpoint. This means the positive encouragement effect cannot fully mitigate the negative free-rider effect in our model. This inefficiency remains even when the discounting is negligible as recorded in part (iii). These results parallel that of Bolton and Harris (1999) who also find that agents experiment too little compared to the socially efficient rate. The last part of Proposition 5 indicates that as the group size grows without bound, the inefficiencies disappear and agents' expected payoffs converge to the socially optimal one. This is intuitive in that in both the incomplete information and the second best cases, the contribution is made by the lowest cost agent and the expected probability of having such an agent converges to 1 in the limit.

5 Complementary Contributions

Until now, I have assumed that agents' contributions are perfect substitutes in that in a given period if at least one of them contributes, the project moves forward. This might seem restrictive in some cases where projects require certain degree of complementarity in agents' efforts. To capture this possibility, the basic model can be easily generalized. Suppose that at least $m \in \{1, ..., N\}$ agents need to contribute in a given period for the project to move forward. We say that a higher m corresponds to a higher degree of complementarity in production with $m = 1$ and $m = N$ being the two polar cases representing perfect substitutes and perfect complements, respectively. An increase in the degree of complemetarity has two opposing effects on the group interaction: (1) It reduces the free-rider problem by making agents' contributions less substitutable, and (2) it exacerbates the coordination problem by requiring others' contributions to make one's own worthwhile. To better highlight the effect of possible coordination problem, I consider the polar case where $m = N$ so that agents' contributions are perfect complements and no free riding incentive is present. Like in perfect substitute case, agent i solves the following dynamic program to decide whether or not to contribute in state T:

$$W_i(T, c_i) = \max_{s_i \in \{0,1\}} \left\{ \begin{array}{c} r(T) + s_i[-c_i + \delta[\lambda^*_{-i}(T) W_i(T+1) + (1 - \lambda^*_{-i}(T)) W_i(T)]] \\ +(1 - s_i)\delta W_i(T) \end{array} \right\} \tag{12}$$

It is clear from (12) that agent i adopts a similar cut-off strategy to (2) with the following cutpoint:

$$x_i(T) = \lambda^*_{-i}(T)\delta\Delta W_i(T) \tag{13}$$

where $\lambda^*_{-i}(T)$ is the equilibrium probability that *all* agents other than i contribute in state T. According to (13), when others increase their contributions in the complementary case, because every agent's contribution is essential for the progress of the project, agent i's contribution becomes more pivotal rather than less, unlike in the substitute case. Thus, agents view their current contributions as strategic complements. In equilibrium, agents optimally choose zero contributions once the project is completed. That is, for $T \geq T^H$ we have $x_i(T) = 0$. For the intermediate states, zero contribution continues to be an equilibrium representing a complete coordination failure–given that at least one agent does not contribute, it is best for agent i not to contribute either. However, there might be other equilibria in which all agents do contribute and which are clearly Pareto improving over the zero-contribution equilibrium. I will assume that agents coordinate on a Pareto improving equilibrium whenever it exists.[29] To simplify the exposition, below I only consider a 2-agent case. To determine the symmetric equilibrium, define the following function:

$$B^C(x; \delta) = \frac{1-\delta}{\delta} \frac{x}{F(x)} + \int_0^x F(c)dc$$

Lemma 4. *(a) For $T < T^H$, $\overline{W}_i(T+1) = B^C(x(T); \delta)$ and $\overline{W}_i(T) = B^C(x(T); 1)$ where $\overline{W}_i(T) = (1-\delta)W_i(T)$.*

(b) $\lim_{x \to 0} B^C(x; \delta) = \frac{1-\delta}{\delta f(0)}$.

[29] Palfrey and Rosenthal (1988, 1991) also consider complementary contributions case and note the multiplicity of equilibria. They use the notion of an equilibrium being *globally expectationally stable*, which can be used in my setting to eliminate the zero-contribution equilibrium whenever another equilibrium exists.

I make the following technical assumption which helps guarantee the uniqueness of an equilibrium:

$$Assumption\ 2 : \frac{d}{dc}\left[\frac{c}{F(c)}\right] \geq 0$$

This assumption implies that $B^C(x;\delta)$ is strictly increasing in x and it is satisfied for $F(c;\lambda) = \left(\frac{c}{\bar{c}}\right)^\lambda$ where $0 < \lambda \leq 1$ and $\lambda = 1$ corresponds to the uniform distribution, for example.[30] The following proposition characterizes the equilibrium:

Proposition 6. *If Assumption 2 holds, there exists a unique symmetric MPE with the following properties: There is a critical state T^* such that*

(i) For $T^* \leq T < T^H$, $x(T) > 0$ and $x(T)$ is strictly increasing in T. Otherwise, $x(T) = 0$.

(ii) For $T^* \leq T < T^H$, $x(T;\delta)$ is increasing in δ and $\lim_{\delta\to 1} x(T;\delta) = x_1$ *where* $B^C(x;1) = v$.

(iii) For $T^* \leq T < T^H$, $W_i(T)$ is strictly increasing at an increasing rate in T.

(iv) For $T^* \leq T < T^H$, $W_i(T;\delta)$ is increasing in δ.

Part (i) of Proposition 6 implies that when agents' contributions are complementary, long-term projects may not start at all. This is unlike the substitute case where with positive probability all projects take off [Part (i) of Proposition 1]. The fact that agent 1's contribution is valuable only when agent 2 contributes creates a coordination problem and raises his "effective" cost of contribution. This cost may prove to be too high compared to the expected value of the project. Refer to Figure 4. Note that even if agent 1 draws a cost close to 0, the effective cost of contribution given by $\frac{1-\delta}{\delta f(0)}$ remains positive and may exceed the expected value of the project. In such a case, agent 1 finds it optimal not to incur even the small cost and thus the project never starts. In the substitute case however, the effective cost tends to 0 as agent 1's cost becomes small. This is because agent 1 knows that for his contribution to be worthwhile, agent 2's contribution is not needed. Part (i) also implies that once the project starts, it is completed with positive probability. This suggests that projects with complementary efforts might require outside help for a number stages to insure that the rest will be finished through voluntary contributions.

This finding has similar flavor to that of Andreoni (1998), who considers the role of

[30] It is clear that the monotonicity of $B^C(.)$ in x is only a sufficient condition for the uniqueness of the equilibrium. When the monotonicity condition is not met, multiple interior equilibria are possible. For simplicity, I abstract from this complication in the analysis.

fund-raising activities in inducing subsequent voluntary contributions. Within a one-shot contribution game with complete information, Andreoni assumes contributions need to exceed some exogenous threshold level to yield any return. He notes that when contributions are perfect substitutes, the zero-contribution equilibrium exists if this threshold is too high. Therefore, an outside help, e.g. an initial pledge or a government grant is needed to break this –rather inefficient– equilibrium and generate future voluntary contributions. The nonconvexity in my model is due to the complementaries in agents' contributions creating a complete coordination failure when agents' values of the project are very low.

Part (ii) indicates that as agents become more patient, they further their contributions like in the substitute case. This implies that projects are more likely to start with more patient agents. In fact, as the discount factor approaches to unity and thus cost of waiting becomes negligible, almost all projects take off with positive probability. Parts (iii) and (iv) are similar comparative static results to those in the substitute case. In Figure 5, I reconsider the numerical example for two-agent case within complementary contributions context. Once again, the cost of contributions come from a uniform distribution in $[0, 3]$ and the final value is $v = 2$ to each agent. Agents discount future returns by $\delta = 0.9$. The noteworthy finding here is that projects with more than 5 stages never start in equilibrium.

6 Discussion

In this paper, I have considered a dynamic and incomplete information model of voluntary provision of a public good whose full returns are to be received upon its completion. In addition to the well-known free riding incentive, I highlight the presence of an encouragement incentive countervailing the former. On the one hand, agents try to free ride on each other to avoid incurring the cost of efforts, and on the other hand they try to make the project worthwhile to others to secure their future efforts. The encouragement effect is stronger in the early stages of the project and loses strength as the project nears completion. I believe this intuition will deepen our understanding into the nature of dynamic interactions in other seemingly unrelated settings.

Consider the following variation of the principal-agent model. Suppose a venture capitalist (principal) and an entrepreneur (agent) both invest in a project. Also, suppose, like in the present model, T^H consecutive subprojects need to be finished before any return can be realized. Let $\mu(T+1|I_v, I_e, T) = aI_v + bI_e$ be the probability that Tth subproject will be finished given venture capitalist's current investment, I_v, and the entrepreneur's current investment, I_e,where a and b are positive constants indicating the productivity of each party's investment. Suppose parties cannot commit to a long-term agreement and therefore they renegotiate how to share the final returns after each subproject is completed. Here, the progress of the project is a public good to which both parties contribute and thus both parties have a free riding incentive. In such a setting, one wonders, for example, how the venture capitalist would change her equilibrium investment schedule if she faced a more productive entrepreneur, i.e., a higher b. The free riding incentive points toward a reduction in I_v as b increases. However, a closer look at the our comparative static result with respect to the group size shows that the venture capitalist might in fact increase her equilibrium investment in some states and decrease in others in response to a higher b. In our public good model above, adding one more agent to the group is as if each agent faced a more productive partner considering the rest of the group. In this case, we know that each agent's contribution is non-monotonic in the group size.

In a finite-horizon differential game context, Lancaster (1973) analyzes a dynamic model of capitalism, where both capitalists and workers simultaneously decide how much to consume from the total output in each period and the rest of the output is invested by capitalists to increase the future output. He shows that both parties will initially sacrifice from their current consumptions to accumulate output and once the output reaches certain amount, they consume it all. Obviously, the total output in Lancaster's framework can be considered as a (imperfect) public good to which both parties contribute. Initially, parties encourage each other by forgoing their consumption and make sure the total output reaches certain level. After this level, the free-riding incentive becomes so severe that no party is willing to contribute anymore. [31]

In general, if parties jointly contribute toward a growing surplus and no long-term contracts are feasible, then we expect there to be a free-riding incentive for each agent to avoid costly contribution or effort. However, in early stages of the relationship when parties' values attached to the relationship very low, they may be reluctant to put high effort. Therefore, it may not be a good strategy for them to purely free ride. In this instance, we expect them to encourage each other by furthering their efforts and making the

[31] Galor (1986) applies Lancaster's framework to a North-South type trade model to highlight a dynamic inefficiency.

relationship worthwhile to others to attract their future efforts. Understanding this latter incentive might prove to be important to explain certain nonmonotonicities in parties' optimal contributions to the total surplus.

7 Future Research

The present model is simplistic in many ways and open to some interesting extensions. For one, I have assumed agents draw their costs from the same distribution. In reality, one researcher's opportunity cost of working on a project, for instance, might come from a stochastically riskier distribution than the other. In such a case, it is possible that the agent with riskier cost distribution may hide behind the risk [e.g., Robledo (1999)] and claim each time he has chosen a higher cost. This gives him additional incentive to further his free riding behavior. It would be interesting to see how equilibrium contributions grow over time with the risk factor. Another possible extension would be to allow for some correlation between costs over time for each agent. In light of Bliss and Nalebuff (1984), and Gradstein (1992), I conjecture that in such a case as the correlation increases, agents would be more reluctant to contribute in order not to leak the valuable information that they have a relatively low cost. This would further slow down the project due to delayed contributions.

The techniques developed in this paper can be fruitfully used to analyze other issues as well. For instance, I have considered here a public good provision setting where an additional agent does not create "congestion". Thus, agents always prefer a larger group-size [Proposition 3]. Consider, however, a teamwork setting where agents share the final return. In such a case, there seems to be a critical relationship between the teamsize and the length of the project. These topics await formal analyses.

8 APPENDIX A

Proof of Lemma 1:

Using (1) and (3) in the text, take the following expectation:

$$\begin{aligned} W_i(T) &= E_c[W_i(T,c)] \\ &= \int_0^{x_i(T)} [-c + \delta W_i(T+1)]dF(c) \\ &+ \int_{x_i(T)}^{\bar{c}} \delta[\lambda^*_{-i}(T)W_i(T+1) + (1-\lambda^*_{-i}(T))W_i(T)]dF(c) \end{aligned} \tag{A1}$$

Integrating the first integral by parts yields:

$$\begin{aligned} W_i(T) &= -x_i(T)F(x_i(T)) + \delta W_i(T+1)F(x_i(T)) + \int_0^{x_i(T)} F(c)dc \\ &+\delta[\lambda^*_{-i}(T)W_i(T+1) + (1-\lambda^*_{-i}(T))W_i(T)][1-F(x_i(T))] \end{aligned} \tag{A2}$$

Recall that

$$x_i(T) = [1-\lambda^*_{-i}(T)]\delta\Delta W_i(T) \tag{A3}$$

Using (A3) successively, (A2) boils down to

$$W_i(T) = \delta W_i(T+1) - \int_0^{x_i(T)} [1-F(c)]dc \tag{A4}$$

(A3) implies that

$$W_i(T) = W_i(T+1) - \frac{x_i(T)}{\delta[1-\lambda^*_{-i}(T)]} \tag{A5}$$

Together with (A5), (A4) reveals that

$$\begin{aligned} \overline{W}_i(T+1) &= (1-\delta)W_i(T+1) \\ &= B(x(T);\delta) \end{aligned} \tag{A6}$$

where I make use of the symmetric equilibrium assumption so that $x_i(T) = x(T)$ for all i in equilibrium and

$$B(x;\delta) \equiv \frac{1}{\delta}\frac{x}{[1-F(x)]^{N-1}} - \int_0^x [1-F(c)]dc \tag{A7}$$

as defined in (5) in the text.

Substituting for $W_i(T+1)$ from (A5) into (A4) also implies that

$$\begin{aligned} \overline{W}_i(T) &= (1-\delta)W_i(T) \\ &= B(x(T);1) \end{aligned} \tag{A8}$$

completing the proof of part (a) of Lemma 1.

Now differentiate $B(x;\delta)$ with respect to x to find:

$$B'(x;\delta) = \frac{1}{\delta}\left[\frac{1}{(1-F(x))^{N-1}} + \frac{(N-1)xf(x)}{(1-F(x))^N}\right] - (1-F(x)) \tag{A9}$$

Since for $x \in [0,\overline{c}]$, $\frac{1}{(1-F(x))^{N-1}} - (1-F(x)) \geq 0$, we have $B'(x;\delta) > 0$. Also, (A9) implies that $B'(0;\delta) = \frac{1}{\delta} - 1. Q.E.D.$

Proof of Proposition 1:

I use backward induction.

Note first that since the payoff does not change for states $T \geq T^H$, we have $\Delta W_i(T) = 0$. Then, (A3) implies that the unique equilibrium is $x(T) = 0$ for $T \geq T^H$, yielding the boundary condition in (4): For $T \geq T^H$,

$$\overline{W}_i(T) = v \tag{A10}$$

Consider $T = T^H - 1$. From (A6), $x(T^H - 1)$ solves the following equation:

$$v = B(x(T^H - 1);\delta) \tag{A11}$$

For $N = 1$, if $v \geq B(\overline{c};\delta)$, then $x(T^H - 1) = \overline{c}$. However, if $v < B(\overline{c};\delta)$, then since $v > 0$ and $B(0;\delta) = 0$, the Intermediate Value Theorem implies there exists $x(T^H - 1) \in (0,\overline{c})$ that solves (A11). For $N \geq 2$, since $v > 0$, $B(0;\delta) = 0$, and $B(\overline{c};\delta) = \infty$, there exists $x(T^H - 1) \in (0,\overline{c})$ that solves (A11). Moreover, $x(T^H - 1)$ is unique due to $B'(x;\delta) > 0$.

Also from (A8),

$$\begin{aligned} \overline{W}_i(T^H-1) &= B(x(T^H-1);1) \\ &< B(x(T^H-1);\delta) \leq v \end{aligned} \tag{A12}$$

Thus, we have $\overline{W}_i(T^H-1) < \overline{W}_i(T^H)$.

Now suppose for some $k \geq 1$, there exists a unique $x(T^H-k) \in (0, \overline{c})$ and $\overline{W}_i(T^H-k) < \overline{W}_i(T^H-k+1)$. Again, from (A6), $x(T^H-k-1)$ solves the equation:

$$\overline{W}_i(T^H-k) = B(x(T^H-k-1);\delta) \tag{A13}$$

Since $\overline{W}_i(T^H-k) > 0$, using a similar argument above, there exists a unique $x(T^H-k-1) \in (0, \overline{c})$ that solves (A13). Furthermore,

$$\begin{aligned} \overline{W}_i(T^H-k-1) &= B(x(T^H-k-1);1) \\ &< B(x(T^H-k-1);\delta) = \overline{W}_i(T^H-k) \end{aligned} \tag{A14}$$

Since $x(T^H-k)$ uniquely solves $\overline{W}_i(T^H-k+1) = B(x(T^H-k);\delta)$ and $x(T^H-k-1)$ solves (A13), given that $\overline{W}_i(T^H-k) < \overline{W}_i(T^H-k+1)$ by the induction hypothesis and $B'(x;\delta) > 0$, we have

$$x(T^H-k-1) < x(T^H-k) \tag{A15}$$

Hence, there exists a unique symmetric MPE where for $T < T^H$, $x(T)$ is strictly positive and increasing in T.

To prove the last part of Proposition 1, note that in equilibrium (A3) can be written as:

$$x(T) = [1 - F(x(T)]^{N-1}\delta\Delta W_i(T) \tag{A16}$$

Since $x(T)$ is strictly increasing for $T < T^H$, we must have

$$\Delta W_i(T-1) < \Delta W_i(T) \tag{A17}$$

This completes the proof. *Q.E.D.*

Proof of Lemma 2.

Recall that the sequence $\{x(T^H-k)\}$ is strictly decreasing in k and it is bounded below by 0. This implies the sequence converges to some $x_l \geq 0$. Suppose $x_l > 0$. This means as k grows large, $x(T^H-k)$ becomes arbitrarily close to x_l. If $x(T^H-k^*) = x_l$ for some k^*, then since the sequence is strictly decreasing, (A6) would engender $x(T^H - k^* - 1) < x_l$. Also, since $B(x;\delta)$ is continuous in x and $\lim_{k\to\infty} x(T^H - k) = x_l$, for sufficiently large k, we would have $x(T^H - k - 1) < x_l$. This contradicts the hypothesis that $x_l > 0$ is the limit point. Hence, $\lim_{k\to\infty} x(T^H - k) = 0$. *Q.E.D.*

Proofs of Proposition 2 and 3.

I start showing by induction that for any $N \geq 1$, $W_i(T, N+1) > W_i(T, N)$ for any $T < T^H$.

Suppose in equilibrium that $W_i(T^H-1, N+1) \leq W_i(T^H-1, N)$. Since $W_i(T^H, N+1) = W_i(T^H, N) = \frac{v}{1-\delta}$, we have $\Delta W_i(T^H-1, N+1) \geq \Delta W_i(T^H-1, N)$. Then, (A16) implies $x(T^H-1, N+1) \geq x(T^H-1, N)$. Since $B(x;1)$ is strictly increasing in x and N, (A8) reveals that $W_i(T^H-1, N+1) > W_i(T^H-1, N)$, which is a contradiction. Hence, $W_i(T^H-1, N+1) > W_i(T^H-1, N)$.

To complete the induction argument, suppose $W_i(T, N+1) > W_i(T, N)$ for some $T \leq T^H - 1$. Suppose, on the contrary, that $W_i(T-1, N+1) \leq W_i(T-1, N)$. Since $W_i(T, N)$ is strictly increasing in T for any given N, we have

$$W_i(T-1, N+1) \leq W_i(T-1, N) < W_i(T, N) < W_i(T, N+1) \tag{A18}$$

(A18) implies that $\Delta W_i(T-1, N+1) > \Delta W_i(T-1, N)$. Again, (A16) implies that $x(T-1, N+1) \geq x(T-1, N)$, which, in turn, implies $W_i(T-1, N+1) > W_i(T-1, N)$, yielding a contradiction. Hence, $W_i(T-1, N+1) > W_i(T-1, N)$. This completes the proof of Proposition 3.

Now I prove part (a) of Proposition 2.

Suppose by way of contradiction that $x(T^H-1, N+1) \geq x(T^H-1, N)$. From the above result, we know that $W_i(T^H-1, N+1) > W_i(T^H-1, N)$. Again, given that $W_i(T^H, N+1) = W_i(T^H, N) = \frac{v}{1-\delta}$, we have $\Delta W_i(T^H-1, N+1) < \Delta W_i(T^H-1, N)$. From here, (A16) implies $x(T^H-1, N+1) < x(T^H-1, N)$, a contradiction. Hence, $x(T^H-1, N+1) < x(T^H-1, N)$.

To prove the last part of Proposition 2, note the following first-order Taylor expansion for x sufficiently close to 0.

$$\begin{aligned} B(x;\delta) &\approx B(0;\delta) + B'(0;\delta)x \\ &\approx \left(\frac{1}{\delta} - 1\right)x \end{aligned} \tag{A19}$$

Since, from Lemma 2, we know that for sufficient large k, $x(T^H - 1, N)$ is arbitrarily close to 0, (A6) and (A19) imply that

$$\begin{aligned}\overline{W}_i(T^H - k + 1, N + 1) &\approx \left(\frac{1}{\delta} - 1\right) x(T^H - k, N + 1) \\ \overline{W}_i(T^H - k + 1, N) &\approx \left(\frac{1}{\delta} - 1\right) x(T^H - k, N)\end{aligned}$$

Furthermore, since $\overline{W}_i(T^H - k + 1, N + 1) > \overline{W}_i(T^H - k + 1, N)$ from Proposition 3, we must have $x(T^H - k, N + 1) > x(T^H - k, N)$ for sufficiently large k.

To prove part (iii), first recall from part (i) that $\{x(T^H - 1; N)\}_{N=1}^{N=\infty}$ is a strictly decreasing sequence. Since it is bounded below by 0, it must converge to some $\alpha \geq 0$. Suppose $\alpha > 0$. For any N, $x(T^H - 1; N)$ uniquely solves the equation $B(x(T^H - 1; N)) = v$. Given $\lim_{N\to\infty} x(T^H - 1; N) = \alpha > 0$, we have $\lim_{N\to\infty} B(x(T^H - 1; N)) = \infty \neq v$. Thus, $\alpha = 0$.

Now take any $T < T^H - 1$. Since $x(T; N)$ is strictly decreasing in T and it is nonnegative, we have

$$0 \leq x(T; N) \leq x(T^H - 1; N)$$

This implies

$$0 \leq \lim_{N\to\infty} x(T; N) \leq \lim_{N\to\infty} x(T^H - 1; N) = 0$$

Thus, $\lim_{N\to\infty} x(T; N) = 0$.

To complete the proof, suppose $\lim_{N\to\infty} [1 - F(x(T; N)]^N > 0$. But given that $\lim_{N\to\infty} x(T; N) = 0$, we have $\lim_{N\to\infty} B(x(T; N)) = 0 \neq v > 0$. Thus, it must be that $\lim_{N\to\infty} [1 - F(x(T; N)]^N = 0$. *Q.E.D.*

Proof of Proposition 4.

Take any $\delta_1$and δ_2 such that w.l.o.g. $0 < \delta_1 < \delta_2 < 1$. I proceed by induction. Suppose by way of contradiction that $x(T^H - 1; \delta_1) \geq x(T^H - 1; \delta_2)$. Since $B(x; \delta)$ is increasing in x and decreasing in δ, (A6) implies $B(x(T^H - 1; \delta_1); \delta_1) = v > v = B(x(T^H - 1; \delta_2); \delta_2)$, which is a contradiction. Thus, $x(T^H - 1; \delta_1) < x(T^H - 1; \delta_2)$.

To complete the induction argument, suppose, for some $k \geq 1$, that $x(T^H - k; \delta_1) < x(T^H - k; \delta_2)$ and on the contrary that $x(T^H - k - 1; \delta_1) \geq x(T^H - k - 1; \delta_2)$. Since $B(x; 1)$ is increasing in x, (A8) yields:

$$\overline{W}_i(T^H - k; \delta_1) < \overline{W}_i(T^H - k; \delta_2) \tag{A20}$$

. Note from (A8) that $x(T^H - k - 1; \delta_1)$ and $x(T^H - k - 1; \delta_2)$ uniquely solve the following equations:

$$\overline{W}_i(T^H - k; \delta_1) = B(x(T^H - k - 1; \delta_1); \delta_1) \quad \text{(A21)}$$

$$\overline{W}_i(T^H - k; \delta_2) = B(x(T^H - k - 1; \delta_2); \delta_2) \quad \text{(A22)}$$

Again, since $B(x; \delta)$ is increasing in x and decreasing in δ, (A21) and (A22) imply that $\overline{W}_i(T^H - k; \delta_1) \geq \overline{W}_i(T^H - k; \delta_2)$, which contradicts (A20). Hence, $x(T^H - k - 1; \delta_1) < x(T^H - k - 1; \delta_2)$. This completes the proof of part (i).

Given that for $T < T^H$, $x(T; \delta)$ is strictly increasing in δ and noting from (A8) that $W_i(T; \delta) = \frac{1}{1-\delta} B(x(T; \delta); 1)$, we conclude that $W_i(T; \delta)$ is strictly increasing in δ.

To prove the last part of Proposition 4, let x_1 be the unique solution to $B(x_1; 1) = v$. Recall that $x(T^H - 1)$ uniquely solves $B(x(T^H - 1); \delta) = v$. Since $B(x; \delta)$ is continuous both in x and δ, $x(T^H - 1) \to x_1$ as $\delta \to 1$. Now suppose for some $k \geq 1$, $x(T^H - k) \to x_1$ as $\delta \to 1$. This means that as $\delta \to 1$, we have $\overline{W}_i(T^H - k) \to v$, again since $B(x; \delta)$ is continuous both in x and δ. Given that $x(T^H - k - 1)$ solves $\overline{W}_i(T^H - k) = B(x(T^H - k - 1); \delta)$, we have $x(T^H - k - 1) \to x_1$ as $\delta \to 1$. *Q.E.D.*

Proof of Lemma 3.

Lemma 3 is proved in exactly the same way as in the proof of Lemma 1 by noting the optimal $x_i(T) = x^{**}(T)$ in (9) and (10). *Q.E.D.*

Lemma A1. For $N > 1$ and $x \in (0, \overline{c}]$, $B(x; \delta) > B^{**}(x; \delta)$.

Proof of Lemma A1. Take any $N > 1$ and $x \in (0, \overline{c}]$. Suppose $B(x; \delta) \leq B^{**}(x; \delta)$. That is,

$$\frac{x}{\delta\left[1 - F(x)\right]^{N-1}} - \int_0^x [1 - F(c)]dc \leq \frac{x}{N\delta\left[1 - F(x)\right]^{N-1}} - \int_0^x [1 - F(c)]dc + \left(1 - \frac{1}{N}\right) x \left[1 - F(x)\right]$$

which implies that

$$0 \leq \left(1 - \frac{1}{N}\right) x \left[(1 - F(x)) - \frac{1}{\delta\left(1 - F(x)\right)^{N-1}}\right] < 0,$$

a contradiction. Thus, $B(x; \delta) > B^{**}(x; \delta)$. *Q.E.D.*

Proof of Proposition 5.

First note the following comparative static by applying the Envelope Theorem on (9):

$$\frac{\partial W^*(T)}{\partial \Delta W^{**}(T)} = \frac{\delta}{1-\delta}\left[1-(1-F(x^{**}(T))^N\right] > 0 \tag{A23}$$

Now consider a backward induction and suppose $W_i(T^H-1) \geq W^{**}(T^H-1)$. Since $W_i(T^H) = W^{**}(T^H) = \frac{v}{1-\delta}$, we must have $\Delta W_i(T^H-1) \leq \Delta W^{**}(T^H-1)$. Suppose for a moment that $\Delta W_i(T^H-1) = \Delta W^{**}(T^H-1)$. In equilibrium, (2), (10), and $N>1$ imply that $x^{**}(T^H-1) > x(T^H-1)$, in particular $x^{**}(T^H-1) \neq x(T^H-1)$. However, choosing $x_i = x(T^H-1)$ is feasible in (9) and it is not chosen. Then, we must have $W_i(T^H-1) < W^{**}(T^H-1)$, contradicting our original supposition. Thus, $W_i(T^H-1) < W^{**}(T^H-1)$. The same argument applies for $W_i(T^H-1) > W^{**}(T^H-1)$ due to (A23).

To complete the induction argument, suppose, for some $k \geq 1$, that $W_i(T^H-k) < W^{**}(T^H-k)$ and, on the contrary, that $W_i(T^H-k-1) \geq W^{**}(T^H-k-1)$. This implies $\Delta W_i(T^H-k-1) < \Delta W^{**}(T^H-k-1)$. Applying a similar argument to that above, we conclude that $W_i(T^H-1) < W^{**}(T^H-1)$,completing the proof of part (i).

To prove part (ii), take any $T < T^H$ and assume that $x(T) \geq x^{**}(T)$. This implies

$$\overline{W}^{**}(T) = B^{**}(x^{**}(T);1) \leq B^{**}(x(T);1) < B(x(T);1) = \overline{W}_i(T)$$

where the first inequality follows from $B^{**}(x;1)$ being strictly increasing in x and the second follows from Lemma A1. However, this contradicts part (i).

The fact that $\lim_{\delta\to 1} x^{**}(T;\delta) = x_1^{**}$ where $B^{**}(x_1;1) = v$ is proved the same way as in the last part of Proposition 4. Also, Lemma A1 further implies that $x_1 < x_1^{**}$.

To prove the last part, recall that $x(T^H-1;N)$ uniquely solves $B(x(T^H-1;N);\delta) = v$. Since $\lim_{N\to\infty} x(T^H-1;N) = 0$ and $\overline{W}_i(T^H-1) = B(x(T^H-1;N);\delta)$, we have $\lim_{N\to\infty} \overline{W}_i(T^H-1) = \delta v$. Using this argument inductively, we find that $\lim_{N\to\infty} \overline{W}_i(T^H-k) = \delta^k v$ or $\lim_{N\to\infty} W_i(T^H-k) = \frac{\delta^k v}{1-\delta}$ for $k \geq 1$. A similar argument for the benchmark case shows $\lim_{N\to\infty} W_i^{**}(T^H-k) = \frac{\delta^k v}{1-\delta}$. *Q.E.D.*

Proof of Proposition 6.

Given that $B^C(x;\delta)$ is strictly increasing in x, the proofs of the existence of a unique MPE and part (i) follow the same lines as the proof of Proposition 1. However, in the complementary case, we have $B^C(0;\delta) = \frac{1-\delta}{\delta f(0)} > 0$ for $\delta < 1$ rather than $B(0;\delta) = 0$ as in the perfect substitute case. Therefore, whenever $\overline{W}(T) \leq \frac{1-\delta}{\delta f(0)}$, the unique symmetric equilibrium is $x(T) = 0$. Define T^* as being the lowest state satisfying $\overline{W}(T^*) > \frac{1-\delta}{\delta f(0)}$.

Given that part (i) holds and that $B^C(x;1)$ is strictly increasing in x, for $T \geq T^*$, $W_i(T)$ strictly increases in T. Furthermore, in equilibrium $\Delta W(T) = \frac{x(T)}{\delta F(x(T))}$. Since, due to Assumption 2, the right hand side is increasing in $x(T)$ and ,due to part (i), $x(T)$ is increasing in T, so is $\Delta W(T)$ in T, proving part (ii).

Proofs of parts (iii) and (iv) follow the same lines as in the proof of Proposition 4. *Q.E.D.*

9 APPENDIX B

Here, I develop a public good model much like the one presented in the text but with continuous contributions and complete information.[32] I show without detailing the full formal argument that the same qualitative results, in particular the encouragement effect can be obtained in such a model.

Suppose agent i contributes a continuous amount $x_i \geq 0$. For simplicity, the marginal cost of contribution is $c > 0$ for all agents and remains constant over time. The project moves from state T to $T+1$ with probability $\Pi = \Pi(\sum_{j=1}^{N} x_j)$ where I assume $\Pi'(.) > 0$, $\Pi''(.) < 0$, $\Pi'(0) = \infty$, and $\Pi(0) = 0$. That is, agents' contributions are perfect substitutes and the production function exhibits diminishing marginal returns.

It is clear that when the project is completed, no agent will contribute and thus in equilibrium:

$$\overline{W}_i(T^H) = v \tag{B1}$$

For states $T < T^H$, agent i solves the following dynamic program:

$$W_i(T) = \max_{x_i} \left\{ -cx_i + \delta W_i(T) + \Pi(\sum_{j=1}^{N} x_j)\delta \Delta W_i(T) \right\} \tag{B2}$$

Maximizing (B2) requires[33]

$$-c + \Pi'(\sum_{j=1}^{N} x_j)\delta \Delta W_i(T) = 0 \tag{B3}$$

[32] Incomplete information can be introduced like in Menezes, Monteiro, and Temimi (2001). However, this would not change the main message of the paper.

[33] The second order conditions are satisfied given that $\Pi''(.) < 0$ and $\Delta W_i(T) > 0$ in equilibrium.

Now define $\widetilde{B}(x;\delta) \equiv \frac{1}{\delta \Pi'(Nx)} - \frac{1-\Pi(Nx)}{\Pi'(Nx)} - x$. Assuming symmetric equilibrium, the equilibrium contributions are found by the following pair of recursive equations:

$$\begin{aligned} \overline{W}_i(T+1) &= c\widetilde{B}(x(T);\delta) \\ \overline{W}_i(T) &= c\widetilde{B}(x(T);1) \end{aligned} \tag{B4}$$

Since $\widetilde{B}(x;\delta)$ is strictly increasing in x with $\widetilde{B}(0;\delta) = 0$, (B4) generates the unique symmetric MPE. From here, it immediately follows that $x(T)$ is increasing in T for $T < T^H$, which also highlights the presence of the encouragement effect. To see this, suppose every agent but i increase his contribution in a given state. This has two opposing effects on agent i's decision: First, it reduces the marginal return on agent i's contribution (due to $\Pi''(.) < 0$) and thus facilitates the free-riding incentive. However, second this also brings the future returns closer and encourages agent i to further his contribution. The latter incentive mitigates the former and therefore agent i increases his contribution.

Other results in the paper can be derived following similar lines in the proofs. Detailed formal analysis is available upon request.

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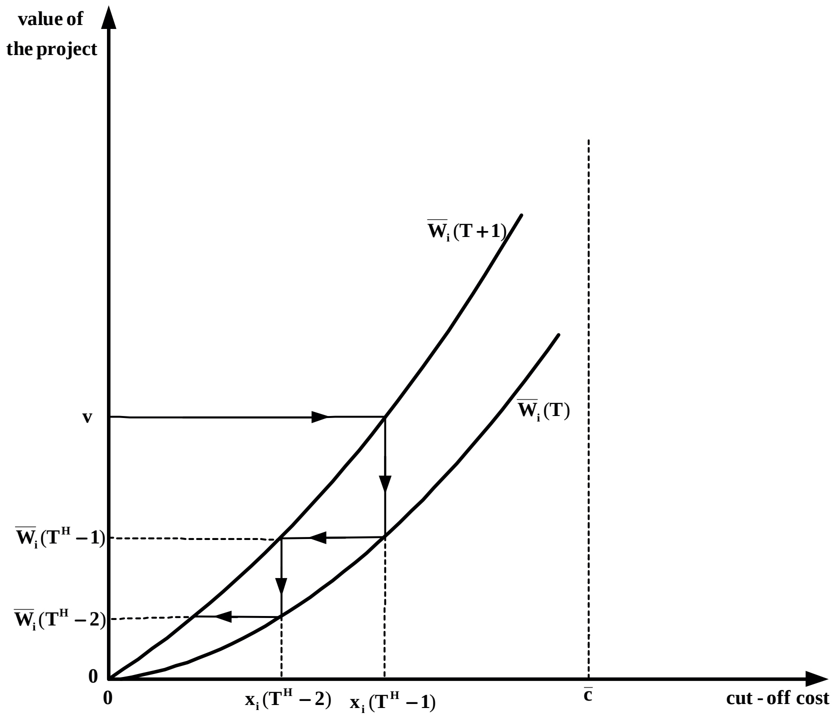
value of
the project
v
$\overline{W}_i(T+1)$
$\overline{W}_i(T)$
$\overline{W}_i(T^H-1)$
$\overline{W}_i(T^H-2)$
0
0
$x_i(T^H-2)$
$x_i(T^H-1)$
$\bar{c}$
cut - off cost

Figure 1

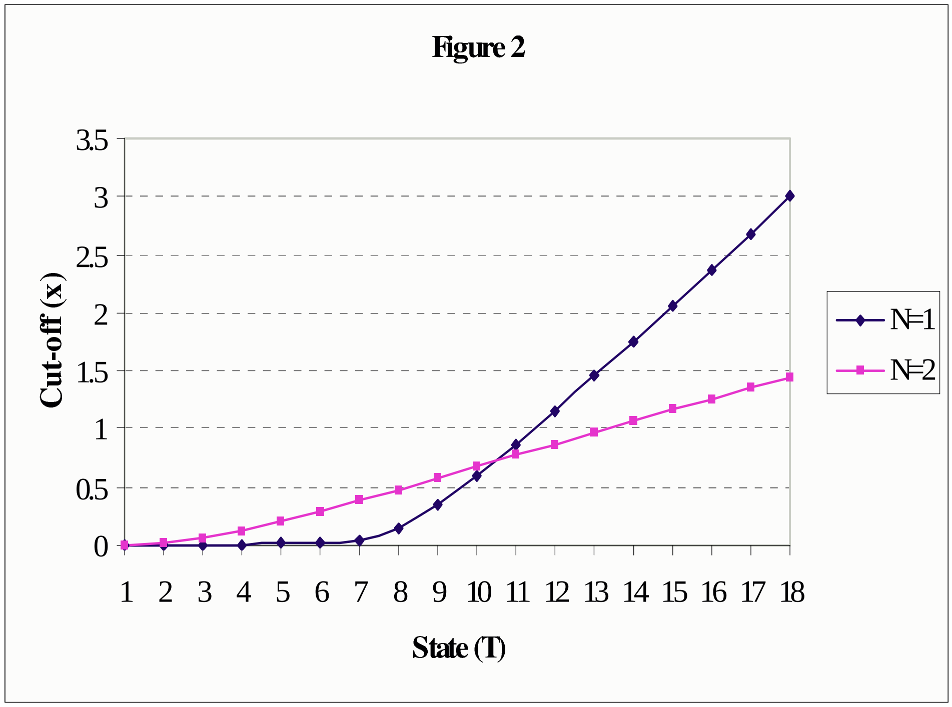
Figure 2
Cut-off (x)
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
State (T)
N=1
N=2

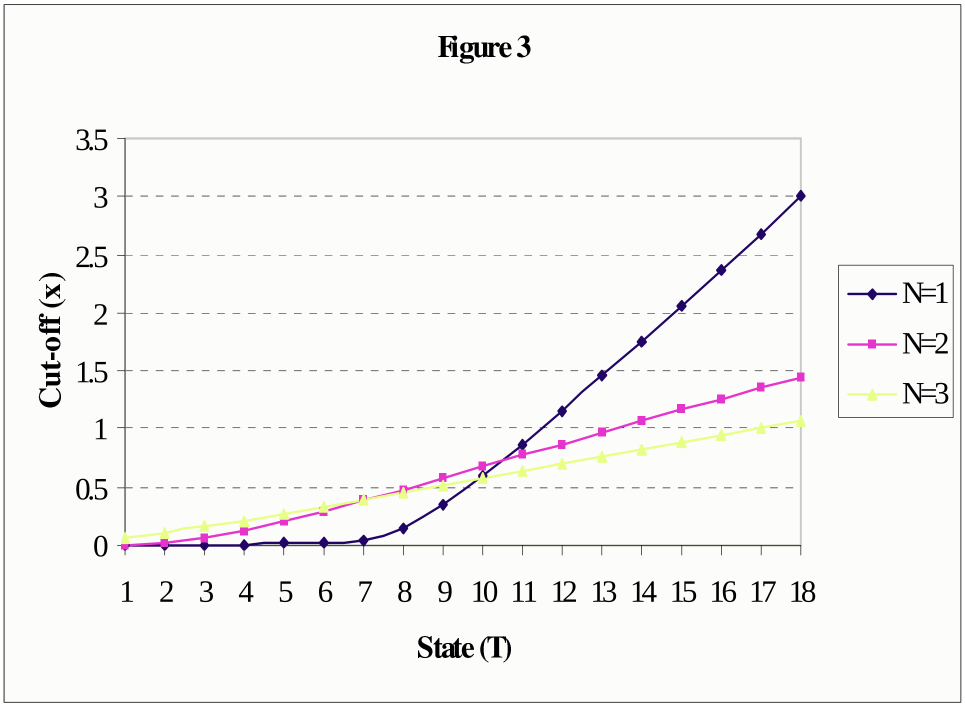
Figure 3
Cut-off (x)
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
State (T)
N=1
N=2
N=3

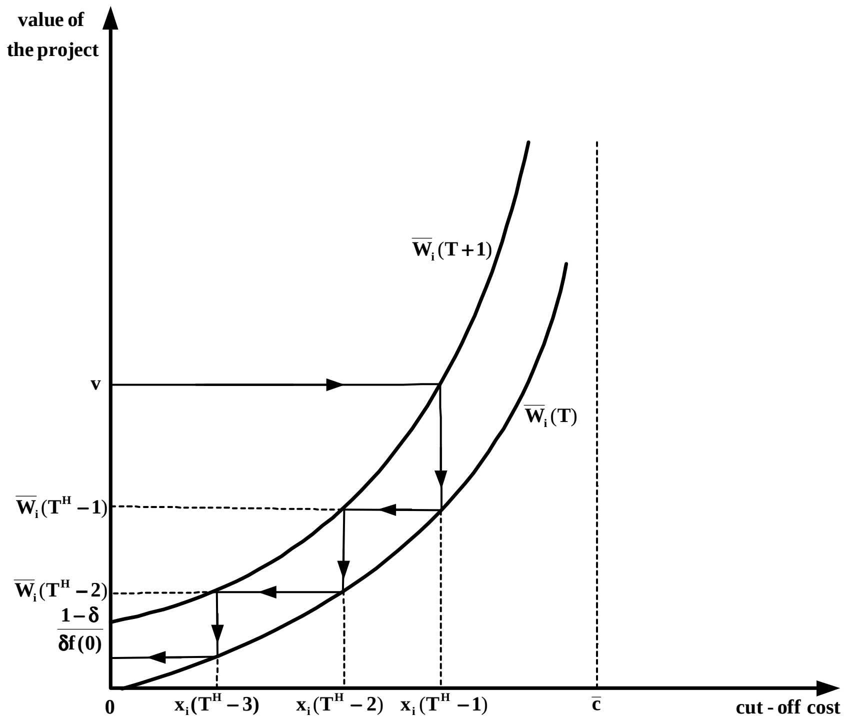
value of
the project
$\overline{W}_i(T+1)$
v
$\overline{W}_i(T)$
$\overline{W}_i(T^H-1)$
$\overline{W}_i(T^H-2)$
$\frac{1-\delta}{\delta f(0)}$
0
$x_i(T^H-3)$
$x_i(T^H-2)$
$x_i(T^H-1)$
$\bar{c}$
cut - off cost

Figure 4

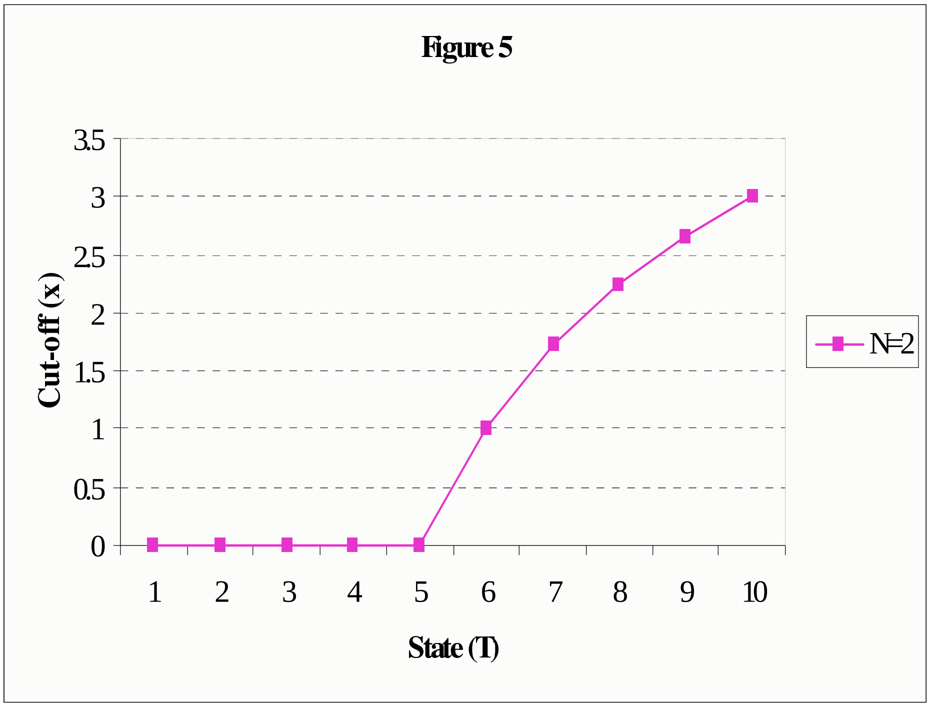
Figure 5
Cut-off (x)
3.5
3
2.5
2
1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10
State (T)
N=2