Estimating Consumption Economies of Scale, Adult Equivalence Scales, and Household Bargaining Power

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Abstract

On average, how much income would a woman living alone require to attain the same standard of living that she would have if she were married? What percentage of a married couples expenditures are controlled by the husband? How much money does a couple save on consumption goods by living together versus living apart? We propose a model of household behavior that permits identification and estimation of concepts like these. We model households in terms of the utility functions of its members and a consumption technology function. We demonstrate generic identification of the model, and hence of equivalence scales, consumption economies of scale, household members' bargaining power and other related concepts. Our empirical application of the model uses data from the UK Family Expenditure Surveys.

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1 Introduction

We propose a model of household consumption behavior in which z, the n vector of consumed quantities of goods, is first transformed into a n vector of private good equivalents x. These private good equivalents are then divided up between the household members, with each member deriving utility from consuming their share of x.

The transformation from z to x, which we call the household's consumption technology, embodies all of the technological economies of scale and scope that result from living together. For a purely private good k, e.g., clothing, for which there is no shared consumption, x_k could equal z_k. For a good k that is shared, e.g., automobile use, x_k might equal $f_k z_k$, where $f_k - 1$ represents the fraction of time that the good is consumed jointly. More generally f_k could be an arbitrary function of z, implying that the fraction of time that the car is consumed jointly depends on the total quantity of car use, and on the quantity of other goods (e.g., vacations and food consumed away from home).

This consumption technology can also incorporate some kinds of changes in tastes that result from living together. For example, if $x_k = f_k z_k$ where k is food eaten at home, then f_k could embody economies of scale (such as less waste in food preparation) but f_k would also differ from one if couples preferred eating at home more than singles do. We will later describe in more detail the kinds of taste changes that may incorporated into the household's consumption technology.

This type of consumption technology is the motivation behind many existing household consumption models, such as Barten scales. In those models, the utility function of the household is assumed to equal the utility function of a single individual, evaluated at the transformed quantities. Our model differs from this earlier literature in that we explicitly divide the transformed quantities amongst the household members, each of whom has their own utility function.

Like the Barten scaling model, our model provides a way to combine the consumption data of singles and marrieds. However, it additionally allows us to estimate important features of the household's behavior, such as the relative bargaining power of the members. We will also use the model to estimate economies of scale in consumption, and household equivalence scales.

To keep notation as simple as possible, let superscripts refer to household members and subscripts refer to goods. Let $U^i(x^i)$ be the direct utility function for a consumer i, consuming the vector of goods $x^i = (x_1^i, ..., x_n^i)$. We will consider households consisting of two consumers, which we will for convenience refer to as the husband ($i = m$) and the wife ($i = f$). For many applications, it will be convenient to interpret one of these utility functions as a joint utility function for all but one member of the household, e.g., U^f could be the joint utility function of a wife and her children.

The proposed model of household consumption behavior is the optimization program

$$\max_{x^f, x^m, z} \widetilde{U}[U^f(x^f), U^m(x^m), p] \tag{P*}$$

$$z = F(x^f + x^m)$$

$$p'z = 1$$

where z is the vector of quantities of the n goods the household purchases, $p = \widetilde{p}/y$ where $\widetilde{p}$ is the n vector of prices the household faces and y is the household's total expenditures, and $\widetilde{U}$ is either a social welfare function for the household, or a bargaining model. The function F is the inverse of a consumption technology function, and the unobserved n vectors x^f and x^m are private good equivalent vectors, that is, they are the quantities of transformed goods that are consumed by the female and male, respectively.

In addition to changes in utility resulting from living together that are contained in F, the model also permits arbitrary changes in utility that are separable from goods consumption, e.g., one could be generally happier as a result of living together, and e.g., a wife's utility could depend on both her own attained utility level over goods (the value of U^f) and on U^m. These effects are assumed to be absorbed into the function $\widetilde{U}$.

We will show that the model is generically identified for arbitrary technology functions F, but for simplicity our applications and examples will focus on linear consumption technologies, defined by

$$F(x) = Ax + a$$

where A is a nonsingular n by n matrix and a is a n vector. Note the close analogy to Gorman's general linear technologies.

One way to interpret our model is to consider two extreme cases. If all goods were private then $F(x) = x$, so z would equal $x^f + x^m$ and the program P^* would reduce to $\max_{x^f} \widetilde{U}[U^f(x^f), U^m(z - x^f), p]$ such that $p'z = 1$, which is just an ordinary collective model. At the other extreme, imagine a household that has a consumption technology function F, but assume that the household's utility function for transformed goods just equaled $U^m(x)$, as might happen if the male were a dictator that forced the other household member to consume goods in the same proportion that he does. In that case, the model would reduce to $\max_z U^m[F^{-1}(z)]$ such that $p'z = 1$, which for linear F is equivalent to Gorman's general linear technology model, a special case of which is Barten scaling (corresponding to A diagonal and a zero). Our general model, program P^*, combines the consumption technology logic of the Gorman or Barten framework with the model of a household as either a bargaining or a social welfare maximizing group.

Given the private good equivalents vector $x = x^f + x^m$ from the model, one may also calculate $\widetilde{p}'z/\widetilde{p}'x$, which is a measure of the overall economies of scale from living together, since $\widetilde{p}'z = y$ is what the household spends on purchased goods, and $\widetilde{p}'x$ equals the cost of buying the private good equivalents x that are constructed from the purchased goods z.

Let η, which we call the intrahousehold income sharing rule, be the fraction of the household's resources that is consumed by member f, and so can be interpreted as a measure of the relative bargaining power of member f in the household. Unlike Browning and Chiappori (1998), who find using household data alone that η can only be identified up to an arbitrary location, we show in our model that combines data from households and from singles living alone that η is completely identified.

Given x^i, one could also calculate y^{i*}, defined as the minimum expenditures required at prices p to buy a vector of goods that is on the same member i indifference curve as x^i. The ratio of y/y^{i*} is then an adult equivalence scale, since y is what the household spends to get member i onto the same indifference curve over goods that he or she could attain while living alone with an income level of y^{i*}.

We show that the private good equivalents x^f and x^m, the bargaining function η, and the household technology function F can all be identified and estimated from ordinary demand data on singles and couples. In addition, the above defined economies of scale in consumption from living together and adult equivalence scales are also identified, (though these economy of scale and adult equivalence scale measures will include taste changes as well as pure cost comparisons to the extent that F includes taste changes).

Essentially, in our model data from single women and men identify the Marshallian demands associated with the utility functions U^f and U^m. These are then combined with the household's demand functions to recover the technology function F and the sharing rule η, which in turn permits calculation of the rest of the features of the model.

An enormous literature exists on specification and estimation of household bargaining models and demand systems, and on identification (and lack thereof) of equivalence scales and similar household welfare measures. Rather than directly cite volumes of that literature, we refer readers to surveys including Deaton and Muellbauer (1980), Blundell (1988), Browning (1992), Pollak and Wales (1992), Blundell, Preston, and Walker (1994), Lewbel (1997), Jorgenson (1997), Slesnick (1998), and Vermeulen (2000).

The next section summarizes the main theoretical results of the paper, for the most part using the special case of linear technologies for clarity. Later sections then provide more formal derivations, describe our empirical implementation of the model, and discuss possible extensions and variants.

2 The Model and Applications

This section describes the basic form of the proposed household model, along with its specification and identification. Next the role of our consumption technology function is discussed in detail. We then describe application of the model to estimation of the relative bargaining power of the household members, calculation of economies of scale from living together, and adult equivalence scales. The purpose of this section is expository, so results will be stated here without proofs. Formal statements and derivations are provided in later sections.

When living alone instead of in a household, household member i would maximize a utility function $U^i(z^i)$ facing prices $\widetilde{p}$ with income level y^i, and so would solve the optimization program

$$\max_{z^i} U^i(z^i) \qquad (\mathrm{P}^i)$$

$$\widetilde{p}' z^i = y^i$$

Let $z^i = z^i(\widetilde{p}/y^i)$ denote the solution to this individual optimization program, so $z^i = z^i(\widetilde{p}/y^i)$ are the Marshallian demand functions of member i when living alone. Define V^i by

$$V^i(\widetilde{p}/y^i) = U^i[z^i(\widetilde{p}/y^i)] \qquad (1)$$

so V^i is the indirect utility function of member i when living alone. The functional form of individual demand functions z^i can be obtained from a specification of V^i using Roy's identity. The Marshallian demand functions $z^m()$ and $z^f()$ will be estimated using ordinary demand data on observed prices, incomes, and quantities purchased for individuals living alone.

Now consider the household's demand functions, which we denote as $z = z(\widetilde{p}/y) = z(p)$. The household's optimization program is the program P*. The solution of this program yields the household demand functions $z = z(p)$, and also private good equivalent demand functions $x^i = x^i(p)$. The vector valued demand function $z()$ will be estimated using ordinary demand data on observed prices, incomes, and quantities purchased by the household.

2.1 The Consumption Technology Function

If there were no sharing of goods then z would equal $x^f + x^m$ and our model given by program P* would be an ordinary collective model. The additional features we add are that the utility functions of the members U^f and U^m (actually, just their subutility functions over purchased goods) are the same as the utility functions they would have if they were single, and the existence of a consumption technology function F, which is primarily intended to model economies of scale and scope in consumption, or equivalently to describe degrees of publicness or privateness of goods. The function F also permits some kinds of changes in preferences over goods when people live as couples.

The functon $z = F(x)$ is the household's *inverse consumption technology function*, meaning that if the household buys the vector of quantities z, then the members by living together can get from z the equivalent amount of consumption as they would get by splitting up quantities $x = F^{-1}(z)$, and consuming the results privately.

Consider some examples. Let good j be gasoline for automobiles. Suppose the husband used 200 gallons of gasoline driving alone, the wife used 300 gallons driving alone, and the couple used 500 gallons while riding together in the car. Then the couple bought $z_j = 1000$ gallons total, the husband used $x_j^m = 700$ gallons total (200 by himself and 500 jointly), and the wife used $x_j^f = 800$ gallons total (300 by herself and 500 jointly). In this example, $x_j = 1500$ gallons, meaning that, through the economies of scale of joint (shared) consumption, The household buying $z_j = 1000$ gallons of gasoline got the same consumption as if they had purchased $x_j = 1500$ gallons and always drove separately. More generally, if the members travel by car together $100r$ percent of the time, then $x_j = (1 + r)z_j$, so $z_j = F_j(x) = x_j/(1 + r)$. This example is similar to the usual motivation for Barten scales, but it is operationally more complicated, because Barten scales fail to distinguish the separate utility functions of the household members.

More complicated consumption technologies can arise in a variety of ways.

The fraction of time r that the couple share the car could depend on the total usage, resulting in F_j being a nonlinear function of x_j. There could also be economies (or diseconomies) of scope as well as scale in the consumption technology, e.g., the shared travel time percentage r could be related to expenditures on vacations, resulting in $F_j(x)$ being a function of other elements of x in addition to x_j.

We can interpret z as the inputs and x as the outputs in the household's consumption technology, described by the production function F^{-1}, though what is being 'produced' is consumption. Define $x = F^{-1}(z)$ to be the *private good equivalents* to z.

Consumption technologies can also control for differences in preferences between singles and households. For example, married couples may tend eat at home, and to prepare meals from scratch, more often than singles do. This could be because couples desire such meals more than they would as singles, or it could be due to economies of scope and scale in food preparation. Either effect might be included in the consumption technology.

2.2 Identification

The identification question is, given the observable demand functions $z^m()$, $z^f()$, and $z()$, can the consumption technology function F, and the private good equivalent demand functions $x^f(p)$ and $x^m(p)$ be estimated? We show that these functions are nonparametrically 'generically' identified, meaning that identification will only fail if the utility and technology functions are too simple, for example, F is not identified if it is linear and demands are of the linear expenditure system form. This identification in turn implies (albeit with some caveats) identification of features of the model including the sharing rule, economies of scale, and adult equivalence scales.

A question closely related to identification is, given functional forms for the technology, the sharing rule, and the Marshallian demands of singles, what is the implied functional form for household demands? This also is generically identified, and has a simple, explicit solution (given in the next subsection) in the case of linear consumption technologies.

2.3 The Form of Household Demands

Consider the special case of a linear consumption technology, so $F(x) = Ax + a$, where A is a nonsingular n by n matrix and a is an n vector. The household's optimization program is then

$$\max_{x^f, x^m, z} \widetilde{U}[U^f(x^f), U^m(x^m), p]$$

$$z = A(x^f + x^m) + a$$

$$p'z = 1$$

With this model, it will be shown that for any function $\eta(p)$ that lies strictly between zero and one, there exists a utility or bargaining function $\widetilde{U}$ such that the household demand function arising from the above household optimization program is

$$z(p) = Az^f\left(\frac{A'p}{1-a'p}\frac{1}{\eta(p)}\right) + Az^m\left(\frac{A'p}{1-a'p}\frac{1}{1-\eta(p)}\right) + a \qquad (2)$$

The function $\eta(p)$, which we call the intrahousehold sharing rule, is a measure of the relative bargaining power of member f versus m implied by the function $\widetilde{U}$. Given estimates of ordinary demand functions z^f and z^m from singles data, equation (2) may then be estimated from household demand data, where the parameters to be estimated are the technology parameters A and a, and any parameters in the sharing rule $\eta(p)$.

Define shadow prices $\pi(p)$ by

$$\pi(p) = \frac{A'p}{1-a'p}$$

The private good equivalents consumed by the household members are given by

$$x^f = x^f(p) = z^f\left(\frac{\pi(p)}{\eta(p)}\right)$$

$$x^m = x^m(p) = z^m\left(\frac{\pi(p)}{1-\eta(p)}\right)$$

The above definitions of π, x^f, and x^m make shadow income $\pi'(x^f + x^m) = 1$. The household's behavior is equivalent to allocating the fraction $\eta^f = \eta(p)$ of this shadow income to member f, and the fraction $\eta^m = 1 - \eta(p)$ of shadow income to member m. Each member i then maximizes their own utility function U^i given the shadow price vector π and their own shadow income η^i to calculate their desired private good equivalent consumption vectors x^f and x^m. The household purchases the vector of goods $z = F(x^m + x^f)$.

2.4 Relative Bargaining Power

We will show later that the household's objective function

$$\max_{x^f, x^m, z} \widetilde{U}[U^f(x^f), U^m(x^m), p]$$

can in general be rewritten as

$$\max_{x^f, x^m, z} \mu(p)U^f(x^f) + U^m(x^m)$$

for some function $\mu(p)$. The larger the value of $\mu = \mu(p)$ is, the greater is the weight that member f's preferences receive in the resulting household program, and the greater will be the resulting private equivalent quantities x^f versus x^m. A difficulty with using μ as a measure of the weight given to (or the bargaining power of) member f is that the magnitude of μ will depend on the arbitrary cardinalizations of the functions U^f and U^m.

A more direct measure of the weight given to member f is the sharing rule $\eta(p)$ which was defined as the fraction of total shadow expenditures that are devoted to her vector of private good equivalents x^f, and equals a number from zero to one. We will show later that there is a one to one relationship between $\mu(p)$ and $\eta(p)$. Given estimates of scaled shadow prices π and private good equivalents x^f and x^m, the sharing rule η is given by $\eta = \pi' x^f$.

More simply, in our empirical implementation the sharing rule η is directly parameterized and estimated as part of the household's demand functions. The simplest specification would simply let η equal a constant to be estimated. Alternatively, η could be specified as a function of variables that affect bargaining power, such as the relative incomes of the household members. This will imply that the utility weighting function μ depend on such variables.

2.5 A Measure of Economies of Scale in Consumption

Given estimates of the private good equivalents vector $x = x^f + x^m$ from the model, we may calculate $p'z/p'x$, which is a measure of the overall economies of scale from living together, since $p'z$ is what the household spent, and $p'x$ would be the cost of buying the private good equivalents of the purchased goods z.

In the case of the linear household technology, this economy of scale measure takes the form

$$\frac{p'z}{p'x} = \frac{p'\left(Az^f[\pi/\eta] + Az^m[\pi/(1-\eta)] + a\right)}{p'\left(z^f[\pi/\eta] + z^m[\pi/(1-\eta)]\right)}$$

where $\pi = A'p/(1 - a'p)$.

2.6 Adult Equivalence Scales

Equivalence scales seek to answer the question, ”how much money does a household need to spend to be as well off as a single male living alone?” The equivalence scale itself is then the expenditures of the household divided by the expenditures of the single male that enjoys the same ”standard of living” as the household.

Equivalence scales have many practical applications. For example, given a poverty line for single individual, that line may be multiplied by a household equivalence scale to obtain a comparable poverty line for the household. Income inequality measures should be applied to equivalence scaled income to adjust for household composition.(see, e.g., Jorgenson 1997). Calculation of appropriate levels for alimony or life insurance require comparing costs of living for couples versus those of singles. Similarly, Lewbel and Weckstein (1995) show that United States legal statutes regarding appropriate compensation in cases of wrongful death can be interpreted as requiring an equivalence scale calculation.

The early equivalence scale literature attempted to define this ratio of costs of living directly in terms of measurable quantities such as the costs of acquiring a required number of calories, but this was soon replaced by the concept of defining the household and the single male as equally well off to mean that they attain an equal level of utility (see, e.g., Lewbel 1997 for a survey). Just as a true cost of living price index measures the ratio of costs of attaining the same utility level under different price regimes, equivalence scales were supposed to measure the ratio of costs of attaining the same utility level under different household compositions.

Unfortunately, unlike true cost of living indexes, equivalence scales defined in this way can never be identified from revealed preference data (that is, from the observed expenditures of different households under different price and income regimes). The reason is that defining a household to have the same utility level as a single male requires that the utility functions of the household and of the single male be comparable. We cannot avoid this problem by defining the household and the single to be equally well off when they attain the same indifference curve, analogous to the construction of true cost of living indices, because the household and the single have different preferences and hence different shaped indifference curves. Pollak and Wales (1992) describe these identification problems in detail, while Blundell and Lewbel (1991) prove that only changes in traditional equivalence scales, but not their levels, can be identified by revealed preference.

Yet another difficulty with the standard equivalence scale definition is that a household may not possess a traditional utility function, but may instead use some kind of bargaining process to determine its purchases.

We argue that the source of these identification problems is that the equivalence scale question, ”how much income would a single individual need to attain the same utility level as a household,” is badly posed. The appropriate question to ask is, ”how much income would an individual living alone need to attain the same indifference curve that the individual attained as a member of the household?” This latter question avoids issues of interpersonal comparability and differences in indifference curves, and hence is at least in principle answerable from revealed preference data.

To distinguish this concept from traditional equivalence scales, we define a household member i's *intrahousehold based equivalent income (or expenditure)* to be the income or total expenditure level y^{i*} required by household member i, when living alone, to attain the same standard of living (i.e., the same indifference curve over goods) that he or she had while living in the household. Member i's ”*intrahousehold based equivalence scale* would then be y/y^{i*} the ratio of the household's total expenditures to member i's intrahousehold based equivalent total expenditures.

In addition to avoiding unobservable identifying assumptions regarding interpersonally comparable or cardinalizable utility, intrahousehold based equivalence scales also do not assume that all household members are equally well off, that is, they permit households to use bargaining models.

To see the appropiateness of intrahousehold based equivalent income, consider the question of determining an appropriate level of life insurance for a spouse. If the couple spends y dollars per year then for a nonworking wife to maintain the same standard of living after a working husband dies, she will need an insurance policy that pays enough to permit spending y^{f*} dollars per year. Similarly, in cases of wrongful death, juries are instructed to assess damages both to compensate for the loss in ”standard of living,” (i.e., y^{i*}) and, separately for ”pain and suffering,” which would presumably be noneconomic effects that could be embodied in the function $\widetilde{U}$.

If there were no shared consumption goods, we could directly observe the consumption of any household member and simply add up the cost of buying what he or she consumed within the household to determine the money that person would need to spend to as a single to attain the same standard of living. The difficulty in calculating the intrahousehold based equivalence scales arises from shared consumption. We overcome this obstacle by identifying and estimating the household's consumption technology function.

Recall that $V^i(\widetilde{p}/y^i)$ is the indirect utility function of member i when living alone, and the private good equivalent vector consumed by member i while in the household is $x^i(p) = z^i\left[\pi(p)/\eta^i(p)\right]$, where $\eta^f(p) = \eta(p)$ and $\eta^m(p) = 1 - \eta(p)$. Define $y^{i*}(\widetilde{p}, y)$ to be the intrahousehold based total expenditures for member i, defined as the total expenditures member i would require, when living alone, to attain the same standard of living (i.e., the same indifference curve over goods) that he or she attains as a member of the household. It follows immediately from these definitions that $y^{i*}(\widetilde{p}, y)$ for $i = m$ and $i = f$ may be calculated as the solution to the equation

$$V^i\left(\frac{\widetilde{p}}{y^{i*}(\widetilde{p}, y)}\right) = V^i\left(\frac{\pi(\widetilde{p}/y)}{\eta^i(\widetilde{p}/y)}\right) \tag{3}$$

Note that this definition is ordinal, i.e., it does not depend on the chosen cardinalization for member i's utility function. In an application, functional forms for V^i and η^i would be directly specified, and in the example of a linear technology we have $\pi(\widetilde{p}/y) = A'\widetilde{p}/(y - a'\widetilde{p})$.

Given $y^{i*}(\widetilde{p}, y)$, we may calculate equivalence scales $y/y^{i*}(\widetilde{p}, y)$ or the difference in income required, $y - y^{i*}(\widetilde{p}, y)$. For example, at prices $\widetilde{p}$, y would equal the amount of money that the household must spend so that the man in that household has the same standard of living (i.e., attains the same indifference curve over goods) that he would attain if he were to live alone with total expenditures $y^{m*}(\widetilde{p}, y)$. Similarly, in the case of a wrongful death or insurance calculation, a woman (or a mother and her children, if V^f is defined as a joint utility function of a mother and her children) would need income $y^{f*}(\widetilde{p}, y)$ to attain the same standard of living without her husband that she (or they) attained while in the household with the husband and total expenditures y. This would be be sufficient to compensate for the loss of economies of scale and scope from shared consumption (and for any changes in preferences over goods that result from living in the household), but would not account for grieving, loss of companionship, or other components of utility that are assumed to be separable from consumption of goods.

Another interesting equivalence scale to construct might be y^{f*}/y^{m*}, the ratio of how much income a woman needs when living alone to the income a man needs when living alone to make each as well off as they would be in a household. Interestingly, even if men and women had identical preferences, this ratio would not equal one in general, because for example if the sharing rule η is bigger than one half, then the wife received more than half of the household's resources, and hence would need more income when living alone to attain the same standard of living.

To separate bargaining effects from other considerations, other measures of intrahousehold based equivalent income could be calculated. For example, the ratio y^{f*}/y^{m*} might better match the intuition of an equivalence scale if each y^{i*} equation were calculated as the solution to

$$V^i\left(\frac{\widetilde{p}}{y^{i*}(\widetilde{p},y)}\right) = V^i\left(\frac{\pi(\widetilde{p}/y)}{1/2}\right) \tag{4}$$

which gives equivalent incomes assuming equal sharing of resources.

3 The Formal Model

Here some general results regarding the model are stated more formally. Proofs of Lemmas and Theorems provided here are given in the Appendix.

ASSUMPTION A1: The household's consumption technology is described by $z = F(x^f + x^m)$, where z is the vector of consumption goods purchased by the household and F is an invertible, monotonically increasing, differentiable n vector valued function. Each household member i derives utility $U^i(x^i)$ from the vector of private good equivalents x^i. The member utility functions U^i are monotonically increasing, are continuously twice differentiable, and strictly quasiconcave. The corresponding member indirect utility functions V^i are differentiable. The function $\widetilde{U}(U^f, U^m, p)$ is differentiable in U^f and U^m with positive derivatives

The differentiability in assumption A1 simplifies the derivations and results provided here, but is otherwise not required.

ASSUMPTION A2: The household faces the budget constraint $y \geq \widetilde{p}'z$, where $\widetilde{p}$ is a vector of market prices. The household's consumption choice z is Pareto efficient, and does not suffer from money illusion.

Let $p = \widetilde{p}/y$. Assumptions A1 and A2 imply that the household's consumption choice $z = z(p)$ is determined as the solution to an optimization program of the following form, where the function $\widetilde{U}$ is increasing in U^f and U^m.

$$\max_{x^f, x^m, z} \widetilde{U}[U^f(x^f), U^m(x^m), p] \tag{P*}$$

$$z = F(x^f + x^m)$$
$$p'z = 1$$

We denote this optimization as Program P*. Pareto efficiency implies the existence of the function $\widetilde{U}$, which may represent a bargaining model or some kind of family social welfare function. Note that the function $\widetilde{U}$ may depend on income and on market prices, for example, in a bargaining model the relative power of one family member versus the other may depend on income or prices. The absence of money illusion requires the dependence of $\widetilde{U}$ on $\widetilde{p}$ and y to be homogeneous of degree zero, and hence depends on total expenditures and prices only through p. The functions $\widetilde{U}$, U^f and U^m could also depend on other observables, e.g., each U^i might depend on demographic characteristics of member i, and $\widetilde{U}$ could depend on variables that affect the relative bargaining power of the members, such as their relative earnings or wage rates.

The function $\widetilde{U}$ may also implicitly contain non market components of utility that are assumed to be separable from consumption, such as love, altruism, and other benefits that family members obtain from being in the household.

The functions $\widetilde{U}$, U^f and U^m are unobservable. What can be observed are Marshallian demands. We assume that we have observed the consumption patterns both of individuals living alone and of households in a variety of price and income regimes, and so can estimate the Marshallian demand functions of each. Let $z^i(\widetilde{p}/y^i)$ be the Marshallian demand function of individual i living alone with a budget y^i, so $z^i(\widetilde{p}/y^i)$ is the solution to the following optimization program, which we denote Program P^i.

$$\max_{z^i} U^i(z^i) \tag{P^i}$$
$$\widetilde{p}'z^i = y^i$$

ASSUMPTION A3: The household demand function $z(\widetilde{p}/y)$, and the individual Marshallian demand functions $z^f(\widetilde{p}/y^f)$ and $z^m(\widetilde{p}/y^m)$ are known.

Essentially, all Assumption A3 says is that we can observe the demands of singles and households in different price and income states, and so can estimate their Marshallian demand functions. By construction, the functions $z^f(\widetilde{p}/y^f)$ and $z^m(\widetilde{p}/y^m)$ satisfy the usual properties of Marshallian demands, such as Slutsky symmetry. The function $z(p)$ will by construction satisfy the "adding up" and homogeneity properties of ordinary Marshallian demands, but need not satisfy Slutsky symmetry because of the possible presence of p in the bargaining function $\widetilde{U}$. Issues regarding the specification and functional forms for $z(p)$, and its dependence on F, will be discussed later.

The solution to the household's program P* is the household demand function $z(p)$, and the private good equivalent demand functions $x^f(p)$ and $x^m(p)$. The goal is identification (and estimation) of these private good equivalents $x^i(p)$. Given the function $x^i(p)$, define the function $y^{i*}(\widetilde{p}, y)$ by the program

$$y^{i*}(\widetilde{p}, y) = \min_{z^{i*}} \widetilde{p}' z^{i*} \tag{5}$$

$$U^i(z^{i*}) = U^i[x^i(\widetilde{p}/y)] \tag{6}$$

From observable Marshallian demands, the utility function U^i is identified only up to an arbitrary monotonic transformation, but that is sufficient to solve this for y^{i*}.

The function $y^{i*}(\widetilde{p}, y)$ equals the least amount of money that individual i would require, while living alone, to reach the same indifference curve over goods that he or she attained while inside the household, when the household had income y and the price regime is $\widetilde{p}$. As discussed earlier, y^{i*} would be required in a calculation of equivalence scales, or of other economic costs of a divorce, wrongful death, or life insurance requirements (apart from any noneconomic issues e.g., loss of companionship).

Depending on the application, we might also be interested in $\widetilde{p}' x^i$ itself, which is the cost, while living alone, of buying the same bundle that individual i consumes as a household member. So, for example, $y/[\widetilde{p}'(x^f + x^m)]$ is a measure of the economies of scale in consumption achieved by the household.

Lemma 1. Let Assumptions A.1, A.2, and A.3 hold. Define $x(p)$ and $\pi(p)$ by

$$x = x(p) = F^{-1}[z(p)]$$

$$\pi(p) = \frac{DF(x)'.p}{x'p}$$

Then there exists a scalar valued sharing rule $\eta(p)$, with $0 < \eta(p) < 1$, such that

$$x^f(p) = z^f\left(\frac{\pi(p)}{\eta(p)}\right) \tag{7}$$

$$x^m(p) = z^m\left(\frac{\pi(p)}{1 - \eta(p)}\right)$$

$$z(p) = F[x^f(p) + x^m(p)]$$

Lemma 1 above demonstrates the existence of a sharing rule $\eta^f = \eta(p)$, $\eta^m = 1 - \eta(p)$, and shadow prices $\pi = \pi(p)$ that may be used to construct private good equivalents $x^i(p)$.

Lemma 2. Let Assumptions A.1, A.2, and A.3 hold. The household utility or bargaining function $\widetilde{U}[U^f(x^f), U^m(x^m), p]$ is observationally equivalent to $\mu(p)U^f(x^f) + U^m(x^m)$, where the function $\mu(p)$ satisfies

$$\mu(p) = -\frac{\partial V^f(\pi/\eta)/\partial\eta}{\partial V^m[\pi/(1-\eta)]/\partial\eta}$$

Lemma 2 shows that we may without loss of generality posit a household utility or bargaining model of the form $\mu(p)U^f(x^f)+U^m(x^m)$, and shows the relationship between the bargaining weight function $\mu(p)$ and the income sharing rule $\eta(p)$.

3.1 Nonparametric Identification

We now consider nonparametric identification of the model. In particular, we will show that given the observable Marshallian demand functions of singles and households, we can in general recover the private good equivalent functions x^m and x^f, the consumption technology F, and the sharing rule η. As a result, we can identify expressions that are defined in terms of these functions, such as economies of scale to consumption and adult equivalence scales.

Define a good j to be an *exclusive* (or private) good for household member i if $z_j^k(\widetilde{p}/y^k)$ is identically zero for $k \neq i$, and $z_j(p) = x_j(p)$. An exclusive good for member i is a good that is only consumed by member i. Since the other member does not consume any of the good, exclusivity also assumes that there are no economies of scale in that good, so the quantity of that good purchased by the household equals the private equivalent quantity of that good consumed by member i. An example of a private good might be womens clothes, which are only used by the wife. Alternatively if U^f is the joint utility function of the wife and children, and U^m is the utility function of just the husband, then the private good could be a product that is only consumed by children.

Define a good j to be a *monotonic* good for household member i if $z^i_j(\widetilde{p}/y^i)$ is strictly monotonic in y^i. Monotonicity implies for consumer i the good j is either a normal good or an inferior good, but does not permit it to vary, i.e., it does not allow good 1 to be normal at some income levels and inferior at others.

Our generic identification result assumes the existence of a monotonic, exclusive good. This facilitates construction of a mapping used for demonstrating identification of the consumption technology function. In our later empirical application, we do not assume the existence of monotonic, exclusive goods, and instead obtain identification via functional form.

Theorem 1. Let Assumptions A.1, A.2, and A.3 hold. Assume the existence of one monotonic, exclusive good. Then the functions x^m, x^f, F, and η are generically identified.

The meaning of 'generic' identification is provided in the appendix. Essentially, it implies that identification will only fail in demands having a structure that is too simple. For example, a linear technology will not be fully identified if the demand system is the linear expenditure system. A similar case of generic identification, although in a different context, appears in Chiappori and Ekeland (1999).

Given Theorem 1, assume F, η, x^f, and x^m are identified from observable demand functions z^f, z^m, and z. We may therefore immediately construct the economies of scale measure $p'z/p'x$, by

$$\frac{p'z}{p'x} = \frac{p'z(p)}{p'\left[x^f(p) + x^m(p)\right]}$$

this is a measure of the overall economies of scale from living together because y is what the household spent on z, and the denominator would be the cost of buying the private good equivalents of the purchased goods z.

Given the Marshallian demand functions $z^i(\widetilde{p}/y^i)$ for member i, one may by standard methods recover the corresponding member i indirect utility function $V^i(\widetilde{p}/y^i)$, up to an arbitrary monotonic transformation. Given the function V^i, the member i cost function $C^i(\widetilde{p}, u^i)$ is defined by $C^i[\widetilde{p}^i, V^i(\widetilde{p}/y^i)] = y^i$.

The private good equivalents consumed by each member i in the household are given by $x^i(\widetilde{p}/y) = z^i[\pi(\widetilde{p}/y)/\eta^i(\widetilde{p}/y)]$, where $\eta^f = \eta$ and $\eta^m = 1 - \eta$. Based on member i's preferences as a single, at prices $\widetilde{p}$ the least expenditures required for member i to buy a vector of goods that are as good as the private good equivalent vector $x^i(\widetilde{p}/y)$ is given by

$$y^{i*}(\widetilde{p}, y) = C^i \left[\widetilde{p}, V^i \left(\frac{\pi(\widetilde{p}/y)}{\eta^i(\widetilde{p}/y)} \right) \right]$$

which by construction is the minimum expenditure required by member i, while living alone, to attain the same indifference curve over goods that he or she attains as a member of the household. We may then calculate adult equivalence scales such as y/y^{m*} and y^{f*}/y^{m*}.

3.2 Model Specification

An issue related to identification is that of functional form specification. Identification concerns recovery of structural functions, such as F and η, from observable demand functions. Model specification essentially goes in the opposite direction, i.e., given the specification (e.g., functional forms) of the household's program and technology, what is the specification of z?

Analytic solutions of the household's original program P^* are difficult to derive, except for extremely simple cases such as Cobb-Douglas utility functions. The same problem exists in ordinary utility maximization, and the standard solution is to exploit duality, i.e., specifying functional forms for indirect utility functions instead of direct utility functions. We describe here how similar methods can be used for specifying household demands $z(p)$.

Individual member demand functions $z^i(\widetilde{p}/y^i)$ can be constructed by specifying functional forms for the member indirect utility functions $V^i(\widetilde{p}/y^i)$ and applying Roys identity. Unlike traditional equivalence scale calculations, the choice of cardinalization for V^i will not affect our calculation of intrahousehold based equivalence scales.

The remainder of the model entails specification of a technology F and a household utility or bargaining function $\widetilde{U}$. Based on Lemmas 1 and 2, instead of specifying $\widetilde{U}$, consider specifying the sharing rule η. Given specifications (e.g., functional forms) of z^m, z^f, F, and η, then based on Lemma 1 the corresponding household demand function $z(p)$ can be implicitly defined as a fixed point, as follows. Let $\overline{z}(p)$ be a candidate specification for $z(p)$. Let $x(p) = F^{-1}(\overline{z})$, $\pi(p) = DF(x)'.p/[x'.DF(x)'.p]$, and then define the mapping $\widetilde{z} = \mathfrak{S}(\overline{z})$ by $\widetilde{z} = F[z^f(\pi/\eta) + z^m(\pi/(1-\eta))]$. Then $z(p)$ is a fixed point of this mapping.

For the case of linear technologies, we may make this procedure explicit, as follows. Define a Marshallian demand function z^i to be regular if it comes from the program P^i (that is, maximization of a utility function U^i under a linear budget constraint) where U^i is monotonically increasing, continuously twice differentiable, and strictly quasiconcave.

Theorem 2. The household's consumption technology is described by $F(x) = Ax + a$, where A is a nonsingular n by n matrix and a is a n vector. Given any regular Marshallian demand functions $z^m(\widetilde{p}/y^m)$ and $z^f(\widetilde{p}/y^m)$, and any function $\eta(p)$ that always lies between zero and one, there exists a corresponding program P^* that possesses these functions, and the corresponding household demand functions $z(p)$ is given by

$$z(p) = A\left[z^f\left(\frac{A'p}{(1-a'p)\eta(p)}\right) + z^m\left(\frac{A'p}{(1-a'p)[1-\eta(p)]}\right)\right] + a \tag{8}$$

The proof of Theorem 2 in the Appendix is constructive, deriving an explicit expression for the required program P^*. The scaled shadow price vector for this model is

$$\pi(p) = \frac{A'p}{1-a'p} \tag{9}$$

and the private good equivalents are $x^f = z^f[\pi/\eta]$ and $x^m = z^m[\pi/(1-\eta)]$.

Theorem 2 shows that, with a linear household consumption technology, the following method can be used to construct functional forms for estimation. First, choose ordinary indirect utility functions for members m and f, and let z^m and z^f be the corresponding ordinary Marshallian demand functions. Next choose a functional form for the sharing rule η, which could simply be a constant, or a function of measures of bargaining power such as relative wages of the household members. This sharing rule function must lie between zero and one. Theorem 2 then provides the resulting functional form for the household demand function $z(p)$, and ensures that a corresponding household program exists that rationalizes the choice of functions z^m, z^f, and η.

In addition to providing a recipe for empirical application, Theorem 2 also illustrates the generic identification provided by Theorem 1. Theorem 2 shows that with a linear household technology the household demand functions are $z = Az^f[\pi/\eta] + Az^m[\pi/(1-\eta)] + a$. Here π/η and $\pi/(1-\eta)$ can take on any value in a non measure zero subset of the positive orthant, so as long as the resulting range of the private good equivalents $z^f[\pi/\eta] + z^m[\pi/(1-\eta)]$ is not severely limited, the matrix A and the vector a (and hence the consumption technology function F)

will be identified. This identification is obtained by specifying functional forms for F and η, instead of by assuming the existence of an exclusive good as in Theorem 1.

4 Additional Results

4.1 Technology and Preference Changes

The consumption technology function F may incorporate some changes in preferences resulting from living in a household versus living alone, as well as pure consumption economies of scale or scope (i.e., sharing). To illustrate this point, consider linear consumption technologies. Let $U^i()$ denote the utility function of household member i when living alone as before, but now let $\widetilde{U}^i()$ denote the corresponding utility over goods of member i while living inside the household. Both U^i and $\widetilde{U}^i$ can be interpreted as subutility functions over goods, separable from other attributes, such as companionship, that affect total utility. Assume that the relationship between U^i and $\widetilde{U}^i$ is $U^i(z^i) = \widetilde{U}^i(A^i z^i + a^i)$ for some nonsingular n by n matrix A^i and some n vector a^i. The parameters A^i and a^i represent only changes in tastes for goods, resulting from living with another versus living alone. Assuming that the household also has a linear consumption technology arising from economies of scale and scope in consumption (with parameters $\widetilde{A}$ and $\widetilde{a}$), we may write this household's optimization program as

$$\max_{\widetilde{x}^f, \widetilde{x}^m, z} \widetilde{U}[\widetilde{U}^f(\widetilde{x}^f), \widetilde{U}^m(\widetilde{x}^m), p/y]$$

$$z = \widetilde{A}(\widetilde{x}^f + \widetilde{x}^m) + \widetilde{a}$$

$$p'z = y$$

If $A^f = A^m$, then this program is equivalent to the program P* with technology $z = A(x^f + x^m) + a$, as can be seen by letting $\widetilde{x}^i = A^i x^i + a^i$ for $i = m$ and $i = f$, and defining $A = \widetilde{A}A^i$ and $a = \widetilde{A}a^m + \widetilde{A}a^f + \widetilde{a}$.

The implication of the fact that F may contain preference changes as well as purely technological considerations means that some calculations, in particular economies of scale and adult equivalence scales, must be interpreted with caution. For example, a change in taste for dancing as a result of marriage could show up as economies or diseconomies of scale in the consumption of dancing shoes.

If we wish to interpret the model's estimates of economies of scale as being due to the technology of sharing and not on taste changes, then it must be assumed that the dollar effect of any change in tastes for goods is small, so that F largely represents true technological changes in consumption. For example, joint consumption of heating is likely to have a much larger effect on measured cost savings of living together than on any change in tastes for heat.

The same caveat applies to our intrahousehold based adult equivalence scales, if they are to be interpreted as purely based on consumption technology. However, many applications of equivalence scales deal with appropriate compensation for an change in status, for example, calculation of the appropriate level of life insurance on a spouse, or compensation in legal cases of wrongful death. In these circumstances, it may be completely appropriate to compensate for all the effects embodied by F, both changes in tastes and the loss of purely technological economies of scale in consumption. This would still be separate from to any consideration of noneconomic effects such as grief and loss of companionship.

For yet other calculations based on the model, such as demand elasticites, or the interpretation of the sharing rule η as a measure of bargaining power, the portion of F that corresponds to taste changes versus consumption technology is largely irrelevant.

Future work may lead to additional data or alternative assumptions that allow for separate identification and estimation of taste changes and purely technological economies from sharing. This could include direct measurement of some sharing within the household (e.g., the percentage of time that couples use their car jointly) or functional form restrictions. An examples of the latter would be to impose the assumption that some goods (e.g., clothing) are purely private, which would imply that any difference between purchased and private equivalent quantities for such goods must be purely changes in taste.

While our particular identifying assumptions are open to debate and (we hope) improvement, we believe that in any case the model we have provided is an appropriate framework for analyzing these issues.

4.2 Barten and Gorman Scales

Assume F is linear, and let π be given by the equation (9). Gorman's general linear technology model assumes that household demands are given by

$$z = A\left[z^m\left(\pi/y\right)\right] + a \tag{10}$$

Barten scaling (also known as demographic scaling) is the special case of Gorman's model in which $a = 0$ and A is a diagonal matrix. Demographic translation is Gorman's linear technology with A equal to the identity matrix and a non zero, and what Pollak and Wales call the Gorman and reverse Gorman forms have both A diagonal and a nonzero. These are all standard models for incorporating demographic variation (such as the difference between couples and individuals) into demand systems. The motivation for these models is identical to the motivation for our linear technology F, but they fail to account for the structure of the household's program. Even if the household members have identical preferences ($z^f = z^m$) and identical private equivalent incomes ($\eta = 1/2$), comparison of equations (8) and (10) shows that household demand functions will still not actually be given by the Gorman or Barten model. In fact, comparison of these models shows that household demands will take the form of Gorman's linear technology, or some special case of Gorman such as Barten, only if either one household member consumes all the goods (η is zero or one), or if demands are linear in prices (i.e., the linear expenditure system).

5 Empirical Application

5.1 Data

We use Canadian FAMEX data from 1974, 1978, 1982, 1984, 1986, 1990 and 1992. The Canadian FAMEX is a multistaged stratified clustered survey that collects information on annual expenditures, incomes, labour supply and demographics for individual ('economic') households. The survey is *not* nationally representative. In particular, in most years only information from respondents living in large cities is collected (hence the hgih proportion of city dwellers in our samples below). The survey is run in the Spring after the survey year (that is, the information for 1978 was collected in Spring 1979). All of the information is collected by interview so that the expenditure and income data are subject to recall bias. Although this may give rise to problems, the FAMEX surveying method has the great advantage that information on annual expenditures is collected. Thus the FAMEX has much less problem with infrequency bias than do surveys based on short diaries. It is also the case that since the survey year coincides with the tax year (January to December) the income information is thought to be unusually reliable since it is collected at about the time that Canadians are filing their (individual) tax returns. These are often explicitly referenced by the enumerators.

Prices are taken from Statistics Canada. When composite commodities are created, the new composite commodity price is the weighted geometric mean of the component prices (a Stone price index) with budget shares averaged across the strata (couples, single males and single females) for weights. Thus, the weights are not the individual household budget shares.

We have three strata: single females, single males and couples with no on else present in the household. We sample only younger agents: the single females and wives in couples are aged less than or equal to 42; single males and husbands are all aged less than or equal to 45. All agents are in full year full time employment and we have excluded any household with non–negative net income or non-negative individual gross incomes. This results in sample sizes of 1393, 1574 and 1692 for single females, single males and couples respectively. The (non-durable) goods we model are: food at home, restaurant expenditures, clothing, recreation, transport, services and vices (alcohol and tobacco).

	Single females	Single males	Couples Husband	Couples Wife
Net income*	26, 137	30, 890	56, 481	
Total expenditure*	11, 855	13, 645	23, 259	
Wife's share of gross income	-	-	0.43	
bs(food at home)	0.18	0.16	0.19	
bs(restaurant)	0.11	0.15	0.11	
bs(clothing)	0.16	0.09	0.14	
bs(recreation)	0.11	0.13	0.11	
bs(transport)	0.21	0.25	0.25	
bs(services)	0.17	0.10	0.12	
bs(vices)	0.07	0.12	0.08	
car	0.64	0.78	0.95	
home owner	0.13	0.23	0.57	
city dweller	0.85	0.81	0.83	
age	29.9	31.1	30.1	28.1
higher education	0.20	0.25	0.21	0.19
Francophone	0.19	0.17	0.21	0.18
Allophone	0.08	0.08	0.09	0.07
White collar	0.43	0.40	0.39	0.37
Notes. * mean for 1992 (when prices are unity).				

Major remarks:

- Income and total expenditures are lowest for single females, highest for couples.

- Total expenditure on non-durables is between 41% and 45% of net income.
- Car ownership and home ownership are significantly higher for couples than for singles.
- Single males have significantly higher budget shares for restaurant, transport and vices than single females. Single females have higher budget share for food at home, clothing and services. Of course, this does not necessarily mean that single men and women have different preferences since these are conditional on demographics and real total expenditure which differ across the two strata. Below we shall present tests for whether single men and women have the same preferences.
- Apart from transport and services, couples' budget shares resemble single female's shares much more closely than those of single males.

5.2 Barten Technology, Overheads, and Budget Shares

Our empirical application will use a Barten type technology function with overheads, and express demands in budget share form. This technology function is defined as

$$z_k = A_k x_k + a_k \tag{11}$$

for each good k, and so is equivalent to the general linear technology $z = Ax + a$ when the matrix A is diagonal. The shadow prices for this technology are

$$\pi_k = \frac{A_k \widetilde{p}_k}{y - a'\widetilde{p}} \tag{12}$$

where the couple faces prices $\widetilde{p}$ and has total expenditure level y. This technology function is particularly convenient for budget share models. Let $w_k = \omega_k(\widetilde{p}/y)$ be the budget share of good k for the household, so $\omega_k(p) = \widetilde{p}_k z_k(\widetilde{p}/y)/y$. Similarly, let $w_k^i = \omega_k^i(\widetilde{p}/y^i)$ be member i's budget share of consumption of good k when living as a single, for $i = f$ or m. With the technology function (11) and corresponding shadow prices (12), based on Theorem 2 we obtain the following simple expression for the couple's budget shares

$$\omega_k(p) = \left(1 - \frac{a'\widetilde{p}}{y}\right)\left[\eta\omega_k^f\left(\frac{\pi}{\eta}\right) + (1-\eta)\omega_k^m\left(\frac{\pi}{1-\eta}\right)\right] + \frac{a'\widetilde{p}}{y} \tag{13}$$

Apart from adjusting for overheads, equation (13) shows that the budget shares of the couple equal a weighted average of the budget shares of the members, with weights given by the income sharing rule η and $1 - \eta$.

Couple's budget shares may be estimated by substituting into equation (13) the estimated single's budget share functions ω_k^i, the shadow price equation (12), and a specification for the income sharing rule η. The singles data identify the parameters of ω_k^i, so the only additional parameters that require the household data are the parameters in A, a, and η.For efficiency, the singles and couples models will be estimated simultaneously rather than sequentially.

Given estimates of the budget share systems for singles, $w_k^i = \omega_k^i(\widetilde{p}/y^i)$, and for households (equation 13), the private good equivalent quantities for each household member may be calculated as

$$x_k^f = \frac{\eta}{\pi_k}\omega_k^f\left(\frac{\pi}{\eta}\right) \tag{14}$$

$$x_k^m = \frac{1-\eta}{\pi_k}\omega_k^m\left(\frac{\pi}{1-\eta}\right), \tag{15}$$

the economies of scale to consumption are

$$\frac{\widetilde{p}'z}{\widetilde{p}'x} = \frac{y}{\widetilde{p}'(x_k^f + x_k^f)}, \tag{16}$$

and adult equivalence scales are obtained as follows. Let $V^i(\widetilde{p}/y^i)$ be the indirect utility function (up to any arbitrary monotonic transformation) that gives rise to member i's budget share demand functions $w_k = \omega_k(\widetilde{p}/y^i)$. Then the minimum expenditure required by member i, while living alone, to attain the same indifference curve over goods that he or she attains as a member of the household is the number y^{i*} that solves the equation

$$V^f\left(\frac{\widetilde{p}}{y^{f*}}\right) = V^f\left(\frac{\pi}{\eta}\right) \tag{17}$$

for $i = f$, and

$$V^m\left(\frac{\widetilde{p}}{y^{m*}}\right) = V^m\left(\frac{\pi}{1-\eta}\right) \tag{18}$$

for $i = m$. We may then calculate the male adult equivalence scale for the household as y/y^{m*} and for a female as y^{f*}/y^{m*}. Alternatively, to separate issues of bargaining power from other considerations, we could evaluate these equations and the resulting equivalence scales substituting $\eta = .5$ into equations (17) and (18).

Note in all of the above that π, η, x_k^f, x_k^m, y^{m*}, and y^{f*} are implicitly functions of $p = \widetilde{p}/y$, and so may vary as prices and income varies.

5.3 Functional Form

For our empirical application, we assume singles have preferences given by the Integrable QUAIDS demand system of Banks, Blundell and Lewbel (1997). For $i = f$ or m, let $w^i = \omega^i(\widetilde{p}/y^i)$ denote the n-vector of member i's budget shares w_k^i $(k = 1, ..., n)$ when living as a single, facing prices $\widetilde{p}$ and having total expenditure level y^i. The QUAIDS demand system we estimate takes the vector form

$$\omega^i(\widetilde{p}/y^i) = \alpha^i + \Gamma \ln \widetilde{p} + \beta^i \left[\ln\left(y^i\right) - c^i\left(\widetilde{p}\right)\right] + \frac{\lambda}{b^i\left(\widetilde{p}\right)}\left[\ln\left(y^i\right) - c^i\left(\widetilde{p}\right)\right]^2 \quad (19)$$

where $c^i(\widetilde{p})$ and $b^i(\widetilde{p})$ are price indices defined as

$$c^i\left(\widetilde{p}\right) = \delta^i + (\ln \widetilde{p})'\alpha^i + \frac{1}{2}(\ln \widetilde{p})'\Gamma \ln \widetilde{p} \quad (20)$$

$$\ln[b^i\left(\widetilde{p}\right)] = (\ln \widetilde{p})'\beta^i. \quad (21)$$

Here α^i, β^i and λ are n-vectors of parameters, Γ is an $n \times n$ matrix of parameters and δ^i is a scalar parameter which we take to equal zero, based on the insensitivity reported in Banks, Blundell, and Lewbel (1997). Adding up implies that $e'\alpha^i = 1$ and $e'\beta^i = e'\lambda = \Gamma e = 0$ where e is an n-vector of ones. Homogeneity implies that $\Gamma' e = 0$ and Slutsky symmetry is equivalent to Γ being symmetric.

With these restrictions, these singles demand functions $\omega^i(\widetilde{p}/y^i)$ can be obtained by applying Roy's identity to the Integrable QUAIDS indirect utility function

$$V^i\left(\frac{\widetilde{p}}{y^i}\right) = \left[\left(\frac{\ln\left(y^i\right) - c^i\left(\widetilde{p}\right)}{b^i\left(\widetilde{p}\right)}\right)^{-1} + \lambda' \ln\left(\widetilde{p}\right)\right]^{-1} \quad (22)$$

for $i = f$ and $i = m$.

The vectors of parameters α^i and β^i are specified to depend on demographics. Specifically, we take:

$$\alpha_k^i = \alpha_{k0}^i + \sum_{m=1}^{M_\alpha} \alpha_{km}^i z_m \quad (23)$$

$$\beta_k^i = \beta_{k0}^i + \sum_{m=1}^{M_\beta} \beta_{km}^i z_m \tag{24}$$

where $M_\beta = 2$, with the demographics z_m being dummies for owning a car and being a home owner. For the intercept demographics we have $M_\alpha = 13$. These demographics z_m include four regional dummies; a dummy for owning a car, a house ownership dummy and a dummy for living in a city. As well we include individual specific demographics: age and age squared, a dummy for having more than high school education, dummies for being French speaking or neither English nor French speaking and an occupation dummy for being in a white collar job. The model results in 24 parameters per good/equation for singles

We will also estimate a separate QUAIDS model for couples, to use for comparison to our formal household model. This comparison QUAIDS model is identical to the above specifications for singles, except that the individual specific demographics for both members of the household are included, resulting in $M_\alpha = 19$ and hence in 30 parameters per good/equation for couples.

Given the demand functions for singles, our household model for couples is given by equations (12), (13), and specifications for the income sharing rule η and the technology parameters A_k and a_k, $k = 1, ..., n$. The Barton type scales A_k we specify as

$$A_k = \exp\left(A_{k0} + \sum_{m=1}^{M_A} A_{km} \widetilde{z}_m \right) \tag{25}$$

where $M_A = 3$, with the demographics characteristics $\widetilde{z}_m$ being dummies for couples car and home ownership (which differ greatly between singles and couples and have a strong impact on transport and services expenditures), and on a dummy for the couple having different education levels (where there are only two education levels: 'high school or less' and 'more than high school'). The exponent form ensures that the scales are positive.

We model overheads a_k as constants. To ensure that $a'p$ remains less than y, so that we can take logs of $y - a'p$, we introduce a penalty function into the optimization criterion. The optimal estimates are not close to the threshold.

Based on Browning, Bourguignon, Chiappori and Lechene (1994) and Browning and Chiappori (1998), our model for the sharing rule is

$$\eta = \eta_0 + \eta_1 \frac{Y^f}{Y^f + Y^m} + \eta_1 \ln\left(\frac{y}{P}\right) \tag{26}$$

where Y^i is the gross income of household member i and P is a Stone price index for the couple. This sharing rule depends on the wife's share of total gross income (a potential measure of bargaining power) and on the couple's total real expenditures.

5.4 Estimation

Our estimation procedure proceeds in two stages. First we estimate the parameters of the system for single males and single females separately. We drop the last equation to accomodate adding up and we impose homogeneity by including only relative prices in each equation. To take acccount of the endogeneity of total expenditure we estimate by GMM. For instruments we take all of the demographics plus log real net income, (defined as the log of nominal net income divided by a Stone price index computed for our seven non-durables goods) and its square, the product of real net income with the car and home ownership dummies and absolute log prices. Thus there is one degree of over-identification for each equation. The $\chi^2(6)$ statistics for the over-identifying restrictions are 5.8 (probability = 44%) and 12.4 (5.4%) for single females and single males respectively. We then impose symmetry on both sets of estimates; the respective $\chi^2(15)$ statistics were 25.3 (4.6%) and 25.3 (4.6%). We denote the symmetry constrained sets of estimates by $\hat{\theta}_f$ and $\hat{\theta}_m$ for females and males respectively. Since so much of our analysis is based on single men and women having different preferences, we present tests for the sets of parameters being the same. For the six equations modelled we have the following $\chi^2(24)$ statistics for equality: 52 (food), 118 (restaurant), 686 (clothing), 95 (recreation), 117 (transport) and 622 (services). Thus we decisively reject that single men and women have the same preferences.

In the second step we take the singles' estimates, $\hat{\theta}_f$ and $\hat{\theta}_m$, the data on couples and trial values for the parameters in equation (13). The latter are the parameters of the sharing function η, the Barten terms $A_1, ...A_n$ and the fixed costs elements $a_1...a_n$. We shall allow that these can depend on household characteristics (details are given below) and we denote the set of them by ζ. From these we can form predicted budget shares for each couple in our sample, based on their prices, total expenditure and demographics, denote these $\hat{\omega}(\zeta)$. We then minimise, with respect to the trial parameter values, a weighted sum of squares of the difference between the predicted budget shares and the actual budget shares, ω:

$$\min_{\zeta} \left(\hat{\omega}(\zeta) - \omega\right)' W \left(\hat{\omega}(\zeta) - \omega\right) \tag{27}$$

For the weighting matrix W we use the weighting matrix from GMM estimates of the QUAIDS system for couples:

$$W = (I_{n-1} \otimes Z)\hat{V}(I_{n-1} \otimes Z)' \tag{28}$$

where Z is an $(H \times g)$ matrix of instruments and $\hat{V}$ is derived from the first round of GMM estimates in the usual way. Testing is based on the minimised criterion having a χ^2 distribution with degrees of freedom equal to $((n-1)g - \dim(\zeta))$. To derive the weighting matrix we estimate the QUAIDS for couples. For instruments we use the instruments used for singles (with individual specific values for husbands and wives, where appropriate). We also include logs of the individual gross incomes and the ratio of the wife's gross income to total gross income (we shall see why below). In all we have $g = 34$.

Before presenting estimates with our restrictions imposed, we graph the fits for the different goods for the three strata. To do this we first estimate QUAIDS system for each strata (with homogeneity and symmetry imposed) and then caculate predictions for agents who live in Ontario, living in city, owning a car, renting their accmodation, aged 30, with high school education, Anglophone and all having a blue collar job. We predict for 1992, the year in whcih prices are close to unity. For total expenditure we take a range from the first decile to the ninth decile. For couples we divide total expenditure by 1.7, the scale recommended by the OECD, to make the utility levels roughly constant (later we present.our own estimate of the required scale). Figures 1 to 7 present the plots for our seven goods.

5.5 Estimates of Couples Model

We start with an extremely parsimonious specification and add extra parameters until we find a satisfactory fit. As a benchmark, we first estimate with all scales A_k equal to one, all overheads a_k equal to zero, and the sharing rule η constant, so $\eta = \eta_0$ Our criterion is minimised at a value of $\eta = 0.62$ with a criterion value of 1270. The criterion value for setting $\eta = 0.5$ (equal sharing) is 1583, so that we reject equal sharing for this simplest model (formally we have a χ^2 (1) statistic of $1583 - 1270 = 313$). The value of η implies that the budget shares of couples are somewhat closer to those of single females than to those of single males.

The first model we estimate (model 1) has adds constant Barten terms to the benchmark model; thus we have only the parameters $\eta_0, A_{01}, ...A_{0n}$. The results are given in the first column of the Table below. As can be seen, adding the Barten terms improves the fit dramatically: the χ^2 (7) statistic for them all being zero is 733. However we still reject the overall model.

Model 2 adds some characteristics to the sharing function, with η given by equation (26). In the Table we present the value of η evaluated at the means of its arguments. The fit improves a small amount; the χ^2 (2) for excluding the three new parameters is 6.5 which has a probability value of 3.9%. Consistent with this, the mean value of η is unchanged and the other parameter estimates are not much changed.

In model 3 we introduce fixed costs into model 2. The extra parameters $a_1, ..., a_n$ are highly significant (a χ^2 (7) of 84) but induce only small changes in the other parameters.

Model 4 introduces demographics into the Barten scales, with each A_k given by equation (25). The new parameters are highly significant (a χ^2 (21) of 109). The Barten scale for 'vices' is rather high, but it is probably not significantly different from a more reasonable value.

	1	2	3	4	5
$\hat{\eta}$	0.55	0.55	0.52	0.54	
$\hat{A}_1$	1.12	1.21	0.99	1.35	
$\hat{A}_2$	0.52	0.52	0.33	0.33	
$\hat{A}_3$	2.47	2.54	3.21	4.75	
$\hat{A}_4$	0.75	0.76	0.74	0.82	
$\hat{A}_5$	0.39	0.39	0.42	0.48	
$\hat{A}_6$	1.00	0.99	0.79	0.94	
$\hat{A}_7$	1.69	1.84	6.93	15.2	
χ^2	536.9	530.4	446.9	337.9	
df	196	194	187	166	

Additional models to try: restricting Barton scales to less than two, or restricting overheads to be nonnegative, allowing δ^i (the intercept in the c functions) to be nonzero and possibly contain demographics, estimating couples using an ordinary GMM weighting matrix, doing a full optimization to simultaneously estimate the parameters for singles and couples, and trying some additional demographic variables .

5.6 Implications of parameter estimates.

We present some implications of our parameter estimates. We evaluate everything at the mean total expenditures in 1992 (see table 1) using unit prices. This includes mean values of the Barton scales, overheads, and the income sharing rule. We also use equations (14), (15), (17),and (18) to construct private good equivalents

x_k^i and equivalent incomes y^{i*} for $i = f$ and $i = m$. We may then construct the household economies of scale by equation (16), and household equivalence scales for the household as y/y^{m*}, and for a female as y^{f*}/y^{m*}.We also report the what the economies of scale and adult equivalence scales would equal using a sharing parameter of $\eta = 1/2$, to remove the role of bargaining power from these calculations.

5.7 Figures.

This section and additional empirical work is incomplete.

6 Conclusions

Many previous attempts to identify equivalence scales are based on assumptions such as Independence of Base (IB). For example, Jorgenson and Slesnick's model is a special case of an IB assumption. IB imposes testable restrictions on demands, but it also requires untestable (that is, cardinal or comparable utility type) restrictions. IB is an example of the old paradigm of comparing an individuals's utility to that of a household, which is intrinsically impossible without some at least partial cardinalization of utility.

We instead estimate the cost of getting an individual, living alone, to the same indifference curve that the individual attained while in the household. No cardinality or interpersonal comparability is required, since we are in principle looking at the same individual in two different states, rather than comparing the well being of an individual to other individual or group.

6.1 Appendix A: Proofs

Here we provide proofs of lemmas and theorems in the paper, and some additional related results.

Proposition 1. There exist shadow prices $\widetilde{\pi}$ and shadow budgets ρ^f and ρ^m such that Program P* is equivalent to the following Program P

$$\max_{x^f, x^m} \widetilde{\pi}' \left(x^f + x^m\right) - p'F\left(x^f + x^m\right) \qquad \text{(P)}$$

$$x^i = z^i(\widetilde{\pi}/\rho^i) \tag{29}$$

This proposition follows immediately from the second welfare theorem.

The interpretation of the decentralization Program P is that the household can satisfy its objective function, Program P*, by telling each member i to choose a utility maximizing private consumption vector x^i (program P^i), but subject to the budget constraint $\widetilde{\pi}'x^i = \rho^i$. Member i chooses $x^i = z^i(\widetilde{\pi}/\rho^i)$, and the household then actually purchases the vector $z = F\left(x^f + x^m\right)$.

Proof of Lemma 1. Define x by

$$x = x^f + x^m$$

The first order conditions for program P include

$$\widetilde{\pi} = DF(x)'.p \tag{30}$$

where x is evaluated at

$$x = x(p) = F^{-1}[z(p)] \tag{31}$$

This shows that $\widetilde{\pi} = \widetilde{\pi}(p)$ is fully determined by the functions $F()$ and $z()$. Next define the functions $\rho^i(p) = \widetilde{\pi}(p)'x^i(p)$ for $i = m, f$. We may now define scaled shadow prices $\pi(p)$ by

$$\pi(p) = \frac{\widetilde{\pi}(p)}{\rho^f(p) + \rho^m(p)}$$

Just as the scaled price vector p has the property that $p'z = 1$, the scaled shadow price vector $\pi = \pi(p)$ satisfies $\pi'x = 1$. Next define the *intrahousehold income sharing rule* $\eta(p)$ by

$$\eta(p) = \eta^f(p) = \frac{\rho^f(p)}{\rho^f(p) + \rho^m(p)} \tag{32}$$

$$\eta^m(p) = 1 - \eta^f(p)$$

The lemma follows immediately.

The sharing rule $\eta = \eta^f$ is the fraction of the household's shadow income that is allocated to member f, and so can be interpreted as a measure of her weight or bargaining power in the household's program. She then uses this income to 'buy' her vector of private good equivalents x^f, paying shadow prices π. Similarly, member m allocates income $1 - \eta = \eta^m$ to obtain private good equivalents x^m, paying shadow prices π.

Proof of Lemma 2. Given Proposition 1, a first order condition of the household's program is

$$\frac{\partial \widetilde{U}}{\partial U^i} \frac{\partial U^i(x^i)}{\partial x_k^i} = \lambda_0 \pi_k$$

for each member i and good k, where λ_0 is a lagrange multiplier. It follows that

$$\frac{\partial \widetilde{U}}{\partial U^i} \frac{\partial V^i(\pi/\eta^i)}{\partial \eta^i} = \frac{\partial \widetilde{U}}{\partial U^i} \frac{x^{i\prime} D U^i(x^i)}{\eta^i} = \lambda_0$$

so

$$\frac{\partial \widetilde{U}/\partial U^f}{\partial \widetilde{U}/\partial U^m} = \frac{\partial V^f(\pi/\eta^f)/\partial \eta^f}{\partial V^m(\pi/\eta^m)/\partial \eta^m} \tag{33}$$

It can then be directly verified that all of the first order conditions from the program P* are unchanged when $\widetilde{U}$ is replaced with $\mu(p)U^f(x^f) + U^m(x^m)$.

Now we consider the general identification result, Theorem 1. We lead up to the main result with a few simple propositions. The first shows that if the consumption technology F and the sharing rule η can be identified from the observed demand functions, then the individual equivalent consumption vectors are identified.

Propositions 2, 3, and 4 below together yield the proof of Theorem 1.

Proposition 2. Given Assumptions A1, A2, and A3, if the function F is identified, then the functions $x(p)$, $\widetilde{\pi}(p)$, and $\pi(p)$ are identified. If $\eta(p)$ is also identified, then the individual equivalent consumption vectors $x^m(p)$ and $x^f(p)$ are identified.

To prove Proposition 2, we have given the functions F, z, and z^i that $x(p) = F^{-1}[z(p)]$, then $\widetilde{\pi}(p) = DF(x)'.p$, and $\pi(p) = \widetilde{\pi}/(\widetilde{\pi}'x)$. If $\eta(p)$ is also known then $x^f(p) = z^f(\pi/\eta)$ and $x^m(p) = x - x^f$.

Proposition 3. Let Assumptions A.1, A.2, and A.3 hold. Assume the existence of one monotonic, exclusive good. If the function F is identified, then the functions $x^m(p)$, $x^f(p)$, $\widetilde{\pi}(p)$, $\pi(p)$, and $\eta(p)$ are identified.

To prove Proposition 3, we have first by Proposition 2 that, given F, the functions x, $\widetilde{\pi}$, and π are identified. Without loss of generality, assume that good 1 is monotonic and exclusive for member f. Given this monotonicity, we can define the function ϑ_f as the inverse of $z_1^f(\widetilde{p}/y^f)$ with respect to y^f, so $y^f = \vartheta_f(z_1^f, \widetilde{p})$. The first element of $x(p)$ is $x_1(p)$, and so with identification of $x_1(p)$ we have by exclusivity that $x_1(p) = x_1^f(p) = z_1^f[\pi(p)/\eta(p)]$, and therefore $\eta(p) = \vartheta_f[x_1(p), \pi(p)]$ is identified. Finally, again by Proposition 2 since both F and η are identified, x^m and x^f are identified.

Proposition 4. Given Assumptions A1, A2, A3, and A4, the function F is "generically" identified.

For Proposition 4, define the mapping $\mathcal{T}$ within the space of increasing, C^2 functions by the following procedure. Take some increasing, C^2 function $\overline{F}$. Using Proposition 3, construct the functions $x^i(p)$, using $\overline{F}$ in place of the true technology function F. Now define the function $\widetilde{F} = \mathcal{T}\left(\overline{F}\right)$ by

$$\widetilde{F}\left[x^f(p) + x^m(p)\right] = z(p)$$

Then the true function F is a fixed point of the mapping $\widetilde{F} = \mathcal{T}\left(\overline{F}\right)$. The function F is identified if this mapping has a unique fixed point. In general, $\mathcal{T}$ is not a contraction, and so may have several fixed points. However, for suitably regular demand functions (those for which the tangent application to $\mathcal{T}$ is Fredholm) each fixed point will be locally unique as a consequence of Smale's generalized transversality theorem. This provides local generic identification. Regularity conditions such as monotonicity further restrict the space of feasible functions F. Global identification results if, among all fixed points, only one satisfies the required regularity conditions.

The above paragraph describes formally what we mean by 'generic' identification. Informally, a sufficient number of demand functions are identified to generally permit recovery of F, and so F will be identified as long as the demand functions are not too simple, e.g., a linear F will not be identified if the demand functions $x^f(p)$, $x^m(p)$, and $z(p)$ all happen to be linear. However, if the demand functions are nonlinear, then in general a linear F will not only be identified, it will typically be very much overidentified.

The following examples illustrate these points. Suppose F is linear, so

$$z_k(p) = \sum_j A_k^j \left[z_j^f\left(\frac{A'p}{(1-a'p)\eta(p)}\right) + z_j^m\left(\frac{A'p}{(1-a'p)[1-\eta(p)]}\right)\right] + a$$

and that demands have the general form

$$z_j^g\left(\frac{\pi}{\eta_g}\right) = h_j^g\left(\frac{\pi_j}{\eta_g}\right).H^g\left(\frac{\pi}{\eta_g}\right) + \zeta_j^g$$

where the h functions are linearly independent. Then

$$z_k(p) = \sum_j A_k^j\left[H^f\left(\frac{\pi}{\eta}\right)h_j^f\left(\frac{\pi_j}{\eta}\right) + H^m\left(\frac{\pi}{1-\eta}\right)\sum_s h_{j,s}^m\left(\frac{\pi_s}{1-\eta}\right)\right] + \zeta_k^f + \zeta_k^m + a_k.$$

Here

$$\pi_s(p) = \frac{\sum_t A_t^s p_t}{1 - a'p}$$

hence

$$z_k(p) = \sum_j A_k^j\left[H^f\left(\frac{\pi}{\eta}\right)h_j^f\left(\frac{\sum_t A_t^j p_t}{(1-a'p)\,\eta}\right) + H^m\left(\frac{\pi}{1-\eta}\right)h_j^m\left(\frac{\sum_t A_t^j p_t}{(1-a'p)(1-\eta)}\right)\right] + \zeta_k^f + \zeta_k^m + a_k$$

If the h_j^f are functionally different (and independent), identification follows immediately. Even if the h are restricted to satisfy:

$$h_j^g\left(\frac{\pi_j}{\eta_g}\right) = \beta_j^g h\left(\frac{\pi_j}{\eta_g}\right)$$

hence

$$z_j^g\left(\frac{\pi}{\eta_g}\right) = H^g\left(\frac{\pi}{\eta_g}\right)\beta_j^g h\left(\frac{\pi_s}{\eta_g}\right)$$

identification still obtains in general, and in fact in this case only the household demands for one good are required for identification, so the demands of every other good provide overidentifying restrictions. To see this, we have

$$\begin{aligned} z_k(p) &= \sum_j A_k^j\left[H^f\left(\frac{\pi}{\eta}\right)\beta_j^f h\left(\frac{\sum_t A_t^j p_t}{(1-a'p)\,\eta}\right) + H^m\left(\frac{\pi}{1-\eta}\right)\beta_j^m h\left(\frac{\sum_t A_t^j p_t}{(1-a'p)(1-\eta)}\right)\right] + \zeta_k^f + \\ &= H^f\left(\frac{\pi}{\eta}\right)\sum_j A_k^j\beta_j^f h\left(\frac{\sum_t A_t^j p_t}{(1-a'p)\,\eta}\right) + H^m\left(\frac{\pi}{1-\eta}\right)\sum_j A_k^j\beta_j^m h\left(\frac{\sum_t A_t^j p_t}{(1-a'p)(1-\eta)}\right) + \zeta \end{aligned}$$

Given a single household demand function $z_k(p)$ (and the singles' demand functionss for all the goods), we first see that a is identified by the denominator within the h terms, or by the intercept. Next, A is identified up to a permutation of columns from the numerator in the h terms, and finally the outside coefficients pin down the appropriate permutation.

This logic assumes h is nonlinear. If h is linear (and hence no independence). Then

$$z_k(p) = H^f\left(\frac{\pi}{\eta}\right)\left(\frac{\sum_{j,t} A_k^j \beta_j^f A_t^j p_t}{(1-a'p)\,\eta}\right) + H^m\left(\frac{\pi}{1-\eta}\right)\left(\frac{\sum_{j,t} A_k^j \beta_j^m A_t^j p_t}{(1-a'p)\,(1-\eta)}\right) + \zeta_k^f + \zeta_k^m + a_k$$

and the technology is not identified. Only the $2n$ numbers $\sum_j A_k^j \beta_j^f A_t^j, t = 1, \ldots n$ and $\sum_j A_k^j \beta_j^m A_t^j, t = 1, \ldots n$ are available for identifying A. However, even in this linear case the technology can be identified if A is diagonal, indeed, the kth demand identifies A_k^k for $k = 1, \ldots, n$.

To provide a more concrete class of examples, let singles have indirect utility functions of the general form

$$V^i\left(p_1, p_2, p_3\right) = \left(b_1^i h^i\left(p_1\right) + b_2^i h^i\left(p_2\right) + b_3^i h^i\left(p_3\right)\right)^2$$

for an arbitrary differentiable function h^i, where for simplicity we have assumed $n = 3$ goods. Then

$$z_k^i\left(p_1, p_2, p_3\right) = \frac{b_k^i h^{i\prime}\left(p_k\right)}{p_1 b_1^i h^{i\prime}\left(p_1\right) + p_2 b_2^i h^{i\prime}\left(p_2\right) + p_3 b_3^i h^{i\prime}\left(p_3\right)}$$

hence, for linear technologies,

$$\begin{aligned} z_k(p) &= \frac{\sum_j A_k^j b_j^f h^{f\prime}\left(\frac{\sum_k A_k^j p_k}{(1-\sum_k a_k p_k)\eta}\right)}{\pi_1 b_1^f h^{f\prime}\left(\pi_1\right) + \pi_2 b_2^f h^{f\prime}\left(\pi_2\right) + \pi_3 b_3^f h^{f\prime}\left(\pi_3\right)} + \\ &\quad \frac{\sum_j A_k^j b_j^m h^{m\prime}\left(\frac{\sum_k A_k^j p_k}{(1-\sum_k a_k p_k)(1-\eta)}\right)}{\pi_1 b_1^m h^{m\prime}\left(\pi_1\right) + \pi_2 b_2^m h^{m\prime}\left(\pi_2\right) + \pi_3 b_3^m h^{m\prime}\left(\pi_3\right)} + a_k \end{aligned}$$

To make identification more difficult, assume the functions h^i are the same for both household members i. Define $H\left(\pi\right) = h^i\left(\pi\right)$ and $h\left(\pi\right) = h^{i\prime}\left(\pi\right)$. Then

$$z_k(p) = \frac{\sum_j A_k^j b_j^f h\left(\frac{\sum_k A_k^j p_k}{(1-\sum_k a_k p_k)\eta}\right)}{\pi_1 b_1^f h\left(\pi_1\right) + \pi_2 b_2^f h\left(\pi_2\right) + \pi_3 b_3^f h\left(\pi_3\right)} + \frac{\sum_j A_k^j b_j^m h\left(\frac{\sum_k A_k^j p_k}{(1-\sum_k a_k p_k)(1-\eta)}\right)}{\pi_1 b_1^m h\left(\pi_1\right) + \pi_2 b_2^m h\left(\pi_2\right) + \pi_3 b_3^m h\left(\pi_3\right)} + a_k$$

and, as before, we have generically that a is identified from the denominator within the h terms, A is identified up to a permutation of columns from the numerator in the h terms, and the appropriate permutation is determined by the outside coefficients.

For some specific cases, consider first

$$h(x) = x^{\delta}$$

then estimation gives, for instance for $k = 1$:

$$z_1(p) = \frac{\sum_j u_j^1 \left(\frac{\sum_k v_k^j p_k}{w - \sum_k w_k p_k} \right)^{\delta}}{H^f} + \frac{\sum_j \bar{u}_j^1 \left(\frac{\sum_k \bar{v}_k^j p_k}{\bar{w} - \sum_k \bar{w}_k p_k} \right)^{\delta}}{H^m} + a_1$$

where the H are complex functions (sums of δ powers). Then the a_k are identified as

$$a_k = \frac{w_k}{w} = \frac{\bar{w}_k}{\bar{w}}$$

Next, one can map each $\left(\frac{\sum_k v_k^j p_k}{w - \sum_k w_k p_k} \right)^{\delta}$ with one $\left(\frac{\sum_k \bar{v}_k^j p_k}{\bar{w} - \sum_k \bar{w}_k p_k} \right)^{\delta}$ such that the coefficients of the p_k are proportional (there must be a one-to-one mapping). This identifies η and the A_k^j up to a permutation of the j (that is, either $\left(A_1^1, A_2^1, A_3^1\right)$ is proportional to $\left(v_1^1, v_2^1, v_3^1\right)$ or $\left(A_1^2, A_2^2, A_3^2\right)$ is proportional to $\left(v_1^1, v_2^1, v_3^1\right)$ or $\left(A_1^3, A_2^3, A_3^3\right)$ is proportional to $\left(v_1^1, v_2^1, v_3^1\right)$,and so on). Note that the proportionality coefficient has been pinned down by the $\left(\bar{v}_1^1, \bar{v}_2^1, \bar{v}_3^1\right)$, which gives η. Finally, consider the coefficients $\left(u_1^1, u_2^1, u_3^1\right)$ and $\left(\bar{u}_1^1, \bar{u}_2^1, \bar{u}_3^1\right)$. Then one must have that

$$\frac{u_j^1}{\bar{u}_j^1} = \frac{b_j^f}{b_j^m}$$

This will generically be satisfied for one permutation at most, which then must be the correct one.

Another example is the log model,

$$h(x) = \log x$$

$$z_k(p) = \sum_j A_k^j \left[b_j^f h\left(\frac{\sum_k A_k^j p_k}{(1 - \sum_k a_k p_k)\eta} \right) + b_j^m h\left(\frac{\sum_k A_k^j p_k}{(1 - \sum_k a_k p_k)(1 - \eta)} \right) \right] + a_k$$

then estimation gives

$$z_1(p) = \frac{\sum_j u_j^1 \log\left(\sum_k v_k^j p_k\right) + u^a \log\left(1 - \sum_k w_k p_k\right) + n^\eta}{H^f} + \frac{\sum_j \bar{u}_j^1 \log\left(\sum_k \bar{v}_k^j p_k\right) + \bar{u}^a \log\left(1 - \sum_k \bar{w}_k p_k\right) + n^\eta}{H^m} + a_1$$

where the H are sums of logs. Then the same logic again provides identification, this time mapping each $\left(\sum_k v_k^j p_k\right)$ with one $\left(\sum_k \bar{v}_k^j p_k\right)$ such that the coefficients of the p_k are proportional.

Proof of Theorem 2. The proof is by construction. Let U^i be any direct utility function corresponding to z^i, and let V^i be the associated indirect utility function, for $i = f, m$. Define the scaled shadow price function $\pi(p)$ by $\pi(p) = A'p/[1 - a'p]$. Given V^m, V^f, $\pi(p)$, and the chosen function $\eta(p)$, Define the function $\mu(p)$ by

$$\mu(p) = -\frac{\partial V^f(\pi/\eta)/\partial\eta}{\partial V^m[\pi/(1-\eta)]/\partial\eta}$$

Then a suitable program P^* is

$$\max_{x^f, x^m, z} \widetilde{U}(U_f, U_m, p/y) = \mu(p/y)U^f(x^f) + U^m(x^m)$$

$$z = Ax^f + Ax^m + a$$

$$p'z = 1$$

as can be directly verified, using Proposition 1 and the proofs of Lemmas 1 and 2.

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