

## LONG RANGE DEPENDENCE IN DAILY STOCK RETURNS

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*This paper uses the tests of Robinson (1994a) to analyse the degree of dependence in the intertemporal structure of daily stock returns (defined as the first difference of the logarithm of stock prices, where the series being considered is the S&P500 index). These tests have several distinguishing features compared with other procedures for testing unit (or fractional) roots. In particular, they have a standard null limit distribution and they are the most efficient ones when carried out against the appropriate alternatives. In addition, they allow one to incorporate the Bloomfield (1973) exponential spectral model for the underlying  $I(0)$  disturbances. The full sample, which comprises 17,000 observations, is first divided in 10 subsamples of 1700 observations each. These are then grouped two by two, and five by five; finally, the whole sample is considered. The results indicate that the degree of dependence remains relatively constant over time, with the order of integration of stock returns fluctuating slightly above or below zero. On the whole, there is very little evidence of fractional integration, despite the length of the series. Therefore, it appears that the standard practice of taking first differences when modelling stock returns might be adequate.*

**Keywords:** *Fractional Integration, Long Memory, Intertemporal Dependence, Stock Returns*

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## 1. Introduction

An important issue in the empirical analysis of financial time series is whether holding period returns on a risky asset are serially independent, which is required by the efficient market hypothesis. The evidence is mixed. For instance, using a variance-ratio test Lo and MacKinley (1988) and Poterba and Summers (1988), concluded that stock returns exhibit mean reversion. Fama and French (1988), who examined the autocorrelations of one-period returns, also found mean reversion. By contrast, using a generalised form of rescaled range (R/S) statistic, Lo (1991) found no evidence against the random walk hypothesis. Using annual data and allowing for fractional alternatives, Caporale and Gil-Alana (2002) reported that US stock returns are close to being an  $I(0)$  series, and pointed out that their degree of predictability depends on the process followed by the error term.

The study of long range dependence clearly requires sufficiently long series to justify the application of large sample inference rules based on semiparametric models, whilst no finite sample theory yet exists for rules of parametric inference on long memory. A number of recent papers have therefore used the Standard and Poor's (S&P) 500 index of over 17,000 daily observations. In particular, Granger and Ding (1995a,b) focused on power transform or absolute value of the returns (used as proxies of volatility). They estimated a long-memory process to study persistence in volatility, and established some stylised facts (temporal and distributional properties). However, Granger and Ding (1996) found that the parameters of the long-memory model vary considerably from one subseries to the next. Ryden et al (1998) claimed that the temporal higher-order dependence observed in return series are better described by a hidden Markov model, and again reported that the parameter estimates of the model differ depending on the subseries being considered. Their model, though does not account for an important distributional property of absolute returns (i.e. their very slowly decaying autocorrelation function). This is reproduced by Granger and Terasvirta (1999) in the context of a nonlinear model.

The fact that the “long memory” property might reflect the presence of breaks had already been pointed out by Lobato and Savin (1998), though they did not find any evidence to this effect when splitting their sample in two. In a subsequent paper, Aggarwal et al (1999) used a pre-determined break procedure to investigate this issue. Granger and Hyung (1999) applied instead the method of Bai (1997) and Bai and Perron (1998) for estimating multiple breaks at unknown dates, and that of Inclan and Tiao (1994) for changes in variance. They concluded that a series with breaks can mimic the properties of an  $I(d)$  process (such as the autocorrelations), where  $d$  is a fraction whose value depends on the number of breaks for a given sample size. Their simulation results indicate that “long memory” is more likely to be exhibited by absolute returns because of the presence of breaks than their being an  $I(d)$  process. They estimated the parameter  $d$  for the various subperiods as suggested by Geweke and Porter-Hudak (1983) (GPH henceforth), and found strong evidence of long memory in absolute stock returns for all subperiods. However, they also found that the value of  $d$  changes considerably from one period to another, and suggested that a time-varying  $d$  is evidence that a linear model with occasional breaks is appropriate for stock returns.

In this paper we revisit this issue by using Robinson’s (1994a) tests for testing  $I(d)$  statistical models to analyse the degree of dependence in the intertemporal structure of daily stock returns. Specifically, we are interested in establishing whether stock returns can indeed be characterised as an  $I(d)$  (or fractionally integrated) process, and also whether the degree of dependence given by the parameter  $d$  is constant over time. Therefore, we split the full sample, which comprises 17,000 observations, in 10 subsamples of 1700 observations each. These are then grouped two by two, and five by five; finally, the whole sample is considered. The results indicate that the degree of dependence remains relatively constant over time, with the order of integration of stock returns fluctuating slightly above or below zero. On the whole, there is very little evidence of fractional integration, despite the length of the series.

Therefore, it appears that, contrary to what argued in several studies (such as Granger and Ding, 1995a,b), the standard practice of taking first differences when modelling stock returns might be adequate.

The layout of the paper is the following. Section 2 briefly describes a version of the tests of Robinson (1994a) for I(d) statistical models which we apply to daily stock returns. Section 3 presents the empirical analysis. Section 4 offers some concluding remarks.

## 2. Testing for fractional integration

For the purpose of the present paper, we define an I(0) process  $\{u_t, t = 0, \pm 1, \dots\}$  as a covariance stationary process with a spectral density function which is positive and finite at zero frequency. In this context, we can say that  $\{x_t, t = 0, \pm 1, \dots\}$  is I(d) if

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

where L is the lag operator ( $Lx_t = x_{t-1}$ ), and the polynomial in (1) can be expanded using a Binomial expansion such that

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \frac{d(d-1)(d-2)}{6} L^3 + \dots$$

for any real d. This type of models was introduced by Granger and Joyeux (1980), Granger (1980, 1981), and Hosking (1981) (though earlier studies by Adenstedt, 1974 and Taquu, 1975, show an awareness of this representation), and was theoretically justified in terms of the aggregation of ARMA series with randomly varying coefficients by Robinson (1978) and Granger (1980). Similarly, Croccek-Georges and Mandelbrot (1995), Taquu et. al. (1997), Chambers (1998) and Lippi and Zaffaroni (1999) also use aggregation to motivate long memory processes, while Parke (1999) uses a closely related discrete time error duration model.

Following the discussions of Bhargava (1986), Schmidt and Phillips (1992) and others on the parameterisation of unit-root models, we consider the following regression model,

$$y_t = \mathbf{b}' z_t + x_t, \quad t = 1, 2, \dots \quad (2)$$

where  $y_t$  is the time series we observe;  $\beta$  is a  $(k \times 1)$  vector of unknown parameters;  $z_t$  is a  $(k \times 1)$  vector of deterministic regressors that may include, for example, an intercept and a linear time trend if  $z_t = (1, t)'$ ; and  $x_t$  is given by (1). Robinson (1994a) proposed a Lagrange multiplier (LM) test of the hypothesis:

$$H_o : d = d_o, \quad (3)$$

in (1) and (2) for any real value  $d_o$ . Specifically, the test statistic is given by:

$$\hat{r} = \left( \frac{T}{\hat{A}} \right)^{1/2} \frac{\hat{a}}{\hat{\mathbf{S}}^2}, \quad (4)$$

where  $T$  is the sample size and

$$\hat{a} = \frac{-2\mathbf{p}}{T} \sum_{j=1}^{T-1} \mathbf{y}(\mathbf{l}_j) g(\mathbf{l}_j; \mathbf{t})^{-1} I(\mathbf{l}_j)$$

$$\hat{A} = \frac{2}{T} \left( \sum_{j=1}^{T-1} \mathbf{y}(\mathbf{l}_j)^2 - \sum_{j=1}^{T-1} \mathbf{y}(\mathbf{l}_j) \hat{\mathbf{e}}(\mathbf{l}_j)' \times \left( \sum_{j=1}^{T-1} \hat{\mathbf{e}}(\mathbf{l}_j) \hat{\mathbf{e}}(\mathbf{l}_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\mathbf{e}}(\mathbf{l}_j) \mathbf{y}(\mathbf{l}_j) \right)$$

$$\mathbf{y}(\mathbf{l}_j) = \log \left| 2 \sin \frac{\mathbf{l}_j}{2} \right|; \quad \hat{\mathbf{e}}(\mathbf{l}_j) = \frac{\partial}{\partial \mathbf{t}} \log g(\mathbf{l}_j; \mathbf{t}); \quad \mathbf{l}_j = \frac{2\mathbf{p} j}{T}.$$

$I(\lambda_j)$  is the periodogram of  $\hat{u}_t$ , where

$$\hat{u}_t = (1 - L)^{d_o} y_t - \hat{\mathbf{b}} w_t, \quad w_t = (1 - L)^{d_o} z_t; \quad \hat{\mathbf{b}} = \left( \sum_{t=1}^T w_t w_t' \right)^{-1} \sum_{t=1}^T w_t (1 - L)^{d_o} z_t.$$

and  $g$  above is a known function coming from the spectral density of  $u_t$ :

$$f(\mathbf{l}_j; \mathbf{t}) = \frac{\mathbf{S}^2}{2\mathbf{p}} g(\mathbf{l}_j; \mathbf{t}).$$

Thus, for example, if  $u_t$  is white noise,  $g \equiv 1$ , and if  $u_t$  is AR(1) of form:  $u_t = \tau u_{t-1} + \epsilon_t$ ,

$$g(\mathbf{l}_j; \mathbf{t}) = \left| 1 - \tau e^{i\mathbf{l}_j} \right|^{-2}, \quad \text{with } \sigma^2 = V(\epsilon_t), \text{ so that the AR coefficients are function of } \tau.$$

Robinson (1994a) showed that under certain regularity conditions,

$$\hat{r} \rightarrow_d N(0, 1) \quad \text{as } T \rightarrow \infty. \quad (5)$$

Thus, an approximate one-sided  $100\alpha\%$ - level test of  $H_0$  (4) against the alternative:  $H_a: d > d_0$  ( $d < d_0$ ) will reject  $H_0$  (3) if  $\hat{r} > z_\alpha$  ( $\hat{r} < -z_\alpha$ ), where the probability that a standard normal variate exceeds  $z_\alpha$  is  $\alpha$ . Furthermore, he shows that the above test is efficient in the Pitman sense, i.e., that against local alternatives of form:  $H_a: d = d_0 + \delta T^{-1/2}$ , with  $\delta \neq 0$ , the limit distribution is normal with variance 1 and mean which cannot (when  $u_t$  is Gaussian) be exceeded in absolute value by that of any rival regular statistic. Empirical applications based on this version of Robinson's (1994a) tests can be found in Gil-Alana and Robinson (1997) and Gil-Alana (2000a), and other versions of his tests, based on seasonal (quarterly and monthly) and cyclical data, are presented in Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001a) respectively.

Note that the above tests are purely parametric and therefore, they require specific modelling assumptions to be made regarding the short memory specification of  $u_t$ . However, the AR modelling of the  $I(0)$  disturbances  $u_t$  is very conventional, whilst there exist many other types of  $I(0)$  processes, including some outside the stationary and invertible ARMA case. One model that seems especially relevant and convenient in the context of the present tests is that proposed by Bloomfield (1973), in which  $g$  is given by

$$g(\mathbf{I}_j; \mathbf{t}) = \exp\left(2 \sum_{l=0}^k \mathbf{t}_l \cos(\mathbf{I}_j l)\right). \quad (6)$$

Formulae for Newton-type iteration for estimating the  $\tau_1$  are very simple (involving no matrix inversion), updating formulae when  $k$  is increased are also simple, and we can replace  $\hat{A}$  below (4) by the population quantity

$$\sum_{l=k+1}^{\infty} l^{-2} = \frac{\mathbf{p}^2}{6} - \sum_{l=1}^k l^{-2},$$

which indeed is constant with respect to the  $\tau_j$  (unlike what happens in the AR case). The intuition behind this model is the following. Suppose that  $u_t$  follows an ARMA process of the form

$$u_t = \sum_{r=1}^p \mathbf{f}_r u_{t-r} + \mathbf{e}_t + \sum_{r=1}^q \mathbf{q}_r \mathbf{e}_{t-r},$$

where  $\mathbf{e}_t$  is a white noise process and all zeros of  $\phi(L)$  lying outside the unit circle and all zeros of  $\theta(L)$  lying outside or on the unit circle. Clearly, the function  $g$  in the spectrum of this process is then given by

$$g(\mathbf{l}_j; \mathbf{t}) = \left| \frac{1 + \sum_{r=1}^q \mathbf{q}_r e^{i\mathbf{l}_r}}{1 - \sum_{r=1}^p \mathbf{f}_r e^{i\mathbf{l}_r}} \right|^2. \quad (7)$$

Bloomfield (1973) showed that the logarithm of the above function is a fairly well-behaved function and can thus be approximated by a truncated Fourier series. He showed that (6) approximates (7) well, where  $p$  and  $q$  are small values, which usually happens in economics. Like the stationary AR( $p$ ) case, this model has exponentially decaying autocorrelations and thus, using this specification, we do not need to rely on as many parameters as in the ARMA processes, which is always tedious in terms of estimation, testing and model specification.

The Bloomfield (1973) model confounded with fractional integration has not been very much used in econometrics though the Bloomfield model itself is a well-known model in other disciplines (see, e.g. Beran, 1993). Amongst the few empirical applications found in the literature are Gil-Alana and Robinson (1997), Velasco and Robinson (1999) and more recently Gil-Alana (2001b).

### 3. Empirical results

In this section we analyse the degree of dependence in the intertemporal structure of daily stock returns using the tests of Robinson (1994a). The series considered is the S&P daily 500

stock returns which covers the period from January 4, 1928 to August, 30, 1991, for a total of 17,054 observations (see Granger and Ding, 1996, for further details). We drop the first 54 and create 10 subsamples of 1700 observations each, which we label A,B, C.... , I, J.

Denoting the series of interest by  $y_t$ , we employ throughout the following model:

$$y_t = \beta_1 + \beta_2 t + x_t \quad t = 1, 2, \dots, \quad (8)$$

$$(1 - L)^d x_t = u_t \quad t = 1, 2, \dots, \quad (9)$$

treating separately the cases  $\beta_1 = \beta_2 = 0$  a priori,  $\beta_1$  unknown and  $\beta_2 = 0$  a priori, and  $(\beta_1, \beta_2)$  unknown. We will model the disturbance term  $u_t$  both as a white noise and as an autocorrelated process (including the case of a Bloomfield process). We start with the assumption that  $u_t$  is white noise. Thus when  $d = 1$ , for example, the differences  $(1-L)y_t$  behave, for  $t > 1$ , like a random walk when  $\beta_2 = 0$ , and a random walk with drift when  $\beta_2 \neq 0$ .

Table 1 presents the test statistics for the order of integration under the assumption of white noise disturbances. It can be seen that the value of the test statistic decreases monotonically with respect to  $d_0$ , as one would expect, because of the one-sided nature of the test. The results are very similar in the three cases of no regressors, an intercept and a linear trend, suggesting that deterministic regressors might not be needed. In fact, the non-rejection values coincide in all three cases. The hypothesis  $d = 0$  cannot be rejected for subsamples A and B. For C and D the order of integration seems to be between 0 and 0.10. For E and F,  $d = 0.10$ , and for G and H the corresponding value is 0.20, whilst it is 0.10 for I and 0 for J. Therefore, in all cases the order of integration is between 0 and 0.20.

**(Table 1 about here)**

Table 2 reports the results when weakly autocorrelated disturbances are considered. Given the similarities in the results when using no regressors, an intercept and an intercept and a linear time trend, we report only the results based only on the third case, i.e., using (8) and (9), and modelling the  $I(0)$  disturbances  $u_t$  in terms of AR(1), AR(2) and a Bloomfield

process. It can be seen that whether  $u_t$  is an AR(1) process or follows a Bloomfield specification, the non-rejection values stay the same, and non-rejections occur with  $d = 0$  for A,B,C,E,F,G, and I. For the remaining subperiod  $d = 0$  is rejected in favour of  $d < 0$ , and the non-rejection occurs when  $d = -0.10$ . The results are similar when the disturbances are modelled as an AR(2) process, though here one also observes non-rejections when  $d = -0.10$  and  $d = 0.10$ , the case of  $d = 0$  being the exception.

**(Tables 2 and 3 about here)**

Table 3 refers to two-by-two groupings of the subperiods, and again reports the test statistics for white noise, autocorrelated and Bloomfield disturbances. Practically all non-rejections occur when  $d = 0$ . We only observe four cases where  $d$  is higher than 0, namely the grouping E+F with AR(2) and Bloomfield (2) disturbances, and C+D with Bloomfield (2) disturbances. It would appear, therefore, that the AR and Bloomfield models are very similar. The same non-rejection values occur with AR(1) and Bloomfield (1) disturbances, and also with AR(2) and Bloomfield (2) disturbances, which suggests that the Bloomfield model approximates very well the AR structure. Further, it has a number of advantages in terms of computation, as explained in the previous section.

In Table 4 the first and the last five subperiods are grouped, and again alternative models for the disturbances are considered. Starting with the white noise specification, it can be seen that the null is always rejected, though non-rejection values may occur when  $d$  is between 0 and 0.10. The same applies to weakly autocorrelated disturbances. The non-rejection value is  $d = 0$  for both the first grouping with AR(1) and Bloomfield (1) disturbances, and the second one with AR(2) and Bloomfield (2) disturbances. In general, the non-rejection values are around 0.

**(Tables 4 and 5 about here)**

Similar considerations can be made when looking at Table 5, where the full sample is considered. If  $u_t$  is a white noise, the hypothesis  $d = 0$  is rejected against  $d > 0$ , and  $d = 0.10$  is

also rejected against  $d < 0.10$ , implying that the order of integration should be between these two numbers. If  $u_t$  is modelled as an AR or Bloomfield process, the only non-rejections occur when  $d = 0$ .

#### **4. Conclusions**

In this paper we have used a version of the tests of Robinson (1994a) to analyse the degree of dependence in the intertemporal structure of daily stock returns (defined as the first difference of the logarithm of stock prices, where the series being considered is the S&P500 index). These tests offer two main advantages. First, they have a standard null limit distribution, unlike other procedures for testing unit (or fractional) roots. In addition, they are the most efficient ones when carried out against the appropriate (fractional) alternatives. Finite-sample critical values of this version of Robinson's (1994) tests have been calculated in Gil-Alana (2000b). However, it was shown in that paper that the asymptotic values perform relatively well for samples of sizes considered here. We have taken various subsamples (the full sample going from 1928 to 1991), and found that the degree of dependence remains relatively constant over time, with the order of integration of stock returns fluctuating slightly around zero. This is in contrast to the widespread notion that many series that are  $I(1)$  become  $I(d)$  with  $d$  smaller than or greater than 1 when using a very long span of data. Therefore, it appears that, despite the length of the series, a standard model in first differences rather than a fractionally integrated one might be appropriate for stock returns. This has obvious implications for investment strategies, since knowing where the expected mean is for a stationary and long memory series enables one to devise rules to buy and sell at the optimal time.

The approach we have taken following Robinson (1994a) is based on a fully specified parametric model. Given the length of the series, it would have been possible to employ semiparametric procedures (see Robinson, 1994b,c 1995a,b). However, these semiparametric

techniques can be very sensitive to the choice of the bandwidth parameters. The fact that the results we have obtained are robust to the model chosen for the disturbances, though, suggests that semiparametric methods would produce very similar results to ours. Other issues such as heteroscedasticity or the potential strong dependence in volatility will be addressed in future papers.

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TABLE 1											
Testing the order of integration of the series with white noise disturbances											
1) With no regressors											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A	103.65	69.27	40.41	20.68	8.43	<b>0.92'</b>	-3.83	-7.02	-9.27	-10.93	-12.22
B	87.59	57.35	32.94	16.41	6.02	<b>-0.51'</b>	-4.82	-7.81	-9.98	-11.62	-12.89
C	97.73	67.30	41.34	22.92	10.92	3.16	-2.01	-5.61	-8.20	-10.14	-11.62
D	84.73	57.16	34.48	18.67	8.47	1.92	-2.41	-5.43	-7.64	-9.33	-10.67
E	116.02	76.71	44.78	24.09	11.53	3.78	<b>-1.22'</b>	-4.64	-7.09	-8.93	-10.37
F	84.50	61.34	39.99	23.31	11.66	3.91	<b>-1.25'</b>	-4.82	-7.38	-9.29	-10.78
G	86.73	65.94	46.41	30.52	18.75	10.38	4.42	<b>0.10'</b>	-3.12	-5.60	-7.56
H	88.05	62.75	41.09	25.19	14.39	7.13	2.08	<b>-1.57'</b>	-4.34	-6.52	-8.28
I	76.83	52.31	33.30	19.59	10.01	3.32	<b>-1.41'</b>	-4.87	-7.47	-9.48	-1106
J	60.81	40.47	24.57	12.96	4.90	<b>-0.58'</b>	-4.38	-7.09	-9.10	-10.65	-11.88
2) With an intercept											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A	96.15	65.51	39.13	20.38	8.39	<b>0.92'</b>	-3.83	-7.02	-9.27	-10.93	-12.22
B	88.10	57.64	33.05	16.44	6.03	<b>-0.51'</b>	-4.82	-7.81	-9.98	-11.62	-12.89
C	96.76	66.48	40.93	22.79	10.89	3.16	-2.01	-5.61	-8.20	-10.14	-11.62
D	85.97	57.72	34.71	18.75	8.48	1.92	-2.41	-5.44	-7.65	-9.34	-10.67
E	94.73	66.83	41.93	23.57	11.48	3.78	<b>-1.22'</b>	-4.64	-7.10	-8.94	-10.39
F	93.07	66.15	42.02	23.94	11.79	3.91	<b>-1.27'</b>	-4.83	-7.38	-9.29	-10.77
G	87.93	66.49	46.61	30.58	18.76	10.38	4.42	<b>0.10'</b>	-3.12	-5.60	-7.56
H	87.86	62.63	41.04	25.17	14.39	7.13	2.09	<b>-1.57'</b>	-4.34	-6.52	-8.28
I	72.74	51.33	33.24	19.65	10.01	3.32	<b>-1.42'</b>	-4.88	-7.47	-9.48	-1106
J	67.81	44.23	26.02	13.37	4.98	<b>-0.58'</b>	-4.39	-7.10	-9.11	-10.65	-11.88
3) With an intercept and a linear time trend											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A	93.90	64.57	38.83	20.30	8.37	<b>0.92'</b>	-3.83	-7.02	-9.27	-10.94	-12.22
B	81.96	54.13	31.52	15.90	5.86	<b>-0.56'</b>	-4.83	-7.81	-9.98	-11.62	-12.89
C	73.77	52.66	34.33	20.17	9.99	2.88	-2.09	-5.63	-8.21	-10.14	-11.62
D	82.54	55.28	33.41	18.21	8.30	1.87	-2.42	-5.43	-7.64	-9.33	-10.67
E	94.88	67.13	42.06	23.57	11.44	3.75	<b>-1.25'</b>	-4.66	-7.11	-8.95	-10.39
F	89.91	64.46	41.27	23.66	11.69	3.88	<b>-1.28'</b>	-4.83	-7.38	-9.29	-10.77
G	86.98	66.13	46.51	30.56	18.76	10.38	4.42	<b>0.10'</b>	-3.12	-5.60	-7.56
H	87.69	62.62	41.06	25.19	14.39	7.13	2.08	<b>-1.57'</b>	-4.34	-6.52	-8.28
I	72.82	51.37	33.26	19.65	10.01	3.32	<b>-1.42'</b>	-4.88	-7.47	-9.48	-1106
J	62.79	41.86	25.09	13.05	4.88	<b>-0.61'</b>	-4.40	-7.10	-9.11	-10.65	-11.88

' and in bold: Non-rejection values of the null hypothesis at the 95% significance level.

<b>TABLE 2</b>											
Testing the order of integration of the series											
1) With AR(1) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A	20.65	16.69	15.87	10.19	4.42	<b>-0.03'</b>	-3.27	-5.61	-7.35	-8.67	-9.72
B	15.46	14.91	13.67	8.52	3.69	<b>-0.06'</b>	-2.92	-5.13	-6.90	-8.35	-9.56
C	20.92	19.68	12.32	8.77	4.45	<b>0.76'</b>	-2.19	-4.56	-6.46	-8.01	-9.29
D	14.75	12.29	10.79	5.79	<b>1.14'</b>	-2.38	-4.96	-6.85	-8.28	-9.40	-10.29
E	12.06	11.53	13.58	8.24	3.00	<b>-0.97'</b>	-3.87	-6.01	-7.63	-8.90	-9.91
F	15.76	12.39	10.56	7.71	3.50	<b>-0.25'</b>	-3.20	-5.45	-7.19	-8.57	-9.68
G	10.65	7.01	5.76	4.85	2.20	<b>-0.51'</b>	-2.85	-4.78	-6.36	-7.67	-8.78
H	9.32	8.34	7.23	4.24	<b>0.83'</b>	-1.91	-3.99	-5.58	-6.87	-7.94	-8.87
I	10.54	6.49	7.20	4.96	2.18	<b>-0.34'</b>	-2.49	-4.27	-5.76	-7.02	-8.11
J	10.27	9.29	7.62	4.03	<b>0.41'</b>	-2.59	-4.93	-6.72	-8.11	-9.21	-10.11
2) With AR(2) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A	15.27	14.68	13.96	10.02	5.32	<b>1.32'</b>	-1.80	-4.20	-6.05	-7.50	-8.67
B	20.93	18.35	10.38	7.12	3.68	<b>0.88'</b>	<b>-1.31'</b>	-3.08	-4.55	-5.82	-6.95
C	12.63	11.34	9.77	7.71	4.78	2.10	<b>-0.18'</b>	-2.14	-3.85	-5.32	-6.61
D	16.19	13.96	12.26	8.01	3.79	<b>0.30'</b>	-2.40	-4.50	-6.14	-7.44	-8.50
E	13.76	13.16	14.59	10.10	5.65	2.03	<b>-0.80'</b>	-3.03	-4.82	-6.27	-7.47
F	19.64	16.65	10.16	8.42	5.23	2.07	<b>-0.60'</b>	-2.77	-4.53	-5.99	-7.21
G	9.30	8.58	7.55	6.23	3.94	<b>1.57'</b>	<b>-0.54'</b>	-2.39	-4.00	-5.40	-6.63
H	10.62	9.90	8.82	5.49	2.23	<b>-0.35'</b>	-2.31	-3.80	-4.97	-5.94	-6.78
I	8.33	5.78	5.58	3.39	<b>0.98'</b>	<b>-1.16'</b>	-2.97	-4.46	-5.71	-6.75	-7.65
J	9.32	8.43	7.57	4.88	1.80	<b>-1.00'</b>	-3.33	-5.19	-6.66	-7.84	-8.80
3) With Bloomfield (1) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A	56.23	37.81	22.75	12.14	4.59	<b>-0.07'</b>	-3.14	-5.34	-7.07	-8.22	-9.30
B	46.60	31.39	19.22	10.18	4.07	<b>-0.31'</b>	-2.90	-4.88	-6.17	-7.39	-8.28
C	50.81	33.52	21.03	11.06	5.03	<b>0.68'</b>	-2.33	-4.21	-6.17	-7.22	-8.24
D	39.78	26.08	14.08	6.51	<b>0.74'</b>	-2.97	-5.13	-7.09	-8.39	-9.25	-10.11
E	58.32	37.61	20.15	9.57	2.65	<b>-1.46'</b>	-4.14	-6.06	-7.60	-8.62	-9.60
F	40.10	27.80	18.10	9.82	3.64	<b>-0.51'</b>	-3.39	-5.52	-7.25	-8.43	-9.26
G	38.15	26.47	16.88	8.75	3.28	<b>-0.44'</b>	-3.12	-5.19	-6.92	-8.14	-9.01
H	39.15	25.92	14.36	7.13	<b>1.55'</b>	-2.04	-4.12	-6.10	-7.43	-8.35	-9.01
I	37.92	25.04	15.49	7.98	3.58	<b>-0.04'</b>	-2.78	-4.56	-6.13	-7.24	-8.02
J	28.32	18.07	10.50	4.51	<b>0.25'</b>	-2.82	-5.13	-6.96	-8.19	-9.03	-9.89

' and in bold: Non-rejection values of the null hypothesis at the 95% significance level.

<b>TABLE 3</b>											
Testing the order of integration of the series											
1) With white noise disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A + B	143.32	94.99	54.83	27.63	10.90	<b>0.67'</b>	-5.82	-10.20	-13.32	-15.64	-17.44
C + D	145.72	95.65	55.98	29.86	13.72	3.57	-3.11	-7.76	-11.13	-13.67	-15.66
E + F	151.15	99.69	59.84	33.16	16.15	5.31	-1.82	-6.71	-10.24	-12.88	-14.93
G + H	124.59	90.70	60.84	38.20	22.40	11.61	4.08	<b>-1.37'</b>	-5.49	-8.70	-11.26
I + J	125.65	74.49	42.14	22.33	9.65	<b>1.20'</b>	-4.63	-8.82	-11.95	-14.35	-16.25
2) With AR(1) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A + B	32.82	23.07	22.95	14.28	6.09	<b>-0.08'</b>	-4.54	-7.80	-10.26	-12.18	-13.72
C + D	21.14	19.92	18.38	11.00	4.09	<b>-1.18'</b>	-5.14	-8.16	-10.51	-12.38	-13.90
E + F	20.83	19.24	17.81	11.52	4.63	<b>-0.91'</b>	-5.08	-8.22	-10.63	-12.53	-14.06
G + H	12.75	10.71	10.21	6.69	1.99	-2.00	-5.10	-7.53	-9.49	-11.13	-12.53
I + J	20.62	17.02	13.08	7.06	1.82	-2.36	-5.65	-8.23	-10.28	-11.94	-13.32
4) With AR(2) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A + B	21.95	20.62	18.25	12.94	6.77	<b>1.62'</b>	-2.37	-5.45	-7.87	-9.82	-11.43
C + D	19.87	17.33	16.62	11.48	6.17	1.76	-1.78	-4.65	-7.00	-8.96	-10.61
E + F	20.38	18.97	16.82	12.72	7.54	2.86	<b>-0.98'</b>	-4.09	-6.62	-8.71	-10.47
G + H	19.42	18.38	11.61	8.05	3.99	<b>0.47'</b>	-2.34	-4.59	-6.43	-7.99	-9.35
I + J	15.15	14.23	11.18	6.55	2.31	<b>-1.30'</b>	-4.28	-6.69	-8.65	-10.25	-11.59
5) With Bloomfield (1) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A + B	77.10	50.61	30.39	16.04	6.73	<b>-0.37'</b>	-4.46	-7.43	-9.79	-11.39	-12.50
C + D	74.26	47.03	25.66	12.85	4.02	<b>-1.38'</b>	-5.03	-8.32	-10.05	-11.62	-13.14
E + F	74.93	47.02	26.45	13.43	4.25	<b>-1.50'</b>	-5.37	-8.22	-10.53	-12.11	-13.21
G + H	55.82	36.44	21.81	10.41	3.19	-2.19	-5.38	-7.81	-9.88	-11.32	-12.36
I + J	64.73	36.76	18.73	8.58	2.52	-2.20	-5.69	-8.47	-10.36	-11.67	-12.57
3) With Bloomfield (2) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A + B	54.27	36.79	24.85	15.23	7.38	<b>1.54'</b>	-2.50	-4.76	-7.53	-7.91	-9.35
C + D	56.02	39.08	23.37	14.54	7.58	<b>1.31'</b>	<b>-1.38'</b>	-3.54	-6.36	-6.82	-8.40
E + F	58.94	39.92	24.07	15.20	7.92	2.32	<b>-0.83'</b>	-3.29	-6.32	-6.87	-8.52
G + H	39.97	29.62	19.37	10.10	5.17	<b>1.03'</b>	-2.01	-5.34	-6.00	-7.68	-8.90
I + J	45.65	25.49	<b>13.53</b>	5.76	2.18	<b>-1.09'</b>	-3.65	-6.11	-6.78	-8.49	-10.14

' and in bold: Non-rejection values of the null hypothesis at the 95% significance level.

<b>TABLE 4</b>											
Testing the order of integration of the series											
1) With white noise disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A B C D E	273.87	174.56	95.87	46.96	18.87	2.33	-8.07	-15.09	-20.09	-23.82	-26.72
F G H I J	244.18	152.74	88.46	48.44	23.79	8.05	-2.52	-10.00	-15.53	-19.77	-23.13
2) With AR(1) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A B C D E	52.68	43.07	41.34	23.88	9.63	<b>-0.34'</b>	-7.35	-12.45	-16.31	-19.33	-21.76
F G H I J	34.53	28.90	26.57	14.88	4.90	-2.54	-8.07	-12.29	-15.61	-18.32	-20.58
3) With AR(2) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A B C D E	33.41	27.64	22.80	21.80	11.06	2.80	-3.41	-8.19	-11.97	-15.04	-17.59
F G H I J	244.18	152.74	88.46	48.44	23.79	<b>-0.04'</b>	-4.98	-8.87	-15.53	-19.77	-23.13
3) With Bloomfield (1) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A B C D E	150.19	94.14	51.97	26.49	9.13	<b>-0.83</b>	-7.19	-11.76	-15.41	-17.90	-19.62
F G H I J	122.88	73.52	39.22	18.68	6.03	-2.80	-7.93	-12.66	-15.06	-18.04	-19.57
3) With Bloomfield (2) disturbances											
Series	Values of $d_0$										
	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
A B C D E	76.43	45.32	39.05	20.09	9.43	2.11	-2.95	-10.98	-11.56	-14.09	-22.43
F G H I J	166.54	109.10	43.67	18.43	6.54	<b>1.02'</b>	-5.23	-6.87	-9.03	-12.56	-17.34

' and in bold: Non-rejection values of the null hypothesis at the 95% significance level.

<b>TABLE 5</b>											
Testing the order of integration of the series											
Series	Values of $d_0$										
ABCDEFGHIJ	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50
White Noise	459.56	276.68	142.12	68.75	28.94	5.64	-9.17	-19.28	-26.57	-32.05	-36.33
AR (1)	323.23	126.75	89.90	54.63	12.34	<b>1.14'</b>	-4.53	-8.35	-14.53	-29.08	-41.02
AR (2)	124.32	100.92	54.76	43.12	10.03	<b>0.86'</b>	-2.34	-6.75	-15.43	-30.01	-39.43
Bloomfield (1)	208.65	176.34	101.43	76.37	20.54	<b>1.21'</b>	-3.54	-8.75	-10.98	-24.32	-42.32
Bloomfield (2)	100.54	65.76	34.32	12.09	7.56	<b>1.19'</b>	-2.45	-6.54	-9.65	16.45	-31.21

' and in bold: Non-rejection values of the null hypothesis at the 95% significance level.